

Evap. Memo. No 16

COMPARISONS OF THE RESULTS OF ESTIMATING DAILY MEAN VALUES OF TEMPERATURE,
SATURATION VAPOUR PRESSURE, VAPOUR PRESSURE AND WIND SPEED FOR USE IN THE
PENMAN FORMULA FOR POTENTIAL EVAPORATION

by A G Seaton and B G Wales-Smith

Wales-Smith and Wright (Met O 8 Evap. Memo No 12) compared the results of evaluating the aerodynamic term in the Penman formula on a monthly basis using various selections of input data obtained from London Airport data for 1965.

The investigations now described use daily data from several stations and deal with each of the three meteorological variables (temperature, vapour pressure and wind speed) separately and in combination.

1 ESTIMATING MEAN AIR TEMPERATURE (in the screen)

In the Penman formula an error of 1°C in the daily mean temperature will lead to an error of $\frac{1}{4}\text{mm}$ in the daily mean potential evaporation; ie $\partial E/\partial T = +0.25\text{mm}/^{\circ}\text{C}$ (this was calculated for a Summer month). A study was carried out to test how good an estimate of the mean temperature in a day is provided by (i) meaning the 03, 09, 15, & 21 hrs temperatures and (ii) meaning the day time maximum & night minimum temperatures; a computer programme calculated the daily and monthly errors in the two methods of meaning temperatures and the standard deviations of (a) the daily errors and (b) the mean errors over a pentad.

Table 1: MEAN ERRORS & SDs OF PENTAD MEAN TEMPS

Station	Year	Error of mean of four temps			Error of mean of max & min temps		
		M4	S4	X4	Mm	Sm	Xm
Boscombe Down	1967	+0.03	0.10	+0.09	+0.26	0.29	+0.51
	1968	+0.04	0.10	+0.16	+0.22	0.31	+0.49
Mildenhall	1968	0	0.09	+0.07	+0.21	0.32	+0.52
Manchester	1968	+0.02	0.10	+0.12	+0.26	0.27	+0.52

M = mean daily error $^{\circ}\text{C}$

S = standard deviation of pentad daily means $^{\circ}\text{C}$

X = daily mean error in month of greatest error $^{\circ}\text{C}$

The Accuracy of the Mean of Four Temps

The error over a year caused by meaning only the four selected temperatures is virtually nil; this is shown in column M4 of Table 1, where the 1968 error at Boscombe Down would lead to an error of less than 3mm in the total PE for the year. The standard deviation of these errors meaned over a pentad (the minimum period for which Penman's formula is assumed to be used) is about 0.1 (see column S4); assuming a normal distribution, (see Fig 1, left-handed diagrams). This means that on no occasion ($<0.001\%$ in fact) will the error caused by meaning only these four temperatures result in a greater than 10% ($\pm 0.3\text{mm}$ in the Summer) error in the value of the PE. Column X4 of Table 1 shows that if monthly PEs are being calculated the greatest error in any month of the four station-years will be 1.2mm (or 0.04mm/day).

The graphs of Fig 2 (ignore ringed symbols), drawn for the two years at Boscombe Down, show no regular month-to-month pattern of errors.

The Accuracy of the Mean of the Max & Min Temps

Meaned over a year the daily means produced from the maximum & minimum temperature

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show an excess over the arithmetic mean of 24 hourly values of about $\frac{1}{4}^{\circ}\text{C}$ (column Mm Table 1). Using the criterion of 10% required accuracy and a standard deviation of the pentad errors of 0.3 (see col Sm) only on 0.07% of occasions will the error be unacceptable (this always an overestimate of the mean). On 5% of occasions there will be an error of $> \frac{3}{4}^{\circ}\text{C}$ ($\approx 0.2\text{mm}$).

Column Xm of Table 1 shows that in the months in which the means of max and min temps give their worst results an error over the month of about 4mm occurs.

The ringed symbol graphs of Fig 2 show a random month-to-month distribution of the mean errors and their standard deviations. The right-hand diagrams of Fig 1 show the distribution of errors involved in regarding the mean of the daily maximum and minimum temperatures as an estimate of the arithmetic mean of 24 separate, hourly temperatures.

2. ESTIMATING MEAN VAPOUR PRESSURE (in the screen)

Table 2 MEAN ERRORS & SDs OF PENTAD MEAN VPs

Station	Year	Error of mean of four VPs			Error of 09 hrs VP		
		M4	S4	X4	M09	S09	X09
Boscombe Down	1967	+0.03	0.09	+0.10	+0.20	0.43	+0.80
	1968	+0.02	0.09	+0.09	+0.14	0.40	+0.47
Mildenhall	1968	+0.02	0.08	+0.10	+0.13	0.35	+0.47
Manchester	1968	+0.01	0.09	+0.10	+0.01	0.33	+0.28
Manchester	1967				-0.04	0.39	+0.43
Boscombe Down	1959				+0.30	0.60	+0.97

M = mean daily error mb

S = standard deviation of pentad daily means mb

X = daily mean error in month of greatest error mb

When the Penman formula is used, the error introduced into the estimate of potential evaporation by an error in the value of the vapour pressure is illustrated by the two sets of data in Table 3.

Table 3: VALUE OF dE/ded FOR 2 TYPICAL MONTHS

	U	T	ed	n/N	Δ	E	dE/ded
	mls/day	$^{\circ}\text{C}$	mbs	-	mbs/ $^{\circ}\text{C}$	mm/day	mm/day/mb
Feb 1970	197	2.5	6.10	0.40	0.521	0.42	-0.71
Jun 1970	112	16.0	16.60	0.50	1.161	2.97	-0.28

The magnitude of dE/ded is principally dependent on the wind speed and the value of Δ (where Δ is an increasing monotonic function of temperature); low winter temperatures and high winter wind speeds combine to increase the value of dE/ded .

A study has been carried out to compare the mean of 24 hourly vapour pressures with estimates of this mean provided by (i) the mean of the 03, 09, 15 & 21 hrs vapour pressures, and (ii) the 09 hrs vapour pressure. A computer programme calculated the daily and monthly errors in the two methods of estimating mean daily vapour pressure and the standard deviations of (a) the daily errors in each month (see Fig 3) and (b) the pentad mean daily errors in a year.

The Accuracy of the Mean of Four VPs

Table 2 indicates that the mean of the chosen four vapour pressures provides a good estimate of the mean of 24 hourly values. It is fortunate that the greatest errors and variability of error (Fig 3) occur during the summer when as shown in Table 3 the potential evaporation is less sensitive to VP errors. Roughly speaking only

on 1% of occasions would the error in the PE calculated for a pentad exceed 0.06 mm/day (in the summer - ie an error of approx 2%); in no month of the four station-years studied would the error in a monthly PE have reached this value. The left-hand diagrams of Fig 4 show the distribution of errors.

The Accuracy of the Use of the 09hrs VP as an Estimate

As above the period of least accuracy is the summer. Totalled over a year this method will always tend to overestimate the VP which means that the PE will be underestimated. In the worst summer months a monthly PE may be underestimated by 4mm; in December & January the monthly total will be overestimated by the same amount (which may be 100% of the actual total). The size of the standard deviation of pentad errors indicates that although the majority of the summer pentad estimates of PE would be acceptable a large proportion of those in the winter half-year would not. Although the error at Manchester meaned over a year is much less than the other two stations studied (due perhaps to its western position) the standard deviations of the monthly and pentad means are just as high.

The right-hand diagrams of Fig 4 show the distribution of errors.

3. THE CALCULATION OF A DAY'S MEAN SVP

A comparison was made of the mean of 24 hourly SVPs and the SVP at the mean temperature calculated from 24 hourly temperatures.

Table 4 DIFFERENCES BETWEEN METHODS OF CALCULATING DAILY MEAN SVP

		No of daily errors			M	S	X	XP
		0 to $-\frac{1}{2}$	$-\frac{1}{2}$ to -1	-1 to $-1\frac{1}{2}$				
Boscombe Down	1967	343	21	1	-.16	0.14	-.38	-.8
	1968	351	14	1	-.15	0.12	-.29	-.5
Mildenhall	1968	342	20	4	-.17	0.16	-.34	-.9
Manchester	1968	356	10	0	-.11	0.09	-.25	-.5

XP = daily mean error in worst pentad

Table 5 VALUE OF dE/de_a FOR TWO TYPICAL MONTHS

	U mls/day	$mbs/^{\circ}C$	E mms/day	dE/de_a mms/day/mb	$dE/de_a = f(u, \frac{1}{\Delta})$
Feb 1970	197	0.521	0.42	+.58	}
Jun 1970	112	1.161	2.97	+.27	

In perhaps one pentad in the station-years studied did the difference result in a 10% error in the value of the potential evaporation but several other errors were nearly of this magnitude. Using the SVP calculated from the mean of 24 temperatures leads to a persistent underestimation of the PE which in a year may amount to roughly 20mm.

Fig. 5 indicates how much more unsatisfactory the use of the one temp becomes in the summer and although dE/de_a is smaller it is not sufficiently so to compensate. The plotted points show mean SVP from \bar{T} — SVP for each month for 4 station-years.

4 COMPARISONS OF 5-DAY PE ESTIMATES USING TWO VERSIONS, EACH, OF THE MEAN AIR TEMPERATURE AND MEAN VAPOUR PRESSURE

Fig 6 a, b, c and d show comparisons of 5-day PE estimates obtained as follows:

Fig	Station	Measure of Radiation	Wind	Saturation Vapour Pressure	Vapour Pressure
6a	Kew	Net R	Mean	(SVP from mean of temps at 03, 09, 15, 21 hrs	Mean of VPs at 03, 09, 15 and 21 hrs
b	Kew	Incoming R	Mean	(VS	VS
c	Kew	Duration of bright sunshine	Mean	(SVP from mean of max and min temps	VP at 0900 hrs GMT
d	Boscombe Down	Duration of bright sunshine	Mean	(

The correlation is high for all four comparisons.

Five-day PE estimates made with temperature and humidity data obtained from stations where the observer visits the screen only once per day (at 0900 hrs GMT) were very close to those made from data obtained four times a day, on nearly all occasions.

5 ESTIMATING MEAN WIND SPEED

Table 6 ERRORS IN ESTIMATING MEAN WIND SPEED FROM SPOT WINDS

		mean of 03, 09, 15, 21 hrs			09 hrs wind			SMR
		M4	S4	X4	M09	S09	X09	
Boscombe Down	1968	+.06	.068	+.5	+.31	.160	+1.6	+.77
	1969	+.13	.064	+.4	+.64	.144	+1.5	+1.21
Mildenhall	1968	+.30	.077	+.6	+.48	.191	+1.6	+0.79
Manchester	1968	-.13	.062	-.5	+.07	.146	+1.0	+.53

M = Mean daily error in knots over a year

S = standard deviation of pentad ratios

X = mean daily error in least accurate month

SMR = mean daily error in knots during summer

Table 7 VALUE OF dE/du FOR 2 TYPICAL MONTHS

	ea mbs	ed mbs	Δ mbs/ $^{\circ}$ C	dE/du mm/day per mls/day kts	
Feb 1970	7.31	6.10	0.521	.0018	.050
Jun 1970	18.17	16.60	1.161	.0015	.041

Table 7 shows how given changes in mean wind speed would affect the PE estimates for the months of least and greatest PE at a typical English Station. The monthly mean values of e_a , e_d and Δ used in the calculations are set out in the first three columns.

It will be noted that the values of saturation deficit (Feb 1.21 mb and Jun 1.57 mb) are small compared with those used by P E Waggoner ("Meteorological data and the agricultural problem") and hence the changes in PE corresponding to given wind speed changes are smaller.

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The Penman formula requires a measure of mean wind speed (or total wind run). Where there is not an anemograph and wind run is not measured, the spot winds noted at fixed hours must be used. For several station-years the mean wind speeds (Fig 7) taken from anemographs were compared with (A) the 09 hrs spot wind, (B) the mean of the 03, 09, 15, 21 hrs spot winds, and (C) the mean of the 03, 06, 09, 12, 15, 18, 21, 00 hrs winds.

Figures 8 and 9a, b and c indicate that using the 8 chosen spot winds provides little improvement over using only the four, therefore in the following paragraphs only the use of four and one spot winds will be compared.

The Accuracy of the Mean of Four Spot Winds

The error over the year given in column M4 of Table 6 and illustrated by Fig 9a, b and c is probably due to the coarseness of the ratio used to reduce spot wind speeds to 10m; Mildenhall's mean daily error of +.30 gives an error in the calculated PE over the year of about 5 mm.

Using Mildenhall again (as its figures show the greatest variability) there is only a one in a hundred chance that the error in the estimate of the daily mean wind speed -

- (1) over a pentad will exceed $+1\frac{1}{2}$ kts or,
- (2) over a month will exceed +1 kt (this is based on an estimated sd of .033).

By referring to Table 7 it can be seen that the errors in the PE caused by the above are quite acceptable.

The Accuracy of the Use of the 09 hrs Wind

The 09 hrs spot wind provides a considerably less good estimate of mean daily wind speed than the mean of 03, 09, 15, 21 hrs; this is shown by Figs 8 and 9. Fig 9 shows that most of the error is concentrated in the summer months.

Boscombe Down (1969) with the worst daily mean error for a year (Table 6) would have an error of about 12mm for the year's total PE. Mildenhall with the worst monthly (± 0.083) and pentad standard deviations (Fig 8) will on one occasion in a hundred have -

- (1) its pentad mean wind speed over-estimated by $3\frac{1}{2}$ knots, and
- (2) its mean wind speed over a month over-estimated by 2 kts.

There is a one in twenty-five chance of errors of $+2\frac{3}{4}$ & $+1\frac{1}{2}$ knots respectively. The equivalent figures for Manchester are $+3\frac{1}{2}$ & $+1\frac{1}{2}$ knots and $+2\frac{3}{4}$ & $+1\frac{1}{4}$ knots; the error totalled over a year would result in virtually no error ($< 2\text{mm}$) in the PE.

The errors in the PE caused by using only the 09 hrs wind are acceptable when used on a monthly basis and over the year they tend to cancel each other out (particularly in the West). What has been said above about pentad errors implies that on 3 occasions in a year (1:25 \pm 3:73) the PE calculated for a pentad will have an error of $\frac{2}{3}$ mm; in the South and East these will be summer over-estimates and in the West they may be summer over-estimates or winter under-estimates.

6 CONCLUSIONS

(i) Data from four observations per day

The above has indicated that the use of the 03, 09, 15, 21 hrs spot winds, temperatures, and vapour pressures provide a perfectly good estimate of the daily mean values of these elements for use in the Penman formula.

(ii) Data from one observation per day

TABLE 8 ACCUMULATED ERROR OVER A YEAR (1 wind, 1 VP, 1 SVP from max & min temp)

		mm
Boscombe Down	1967	-30
	1968	-16
Mildenhall	1968	-21
Manchester	1968	+ 7

Table 8 shows the approximate error in total annual PE caused by using the 09hrs spot wind, the 09hrs vp, the mean of the max and min temp and the SVP calculated from one mean temp. Except for Manchester these errors would be decreased by calculating the SVP more rigorously. Using the criterion of 10% accuracy of PE most of the monthly PEs calculated from the above data will be acceptable. The pentad PEs calculated for the summer should satisfy the criterion but five days is probably too short a period in which to use these data during winter.

(iii) Data from one observation per day and a run-of-wind anemometer or an anemograph

It must be remembered that the comparisons of Section 4 (Figs 6a, b, c & d) can not be included in Part (ii) of these conclusions, since the wind input is mean hourly wind speed (a good approximation to run-of-wind, when multiplied by 24). The climatological station with only one observation of temperature, humidity and maximum and minimum temperature per day but where run-of-wind is available rather than just one instantaneous wind measurement or estimate provides data which, although not of as good quality (for PE calculations) as those from a station with several observations per day, is still sufficiently good for use in making PE estimates for ≥ 5 days.

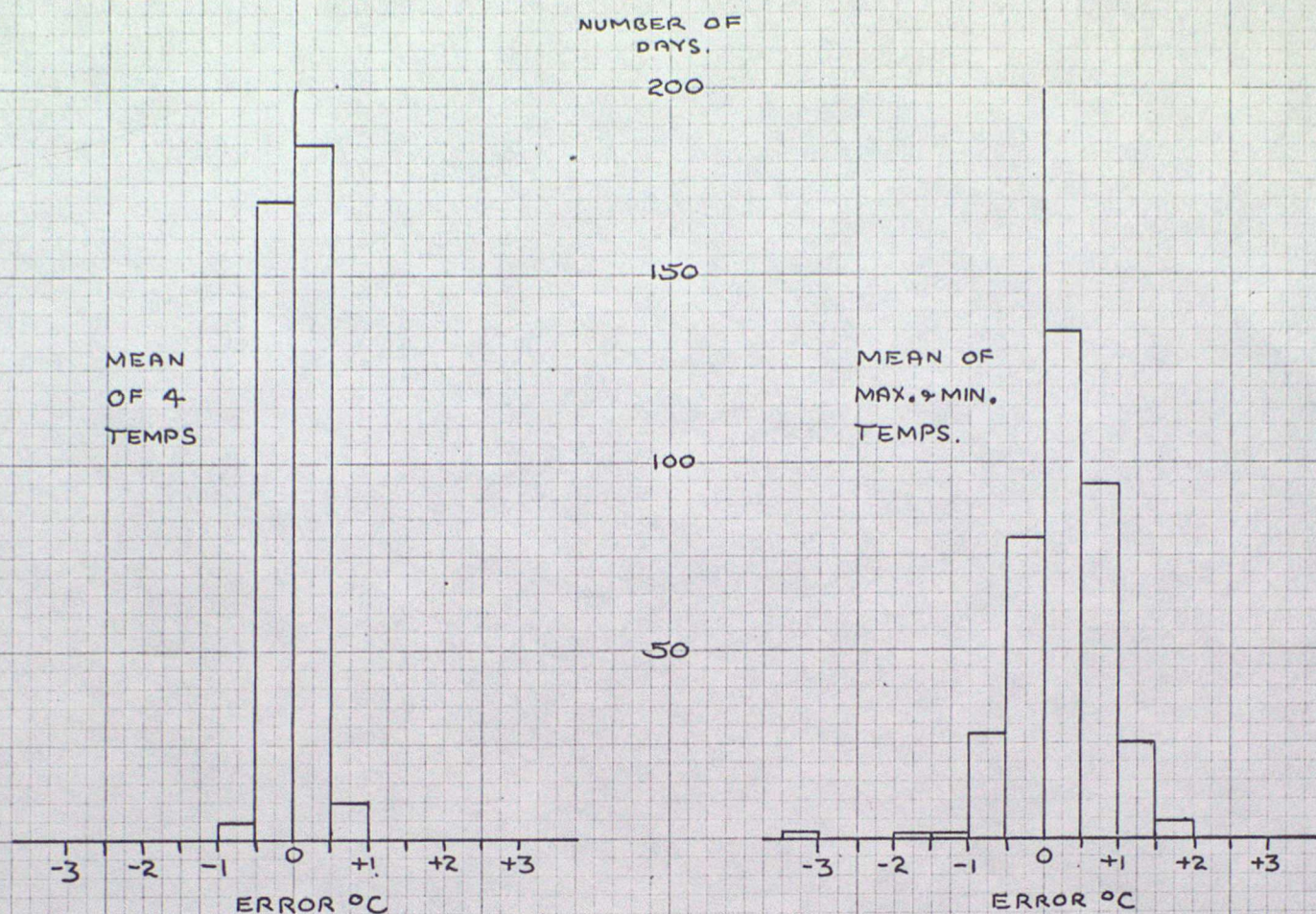
All comments assume the data to be of good quality, i.e. that the measurements are correctly made from good, well exposed instruments and that wind speed estimates are expertly made.

METEOROLOGICAL OFFICE (MET O.8B),
LONDON ROAD,
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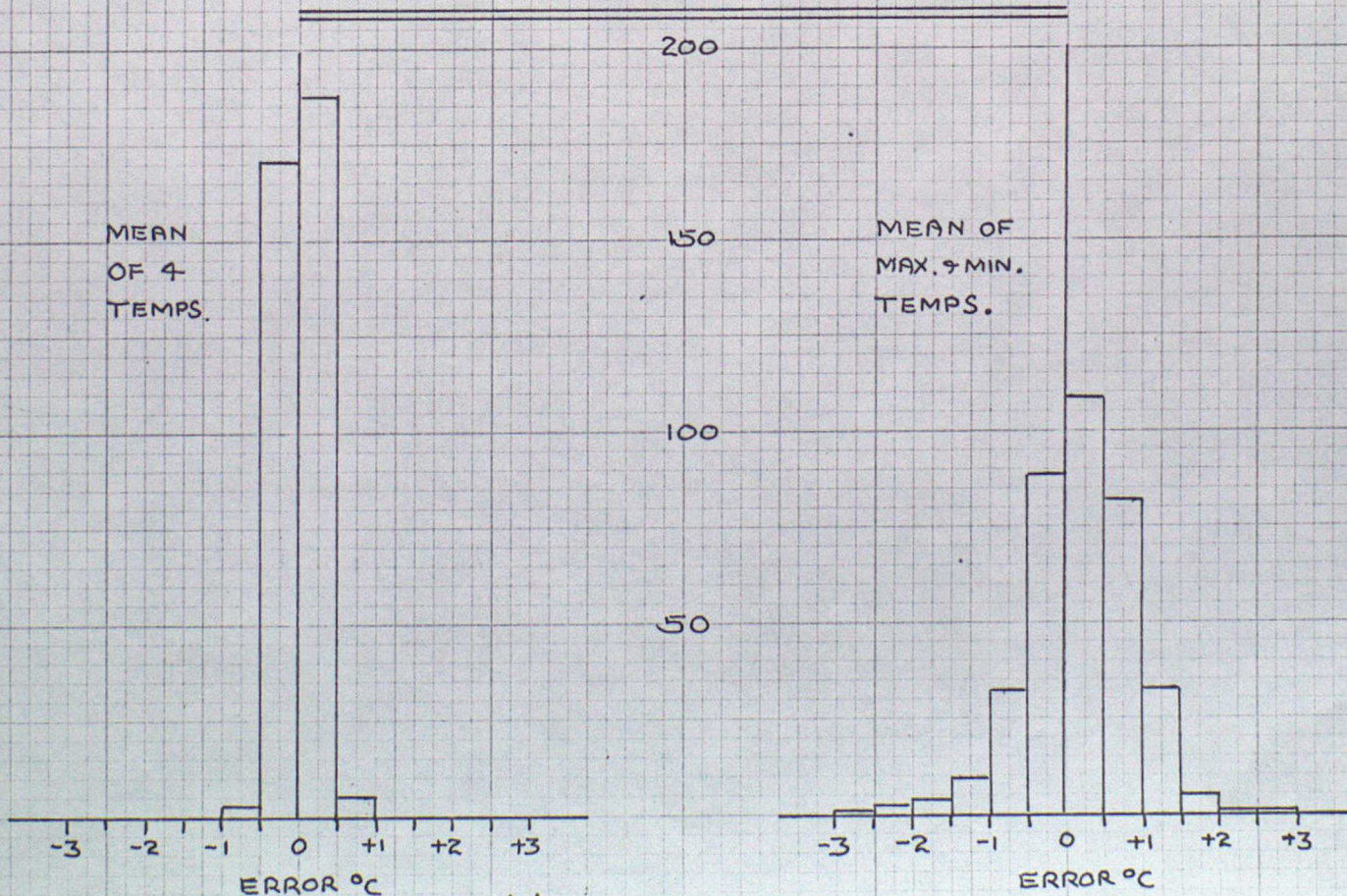
13 MAY 1971

13 MAY 1971

Fig. 1.



MANCHESTER 1968

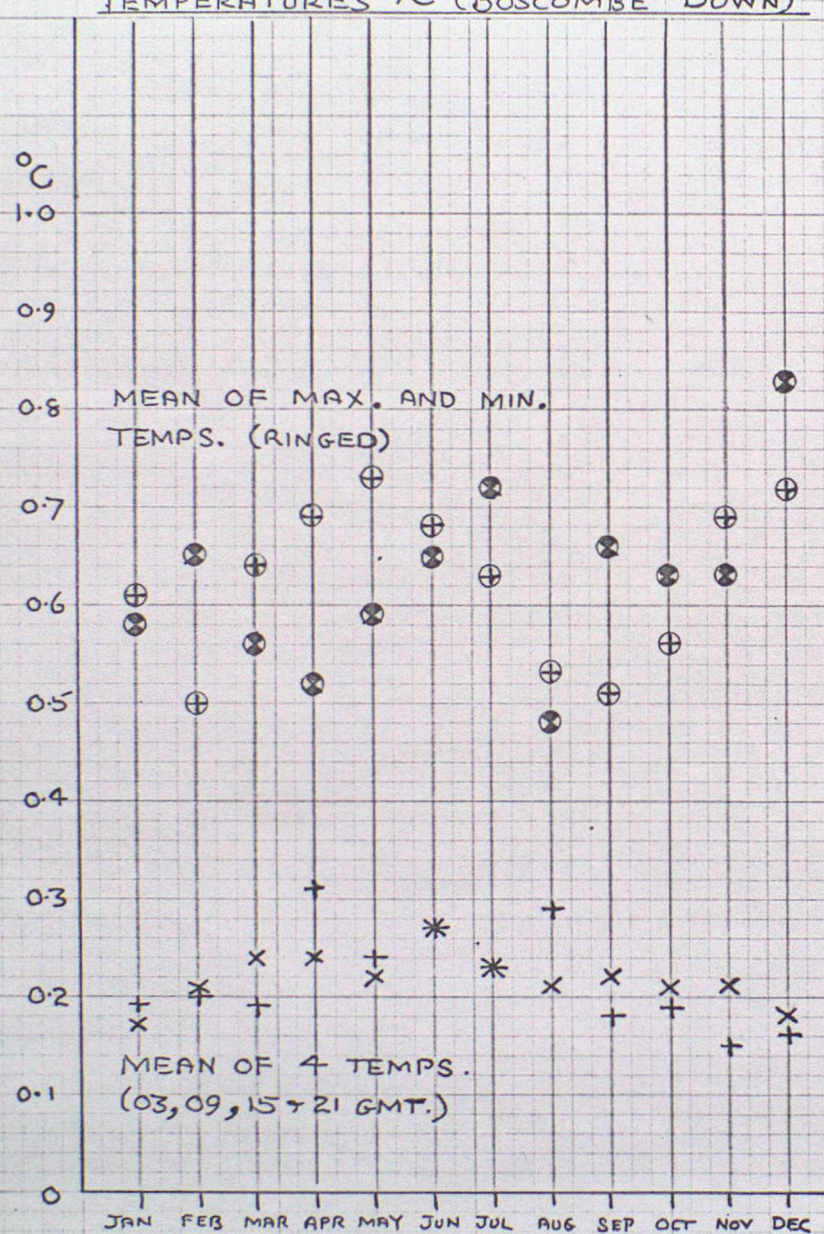


MILDENHALL 1968.

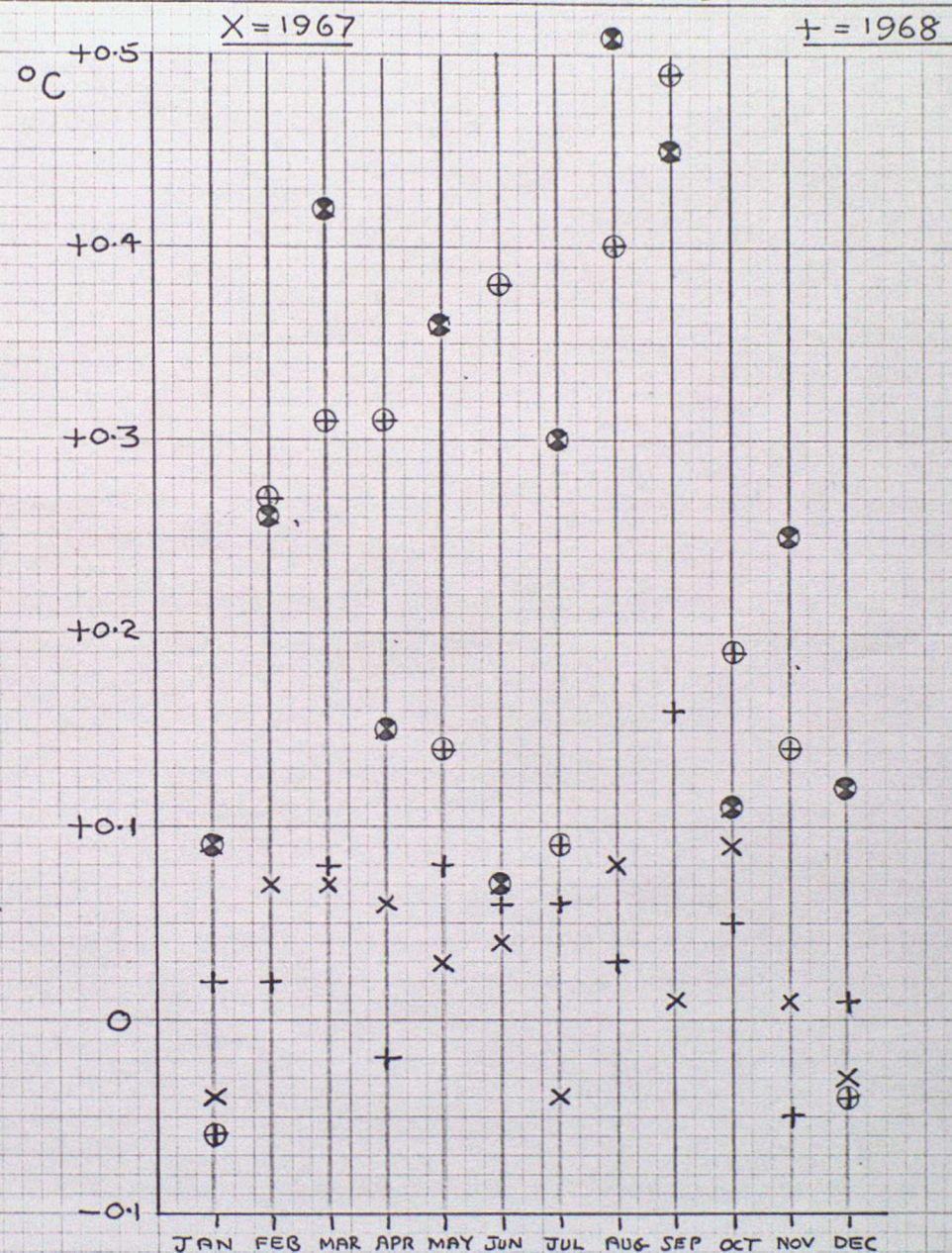
NUMBERS OF DAYS WITH GIVEN ERROR IN TAKING THE MEAN OF TEMPS. A 03,09,15 & 21 HRS. GMT. AND THE MEAN OF THE DAILY MAX. AND MIN. TEMPS AS ESTIMATES OF THE MEAN OF 24 HOURLY (SCREEN) AIR TEMPERATURES.

Fig. 2.

COMPARISON OF THE MEAN OF DAILY MAXIMUM AND MINIMUM TEMPERATURES AND THE (ARITHMETIC) MEAN OF TEMPERATURES AT 03, 09, 15 & 21 GMT. AS ESTIMATES OF THE (ARITHMETIC) MEAN OF 24 HOURLY TEMPERATURES ~ (BOSCOMBE DOWN)



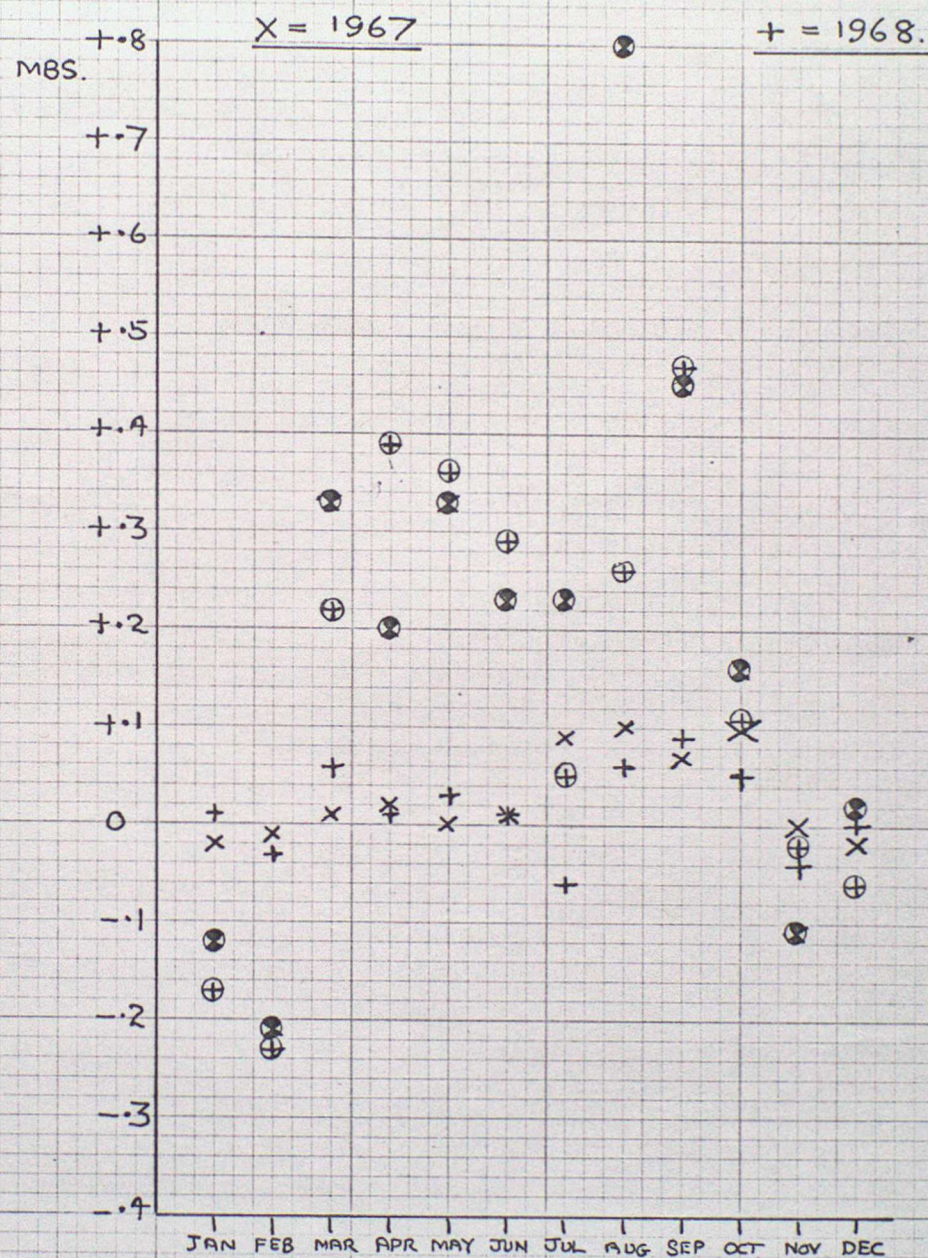
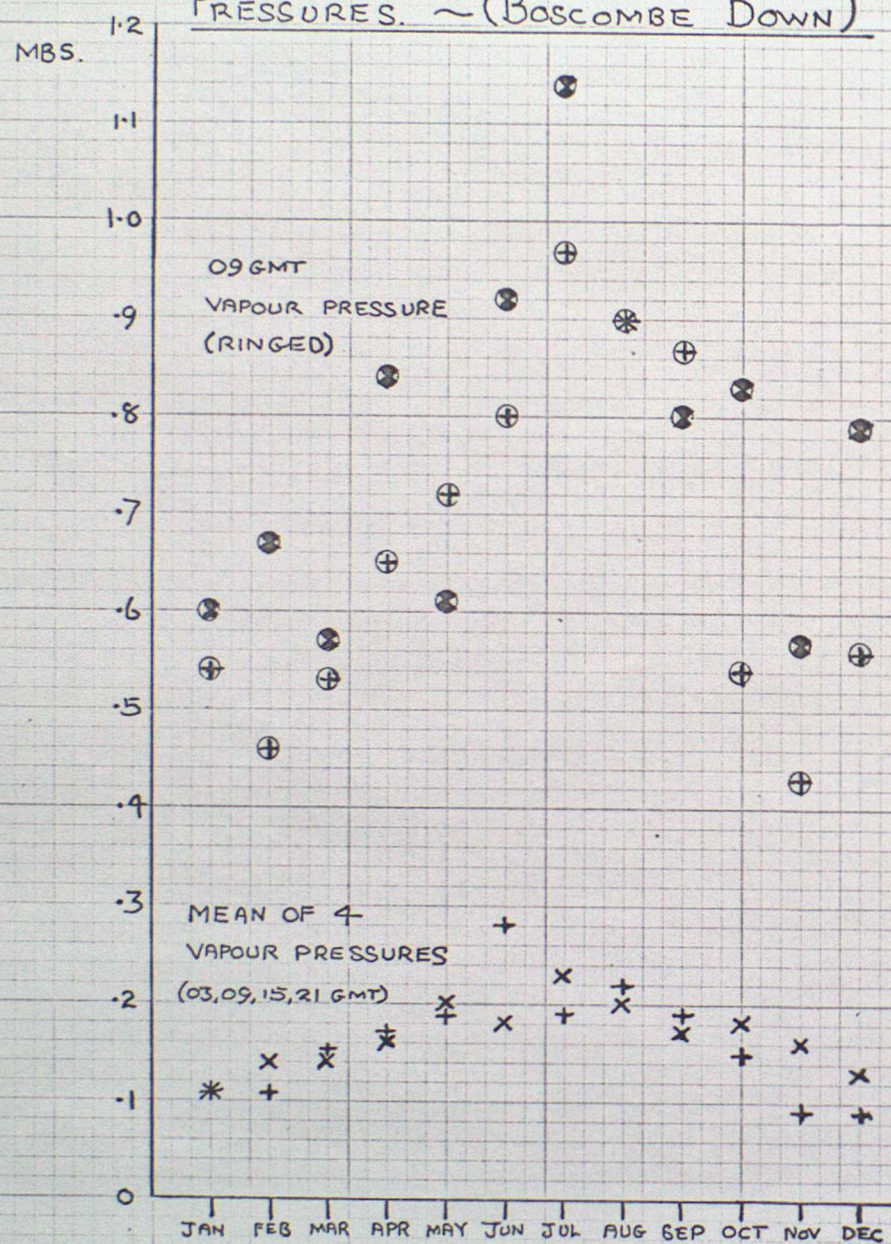
STANDARD DEVIATIONS OF DAILY ERRORS
WITHIN EACH MONTH.



MEAN DAILY ERRORS.

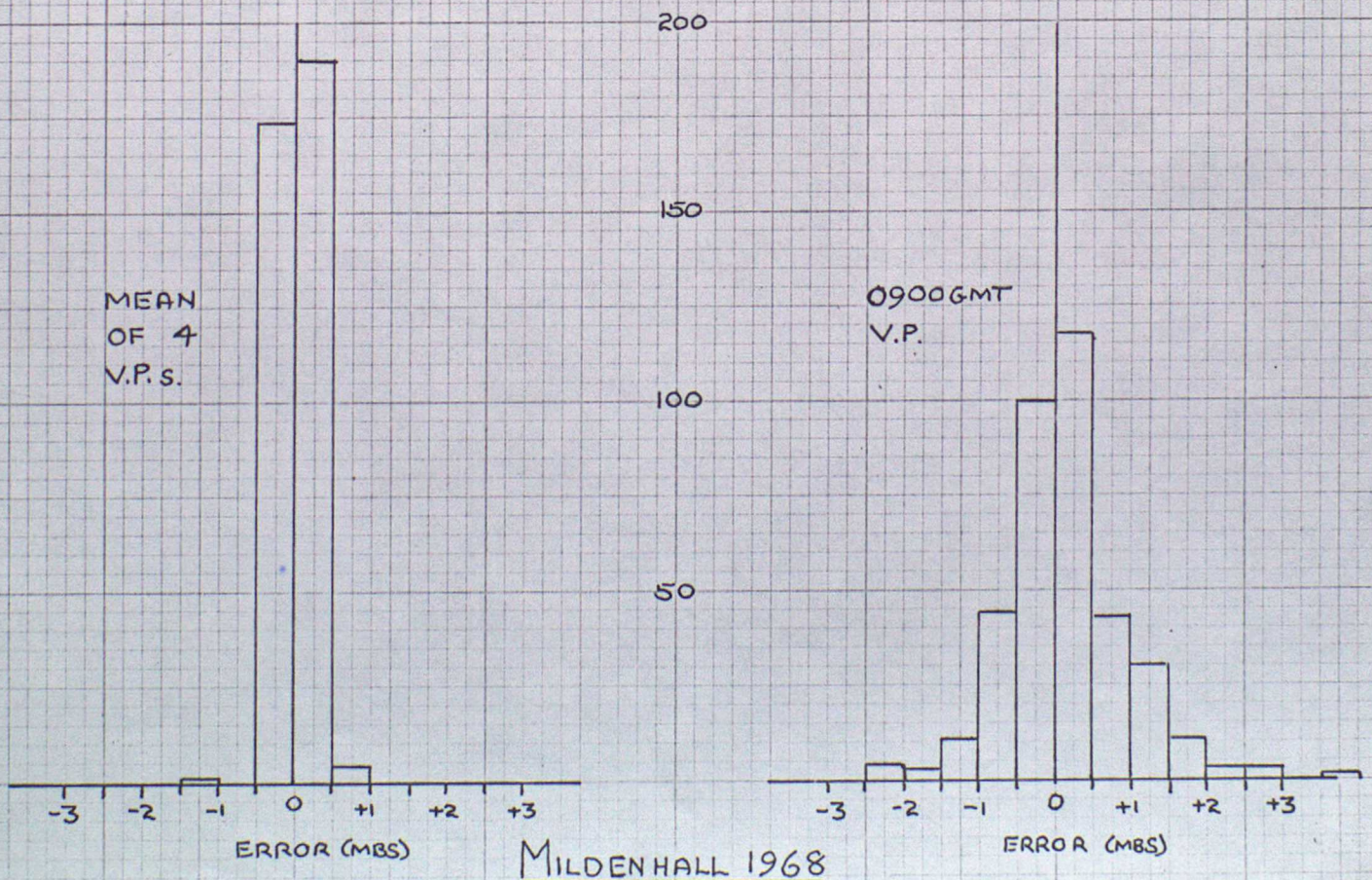
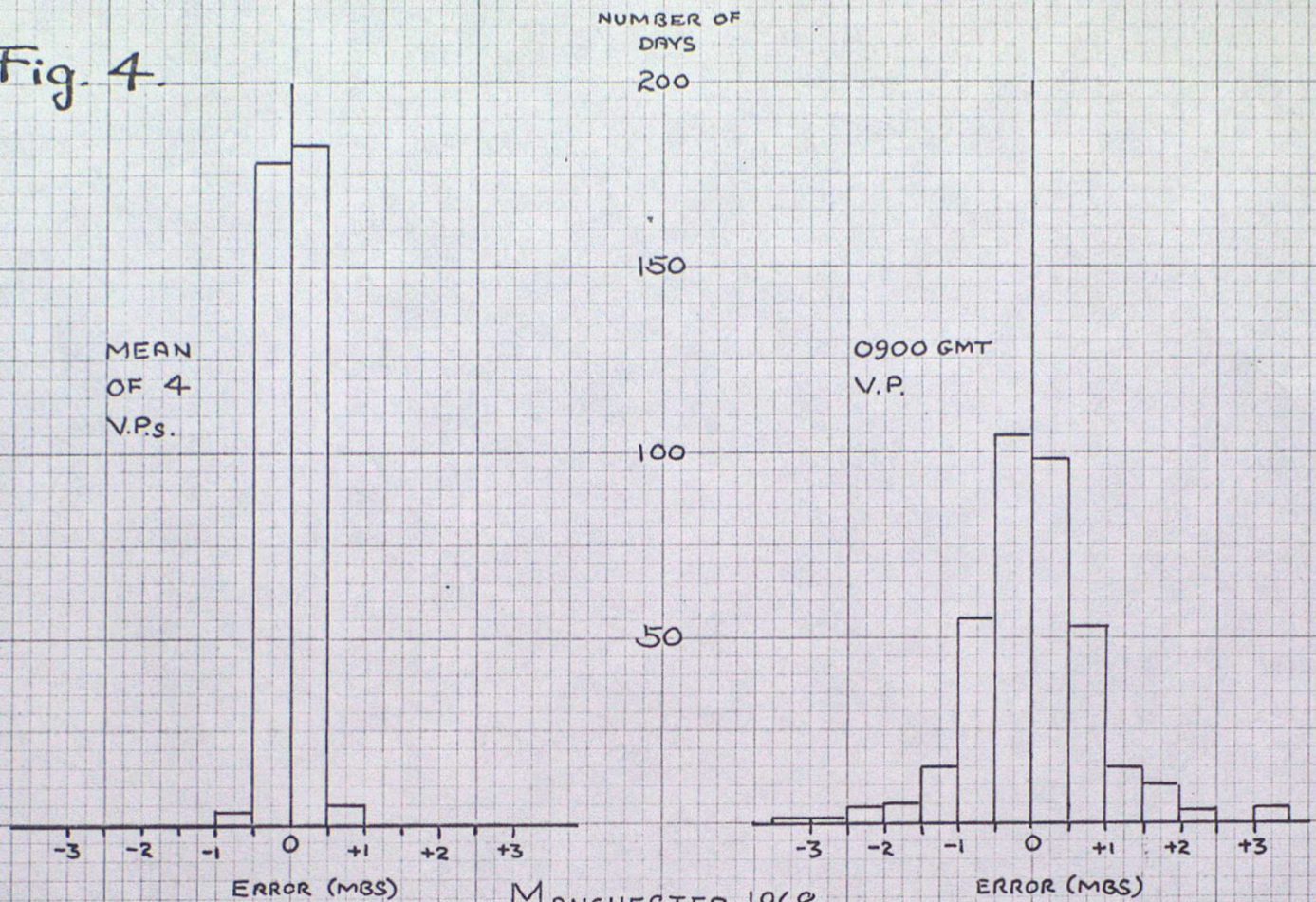
Fig. 3.

COMPARISON OF THE VAPOUR PRESSURE AT 0900 GMT AND THE (ARITHMETIC) MEAN OF VAPOUR PRESSURES AT 03, 09, 15 AND 21 GMT AS ESTIMATES OF THE (ARITHMETIC) MEAN OF 24 HOURLY VAPOUR PRESSURES. ~ (BOSCOMBE DOWN)



MEAN DAILY ERRORS.

Fig. 4.



NUMBERS OF DAYS WITH GIVEN ERROR IN TAKING THE MEAN OF V.P.s AT 03, 09, 15 & 21 HRS. GMT AND THE V.P. AT 0900 GMT (ONLY) AS ESTIMATES OF THE MEAN OF 24 HOURLY VAPOUR PRESSURES.

Fig 5. MEAN DAILY DIFFERENCES BETWEEN THE MEAN OF 24 SEPARATE, HOURLY VALUES OF SATURATION VAPOUR PRESSURE AND THE SVP APPROPRIATE TO THE MEAN OF 24 TEMPERATURES.

MBS. — 0.40

BOSCOMBE DOWN { 1967 ⊗
1968 +
MILDENHALL 1968 •
MANCHESTER 1968 ⊙

— 0.35

— 0.30

— 0.25

SVP of T — SVP

— 0.20

— 0.15

— 0.10

— 0.05

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F

M

A

M

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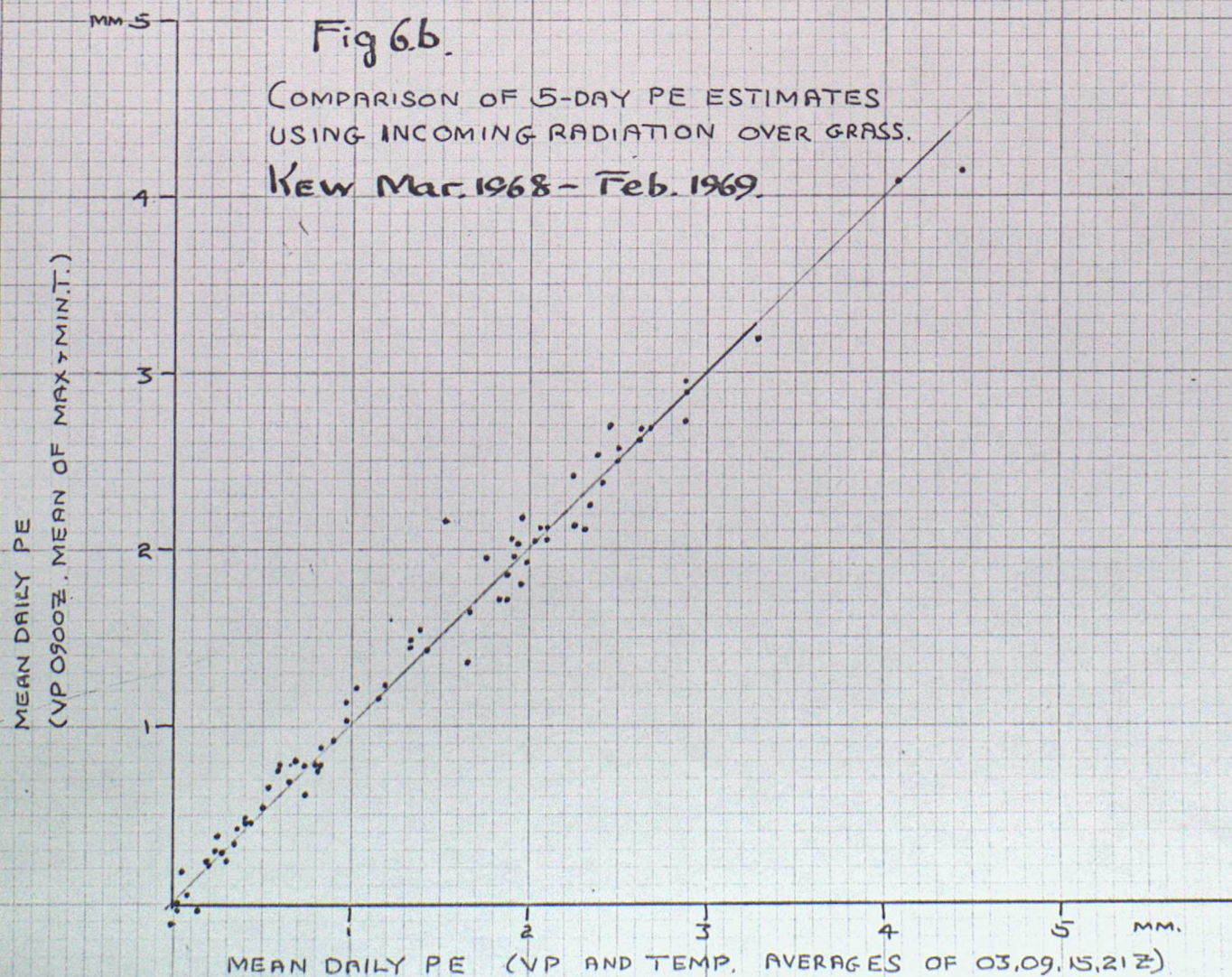
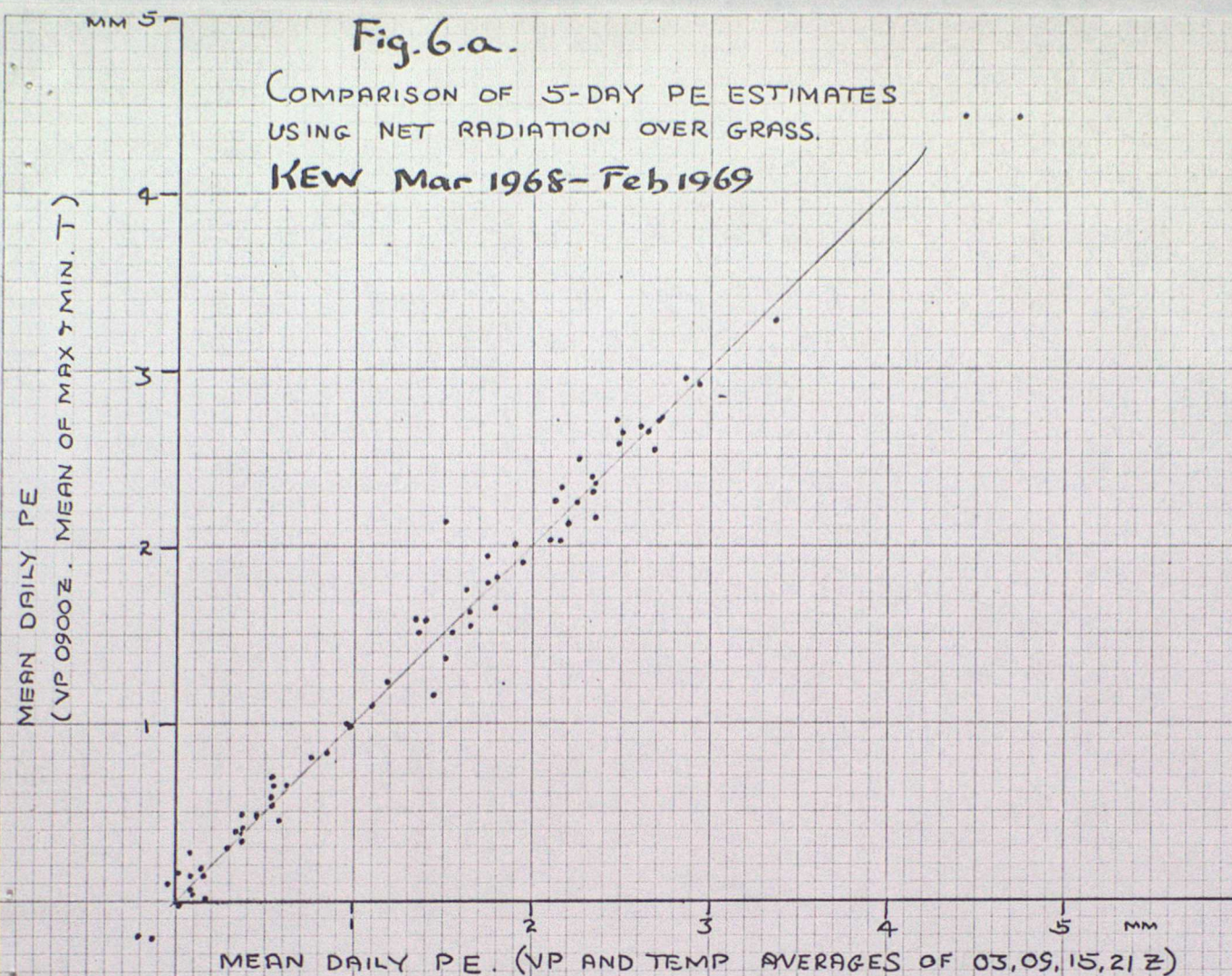
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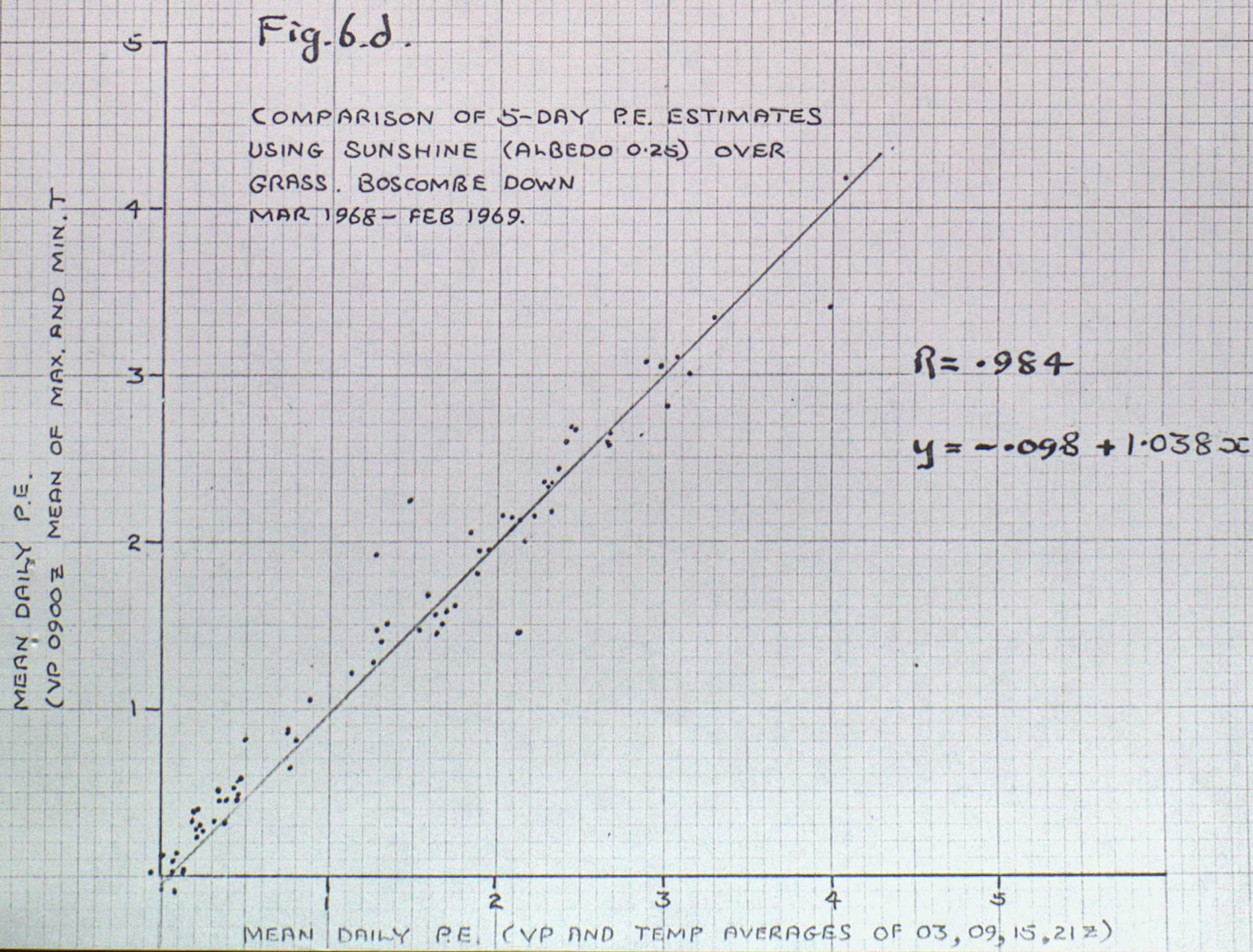
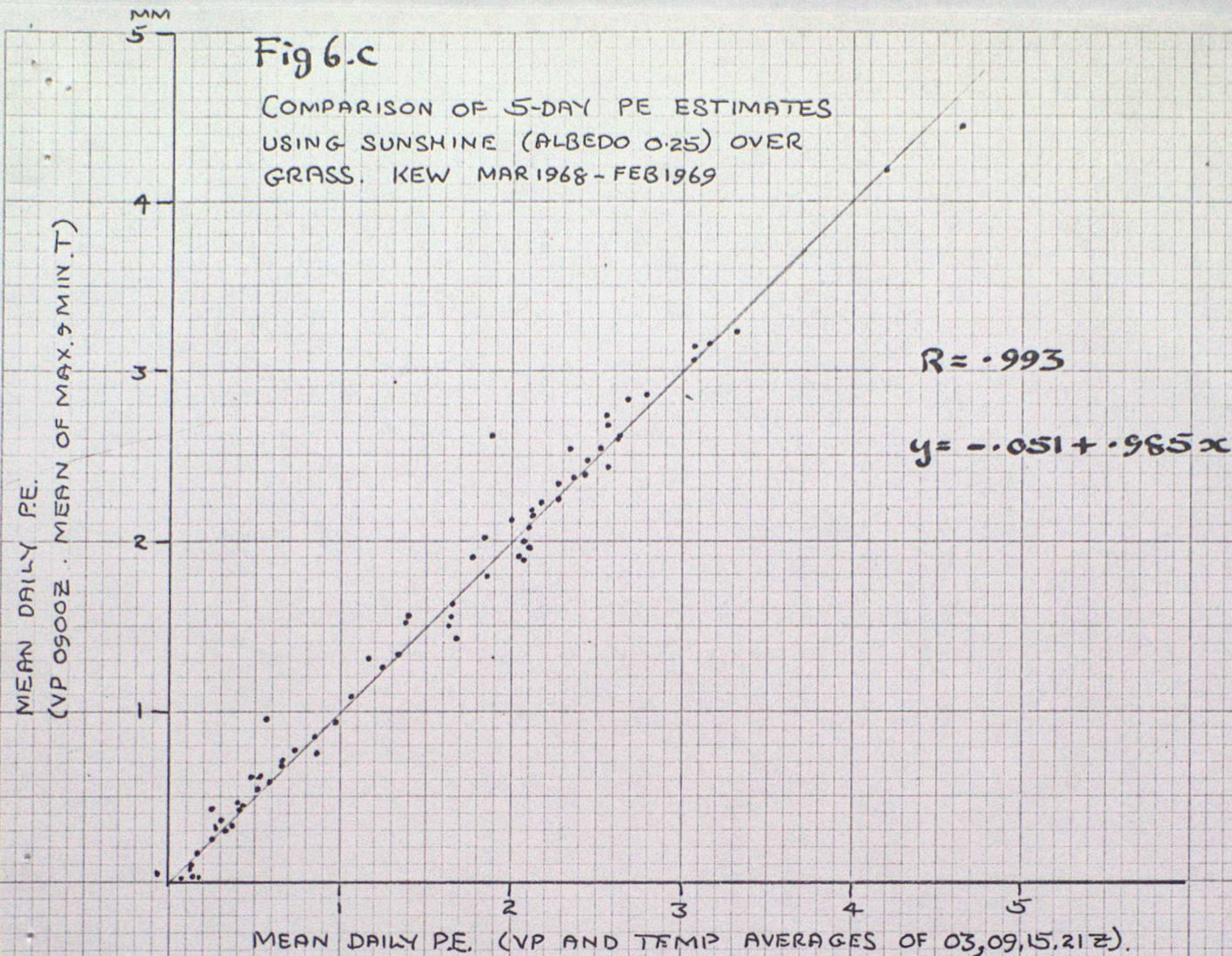


Fig. 7.

MONTHLY MEAN WIND SPEEDS (REDUCED TO 10 METRES)

+ BOSCOMBE DOWN
 x MILDEN HALL
 • MANCHESTER

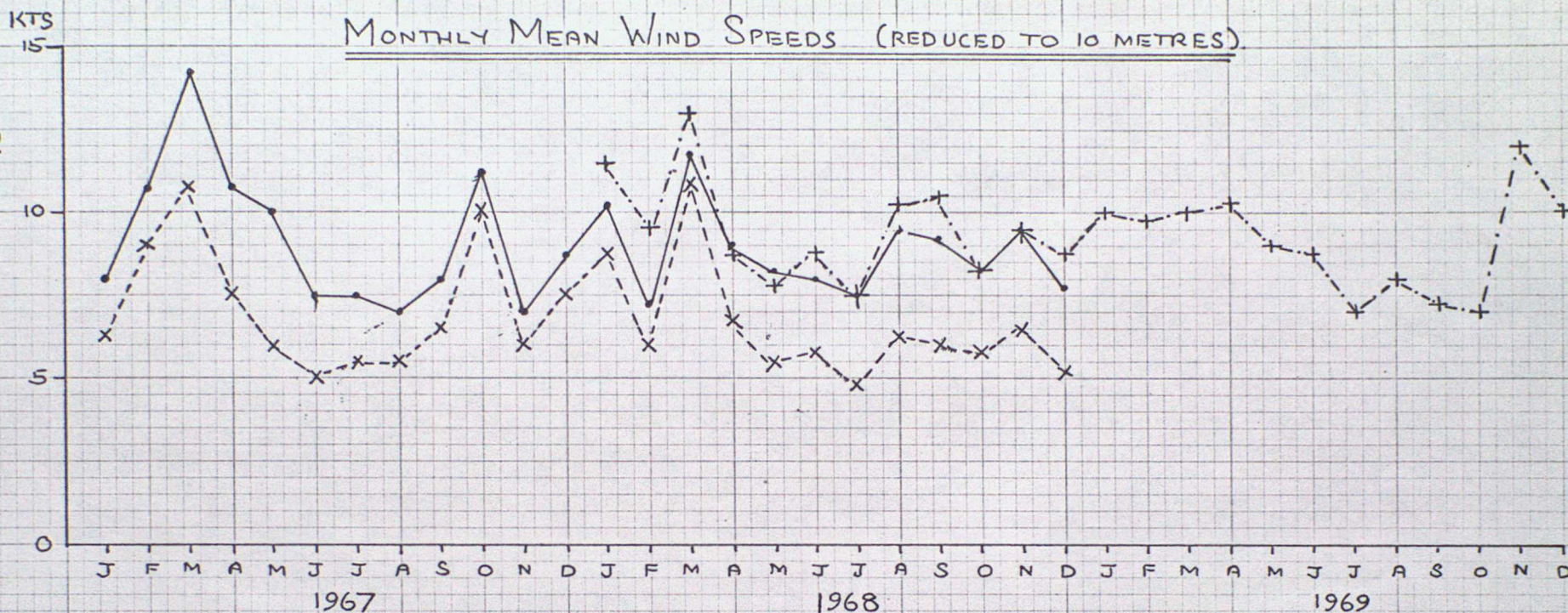
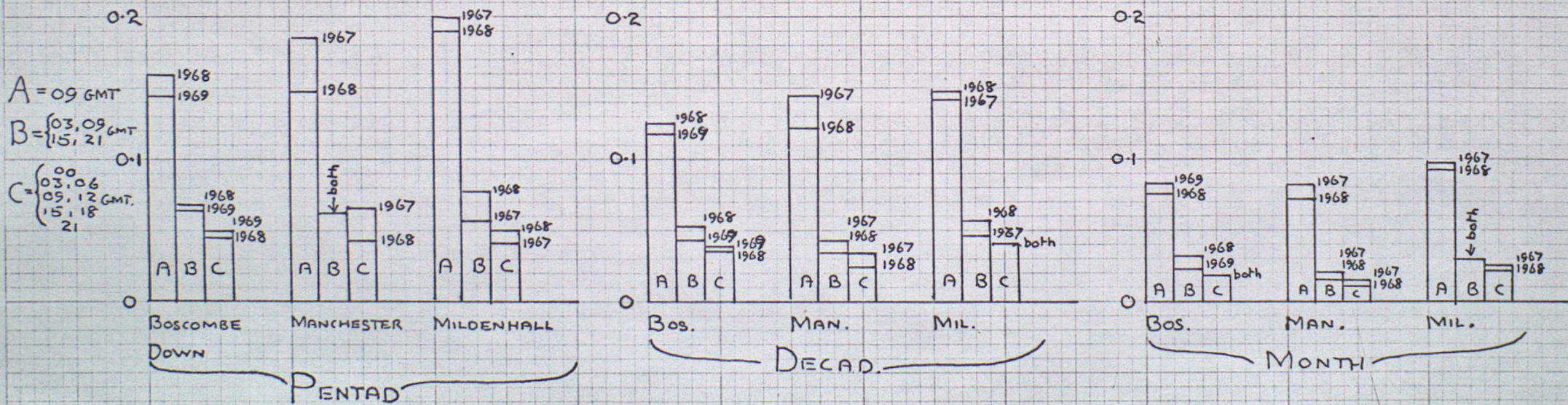


Fig. 8.



STANDARD DEVIATIONS IN RATIOS FOR EACH YEAR.

Fig 9a

MONTHLY RATIOS OF $\left\{ \begin{array}{l} 0900 \text{ GMT WIND} \\ \frac{1}{4}(03+09+15+21 \text{ GMT WINDS}) \\ \frac{1}{8}(00+03+06+09+12+15+18+21 \text{ GMT WINDS}) \end{array} \right\}$ TO TRUE MEAN WIND SPEED

BOSCOMBE DOWN.

+ = 1968
• = 1969

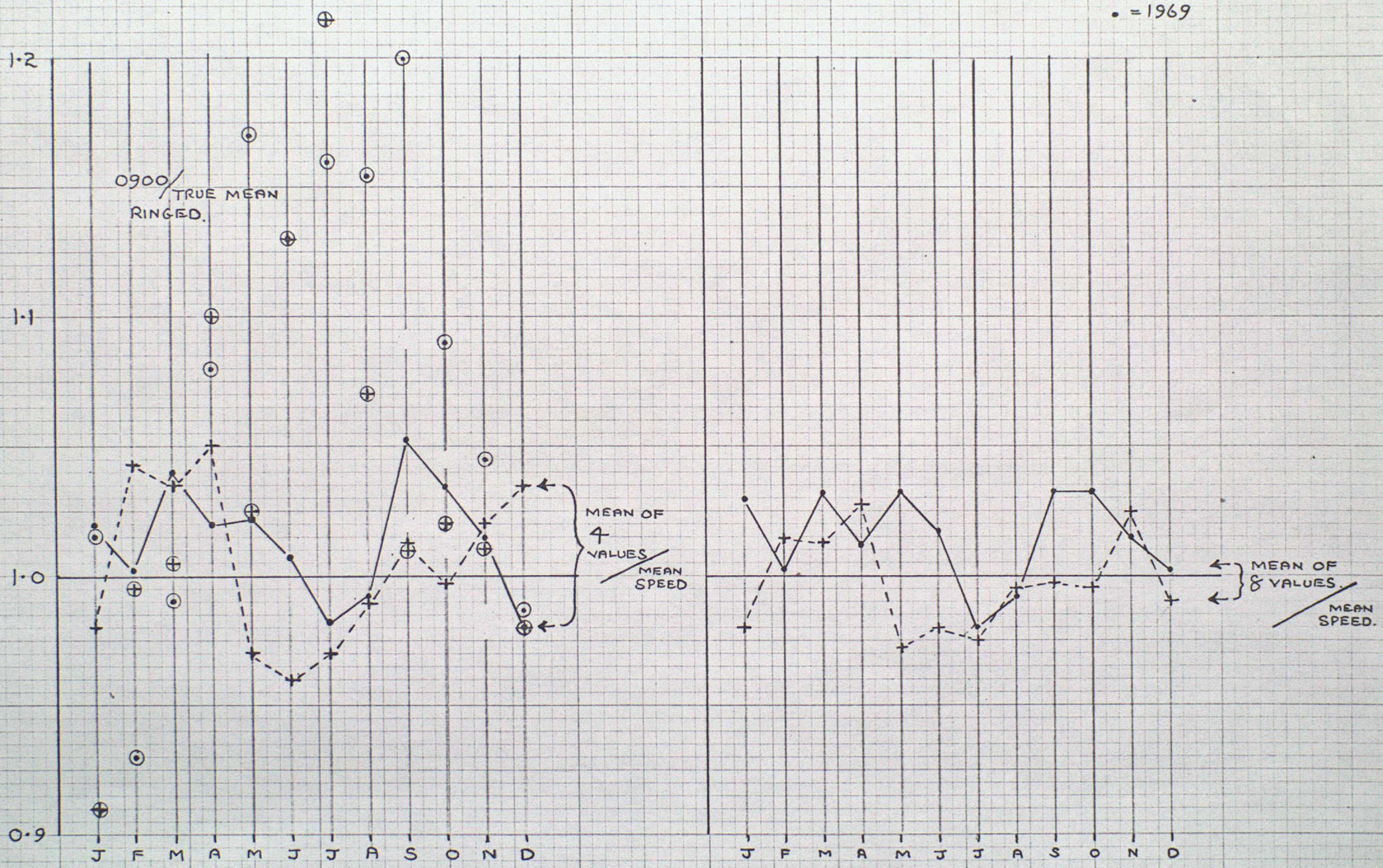


Fig. 9b.

MANCHES

X = 1967

+ = 1968

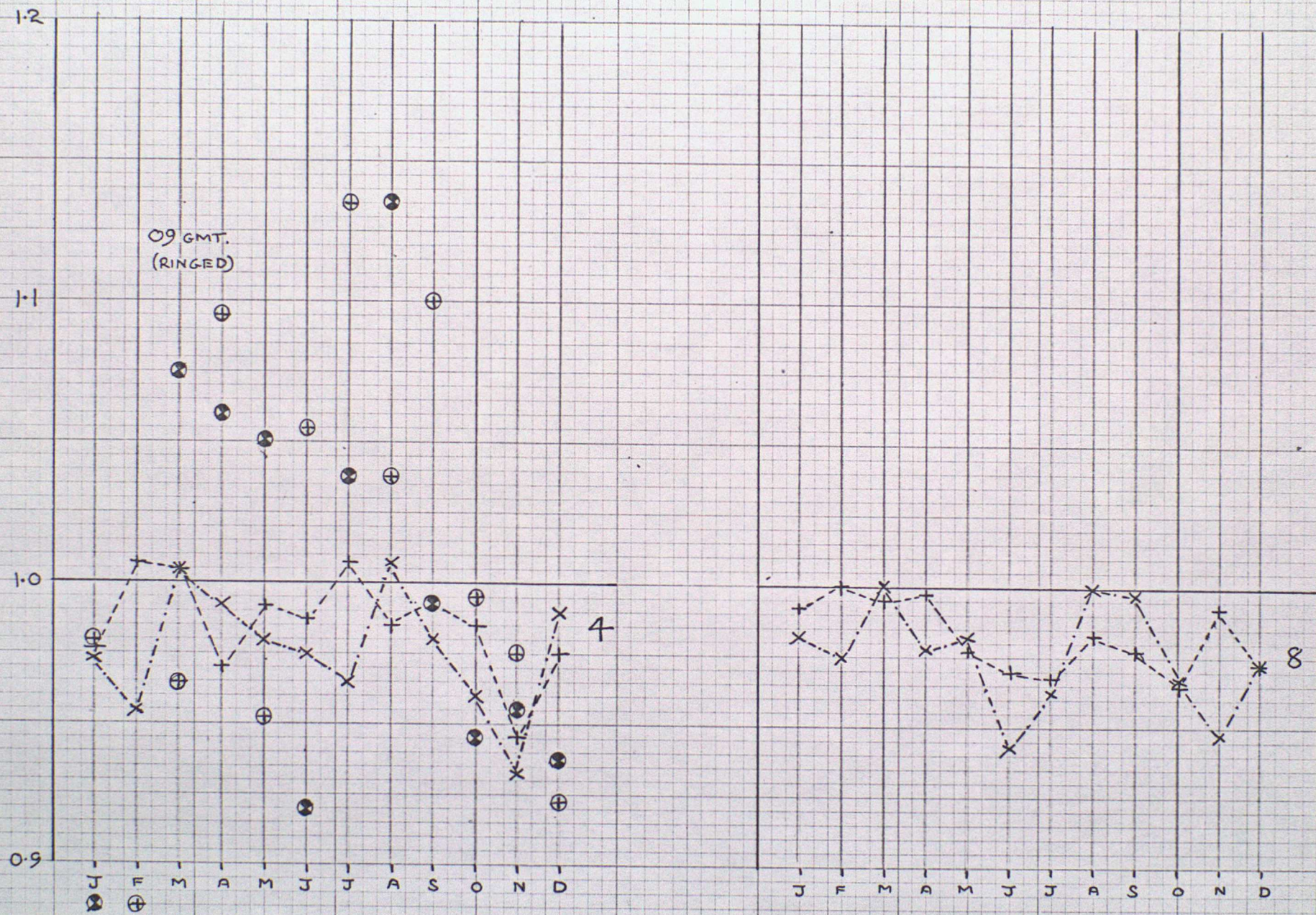


Fig. 9c.

MILDENHALL
X 1967
+ 1968

