(287)

METEOROLOGICAL OFFICE

Forecasting Research

Met O 11 Technical Note No. 33

Spatial diagnostics of operational assimilations using the observation processing database

by

Patrick Jemmer

August 1989

Meteorological Office (Met O 11)
London Road, Bracknell, Berkshire RG12 2SZ, England

MET O 11 TECHNICAL NOTE NO 33

SPATIAL DIAGNOSTICS OF OPERATIONAL ASSIMILATIONS USING THE OBSERVATION PROCESSING DATABASE

by

P. W. Jemmer

Met O 11,
Meteorological Office,
London Road,
Bracknell, Berkshire,
England, RG12 2SZ.

August 1989

ABSTRACT

This study investigates the ability of the Meteorological Office operational coarse mesh assimilation scheme to use efficiently the information contained in upper air temperature observations. The data considered are radiosonde reports from North America, for the winter and summer seasons during the year prior to the introduction of the analysis correction (A/C) scheme, in November, 1988; and for the corresponding seasons after its introduction. The method of diagnosing the efficiency of the assimilation uses an adaptation of the semi-empirical technique of Hollingsworth and Lönnberg (1989), and relies on the observation processing database for the extraction of the required statistics. Results suggest that the new assimilation scheme is rather less efficient than the old scheme in extracting information from the observations, although these results may also reflect the presence of initial imbalances which will lessen with time. There is little seasonal influence on the results.

1. INTRODUCTION

A major part of nwp systems is the taking, transmitting and assimilating of observational information. Thus these observations must be processed carefully to make best use of available data about the real atmosphere, to provide reasonably accurate, gridded initial values to the model of potential temperature, relative humidity and winds. These observations are ideally combined with the *background* from the previous forecast, in an optimal way, to produce the *analysis*, for example, using the theoretical framework provided by *optimum interpolation* techniques (Eliassen, 1954; Gandin, 1963). This requires knowledge of the statistical distribution in space of the background errors, which are normally represented by parametrised, analytic functions that model their covariance profiles. A knowledge of the background error structures also helps to impose certain constraints (such as geostrophy) on the analysis, thus suppressing the tendency of the observations to stimulate unrealistic dynamical modes of the model during the subsequent forecast (Daley, 1985).

For an assimilation scheme based explicitly on optimum interpolation it is important that the covariance profiles used faithfully reflect the actual error distributions typical of the background field and of the observations. Combining verification statistics over an extended period with curve fitting procedures, Thiébaux (1975) and Julian and Thiébaux (1975), have demonstrated the feasibility of characterising background variability (though in their case it was climatological variability) by a simple model with few parameters. In cases where, *a priori*, the shapes or amplitudes of covariance profiles are poorly known it has been proposed to incorporate the problem of analysis within the larger exercise of statistical estimation that includes profile model parameters together with the analysis field values as the set of estimands. Examples of the latter approach are the method of 'generalised cross-validation' (Wahba and Wendelberger, 1980), and 'Bayesian' or 'Likelihood' validation proposed in Purser (1988). Such methods have attractive theoretical properties but are computationally costly, and ill-adapted to accommodate implementations of optimal interpolation that, for perfectly justifiable practical reasons, depart from the strict rigour of the covariance-matrix formalism.

For an assimilation scheme like that of the Meteorological Office, which is not explicitly based on optimum interpolation, covariances of background and observational error are not of primary importance. However, there is usually an implied intention to match the theoretical optimal analysis as closely as the computationally less demanding empirical methods employed allow. As prerequisites for faithful correspondence with the truly optimal analysis fields, certain statistical relationships amongst the analysis, background and observations are expected to be obeyed, at least approximately, in a large enough sample of assimilations. Inspired largely by the recent studies of analysis error by Lönnberg and Hollingsworth (1986), and by Hollingsworth and Lönnberg (1989), this paper investigates the validity of some of these statistical relationships for data archived from the Meteorological Office's global assimilation scheme.

Crucially, the study assumes that the background and observational errors may be represented by simple covariance models; it is not assumed that the true values of these covariances be known with precision. The aim is to use diagnostics extracted from the operational assimilations over a period of weeks, as a framework for the inferences about the actual covariances. This method is basically that of Hollingsworth and Lönnberg (1989), which examines the behaviour of sample covariance statistics as functions of geographical separation of the associated observations. A preliminary network study using similar techniques is detailed in Ingleby and Bromley (1989).

The data studied are from North American radiosondes, during winter and summer seasons both before and after the operational implementation of the analysis correction (A/C) assimilation scheme. In all cases, the results suggest that assimilations have not extracted the maximum useful information from the observations, and that further tuning of the covariance models would be of benefit. Moreover, the assimilations are apparently fitting the data too closely, given their assumed error variances.

2. ANALYSIS THEORY

2.1 Assimilation systems

In this paper, assimilation diagnostics from the coarse-mesh (cm) model of the UK Meteorological Office are used. In mid-latitudes, the cm model has a resolution of about 150km. It must be noted that in November 1988, the model analysis scheme was changed from the old scheme described by Bell and Dickinson (1987), and based on the experimental technique of Lyne *et al.* (1982), to the new A/C scheme devised by Lorenc *et al.* (1989)

The salient distinctions between the schemes are as follows:

1. **the old scheme**;

Relatively few observations were permitted to influence any given grid point, the limitation being caused by the computational cost of performing the linear inversion associated with the local optimum interpolation there.

The observations were grouped to the nearest synoptic time and then assigned collectively the same weight profile in time.

Gaussian correlation functions were used, of the form:

$$\mu = \exp(-R^2/2S^2); \qquad [S = 389\text{km}]$$

2. **the new A/C scheme**;

Here, the corrections are not computed using optimum interpolation explicitly but by taking an empirical approximation that more closely resembles the technique of successive corrections (Bengthorsson and Döös, 1955; Cressman, 1959). This allows observations to exert their influence over a larger range than was typical of the old scheme.

The observations are assigned weights whose profile in time accounts for the actual validity time of each datum, even if this time is asynoptic.

Markov correlation functions are used:

$$\mu = (1+R/S)\exp(-R/S),$$

where S varies during the six hours of continuous assimilation between 360km and 250km, with a time mean of 292km, as described in Lorenc *et al.*(1989).In both cases, R represents horizontal separation, and S is a scale parameter.

The parameter, S, is interpreted differently in both cases, but both Gaussian and Markov profiles are equivalent to second order in (R/S), and behave similarly at small separations. The *Markov* function is a special form of the second order autoregressive function.

It should be noted here that, because the forecasting model used during the assimilation involves non-linear dynamics, the analysis resulting from the assimilation process is not strictly a linear function of the observations. However, the presumably small non-linear influences on the analysis statistics are neglected in this study.

2.2 The observation processing database

The observation processing database (opd) represents an archive of meteorological data, designed for ease of selection and manipulation of records. In addition to each observation, the corresponding values of the background and of the analysis at the synoptic time are stored.

In the operational database, records are stored as described in detail in Smith and Ashcroft (1988). The basic format is:

1. *observation key:* this identifies the observation with which the data are associated; it lists station identifier, observation time, receipt time at the synoptic databank, latitude, longitude and other essential information.

2. *event key:* this describes the state of attached data, and distinguishes between different records.

3. *data record:* this is specified by the event originator, and contains the data.

The physical realisation of the database results in four levels of archiving: short term, medium term, and two long term records.

There are two long term records, but this study used only one of these, called 'individual record type', which is released to permanent tape datasets on the first of each month.

2.3 Statistical Basis of Optimum Interpolation Diagnostics

Note: in this section, implicit Summation Convention is used.

Roman suffices = grid points;

Greek suffices = data points.

Repeated dummy suffices imply summation over their range.

Brackets, <q>, signify the expectation of quantity, q.

Dashes, q', signify the error in quantity, q.

This subsection deals with the statistical implications of methods used to 'spread' observational information to the model grid. In carrying out this process, three sets of quantities are considered:

1. *analysis field, **A**:* a gridded analysis field of the atmospheric state may be constructed for a given validation time, by suitable combination of information from new observations and an existing background field. This analysis provides initial conditions for the numerical weather prediction model;

2. *background field, B:* previous observations are implicitly allowed to influence the analysis, via the latest forecast. The gridded output of this forecast at the validity time of the new observations constitutes the background field.

3. *observations, O:* the data as received from observing devices.

Algebraically, these three types of variable may be regarded as three vectors. Then a linear analysis scheme generates analysis, **A**, as a linear superposition of background, **B**, and observation, **O**, using weight coefficient matrix, **W**:

$$A_i = B_i + W_{i\alpha}(O_\alpha - B_\alpha). \qquad (1)$$

Note that evaluation of B_α implicitly requires an interpolation from gridded values, B_i.

If **O** and **B** are unbiased, and, have error covariances defined by the following identities:

$$C_{ij} = \langle B_i' \, B_j' \rangle, \tag{2.1}$$

$$C_{\alpha\beta} = \langle B_\alpha' \, B_\beta' \rangle, \tag{2.2}$$

$$C_{i\alpha} = \langle B_i' \, B_\alpha' \rangle, \tag{2.3}$$

$$0 = \langle O_\alpha' \, B_i' \rangle, \tag{2.4}$$

$$E_{\alpha\beta} = \langle O_\alpha' \, O_\beta' \rangle, \tag{2.5}$$

then the analysis error covariance matrix, **D**, is, in general, given by:

$$\begin{aligned} D_{ij} &= \langle A_i' \, A_j' \rangle \\ &= C_{ij} - W_{i\alpha}C_{\alpha j} - C_{i\alpha}W^T{}_{\alpha j} + W_{i\alpha}(C + E)_{\alpha\beta}W^T{}_{\beta j}, \end{aligned} \tag{3}$$

and, the result of optimising the weights in (1), to minimise *trace*(**D**), gives a solution for **W** of the form:

$$\begin{aligned} W_{i\beta} &= C_{i\alpha}(C+E)^{-1}{}_{\alpha\beta} \\ &= C_{i\alpha}Q^{-1}{}_{\alpha\beta}, \end{aligned} \tag{4}$$

where **Q** is the sum of observational and background error covariances.

By substituting (4) in (3), the corresponding identity for the covariance of the optimal analysis results:

$$\begin{aligned} D_{ij} &= \langle A_i'A_j' \rangle \\ &= C_{ij} - C_{i\alpha}Q^{-1}{}_{\alpha\beta}C_{\beta j}. \end{aligned} \tag{5}$$

It is now possible to derive some simple, but useful analytical tools, from the definition of optimal analysis, dealing only with observation positions. Sample covariance matrices can be constructed from *N* distinct assimilations, (indexed by *t*) for the same pair, (α, β).

Defining:

$$\hat{\mathbf{O}} = (\mathbf{O} - \mathbf{B}), \tag{6}$$

three such matrices result, for:

1. *(observation - background) autocovariance*

$$\hat{Q}_{\alpha\beta} = (1/N)\Sigma_t\ (O - B)_\alpha (O - B)_\beta$$
$$= (1/N)\Sigma_t\ \hat{O}_\alpha \hat{O}_\beta. \qquad (7.1)$$

$\hat{\mathbf{Q}}$ is independent of the assimilation scheme, and has expectation:

$$\langle\hat{\mathbf{Q}}\rangle = \mathbf{Q}. \qquad (7.2)$$

2. *(observation - analysis) autocovariance*

$$\hat{P}_{\alpha\beta} = (1/N)\Sigma_t\ (O - A)_\alpha (O - A)_\beta$$
$$= (1/N)\Sigma_t\ (I - W)_{\alpha\gamma}\hat{O}_\gamma \hat{O}_\delta (I - W)^T{}_{\delta\beta}; \qquad (7.3)$$

taking the expectation, for optimal weights:

$$\langle\hat{P}_{\alpha\beta}\rangle = (EQ^{-1}E)_{\alpha\beta}$$
$$= E_{\alpha\beta} - (EQ^{-1}C)_{\alpha\beta}. \qquad (7.4)$$

3. *(observation - analysis).(observation - background) crosscovariance*

$$\hat{R}_{\alpha\beta} = (1/N)\Sigma_t\ (O - A)_\alpha (O - B)_\beta$$
$$= (1/N)\Sigma_t\ (I - W)_{\alpha\gamma}\hat{O}_\gamma \hat{O}_\beta. \qquad (7.5)$$

with expectation, for optimal weights:

$$\langle\hat{R}_{\alpha\beta}\rangle = \langle\ (O - A)_\alpha \hat{O}_\beta\rangle$$
$$= E_{\alpha\beta}. \qquad (7.6)$$

Using these sample covariances, it is possible to check the weight matrix, $\mathbf{W}$, given by (1), and, if necessary, improve it by suitable adjustments of its parameters.

The sample covariance matrix, $\hat{\mathbf{Q}}$, given in (7.1), is useful as a direct diagnostic since its value does not depend on the weight matrix, $\mathbf{W}$. Quantitatively, in this case, the intercept at zero spatial lag is the summation of the observation and background error variances.

The sample covariance matrix, $\hat{P}$, of (7.3), has, in the case of optimal weights, an expectation given by (7.4), which consists of two parts. One part is the diagonal contribution from **E**, the observation error covariance matrix; the other (i.e., all of it, for spatial lags greater than zero) is the negative of the optimal analysis error covariance, **D**, of (5), which, at the observation points, has components:

$$\langle A_\alpha' A_\beta' \rangle = (EQ^{-1}C)_{\alpha\beta} \quad . \qquad (8)$$

Thus, since a correlated component of error normally survives the analysis process, one should see in statistics $\hat{P}_{\alpha\beta}$, a distinct trend towards a negative intercept as separations between pairs (α, β) decrease (Hollingsworth and Lönnberg, 1989).

Equation (7.5), for $\hat{R}$, suggests a simple diagnostic which, in quantitative studies, avoids the problem of knowing the theoretical form the covariance, and having to calculate its inverse to determine **D**. Also, with non-diagonal **E** (a feature of many remotely sensed data), the presumed form of **E** may be subtracted from the sample statistics to recover quantities of zero intended expectation. Thus, implementation of this method constitutes a *null test*, which, by its inherent sensitivity, is arguably more revealing than using (7.3).

In practice, the weights are typically suboptimal, so the expectations (7.4) and (7.6) are unlikely to be evident in diagnostics from an actual assimilation scheme. However, consistent departures from these expectations constitute potentially valuable information about the performance of the assimilation, which, in principle, may be exploited to improve the scheme's handling of data. The sample covariances (7.1), (7.3) and (7.5) for individiual pairs of data are too numerous and too noisy by themselves to form the basis of discussion about assimilation deficiencies and their possible remedies. Assuming the background errors to be reasonably homogeneous and following Lönnberg and Hollingsworth (1986), the sample covariances are computed for sample 'bins', at 100km spacing. This substantially reduces the noise, since the averaging involved for each bin value is now over very many distinct location pairs. Also, it reduces the statistics to manageable proportions for display in the form of a histogram or profile.

3. THE EXPERIMENTS

3.1 Protocol

Three statistical features were chosen for qualitative investigation. These were the $\hat{Q}$, $\hat{P}$ and $\hat{R}$ sample covariances, given by equations (7.1), (7.3) and (7.5), but computed for each sampling bin representing a range of horizontal separations ,as discussed in section 2.3. The number of observation pairs in each bin were also collected.

The data, which, in this study are considered unbiased, were extracted from the opd individual records for the radiosonde network of the United States, including Alaska (i.e., for block numbers 70, 72, 74). The network of stations used is shown in Fig. 1. An initial selection procedure was used to reject stations reporting less than 50 observations during the time period sampled, as being unrepresentative; few stations fell into this category.

The time periods sampled were:

1. pre-analysis/correction scheme,

1 December 1987 - 29 February 1988;	1 June 1988 - 31 August 1988;
(Experiment A)	(Experiment B)

2. with analysis/correction scheme,

1 December 1988 - 28 February 1989;	1 June 1989 - 31 July 1989.
(Experiment C)	(Experiment D)

The August 1989 data were not available at the time of this study.

In the full investigation, all 15 σ-levels of the coarse mesh model were used, and are here labelled 1 - 15. However, graphically, only representative levels 4, 8, 11, and 13 were used. The values of the levels are given in Table 1.

For each period, statistics for potential temperature data were stratified into 100km bins, on the basis of station separation, ranging from zero, for observation coincidence, to 2000km. The designation of bin labels is such that the label refers to the midpoint of that 100km bin; so, "0" means 0 - 50km, "100" means 50 - 150km. Then, numerical values for each study were plotted to illustrate the features salient to diagnosis of systematic imperfections of the assimilation schemes.

3.2 Results

The results for the seasons used are displayed in Figs. 2, 3, 4 and 5, for experiments A, B, C and D respectively. Each figure is further divided to show (a), binned numbers of observations, and (b), (c), (d), each of the three binned covariances, $\hat{Q}$, $\hat{P}$, and $\hat{R}$ respectively.

3.2.1 General interpretation

It may be seen from the figures., that in all cases, corresponding graphs, (a) through (d), maintain roughly the same shapes:

(a) *Number of observation pairs:* this peaks at the origin, indicating all the self-paired observations. The 100km bin is empty, and the 200km bin contains few data pairs. From this point on, there is a steady increase in the number of observation pairs per bin, which begins to tail off at sufficiently large separations.

(b) $\hat{Q}$ *profile:* this decays from a maximum at zero separation. It is close to zero for large separations. there is also a 'kink' at the 200 km bin. (Note that all sample covariances are linearly interpolated to their values at the empty 100km bin.) The intercepts show distinctly larger values in winter than in summer for both years, particularly at the upper two levels, 11 and 13. The summer values are closer to theoretical expectations.

(c) $\hat{P}$ *profile:* values are generally scattered widely about zero, with a tendency to be positive. At large separations, the scatter subsides towards zero values as expected. Extrapolating to zero separation (from 200km, since the 100km bin is always empty) often suggests positive intercepts, especially for upper levels, which is counter to theoretical expectations.

(d) $\hat{R}$ *profile:* again, neglecting the initial 'jump', this is randomly distributed about zero, with a positive bias in the higher levels. The 200 km bin 'kink' persists here, too. The 'zero bin' values suggest a fit better than that expected, in all cases, except experiment C. The theoretical intercepts should equal the respective observation error variances assumed by the assimilation. These are: 2.6, 1.7, 4.8 and 7.8K^2 respectively, for levels 4, 8, 11, 13. Note here that it may be possible to infer a systematic background bias from the persistence of positive $\hat{R}$ values even at large separations for a few of the plotted profiles.

In order to judge the coherence of the sample statistics with respect to changes in level the statistic $\hat{R}$ is tabulated for a representative sample of the bins at all 15 model levels. Tables 2, 3, 4 and 5 refer respectively to Experiments A, B, C and D. Except for a few levels near the top and bottom, coherence is apparently maintained.

3.2.2 Specific Results

1. Experiment A (Figure 2)

The $\hat{P}$- profile values lie together on zero for separation bins greater than 600 km, but values for different levels diverge below this, with only levels 4 and 8 tending to negative intercepts, when extrapolated back from bin 200km.

The $\hat{R}$-profile statistics are scattered about zero for the lower two levels, but the upper two are significantly positive. Intercept values are substantially smaller than the theoretical figures given in section 3.2.1.

2. Experiment B (Figure 3)

The $\hat{P}$-profile statistics all lie together on zero for separation bins greater than 500km, and all levels trend towards negative intercepts, when extrapolated back from the positive separation values.

The $\hat{R}$-profile statistics are scattered about zero for the lower two levels, but the upper two are positive. Again, intercept values are much too small.

3. Experiment C (Figure 4)

Here, the $\hat{Q}$-profile is noteworthy for the upper two levels, which have very large intercept values, indicative, perhaps, of poor background values in the stratosphere.

The $\hat{P}$-profile statistics lie together on zero for separation bins greater than 800km, and all levels tend to large positive intercepts, except possibly level 13.

The $\hat{R}$-profile statistics are scattered about zero for the lower two levels, except at small separations where they are positive. The upper two are significantly positive out to considerable separations. Moreover, it is only in this experiment, that the $\hat{R}$-statistics' intercept values are quite close to their theoretical values.

4. Experiment D (Figure 5)

The $\hat{P}$-profile statistics are interesting, since they show values at the 200km bin of very near to zero; beyond this bin, the values diverge.

The $\hat{R}$-profile statistics are scattered about zero for the lower two levels, but the upper two are significantly positive, at least for separations greater than 200km. The intercept values for $\hat{R}$ are smaller than theoretically expected, although the discrepancies are not as pronounced as they were for the previous summer case (experiment B), with the old assimilation.

3.3 Conclusions

In general, it may be concluded that the use of a 'null test', crosscovariance method gives more relevant information on the efficiency of data extraction. This is seen in those cases where $\hat{P}$ statistics do not indicate inefficiency (i.e., positive intercept), but where the $\hat{R}$ statistics indicate significant and systematic deviation from random scatter about zero.

The lower levels, 4 and 8 appear to be served consistently rather better by the assimilations.

It also appears from the statistics at positive separations that, at the present time, the A/C assimilation process is acting less efficiently than the previous scheme, although an improvement is apparent in the magnitudes of the $\hat{R}$ statistics at zero separation.

In conclusion, it is important to learn more about the quantitative effects on these statistics of changing the model covariance functions. Then specific remedies can be proposed for the diagnosed departures from expectation of each of the statistical profiles.

4. PROGNOSIS

In the future development of the diagnostics described here, it would be desirable to conduct similar, but geographically stratified surveys. Thus the whole of North America could be contrasted with, for example, Europe and Asia. In this way, geographical features peculiar to one region might be accounted for.

Also of great importance, is the development of the technique to deal with vertical considerations. The use of the null test variant of the statistic, $\hat{R}$, would be of great use in investigating the efficiency of information extraction from satellite sounding data. Here, the observation error covariance matrix is non-diagonal, since the errors are not independent.

Finally, it is well known that the most important source of information about the smaller scales is the wind observations. A simple extension of the technique described in this paper would allow statistics to be examined for transverse and longitudinal components of wind, correlated with, for example, thermal or humidity data. In this way, the multivariate aspects of the assimilation process could be subjected to critical scrutiny.

Acknowledgment

I must express great thanks to Bruce Ingleby, who has offered me help and support in all stages of this project; and to Jim Purser, who acted as my Tutor for the duration of my appointment as summer vacation student at the Meteorological Office.

References

Bell, R. S. and A. Dickinson; 1987: The Meteorological Office Operational Numerical Weather Prediction System; HMSO. Met. Office Sci. Paper No. 41, Met O 979.

Bengthorsson, P. and B. R. Döös; 1955: Numerical weather map analysis; *Tellus*, ***7***, 329 - 340.

Cressman, G. P.; 1959: An operational objective analysis scheme; *Mon. Wea. Rev.*, ***87***, 367 - 374.

Daley, R.; 1975: The analysis of synoptic scale divergence by a statistical interpolation procedure; *Mon. Wea. Rev.*, ***113***, 1066 - 1079.

Eliassen, A., 1954: Provisional report on calculation of spatial covarience and autocorrelation of the pressure field; *Inst. Weather and Climate Res., Acad. Sci. Oslo. Rept.* ***No. 5***.

Gandin, L. S., 1963: Objective analysis of meteorological fields; *Leningrad. Hydromet. Press.*

Hollingsworth, A., and P. Lönnberg, 1989: The verification of objective analyses: diagnostics of analysis system performance; *Meteorol. Atmos. Phys.*, ***40***, 3 - 27.

Ingleby, N. B., and R. A. Bromley, 1989: Model error structure and estimated analysis accuracy with a network of wind profilers; *Met O 11 Technical Note **No. 28**.*

Julian, P. R., and H. J. Thiébaux, 1975: On some properties of correlation functionsused in optimum interpolation schemes.*Mon. Wea. Rev.*, ***103***, 605 - 616.

Lönnberg, P., and A. Hollingsworth, 1986: The statistical structure of short-range forecast errors as determined from radiosonde data. Part II: the covariance of height and wind errors. *Tellus*, ***34A***, 137 - 161.

Lorenc, A. C., R. S. Bell, and B. Macpherson, 1989: The new Meteorological Office data assimilation scheme; *Met O 11 Technical Note **No. 27**.*

Lyne, W. H., R. Swinbank, and N. T. Birch, 1982: A data assimilation experiment and the global circulation during FGGE special observing periods. *Quart. J. R. Met. Soc.*, ***108***, 575 - 594.

Purser, R. J., 1988: A proposal for assimilating detailed aircraft wind data in a local area. *Met O 11 Technical Note **No. 23**.*

Smith, S. G. and J. Ashcroft, 1988: The Observation Processing Database - its Contents and its derivation from the Operational Suite; *Operational Numerical Weather Prediction System, Met O 2b Documentation Paper **2.3**.*

Thiébaux, H. J., 1975: Experiments with correlation representations for objective analysis. *Mon. Wea. Rev.*, ***103***, 617 - 627.

Wahba, G., and J. Wendelberger, 1980: Some new mathematical methods for variational objective analysis using splines and cross validation. *Mon. Wea. Rev.*, ***108***, 1122 - 1143.

Table 1

Sigma-level designations with indices used in this paper.

σ-level index	**full σ-level value**
15	0.025
14	0.065
13	0.125
12	0.190
11	0.250
10	0.310
9	0.390
8	0.490
7	0.590
6	0.690
5	0.790
4	0.870
3	0.935
2	0.975
1	0.997

Table 2: (O - A)(O - B) crosscovariance statistics in K^2;

Experiment A.

σ-level ↓ / bin →	0km	500km	1000km	1500km	2000km
1	5.00	2.33	1.43	0.53	0.68
2	1.67	0.02	0.07	-0.01	-0.04
3	1.17	-0.03	-0.01	0.00	0.01
4	0.87	-0.01	0.03	-0.01	0.01
5	1.03	0.03	0.00	-0.01	-0.02
6	0.76	0.01	0.00	-0.01	-0.01
7	0.99	0.09	0.04	-0.02	-0.03
8	0.80	0.04	0.02	0.00	-0.01
9	1.14	0.11	0.07	0.03	0.05
10	1.47	0.06	0.11	0.09	0.08
11	2.91	0.48	0.26	0.10	0.11
12	3.17	0.72	0.25	-0.08	-0.15
13	2.97	0.41	0.30	0.00	-0.07
14	4.84	1.01	0.65	0.33	-0.14
15	23.75	6.70	3.59	0.69	-0.43

Table 3: (O − A)(O − B) crosscovariance statistics in K^2;
Experiment B.

σ-level ↓ / bin →	0km	500km	1000km	1500km	2000km
1	4.03	1.16	0.19	0.17	0.94
2	1.77	0.08	0.05	-0.08	-0.05
3	1.06	0.04	0.06	-0.01	-0.02
4	0.51	0.02	0.03	0.02	0.01
5	0.43	0.02	0.01	0.01	0.01
6	0.36	0.01	0.01	0.02	0.02
7	0.49	0.01	0.03	0.04	0.03
8	0.40	-0.01	0.01	0.02	0.01
9	0.58	0.00	0.02	0.04	0.05
10	0.71	-0.01	0.02	0.02	0.01
11	1.60	0.27	0.12	0.12	0.09
12	1.48	-0.04	0.01	0.02	0.06
13	1.93	0.29	0.12	0.04	0.16
14	3.68	0.80	1.12	1.16	0.75
15	15.53	3.68	2.35	0.28	1.49

Table 4: (O - A)(O - B) crosscovariance statistics in K^2;
Experiment C.

σ-level ↓ \ bin →	0km	500km	1000km	1500km	2000km
1	7.34	2.54	1.59	0.81	1.20
2	3.81	0.18	0.24	0.13	0.08
3	2.91	0.06	0.22	0.11	0.08
4	1.99	0.66	0.07	0.03	-0.02
5	1.82	0.05	-0.03	-0.05	-0.03
6	1.40	0.04	-0.02	0.02	0.02
7	1.61	0.08	0.03	-0.01	-0.03
8	1.46	0.03	-0.01	-0.01	0.01
9	1.99	0.18	0.03	0.04	0.01
10	2.87	0.29	0.06	0.05	0.02
11	5.88	1.00	0.25	-0.06	0.02
12	5.35	1.40	0.36	-0.03	-0.17
13	5.05	1.32	0.74	0.26	-0.10
14	7.93	2.28	0.93	0.12	-0.92
15	35.70	10.47	4.19	0.19	-1.14

Table 5: (O − A)(O − B) crosscovariance statistics in K^2;

Experiment D.

σ-level ↓ \ bin →	0km	500km	1000km	1500km	2000km
1	5.07	1.17	0.12	0.11	0.75
2	3.15	-0.01	-0.02	-0.09	-0.08
3	2.30	- 0.12	0.09	-0.02	-0.02
4	1.28	-0.06	0.07	0.03	-0.06
5	1.02	0.03	0.03	0.01	0.01
6	0.79	0.04	0.05	0.07	0.06
7	0.86	-0.05	-0.01	-0.01	-0.01
8	0.79	-0.01	0.01	0.02	0.02
9	1.26	0.03	-0.01	0.02	0.00
10	1.59	0.11	0.09	0.08	0.06
11	3.83	0.51	0.35	0.31	0.06
12	3.33	0.31	-0.06	-0.05	0.06
13	3.49	0.49	0.42	0.22	0.37
14	12.29	5.97	6.82	6.10	5.68
15	36.21	11.49	13.36	10.17	13.80

Figure Captions

Figure 1: Map showing the radiosonde network used in the study.

Figure 2: Curves showing:

a. Number of observation pairs;

b. (O - B)(O - B) autocovariance in K^2;

c. (O - A)(O - A) autocovariance in K^2;

d. (O - A)(O - B) crosscovariance in K^2,

for Experiment A.

Figure 3: As figure 2, but for Experiment B.

Figure 4: As Figure 2, but for Experiment C.

Figure 5: As Figure 2, but for Experiment D.

Figure 1.

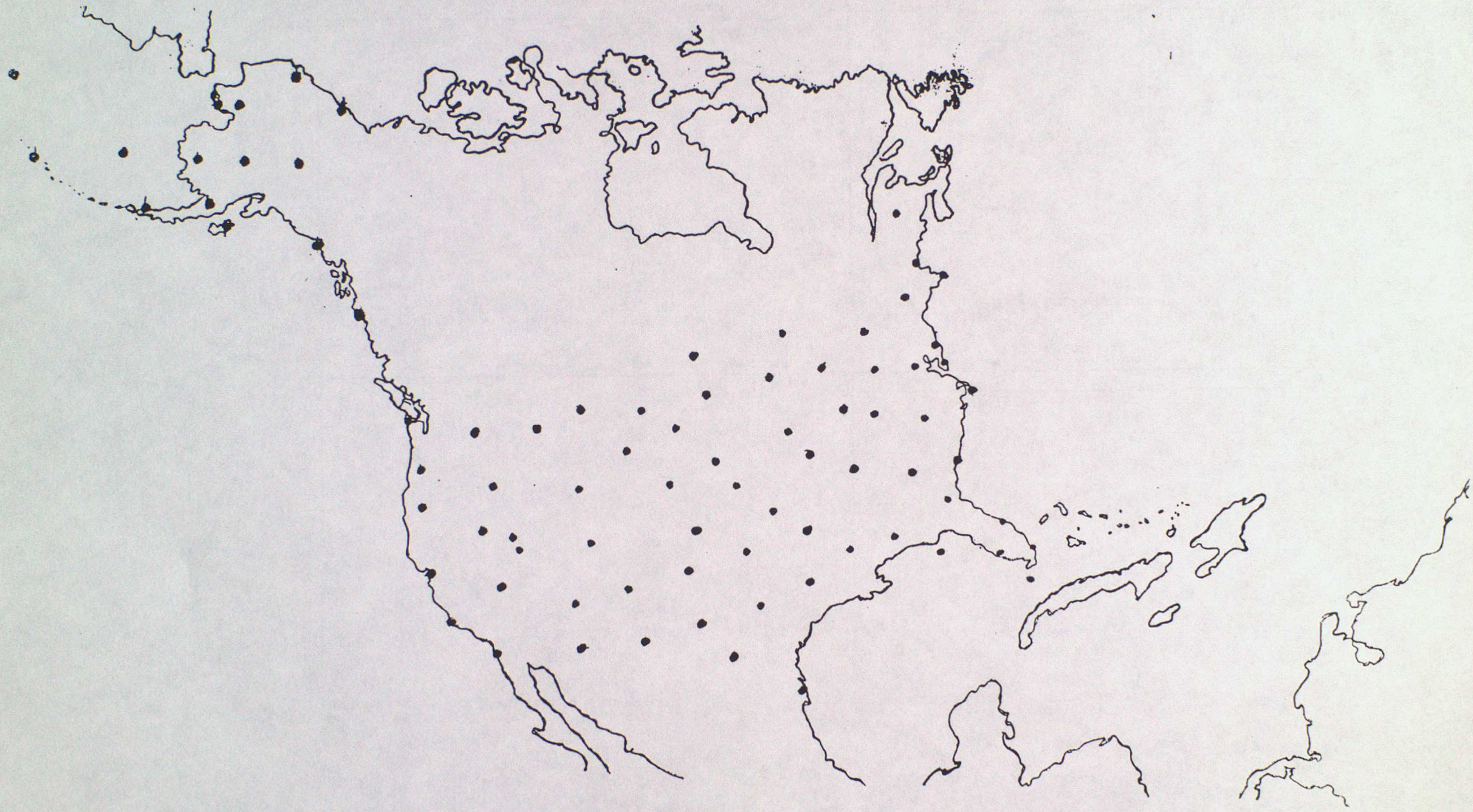
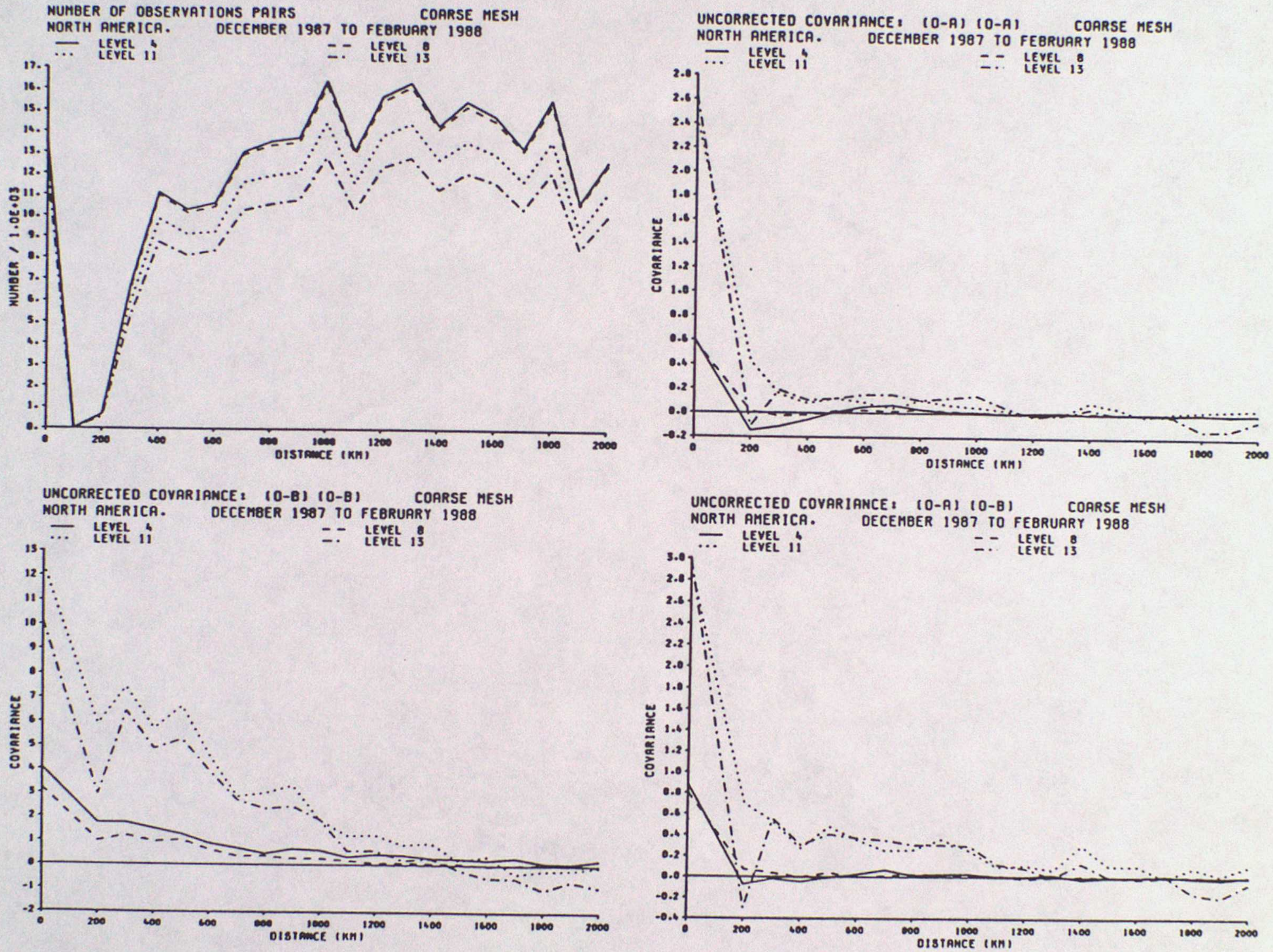


Figure 2.



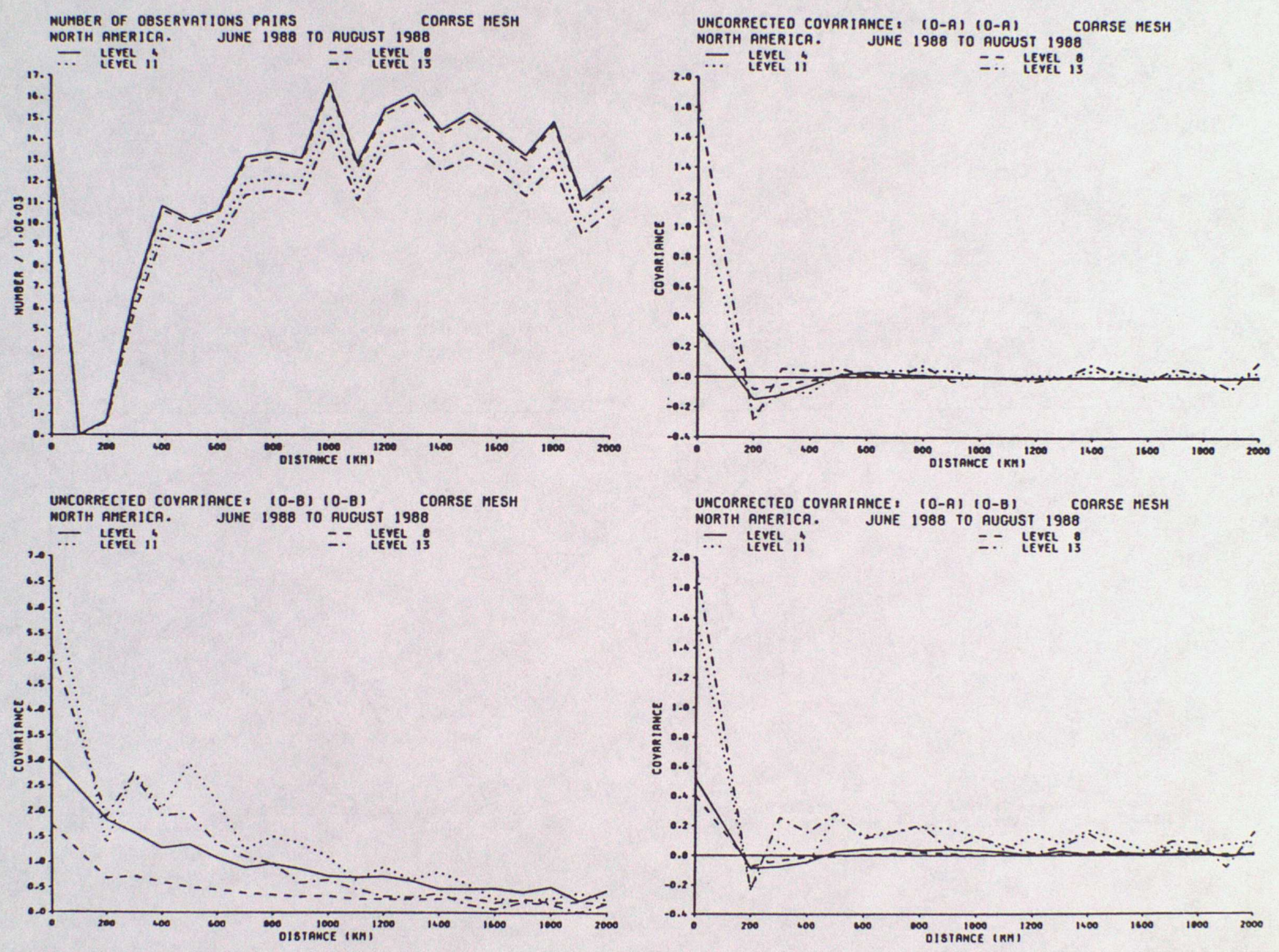
(a)

(c)

(b)

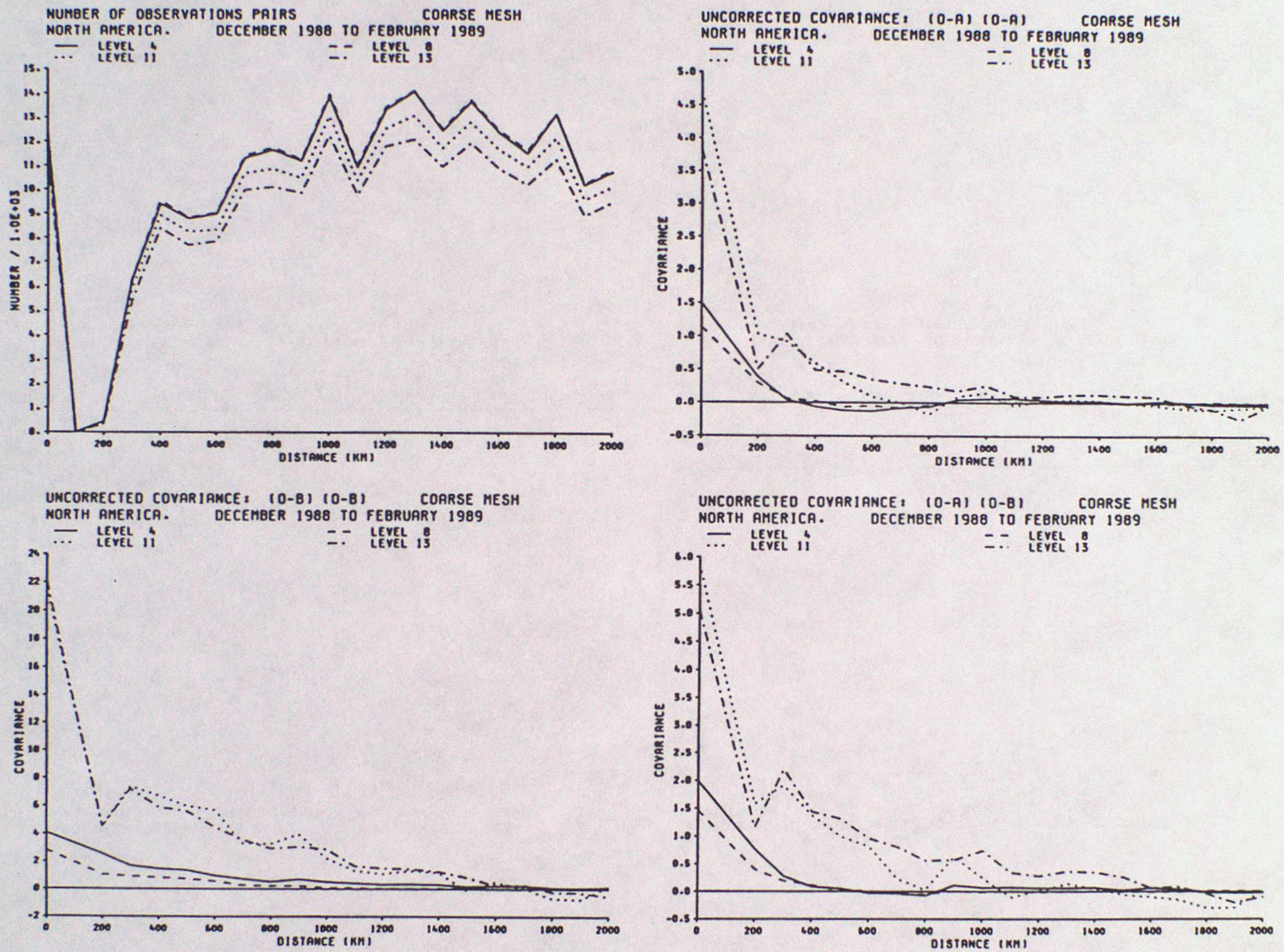
(d)

Figure 3.



(a) (b) (c) (d)

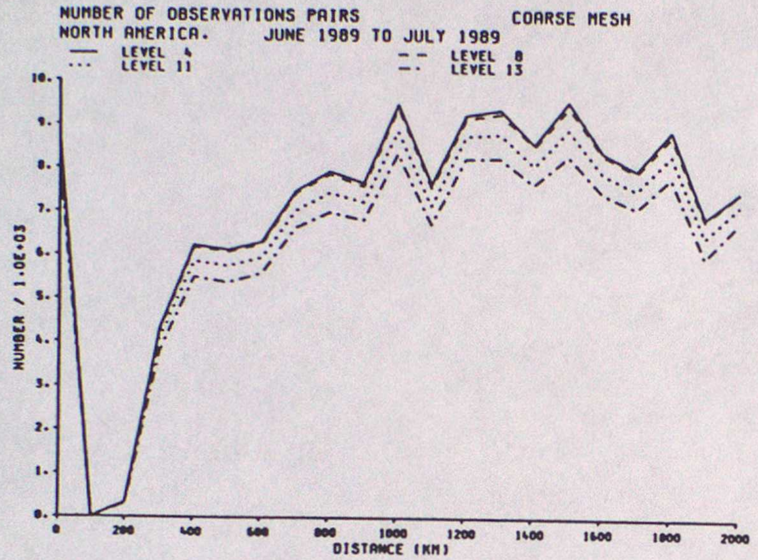
Figure 4.



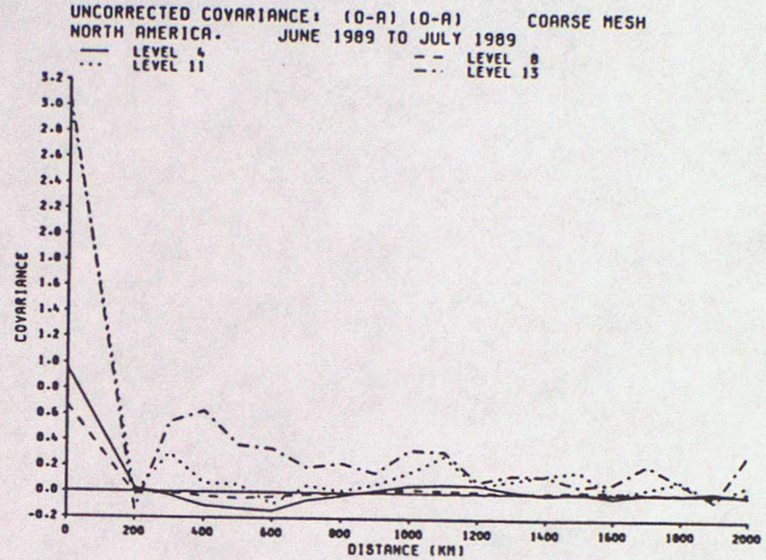
(a) (c)

(b) (d)

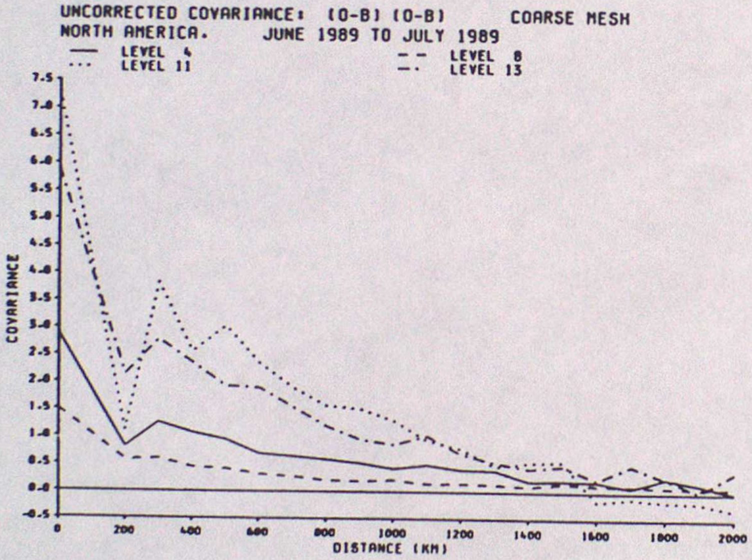
Figure 5.



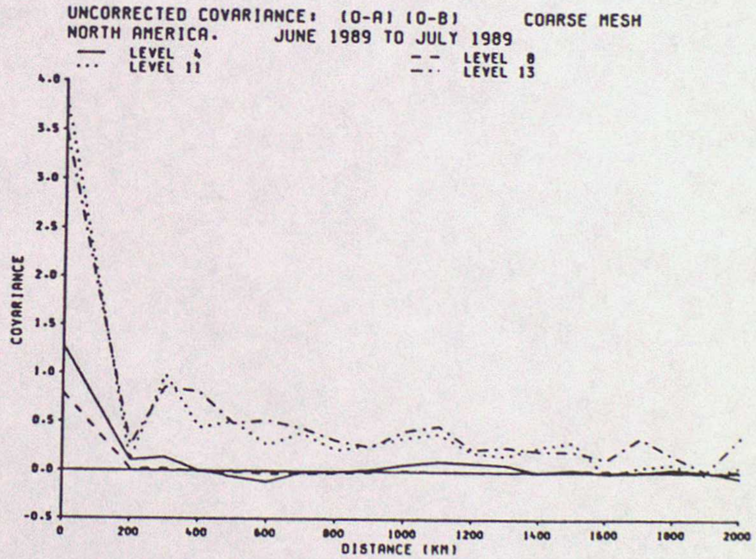
(a)



(c)



(b)



(d)

Met O 11 - Technical Notes

247.	Some experiments with two-dimensional semi-geostrophic and primitive equation models, with sigma as the vertical coordinate.	C.A. Parrett February 1987.
248.	Moist frontogenesis in the geometric model.	M.V. Holt March 1987.
249.	Mesoscale Model Trial of a revised convection scheme and cloud modifications.	O. Hammon May 1987.
250.	Results from the fine-mesh trial of a modified physics package.	O. Hammon July 1987.
251.	Verification of Mesoscale Model forecasts during the Winter, November 1986-February 1987.	O. Hammon March 1987.
252.	Mountain Wave generation by models of flow over synoptic scale orography.	M.J.P.Cullen & C.A. Parrett March 1987.
253.		B. Macpherson
254.	Numerical Modelling of Balanced Atmospheric Flow.	M.J.P. Cullen & G.J. Shutts. April 1987.
255.	Strategies for using Mesoscale data in an operational Mesoscale Model.	B.W. Golding. March 1987.
256.	Experiments with Divergence Damping and reduced diffusion in the mesoscale model.	S. Ballard. May 1987.
257.		A. Lorenc
258.		M. Gange.
259.	Implicit Finite Difference Methods for modelling discontinuous atmospheric flows.	M.J.P. Cullen. June 1987.
260.	A large amplitude gravity wave in the lower stratosphere detected by radiosonde.	G.J. Shutts, M. Kitchen & P.H. Hoare. June 1987.
261.	Modifications to the automatic quality control of ship data and an assessment using case studies.	B.R. Barwell & C.A. Parrett.
262.		
263.	Results from fine mesh model trial using a modified evaporation scheme.	O. Hammon & C.A. Wilson.

Met O 11 - Technical Notes

New Series

1.	An assessment of the results of trials of a new analysis scheme for the operational global model.	R.S. Bell October 1987
2.	A case study showing the impact of analysis differences on medium range forecasts.	R.A. Downton & R.S. Bell January 1987
3	Development of the Analysis Correction Scheme II Inclusion of an observation density analysis.	B. Macpherson
4.	An assessment of a trial to test small changes to the convective scheme in the mesoscale model	O.M. Hammon January 1988
5.	Trial of proposed changes to the mesoscale model for November 1987.	O.M. Hammon December 1987
6.	Assessment of HERMES sounding processed using the NEW cloud clearing scheme	R. Swinbank March 1988
7.	An assessment of the impact of a correction to the mesoscale model turbulence/vertical diffusion scheme implemented in March 1988	S. P. Ballard April 1988
8.	Comparison of algorithms for the solution of cyclic, Block and tri-diagonal systems	M. Mawson May 1988
9.	A comparison of alternating direction implicit methods for solving the 3-D semi-geostrophic equations on a sphere	M. Mawson May 1988
10.	The automatic quality control of surface observations from ships. The final trial, latest statistics, operational implementation & future work	C.A. Parrett May 1988
11.	"Panel-Beater": A proposed fast algorithn for semi-geostrophic finite-element codes.	R.J. Purser June 1988
12.	A 5-day forecast trial of the AC scheme.	R. Downton R.A. Bromley M.A. Ayles September 1988
13.	A theoretical study of the information content of the ERS-1 Scatterometer data	R.J. Purser August 1988
14.	A further global trial of the analysis correction scheme - Christmas 1987	R.S. Bell August 1988
15.	The sensitivity of a medium range forecast with the Analysis correction scheme to data selection in the horizontal.	B. Macpherson R. Downton

Met O 11 - Technical Notes

16.	The sensitivity of fine-mesh rainfall forecasts to changes in the initial moisture fields	R.S. Bell O. Hammon August 1988
17.	Conservative finite difference schemes for the unified forecast/climate model	M.J.P. Cullen T. Davies July 1988
18.	Interpreting results from numerical models	T. Davies August 1988
19.	A comparison of the OWSE assimilation scheme with the operational global assimilation scheme	D.N. Reed M.A. Ayles October 1988
20.	Improvements to low cloud forecasts from the mesoscale and fine-mesh models	O.M. Hammon October 1988
21.	The effect of route choice on aircraft wind observations over the north atlantic	D. Lang N.B. Ingleby October 1988
22.	Maximum likelihood de-aliasing of simulated scatterometer wind fields using adaptive descent algorithms	R.J. Purser
23.	A proposal for assimilating detailed aircraft wind data in a local area	R.J. Purser January 1989
24.	Basic formulation and boundary conditions of the mesoscale model	S.P. Ballard
25.	Development of a new physics package for the global forecast model	C.A. Wilson J.M. Slingo January 1989
26.	The trial of the fine-mesh version of the analysis correction scheme	O.M. Hammon R.A. Bromley B. Macpherson May 1989
27.	The New Meteorological Office Data Assimilation Scheme	A.C. Lorenc R.S. Bell B. Macpherson April 1989
28.	Model error structure and estimated analysis accuracy with a network of wind profilers	N.B. Ingleby R.A. Bromley April 1989
29.	Examples of hybrid vertical co-ordinate systems for the unified forecast/climate model.	R.Swinbank July 1989
30.	The Meteorological Office Experimental Mesoscale Numerical Weather Prediction System:July 1989	B.W.Golding July1989

Met O 11 - Technical Notes

31	Tests of the Heun Advection Scheme.	B.L.Marshall July 1989
32	Not Yet Available	R.A.Bromley R.A.Downton
33	Spatial diagnostics of operational assimiliations using the observation processing database.	P. Jemmer August 1989