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Report on Charney and Eliassen's One-Dimensional Numerical Method for Calculating Motion of Barotropic Disturbances in a Westerly Airstream

by

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Introduction

Numerous writers in America have suggested that the changes in the atmospheric flow in the middle troposphere can be treated as though the atmosphere was a barotropic fluid, i.e. a fluid in which pressure is a function of density alone. Recently Charney and Eliassen (1) have given a numerical method of computing changes caused by small perturbations superimposed on a uniform zonal current. The treatment is essentially one dimensional and was regarded by the authors as a preliminary study to an attack on the more general two-dimensional problem. However, the authors claimed that the one-dimensional method could produce results in forecasting comparable with existing techniques and it was considered worth examination in the Forecast Research Division at Dunstable.

The present report records the first results of such an examination. A summary of the theoretical basis of the original paper is given so that reference to the original is not essential. Charney's method has been applied daily to the 0300 G.M.T. 500 mb. profiles along latitudes 45° North and 60° North for the month of November, 1949, and the results compared with forecasts produced by other methods. An attempt has been made to formulate certain objective rules which would specify the regions in which Charney's method would probably give an erroneous result.

Summary of Theory

The one-dimensional method depends upon several assumptions, the main ones being as follows:-

1. The existence of an equivalent barotropic level at approximately 500 mbs.
2. That the motion can be considered as consisting of small perturbations superimposed upon a zonal current constant with respect to time and longitude.
3. The height perturbation is assumed to have a sine dependency on the North-South co-ordinate.

Charney commences his theoretical approach by starting with the well-known Rossby vorticity equation which relates changes of vorticity to the divergence in a horizontal layer of the atmosphere.

$$\frac{d}{dt}(f + \zeta) \quad + \quad \frac{f}{\rho}\left\{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v)\right\} \quad = \quad 0 \qquad \text{......} \quad (1)$$

where

ζ = vertical vorticity component relative to the earth.

f = Coriolis parameter.

/u =

u = component of velocity in x direction (east).

v = component of velocity in y direction (north).

ρ = density.

Charney averages this equation in the vertical direction with respect to pressure throughout the atmosphere, and utilising the tendency equation, obtains

$$\overline{\frac{d}{dt}(f + \zeta)} = \frac{f}{p_o}\frac{\partial p_o}{\partial t} - \frac{f}{H} w_o \qquad \text{.........(2)}$$

where the bar denotes average value

subscript "o" denotes surface values

p = pressure

T = temperature

H = $\frac{RT_o}{g}$, where R = Specific gas constant.

w = vertical velocity component.

Charney assumes that the shape of the streamlines is the same at all levels and that the increase of wind with height is similar along all verticals. The velocity field can then be written.

$$u = \bar{u}(x,y,t)\,A(p) \qquad v = \bar{v}(x,y,t)\,A(p) \qquad \text{(3)}$$

and (2) becomes

$$\frac{\partial \bar{\zeta}}{\partial t} + \left(\bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y}\right)(f + \overline{A^2}\bar{\zeta}) = \frac{f}{p_o}\frac{\partial p_o}{\partial t} - \frac{f}{H} w_o \qquad \text{.......(4)}$$

There must be a certain level $p = \bar{p}$, where $u = \bar{u}$, $v = \bar{v}$, $\zeta = \bar{\zeta}$ and at this level (4) becomes

$$\frac{\partial \zeta}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)(f + \overline{A^2}\zeta) = \frac{f}{p_o}\frac{\partial p_o}{\partial t} - \frac{f}{H} w_o \qquad \text{.........(5)}$$

In a barotropic fluid $A(p) \equiv 1$, and $\overline{A^2} = 1$, so that equation (5) (with $\overline{A^2} = 1$) holds at all levels, and can be interpreted as indicating that a level $p = \bar{p}$ exists at which the motion of the baroclinic atmosphere corresponds to the motion of an equivalent barotropic atmosphere. This level is defined as the "equivalent barotropic level" and for convenience is taken as being 500 mbs; although Charney (2) has stated that the value lies between 550 and 600 mbs., and a proper value for $\overline{A^2}$ in the Atmosphere is 5/4.

In order that equation (5) should imply the existence of an equivalent barotropic atmosphere, it is necessary to approximate $\overline{A^2}$ to 1. It has been shown in reference (3) that replacing assumption (3) by the more general assumption.

$$u = \bar{u} + B(p)u' \;, \quad v = \bar{v} + B(p)v' \qquad \text{........ (6)}$$

where

/ u' =

u' = thermal wind component in x direction per unit thickness. $\overline{A^2}$ will normally vary between 1 and 4/3, but although locally $\overline{A^2}$ may increase to as much as 2, on those occasions $\frac{\partial \zeta}{\partial t}$ will be small.

The results obtained by Charney would seem to justify this approximation of $\overline{A^2} = 1$, and the consequent existence of an equivalent barotropic level, even though on theoretical grounds the approximation does in certain cases seem to be rather coarse.

Introducing the geostrophic approximation into the vorticity equation (5), considering small perturbations on a constant zonal current U, and taking into account the effect of topography and friction, Charney obtains the following linear differential equation.

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial z} + \mu F\right)\left(\frac{\partial^2 z}{\partial x^2} - m^2 z\right) + \beta\frac{\partial z}{\partial x} - \lambda^2\frac{\partial z}{\partial t} = -\mu\lambda^2 U\frac{\partial h}{\partial x} \qquad (7)$$

where the unsuffixed variable now refers to the 500 mb. level.

U = zonal current.

μ = u_{g_o}/u where suffix g_o refers to top of friction layer.

$F = \frac{\sin 2\alpha}{\sqrt{2}} \frac{\sqrt{Kf}}{H}$

α = angle between isobars and the surface wind.

K = eddy diffusivity.

z = height of 500 mb. surface.

$\beta = \frac{\partial f}{\partial y}$

$\lambda^2 = \frac{f^2}{gH}$

$h(x)$ = mountain profile along latitude circle in question

A sine variation of the z perturbation with y has been assumed, given by

$$\frac{\partial^2 z}{\partial x^2} = -m^2 z \qquad (8)$$

In the computations the units used were x in radians of longitude, t in days. This gives

$\beta = 4\pi \cos^2\phi$

$\lambda^2 \doteqdot 2.5 \sin^2 2\phi$

/ Comparison

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Comparison of observed and computed stationary wavelengths over oceans suggested that m^2 was approximately 15, but the value of m^2 does not make a vital difference to the computations.

Integration of (7) by Fourier Analysis gives

$$z(x + Ut,t) = z(x,o) + \int_o^{2\pi} z(\alpha,o)\ I_{a^2}(x - \alpha,t)\ d\alpha + \mu\lambda^2\int_o^{2\pi} h(\alpha)\ J(x - \alpha,t)\ d\alpha \quad \ldots\ldots (9)$$

where

$$I_{a^2}(x,t) = \frac{1}{2\pi}\sum_{-\infty}^{\infty}\left\{\exp\left(\frac{inbt}{n^2+a^2}\right) - 1\right\}\exp(inx) \quad \text{.......... (10)}$$

$$J(x,t) = \frac{1}{2\pi}\sum_{-\infty}^{\infty}\left\{\exp(inUt) - \exp\left(\frac{inbt}{n^2+a^2}\right)\right\}\frac{\exp(inx)}{n^2-s^2} \quad \text{.......... (11)}$$

$$a^2 = m^2 + \lambda^2 \qquad\qquad b = \beta + \lambda^2 U$$

By computing functions I and J, Charney showed that for t = 1 day J is small compared with I and can be neglected. Also, it is possible to take a limited influence region for the summation of I. This is apparent from graphing I with respect to x. Equation (9) then becomes

$$z(x + Ut,t) = z(x,o) + \int_{x-x_1}^{x+x_2} z(\alpha,o)\ I_{a^2}(x - \alpha,t)\ d\alpha \quad \text{.......... (12)}$$

$a^2 = m^2 + \lambda^2$ and was taken $\doteqdot$ 18.

For a one day forecast

$$z(x + U,1) = z(x,o) + \int_{x-x_1}^{x+x_2} z(\alpha,o)\ I_{18}(x - \alpha,1)\ d\alpha \quad \text{.......... (13)}$$

Charney (2) has calculated and tabulated the function I_{18}

$$\int_{x-x_1}^{x+x_2} z(\alpha,o)\ I_{18}(x - \alpha,1)\ d\alpha$$

can be evaluated by Simpson's Rule from the observed initial profile z(x,o) taking intervals of 10° longitude.

For latitude 45° North, values of $I_{18}(x,1)$ are given in reference (2). These values are reproduced in Table I. In obtaining these values b is taken as 7. Taking x_1 = 80, x_2 = +40, and integrating (13) by Simpson's Rule,

$$z(x + U, 1) = z(x, o) + \sum_{-4}^{8} A_N \, z(x + 10N, o)$$

$$= z(x, o) + \sum_{-3}^{7} A_N \, z(x + 10N, o)$$

as A_8 and A_{-4} are negligible. Coefficients A_N are given in table II.

For latitude 60° North, values of $I_{18}(x,1)$ were computed for a value of $b = 4.5$. (On the month's charts considered, a more accurate value would have been $b = 4$). In the expression $I_{a}2(x,t)$ in (10)b only occurs in a product term containing t. Values for $I_{18}(x,t)$ are tabulated in the Journal of Meteorology for $b = 7$, $t = 1, 2, 3....7$. As a value for I_{18} for $b = 4.5$, $t = 1$, was required, the values of $I_{18}(x,t)$ given for $bt = 7, 14, 21....49$ were interpolated by means of Newton's forward difference formula to obtain $I_{18}(x,t)$ for $bt = 4.5$. In the interpolation use was made of the identity $I_{18}(x,o) \equiv o$. These values are tabulated in Table I.

Integration by Simpson's Rule was again carried out taking $x_1 = 60$, $x_2 = +40$ giving

$$z(x + U, 1) = z(x, o) + \sum_{-4}^{6} B_N \, z(x + 10N, o)$$

$$= z(x, o) + \sum_{-3}^{6} B_N \, z(x + 10N, o)$$

as coefficients B_6 and B_{-4} are negligible. Coefficients B_N are given in Table II.

Table I

	b = 7		b = 4.5	
x	$I^{*}_{18}(x,1)$	$I^{*}_{18}(-x,1)$	$I_{18}(x,1)$	$I_{18}(-x,1)$
0°	-4.301	1.999	-3.02	1.55
10°	-1.506	1.239	-.94	.86
20°	-0.529	0.777	-.45	.50
30°	-0.167	0.457	-.15	.27
40°	-0.048	0.275	-.06	.17
50°	-0.016	0.161	-.02	.09
60°	0.002	0.091	.00	.04
70°	-0.003	0.057	.00	.03
80°	0.003	0.028	.00	.01
90°	-0.001	0.020	.00	.01
100°	0.001	0.008	.00	.00
110°	0.001	0.007	.00	.01
120°	-0.002	0.004	.00	.00
130°	0.002	0.001	.00	.00
140°	-0.002	0.003	.00	.00
150°	0.002	-0.002	.00	.00
160°	-0.001	0.002	.00	.00
170°	0.001	-0.001	.00	.00
180°	0.001	0.001	.00	.00

/ Table II

Table II

N	A^{*}_{N}	B_N
-3	-.039	-.035
-2	-.062	-.052
-1	-.351	-.219
0	-.134	-.086
1	.289	.200
2	.091	.058
3	.106	.063
4	.032	.020
5	.038	.021
6	.011	
7	.013	

*These coefficients are slightly different from those published in Telus Vol.1 No. 2, but are based on figures given in the more complete table in reference (2).

Method of Investigation

The practical application of the forecast formula is a simple mechanical process, which is described in Appendix I. The formula was first applied to the 500 mb. profiles along latitude 45° North for the 0300 G.M.T. charts of November 1949. The forecast change in the profile was plotted on a graph, and compared with the actual change. Also plotted on the graph were the changes forecast by Forecast Division Dunstable on their routine 500 mb. prontours over the range from 35° West to 35° East. The forecast thickness lines for the first days of the Research Division "four day forecasts" were gridded with the actual 1,000 mb. contours for the appropriate day, and the results also plotted. From the results obtained it was found possible to make certain objective rules which specify the synoptic regions in which Charney's formula will not give a reasonably good forecast.

The formula was then applied to the 500 mb. profile along latitude 60° North for the same month's charts, and the results again compared with the Forecast Division's prontours.

Before the numerical forecast changes were examined, the objective rules were applied to specify those areas where an accurate forecast was not expected.

Summary of Results

The results for latitude 45° North were first analysed. A comparison was made between the numerical forecast change and the actual change (See Table III line 1). The following notation was used:-

A. Good agreement over the whole range.

B. Good agreement over most of range.

BC. Good agreement over about half of the range

C. Poor agreement over most of the range

D. Poor agreement over whole range.

Good agreement was taken as meaning a difference of 200 feet or less.

Secondly, a comparison was made of the difference between the numerical forecast change and the actual change, and the difference between the Forecast Division's forecast and the actual change (See Table III line 2). The following notation was used:-

/ A.

A. The numerical method better than Forecast Division over whole range.

B. The numerical method better than Forecast Division over most of range.

BC. Little to choose between the numerical results and Forecast Division's results.

C. Forecast Division better than the numerical method over most of range.

D. Forecast Division better than the numerical method over whole range.

Thirdly a comparison was made between the results obtained by the numerical method and the results obtained by utilising the "pre-thickness" pattern produced on the first days of the "four day" forecast produced in the Research Division at Dunstable (See Table III line 3). These were classified using the notation described for comparing the numerical forecast with the Forecast Division's forecast. It must be remembered, however, that no attempt had been made in the Research Division's Experimental forecasts to forecast the actual contours.

Table III

Comparison	A	B	BC	C	D
Charney's forecast change and actual change	1	12	1	13	3
Charney's forecast change and Forecast Divisions forecast change.	2	4	9	9	5
Charney's forecast change and "pre-thickness" forecast change.	1	4	2	1	1

Charney's method gave a reasonably accurate forecast at about half the positions for which the computations were carried out, although the correlation coefficients for the forecast changes and the actual changes was only .34. The correlation coefficient between the actual change and the forecast change obtained by assuming an eastward displacement of the pressure profile equivalent to one day's zonal flow was .27. Thus the more elaborate features in the theory do improve the forecast appreciably.

Nearly all the cases when the numerical method gave an error greater than 200 ft. occurred when one of the following criteria applied, each of which meant that Charney's original assumptions were not fulfilled. In the first 3 cases the error in the forecast is generally displaced downstream a distance equivalent to one day's zonal flow.

1. The presence of a closed circulation in the 500 mb. contour pattern during forecast period.

2. A trough or ridge in the 500 mb. contour pattern whose axis is inclined at an appreciable angle to the meridian, or a U shaped trough or ridge whose axis is parallel to a maridian, but with strong meridional flow along its sides.

3. A flat area of insignificant pattern in the 500 mb. contour patterns.

4. A strong thermal field with surface isobars at right angles to it (generally wave depressions on an active front).

The closed circulation in the 500 mb. pattern usually has a fairly local effect, and elsewhere along the latitude circle the results are good unless any other factors are present to cause further discrepancies. The presence of a closed circulation means that the motion can no longer be regarded as a small

/ perturbation

perturbation on a constant zonal current, and that there is not a sine dependency on the y (North-South) co-ordinate. The forecast for 0300Z on 3rd November, 1949, based on the 0300Z chart on 2nd November, 1949, shows the discrepancies due to a closed circulation. (See figs. 1a, 2a, 2b).

In the case of the trough or ridge in the 500 mb. pattern whose axis is inclined to the meridian, once again the effect is also usually relatively local. The motion can no longer be regarded as a small perturbation on constant West-East zonal currents, and in a large number of cases there will be an East-West rather than West-East component of the actual motion. There is also no sine dependency on the y co-ordinate of the height perturbation. A U-shaped trough or ridge with strong meridional flow along its sides also cannot be regarded as a small perturbation on a West-East zonal current, and again there is no sine dependency on the y co-ordinate. The forecast for 0300Z on 9th November, 1949, shows a good example of the discrepancies due to an upper trough whose axis is inclined to a meridian. Here the discrepancy in the forecast is about 15 degrees of longitude further East than the axis of the quasi-stationary trough (See figs. 1b, 3a, 3b).

A flat area of insignificant pattern means that the actual zonal flow in that area is very much less than the average flow over the hemisphere. (See figs. 1c, 4a, 4b).

The main errors from 20th November to 25th November, 1949, over the Atlantic were caused by the meridional advection of a strong W-E thermal field on the North side of a frontal belt lying parallel to the 45° North latitude. This advection took place near developing waves on the fronts and did not cause a big distortion in the sinusoidal pattern, but rather a general increase (or decrease) in the 500 mb. level over a wide area. (See figs. 1d, 5a, 5b, 5c, 5d). Charney's method assumes the shape of the streamlines in approximately the same at all levels, and that the increase of wind with height is similar along all verticals. This is certainly not the case here. (See reference (3)).

A correlation coefficient was computed for the Forecast Division's forecast change and the actual change, and compared with a similar correlation coefficient for the numerical method of the same range. The correlation coefficient was .48 for the Forecast Division's method and .37 for the numerical method, but it must be remembered that in several of the cases considered, Charney's original assumptions were not fulfilled.

It was not possible to determine the actual number of independent observations on which the correlation coefficients were based, as there is a certain correlation between changes at neighbouring points along the profile, and also between changes on consecutive days. However, on the data examined, the numerical forecast applied to a total of 500 points, and the Forecast Division's forecast applied to a total of 300 points. Even allowing for a degree of dependence of certain of the forecast changes upon each other, it was estimated that the correlation coefficients obtained were still significant, having an error of not greater than ±.1 from the true value. By examining further data the correlation coefficients could be obtained more accurately, but whilst the present coefficients are not conclusive, it is considered that they are significant.

Before Charney's forecast formula was applied to the 500 mb. profile along 60° North, the surface, 500 mb. and 1,000-500 mb. thickness charts were carefully examined, and in accordance with the four criteria stated above, certain points were specified as being regions where the numerical method was not expected to apply. The correlation coefficient between the numerical forecast change and the actual change for the whole range was .37, whilst the correlation coefficient for the points left after certain regions had been excluded was .47.

/ When

When the Forecast Division's prontours at 60° N were examined, the correlation coefficient between the change they forecast, and the actual change was .56. This was increased to only .59 when the points specified above were left out. The correlation coefficient between the numerical forecast change and the actual change for the points in the Forecast Division's range after the points specified had been left out was .45.

Throughout the range from 75° W to 85° E, it was found that 62% of the the points were points where the numerical method could be expected to apply. As during the month considered the main train was usually further south than 60° North, a higher percentage of points had to be left out than would be the case if the method was applied to latitudes further south.

Conclusion

The results of the test show significant success but the standard obtained is less than that achieved by conventional methods.

From the study of the month's charts it would seem that the assumptions in Charney's formula are too great for the formula to be applied as a routine measure to all areas of the chart. However, by applying the formula to areas of the chart which are not affected by the following criteria:

1. A closed circulation in 500 mb. contour pattern.

2. A trough or ridge whose axis is inclined at an appreciable angle to the meridian, or a U-shaped trough or ridge whose axis is parallel to the meridian, but with a strong meridional flow along its sides.

3. A flat area of insignificant pattern in the contour pattern.

4. A strong thermal field with surface isobars at right angles to it,

one would expect a forecast which was almost as accurate as the routine forecasts produced by conventional methods. Before conclusive results about the relative accuracy of Charney's one-dimensional method, and the conventional thickness method could be obtained, further investigation would be necessary but it is considered that on the results so far obtained the method deserves further study both from the practical and theoretical aspects.

Acknowledgements

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References

1. Charney J.G. and Eliassen A. A numerical method for predicting the perturbations of the middle latitude westerlies. Tellus Volume 1, No. 2, 1949, p.38.

2. Charney J.G. On a physical basis for numerical prediction of large scale motions in the atmosphere. Journal of Meteorology Vol. 6, No. 6, 1949, p.371.

3. Bushby, F.H. M.O.21 Technical Note No. 14.

Table IV

Multiples of coefficients A_N for latitudes 45° North (b = 7)

N	A_N	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000	1050	1100	1150	1200
-3	-.039	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20	-21	-23	-25	-27	-29	-31	-33	-35	-37	-39	-41	-43	-44	-46
-2	-.062	-3	-6	-9	-12	-16	-19	-22	-25	-28	-31	-34	-37	-40	-43	-47	-50	-53	-56	-59	-62	-65	-68	-71	-74
-1	-.351	-17	-35	-53	-70	-88	-105	-123	-140	-158	-176	-193	-211	-228	-246	-263	-281	-298	-316	-333	-351	-369	-386	-404	-422
0	-.134	-7	-13	-20	-27	-33	-40	-47	-54	-60	-67	-74	-80	-87	-94	-101	-107	-114	-121	-127	-134	-141	-147	-153	-160
1	+.289	14	29	43	58	72	87	101	116	130	145	159	173	188	202	216	231	246	260	275	289	303	318	332	346
2	.091	5	9	14	18	23	27	32	36	41	45	50	55	59	64	68	73	77	82	86	91	96	100	105	110
3	.106	5	11	16	21	27	32	37	42	48	53	58	64	69	74	79	85	90	95	101	106	111	117	122	128
4	.032	2	3	5	6	8	10	11	13	14	16	18	19	21	22	24	26	27	29	30	32	34	35	37	38
5	.038	2	4	6	8	9	11	13	15	17	19	21	23	25	27	29	30	32	34	36	38	40	42	44	46
6	.011	1	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13	14
7	.013	1	1	2	3	3	4	5	5	6	7	7	8	8	9	10	10	11	12	12	13	14	14	15	16

Table V

Multiples of coefficients B_N for latitude 60° North (b = 4.5)

N	B_N	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000	1050	1100	1150	1200
-3	-.035	-2	-3	-5	-7	-9	-11	-12	-14	-16	-17	-19	-21	-23	-25	-26	-28	-30	-31	-33	-35	-37	-39	-40	-42
-2	-.052	-3	-5	-8	-10	-13	-16	-18	-21	-23	-26	-29	-31	-34	-36	-39	-42	-44	-47	-49	-52	-55	-57	-60	-62
-1	-.219	-11	-22	-33	-44	-55	-66	-77	-88	-99	-109	-120	-131	-142	-153	-164	-175	-186	-197	-208	-219	-230	-241	-252	-263
0	-.086	-4	-9	-13	-17	-21	-26	-30	-34	-39	-43	-47	-52	-56	-60	-65	-69	-72	-77	-82	-86	-90	-95	-99	-103
1	.200	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240
2	.058	3	6	9	12	15	17	20	23	26	29	32	35	38	41	43	46	49	52	55	58	61	64	67	70
3	.063	3	6	9	13	16	19	22	25	28	31	35	38	41	44	47	50	54	57	60	63	66	69	72	76
4	.020	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	.021	1	2	3	4	5	6	7	8	9	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

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Table VI

Application of Charney's formula to profile along 45° North on 8/11/49, producing a forecast profile for 0300Z on 9/11/49. (See Fig.1b)

x	=	-120	-110	-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100	110	120
$z(x,o)$	=	18200	18600	18550	18450	18200	17900	17750	17850	18150	18550	18850	18600	18150	17600	17750	18500	18800	18700	18500	18200	18100	18500	18450	18150	18000
$z(x,o) - 18000$		200	600	550	450	200	-100	-250	-150	150	550	850	600	150	-400	-250	500	800	700	500	200	100	500	450	150	0
A_{-3}	= -.039				-8	-23	-21	-18	-8	4	10	6	-6	-21	-33	-23	-6	16	10							
A_{-2}	= -.062				-37	-34	-28	-12	6	16	9	-9	-34	-53	-37	-9	25	16	-31							
A_{-1}	= -.351				-193	-158	-70	35	88	53	-53	-193	-298	-211	-53	140	88	-176	-281							
A_0	= -.134				-60	-27	13	33	20	-20	-74	-114	-80	-20	-54	33	-67	-107	-94							
A_1	= .289				58	-29	-72	-43	43	159	246	173	43	-116	-72	145	231	202	145							
A_2	= .091				-9	-23	-14	14	50	77	55	14	-36	-23	45	73	64	45	18							
A_3	= .106				-27	-16	16	58	90	64	16	-42	-27	53	85	74	53	21	11							
A_4	= .032				-5	5	18	27	19	5	-13	-8	16	26	22	16	6	3	16							
A_5	= .038				6	21	32	23	6	-15	-9	19	30	27	19	8	4	19	17							
A_6	= .011				6	9	7	2	-4	-3	5	9	8	5	2	1	5	5	2							
A_7	= .013				11	8	2	-5	-3	7	10	9	7	3	1	7	6	2	0							
Positive					18531	18243	17988	17942	18172	18535	18901	19080	18704	18264	17828	18247	18982	19129	18919							
Negative					-339	-310	-205	-78	-15	-38	-149	-366	-481	-444	-195	-32	-73	-283	-406							
$F/Cz(x+17,1)$					18192	17933	17783	17864	18157	18497	18752	18714	18223	17820	17633	18215	18909	18846	18513							

U (zonal current) = 17° long. per day.

G.269304/JMB/6/50/40

Appendix One

Method of Tabulation

It was found convenient to represent negative values in red and positive values in blue ink, and to use "vertically ruled paper". The simplest method of tabulation is as follows:-

1. Write horizontally along the top row the values of x (longitude) at 10° intervals, beginning with value $x = -120$ (120° W), and ending with the value $x = +120$.

2. Write horizontally along 2nd row the values of z (x,o) along the appropriate latitude circle at 10° intervals from $x = -120$ to $x = +120$.

3. Write horizontally along 3rd row the values z (x,o) - 18,000 under the appropriate longitude.

4. Commencing on the 4th row and to the left of the previous 3 rows, write the values of A_N for $N = -3$ to $N = +7$ vertically beneath each other (For latitude 60° North use coefficients B_N for $N = -3$ to $N = +5$).

5. Commencing with the value (Z (-120,o) - 18,000), multiply it by A_{-3}, and place the result in the row A_{-3}, column $x = -90$.

   Next with the value (z (-110,0) - 18,000), multiply it first by A_{-3}, placing result in row A_{-3}, column $x = -80$. Then multiply (z (-110,0) - 18,000) by A_{-2}, placing result diagonally downwards and to the left of previous result. Continue with value (z (-100,0) - 18,000), multiplying in turn by A_{-3}, A_{-2}, A_{-1}, placing first result in row A_{-3} column $x = -70$ and subsequent results diagonally downwards to the left.

   Continue until table is complete.

6. Add each column, including the value of z(x,o) but not value of (z(x,o) - 18,000). This gives forecast value of z(x + U, 1).

As Northern Hemisphere charts were used, (scale 1:30,000,000) it was only possible to measure z to the nearest 50 feet. In carrying out "operation 5" above, it was found convenient to use a table of multiples of coefficients A_N and B_N at intervals of 50 from 0 to 1200. These tables are given as tables IV and V.

A worked example is shown in table VI, forecasting the 500 mb. profile along latitude 45° North for 0300 G.M.T. on November 9th, 1949, using data from the corresponding chart one day earlier. With practice, a competent assistant should be able to work out a forecast profile along one line of latitude from 70° W to 70° E in 20 minutes.

The zonal current was obtained by measuring zonal indices on the 500 mb. chart on which the forecast was based for the half hemisphere from 120° W to 60° E. The zone 35° North to 55° North was taken to obtain the zonal current for 45° North, and the zone 55° North to 70° North was used to obtain the zonal current at 60° North. If the method were being applied in a forecasting office, this may not be practicable but as the zonal current is fairly conservative the error in using the previous day's value would not be very great.

G.269304(a)/JMB/6/50/40

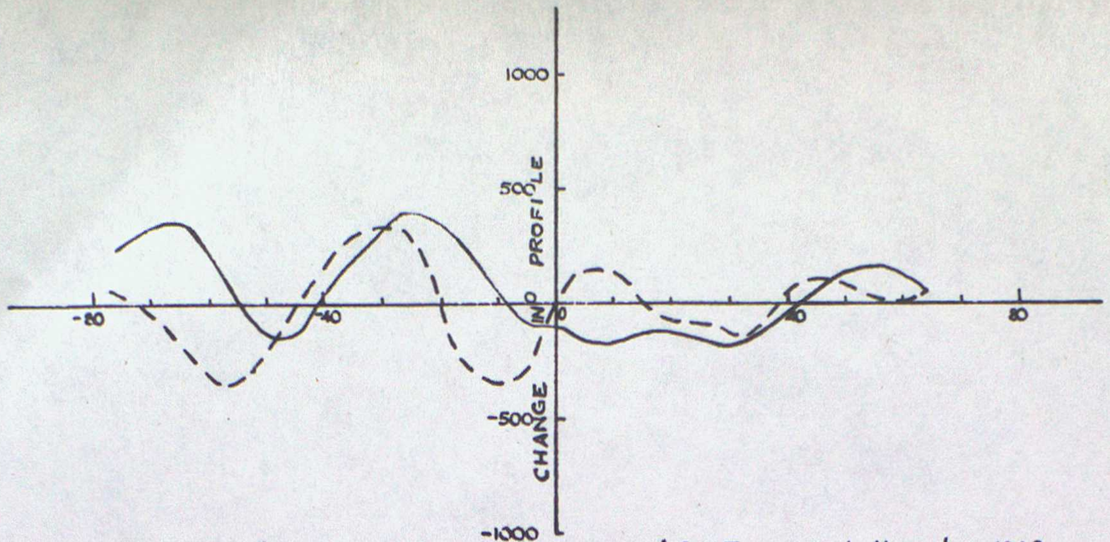


Fig. 1(a) One day changes from 0300h. G.M.T. on 2nd. November 1949.

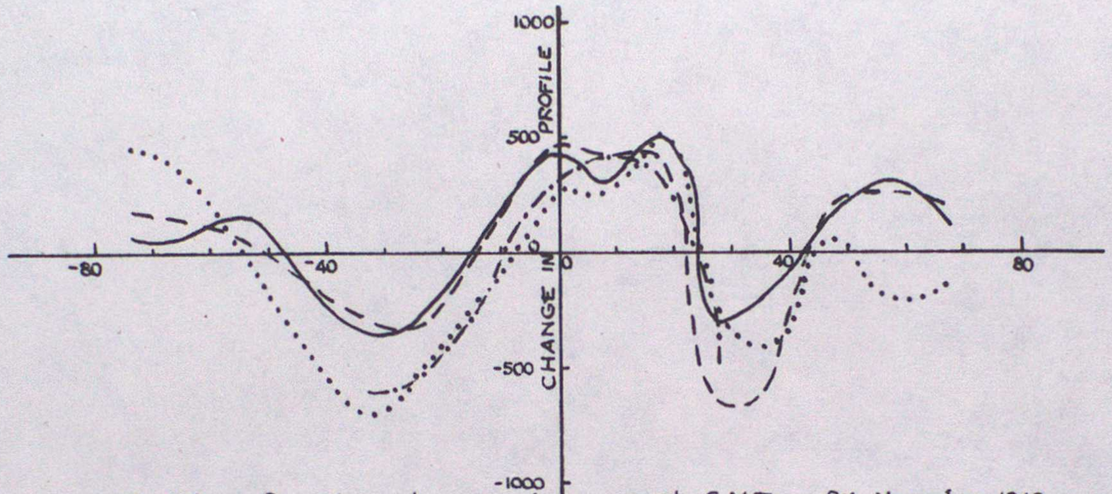


Fig. 1(b) One day changes from 0300h. G.M.T. on 8th. November 1949.

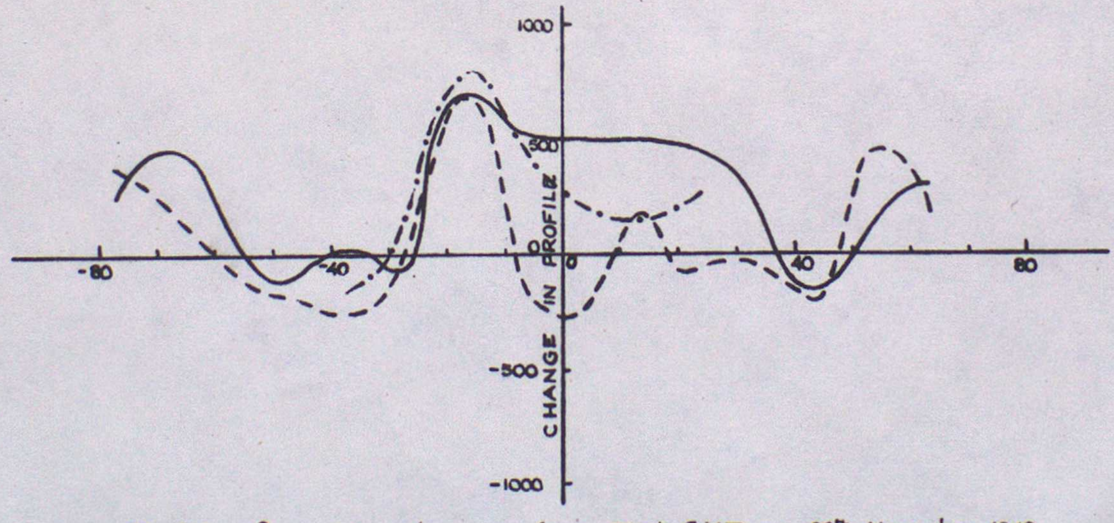


Fig. 1(c). One day changes from 0300h G.M.T. on 28th November. 1949.

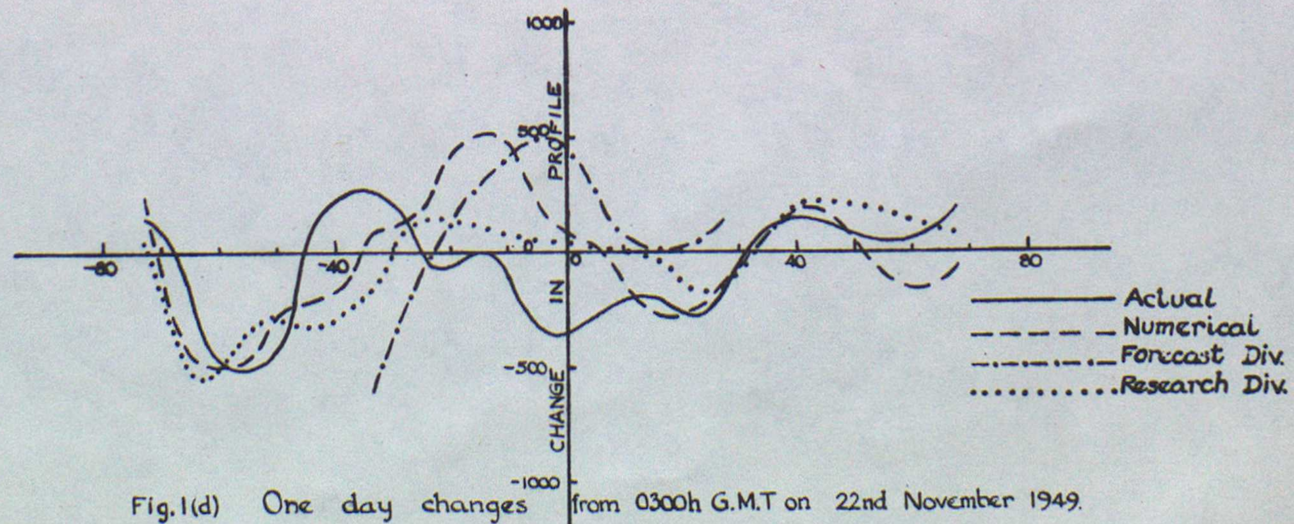


Fig. 1(d) One day changes from 0300h G.M.T on 22nd November 1949.

Fig. 1. Actual and forecast one day changes of some 500mb. profiles.

M.R.P. 566

2·11·49 0300 G.M.T.
500 mb.

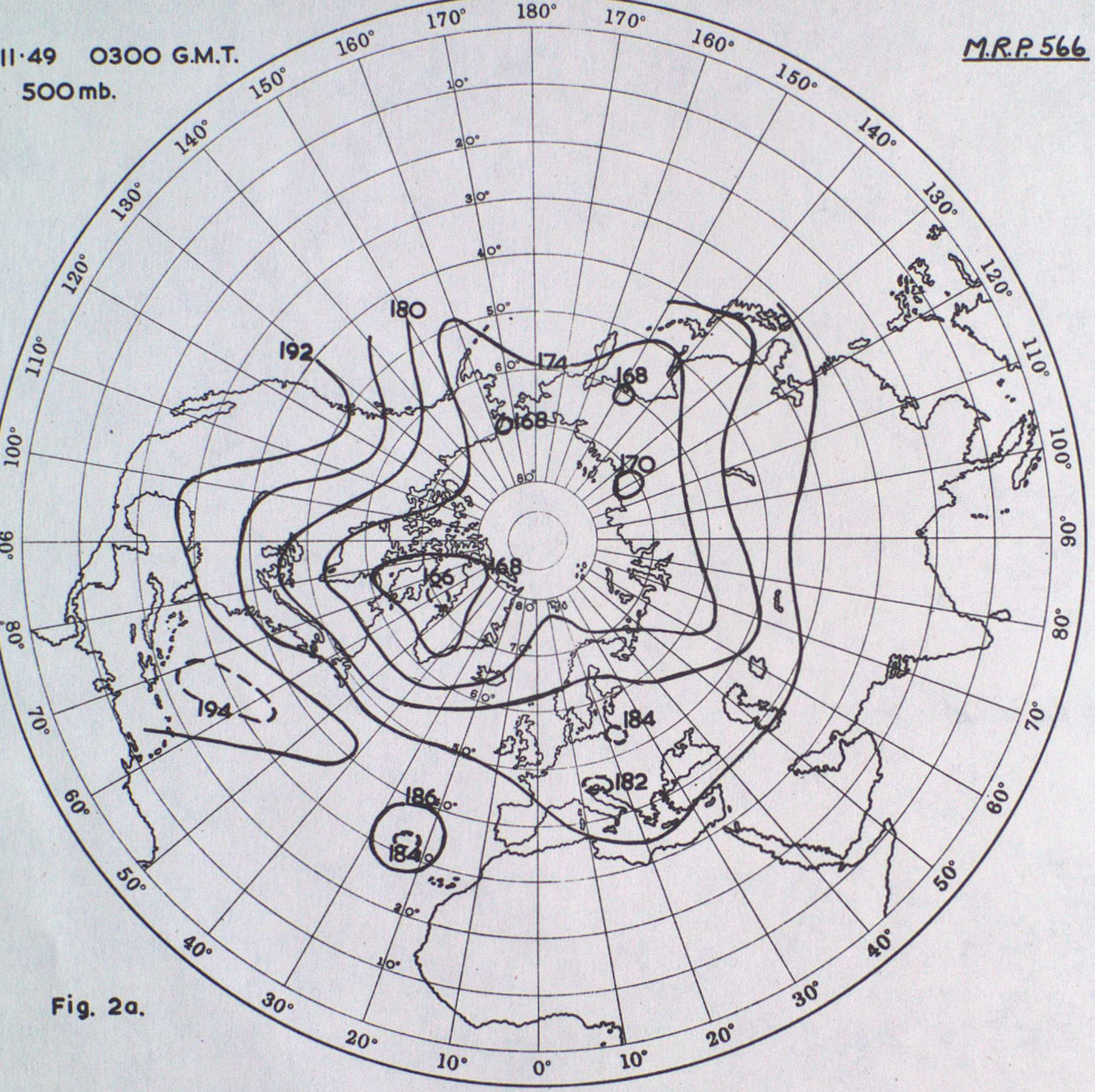


Fig. 2a.

3·11·49 0300 G.M.T.

500mb.

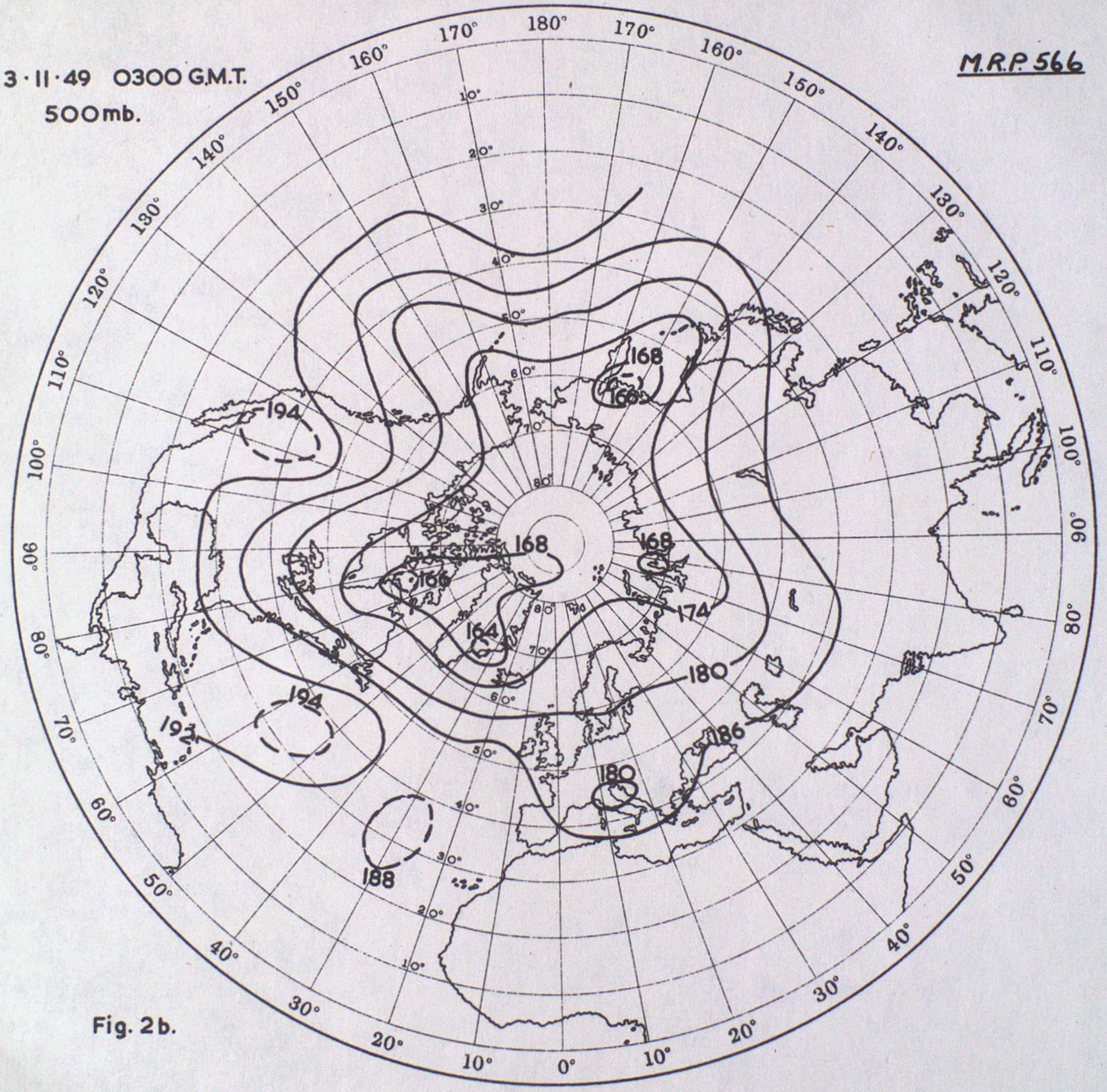


Fig. 2b.

8 · 11 · 49 0300 G.M.T.
500 mb.

M.R.P. 566

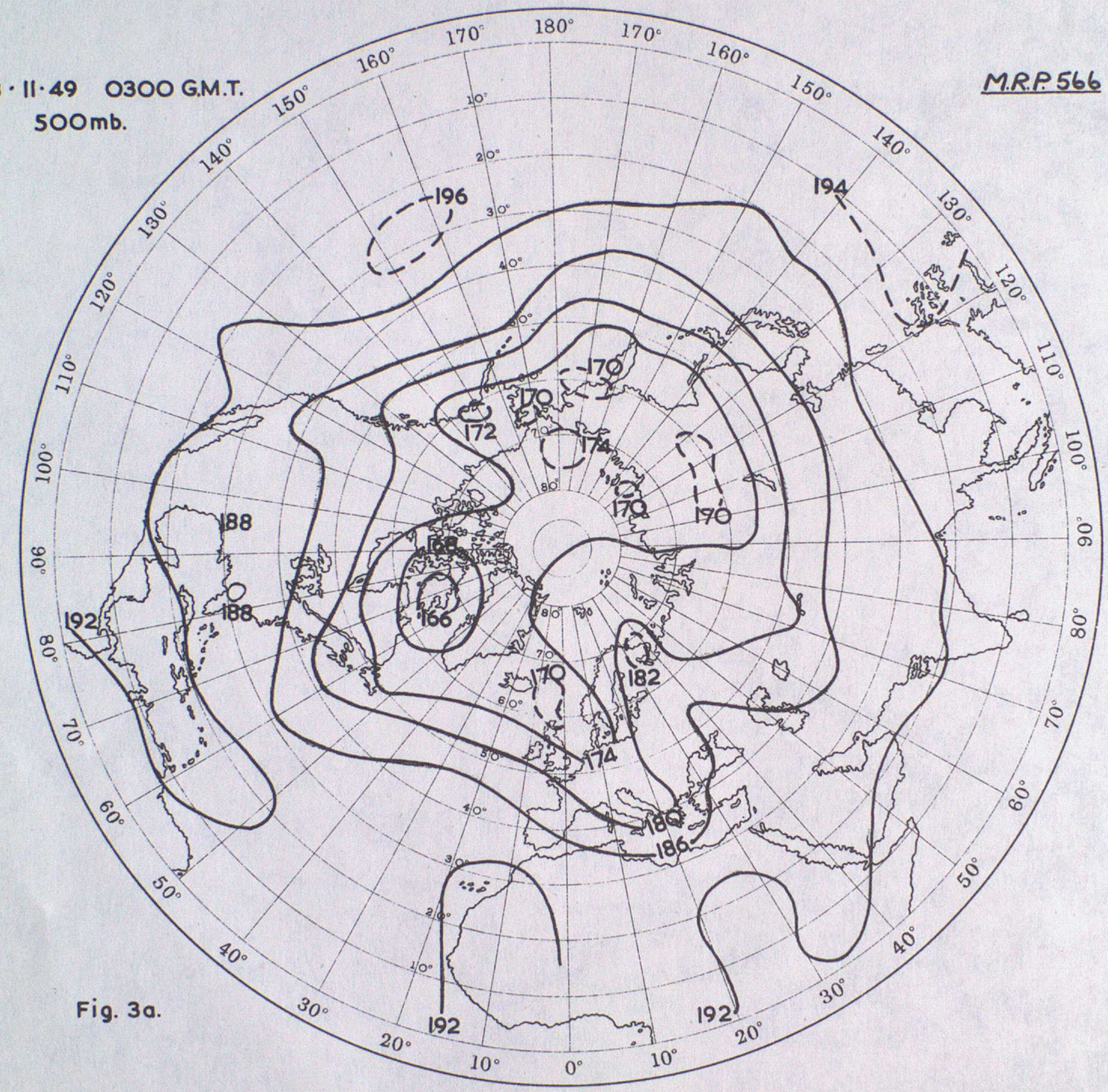


Fig. 3a.

9·11·49 0300 G.M.T.
500 mb.

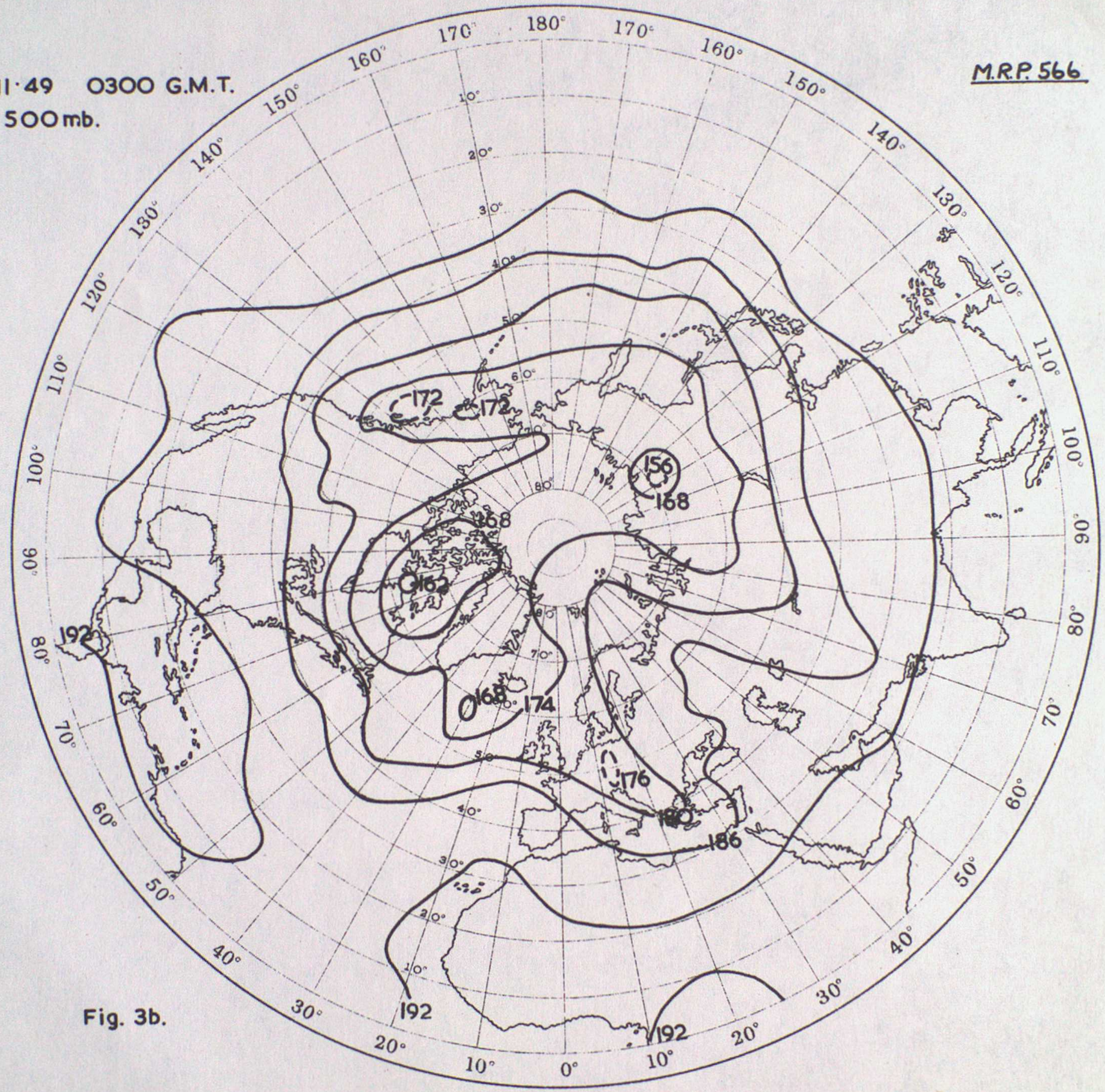


Fig. 3b.

M.R.P. 566

28·11·49 0300 G.M.T.
500 mb.

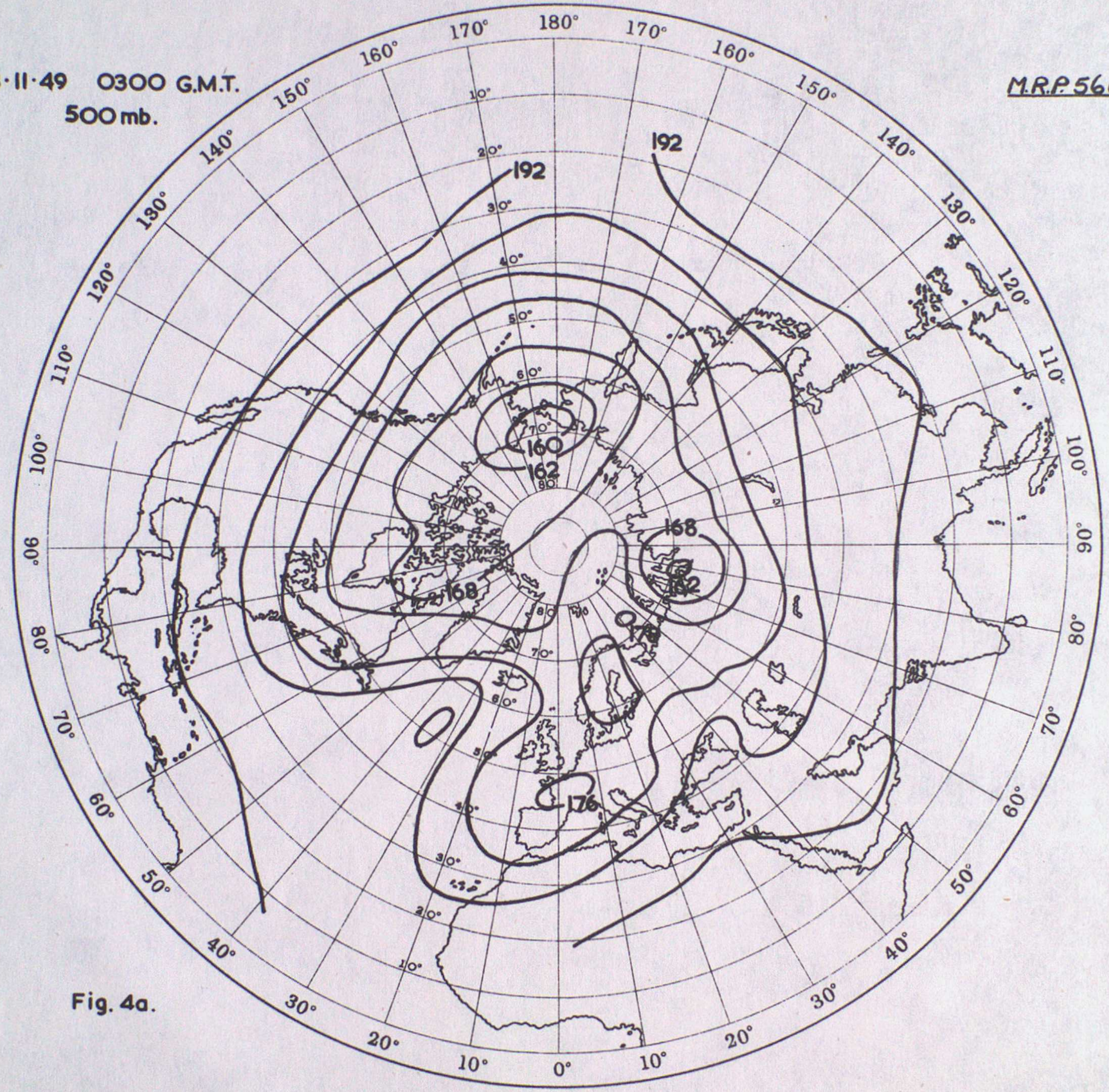


Fig. 4a.

29-11-49. 0300 G.M.T.
500 mb.

M.R.P. 566

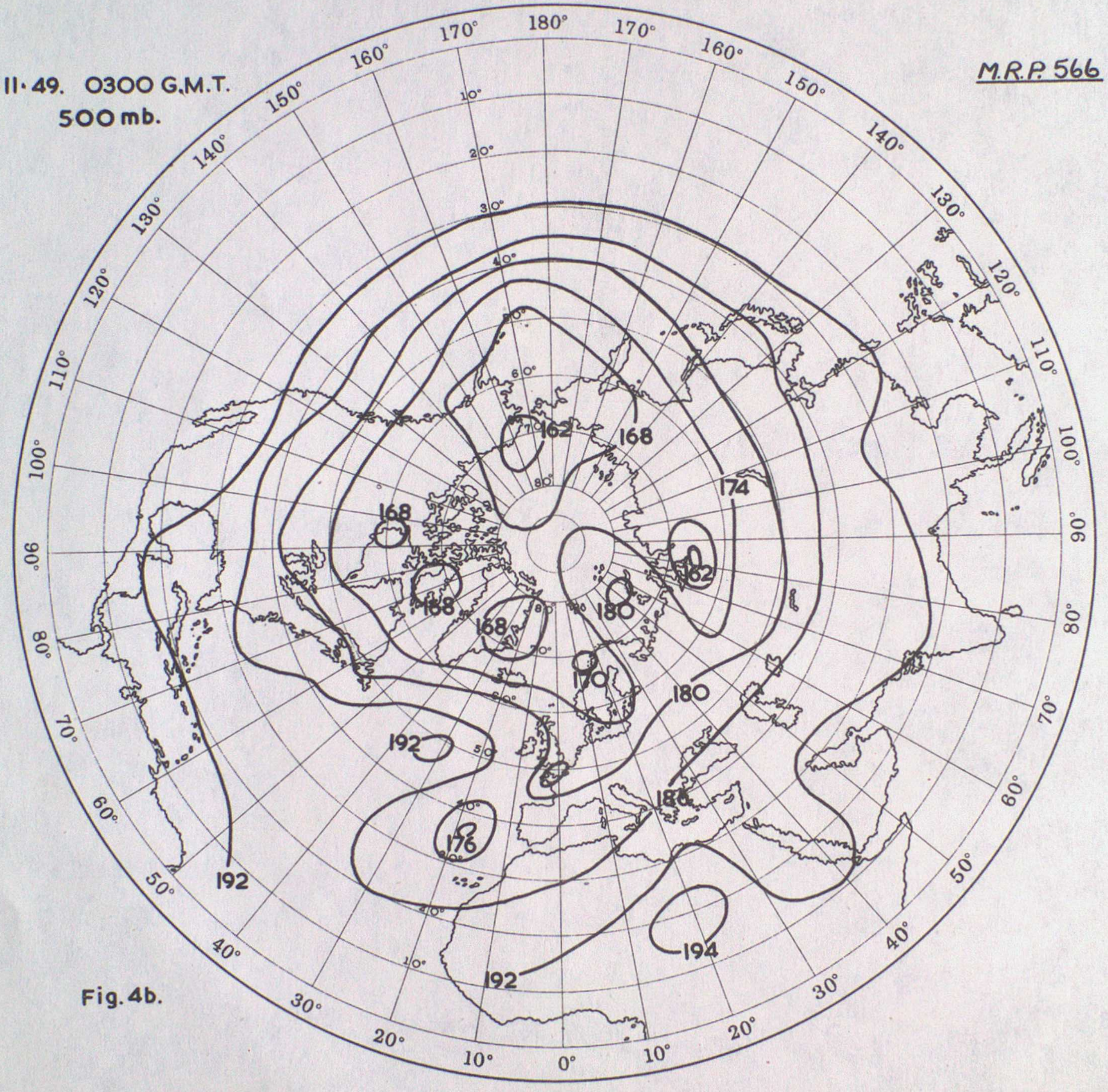


Fig. 4b.

M.R.P. 566

22·11·49 0300 G.M.T.
500 mb.

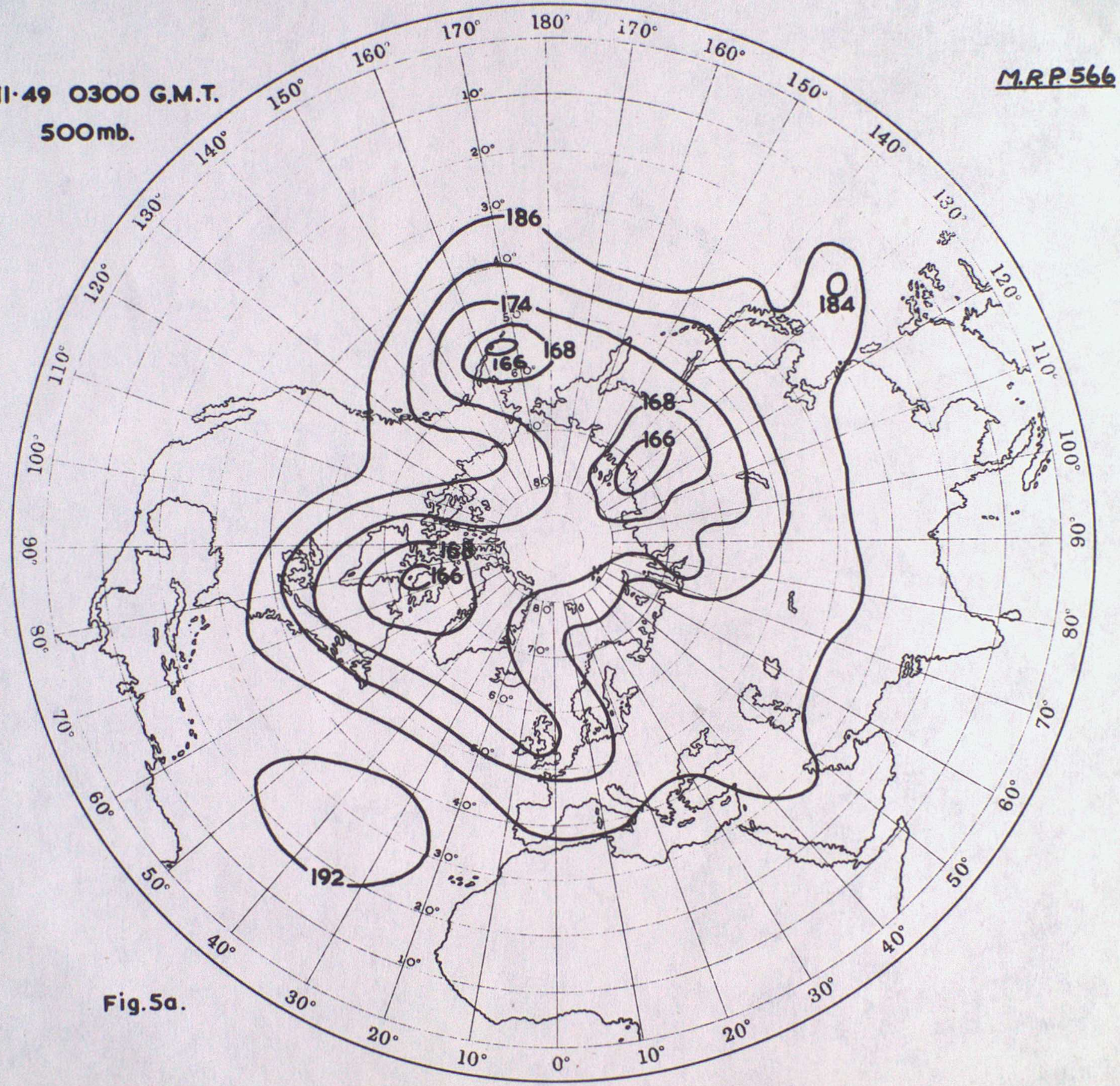


Fig. 5a.

M.R.P. 566

23·11·49 0300 G.M.T.
500mb.

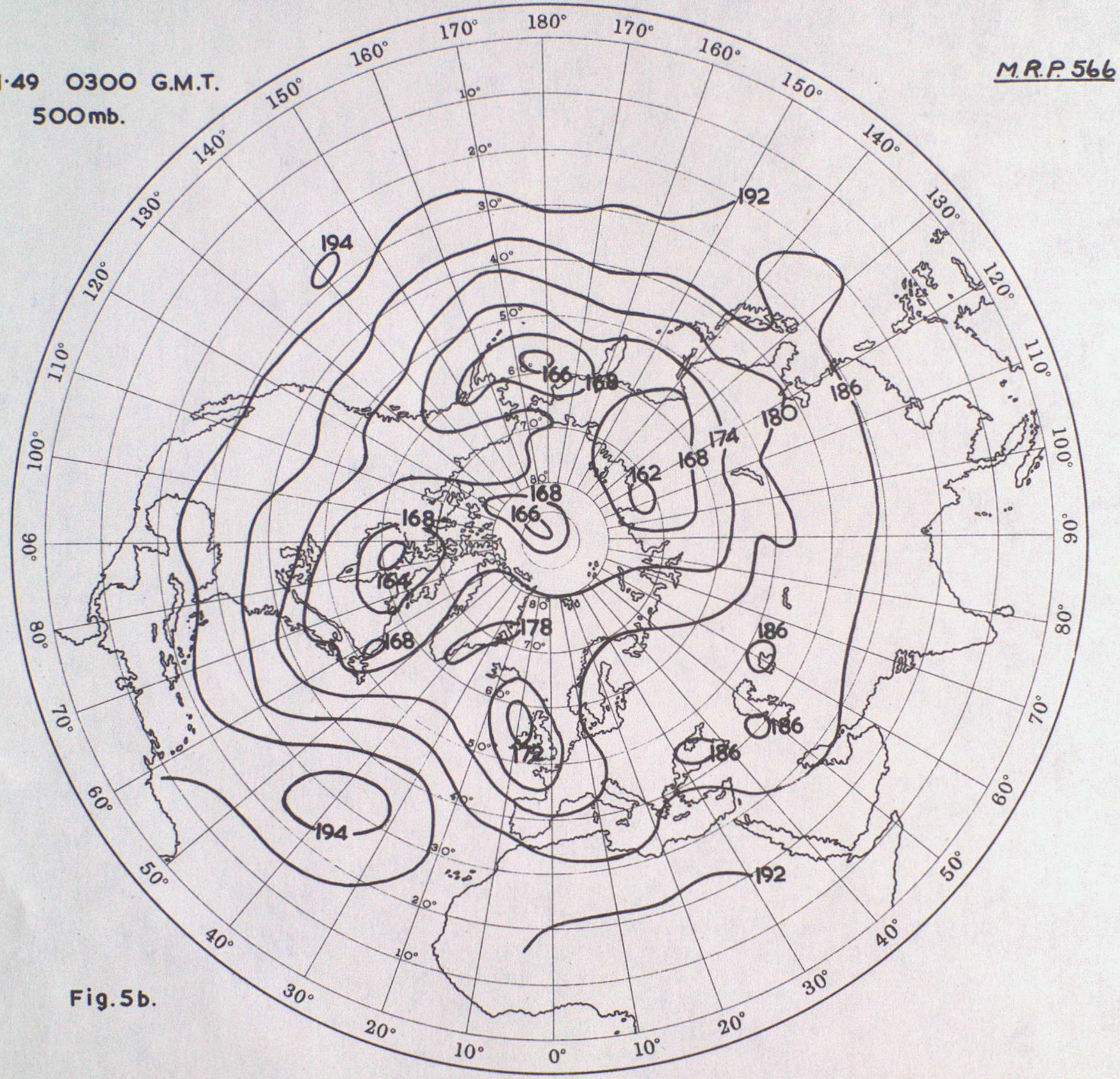


Fig. 5b.

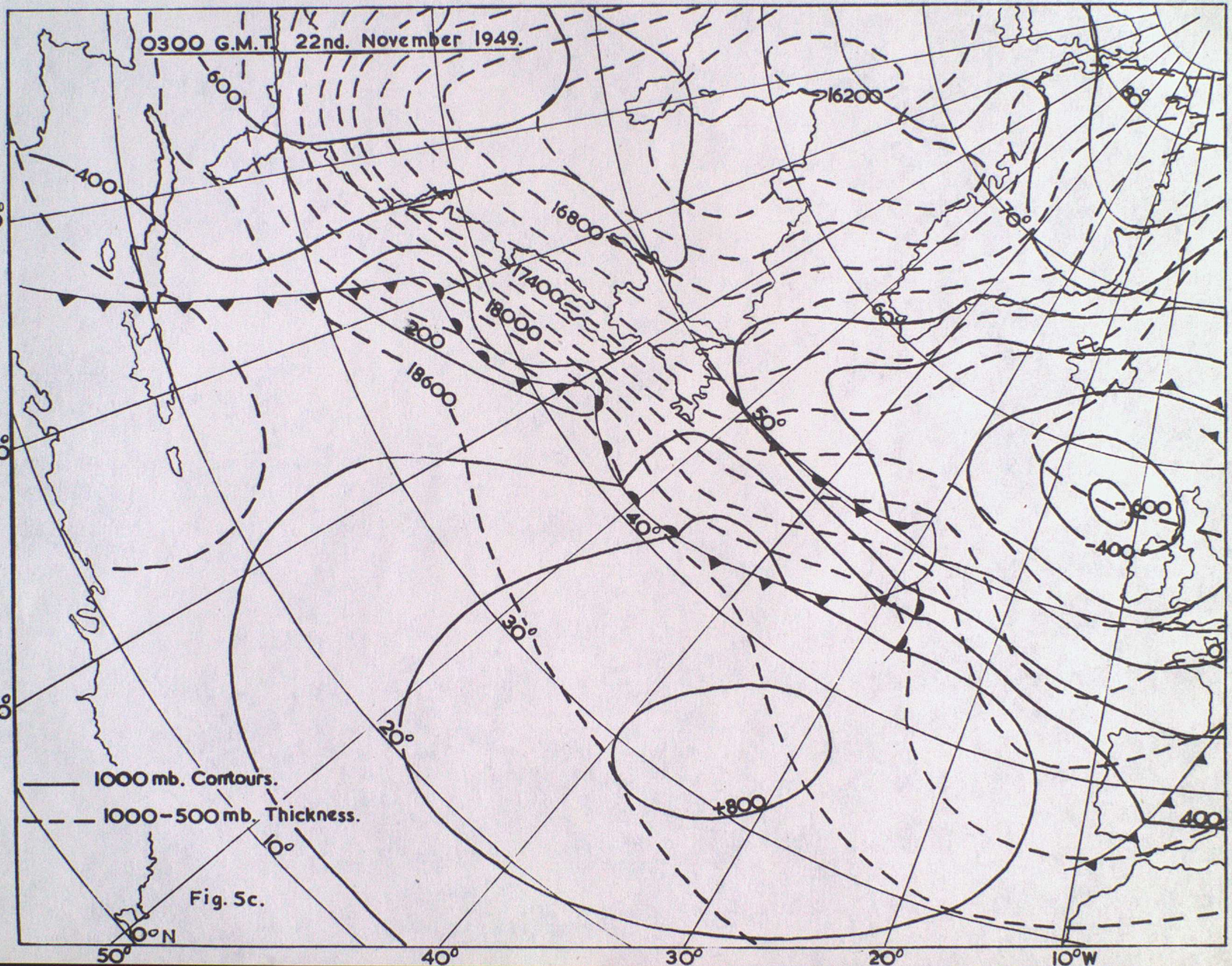
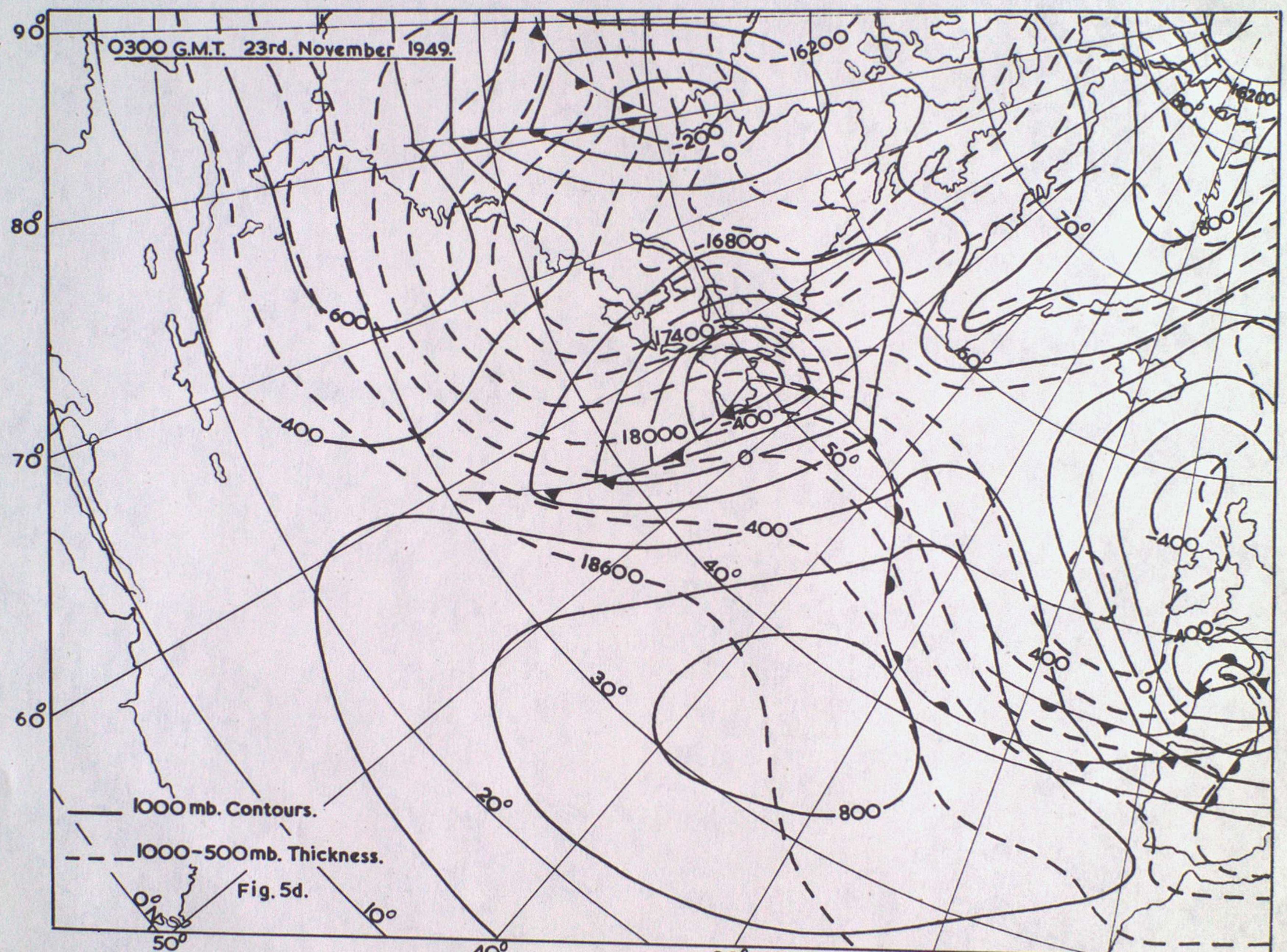

0300 G.M.T. 22nd. November 1949.
600
400
16200
16800
17400
18000
18600
200
-600
400
+800
80°
70°
60°
50°
40°
30°
20°
10°
0°N
50°
40°
30°
20°
10°W
1000 mb. Contours.
1000-500 mb. Thickness.

Fig. 5c.


0300 G.M.T. 23rd. November 1949.
1000 mb. Contours.
1000-500 mb. Thickness.
16200
-200
0
16800
600
17400
-400
18000
400
0
50°
400
18600
40°
30°
20°
10°
800
400
-400
400
0
70°
60°
80°
90°
80°
70°
60°
50°
40°
30°
20°
10°W
Fig. 5d.