LONDON, METEOROLOGICAL OFFICE.

Met.O.15 Internal Report No.59.

A three dimensional cumulonimbus model: formulation, numerical approximation, boundary conditions and coding considerations. By VAN DEN BERGHE,C.S.

London,Met.Off.,Met.O.15 Intern.Rep.No.59, 1985,31cm.Pp.39.34 Refs.

An unofficial document - not to be quoted in print.

METEOROLOGICAL OFFICE
London Road, Bracknell, Berks.

MET.O.15 INTERNAL REPORT

No. 59

A THREE DIMENSIONAL CUMULONIMBUS MODEL: FORMULATION, NUMERICAL APPROXIMATION, BOUNDARY CONDITIONS AND CODING CONSIDERATIONS

by

C S Van den Berghe

May 1985

Cloud Physics Branch (Met.O.15)

BAB

A THREE DIMENSIONAL CUMULONIMBUS MODEL: FORMULATION, NUMERICAL APPROXIMATION, BOUNDARY CONDITIONS AND CODING CONSIDERATIONS

C S van den Berghe

Met O 15

1. Introduction

Three dimensional, time dependent cumulonimbus modelling only became a realistic proposition about a decade ago. The first three dimensional models (Pastushkov (1975) and Miller and Pearce (1974)) suffered from problems of domain size (16 km x 16 km), poor resolution (of order 1 km), crude turbulence parametrization (constant eddy mixing coefficient), crude microphysical parametrizations (no ice, with very simple source/sink terms) and often inappropriate boundary conditions (Pastushkov uses reflective boundaries). Improvements in the capabilities of computers over the past ten years have resulted in models such as those of Clark (1977, 1979), Klemp and Wilhelmson (1978), Schlesinger (1978) and Tripoli and Cotton (1982) with larger domain sizes, non linear turbulence parametrizations (usually a first order closure but occasionally, eg Klemp and Wilhelmson (1978), "$1\frac{1}{2}$ order") and better or more comprehensive microphysics (including an ice phase, eg Cotton et al (1982) and Bennetts and Rawlins (1981)). Theoretical advances have given more appropriate boundary conditions (see Section 4). The grid spacings are still of order 1 km and will remain so for the foreseeable future although techniques such as nesting and stretched coordinates should give improvements of resolution in the region of interest. There is current interest in the development of alternative formulations of the equations of motion and representation of, say, non equilibrium microphysics (eg Hauf, Holler and Schumann (1984)).

The subject has recently been reviewed by Miller and Moncrieff (1983) and Lilly (1979). Cotton (1975) is worth reading for his pessimism while Pielke (1984) makes some comments of pertinence.

For a number of years Met O 15 has had a Cumulonimbus model; a development of the Miller and Pearce (1974) model to include an ice phase (Bennetts and Rawlins (1981)). The model has been used in a number of investigations (eg Bennetts and Ryder (1984)). The model is coded for the IBM computers but to improve the domain size (usually 16 x 16 km) and the resolution it is necessary to use the

capacity of the CYBER 205. The model also has a simple turbulence parametrization and poor boundary conditions so, while changing machines, the opportunity has been taken to reformulate the model to improve in all of these respects.

This note describes the formulation of the new three dimensional comulonimbus model.

The model uses the anelastic equations (Ogura and Phillips (1962)) with a first order closure for turbulence and a parametrization of warm microphysics (described in van den Berghe (1985a), the ice phase will be added later). The boundary conditions are designed to be 'radiating' (see section 4). Height is used as the vertical coordinate. The equations are approximated by second order finite differences and advanced using explicit time integration. It is expected that an integration with 40^3 points will take $\frac{1}{2}$ hr CPU time for 1000 steps allowing significant improvements in domain size (say 45 km x 45 km x 15 km with $\Delta x = \Delta y =$ 1 km, $\Delta z =$ 500 m) or resolution (20 x 20 x 15 km, $\Delta x = \Delta y = \Delta z =$ 500 m). Alternatively since, say, a run with 30^3 points will take 15 mins CPU a larger number of smaller runs could be made enabling a thorough investigation of a particular parameter space.

The remainder of this note describes fully the equations used and approximations made. The detail should be sufficient to enable the model code to be followed.

2. Analytical Equations

The model uses the anelastic equations as derived by Ogura and Phillips (1962) and developed by Clark (1977). These approximate equations do not support sound waves as a solution (and so allow longer time steps to be used) but accurately model internal gravity waves.

The thermodynamic variables are assumed to be decomposed into three elements,

$$\theta(\underline{x},t) = \textcircled{H} + \theta'(z) + \theta''(\underline{x},t) = \textcircled{4}(1+\theta^*) \qquad (1a)$$

$$T(\underline{x},t) = \bar{T}(z) + T'(z) + T''(\underline{x},t) \quad (1b)$$

$$p(\underline{x},t) = \bar{p}(z) + p'(z) + p''(\underline{x},t) \quad (1c)$$

$$\rho(\underline{x},t) = \bar{\rho}(z) + \rho'(z) + \rho''(\underline{x},t) \quad (1d)$$

$$q_v(\underline{x},t) = \quad q_v'(z) + q_v''(\underline{x},t) \quad (1e)$$

Where θ, T, p, ρ, q_v are the potential temperature, temperature, pressure, density and water vapour mixing ratio of the air.

The first expansion is about a dry adiabatic atmosphere in hydrostatic balance, with (large) deviations from the dry adiabat, still in hydrostatic balance, accounted for by the single primed variables. The double primed variables account for the unbalanced and evolving atmospheric motions. This decomposition improves the numerical accuracy of the calculations, allows the linearization of some equations (eg equations (6) and (7)) and reduces an implicit relationship between θ, T and p (through the saturation vapour pressure) to an approximate explicit relation (see van den Berghe (1985a) or Clark (1979)).

The equation of state is,

$$p = \rho R_d \left(1 + \frac{R_v}{R_d} q_v\right) T \quad (2)$$

where R_d is the gas constant for dry air, R_v the gas constant for water vapour.

Potential temperature is defined by

$$\theta = T\left(\frac{p}{p_0}\right)^{-\kappa} \quad (3)$$

$\kappa = R/c_p$, c_p = specific heat of dry air at constant pressure, p_0 = reference pressure.

The initial hydrostatic balances are given by Clark (1979) as,

$$\frac{\partial p}{\partial z} = -\rho g \tag{4a}$$

$$\frac{\partial p'}{\partial z} + \frac{\rho g}{\gamma p} p' = \rho g \left(\frac{\theta'}{\bar{\theta}} + \epsilon q_v' \right) \tag{4b}$$

$\gamma = c_p / c_v$ $\quad c_v =$ specific heat of dry air at constant volume.

$\epsilon = R_v / R_d - 1$

The hydrostatically balanced basic state can be fully defined using equations (2) to (4) and assuming a potential temperature distribution $\bar{\theta} + \theta'(z)$ and a moisture distribution $q_v'(z)$. We then have

$$\bar{T}(z) = \bar{\theta}(1 - z/H_s) \tag{5a}$$

$$\bar{p}(z) = p_0 (1 - z/H_s)^{1/\kappa} \tag{5b}$$

$$\bar{\rho}(z) = \frac{p_0}{R_d \bar{\theta}} (1 - z/H_s)^{\left(\frac{1}{\kappa} - 1\right)} \tag{5c}$$

$$T'(z) = \bar{T} (\theta'/\bar{\theta} + \kappa p'/\bar{p}) \tag{5d}$$

$$\rho'(z) = \bar{\rho} \left(p'/\gamma p - \theta'/\bar{\theta} - \frac{R_v}{R_d} q_v' \right) \tag{5e}$$

$p'(z)$ is found by (numerical) integration of equation (4b). H_s = scale height of an isentropic atmosphere = $c_p \bar{\theta} / g$

Equations (2) and (3) are linearized about the dry adiabatic state to give

$$\frac{\rho' + \rho''}{\bar{\rho}} = -\theta^* + \frac{p' + p''}{\gamma \bar{p}} - \frac{R_v}{R_d} (q_v' + q_v'') \tag{6}$$

$$\theta^* = \frac{T' + T''}{\bar{T}} - \kappa \frac{p' + p''}{\bar{p}} \tag{7}$$

The conservation equations for the three components of momentum, after subtracting off the hydrostatic balances (equations (4)), are

$$\rho \frac{du}{dt} - \rho f v = -\frac{\partial p''}{\partial x} + \frac{\partial \tilde{\tau}_{1j}}{\partial x_j} \qquad (8a)$$

$$\rho \frac{dv}{dt} + \rho f u = -\frac{\partial p''}{\partial y} + \frac{\partial \tilde{\tau}_{2j}}{\partial x_j} \qquad (8b)$$

$$\rho \frac{dw}{dt} = -\frac{\partial p''}{\partial z} - \rho'' g - g\rho\left(q_v'' + \sum_{i=1}^{n} q_i\right) + \frac{\partial \tilde{\tau}_{3j}}{\partial x_j}$$

$$= -\frac{\partial p''}{\partial z} + \rho g\left(\frac{\Theta''}{\Theta} + \epsilon q_v'' - \frac{p''}{p} - \sum_{i=1}^{n} q_i\right) + \frac{\partial \tilde{\tau}_{3j}}{\partial x_j} \qquad (8c)$$

(using equation (6))

Where (u, v, w) are the components of velocity, f the Coriolis parameter, $\tilde{\tau}_{ij}$ the Reynolds stress tensor (repeated suffices imply summation).

We have assumed that the effect of changes of phase of water and the production of rain, snow etc will be represented by the interactions between a number of discrete classes of water substance, each of these classes having a mixing ratio q_i (eg in most warm cumulonimbus models water exists as vapour (q_v), cloud (q_c) or rain (q_R), so $n=2$, $q_1 = q_c$, $q_2 = q_R$

With the deep anelastic assumption the continuity equation is

$$\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w = 0 \qquad (9)$$

To complete the system of equations we need an equation to describe the conservation of heat and equations to describe the conservation of each class of condensed water, eg for the n=2 class system described above,

$$\rho \frac{d\Theta^*}{dt} = \frac{\rho L}{c_p \Pi}\{C_{vc} + C_{vR}\} + \frac{\partial \tilde{\tau}_{\theta j}}{\partial x_j} \qquad (10a)$$

$$\rho \frac{dq_v}{dt} = -\rho C_{vR} - \rho C_{vc} + \frac{\partial \tilde{\tau}_{qvj}}{\partial x_j} \qquad (10b)$$

$$\rho \frac{dq_c}{dt} = -\rho C_{cR} + \rho C_{vc} + \frac{\partial \tilde{T}_{qcj}}{\partial x_j} \qquad (10c)$$

$$\rho \frac{dq_R}{dt} + \frac{\partial}{\partial z}(\rho V_T q_R) = \rho C_{vR} + \rho C_{cR} + \frac{\partial \tilde{T}_{qRj}}{\partial x_j} \qquad (10d)$$

where C_{ij} is the rate of conversion between water fields i and j, V_T the terminal speed of fall of rain. The details of the microphysical scheme used in this model are given in van den Berghe (1985a). For reasons described in that note we sum equations (10b) and (10c) to get

$$\rho \frac{dq_T}{dt} = -\rho C_{cR} - \rho C_{vR} + \frac{\partial \tilde{T}_{qTj}}{\partial x_j}$$
$$q_T = q_v' + q_v'' + q_c \qquad (10e)$$

To advance the model in time we need to evaluate the pressure, which with the anelastic set of equations is evaluated by solving the elliptic diagnostic equation derived as follows.

Writing equations (8) as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p''}{\partial x} + SLU \qquad (11a)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p''}{\partial y} + SLV \qquad (11b)$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p''}{\partial z} - \frac{g}{\rho} p'' + SLW \qquad (11c)$$

Where SLU, SLV, SLW contain the non pressure terms from equations (8).

Taking $\frac{\partial}{\partial x}$ of (11a), $\frac{\partial}{\partial y}$ of (11b), $\frac{\partial}{\partial z}$ of (11c) and using the continuity equation ([9]) the following diagnostic equation for pressure is found,

$$\nabla^2 p'' + \frac{g}{\delta}\frac{\partial}{\partial z}\left(\frac{\rho}{p} p''\right) = \frac{\partial}{\partial x}(\rho SLU) + \frac{\partial}{\partial y}(\rho SLV) + \frac{\partial}{\partial z}(\rho SLW) \quad (12)$$

To close the dry equations we approximate sub-grid scale turbulence by the first order closure of Smagorinsky (1963) and Lilly (1962).

The Reynolds stress tensor is written as,

$$\tilde{\tau}_{ij} = \rho K_M D_{ij} \quad (13)$$

the deformation tensor as,

$$D_{ij} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\left\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right\}\right) \quad (14)$$

and the total deformation as,

$$DEF = \left[\frac{1}{2}\sum_{ij} D_{ij}^2\right]^{1/2} \quad (15)$$

The eddy mixing coefficient for momentum is given by (Clark (1979)),

$$K_M = \begin{cases} (c\Delta)^2 |DEF| (1 - Pr\, Ri)^{1/2} & Pr\, Ri < 1 \\ K_0 & Pr\, Ri \geq 1 \end{cases} \quad (16)$$

where $c = .25$, $K_0 = 147.6\, m^2 s^{-1}$ and Δ is a measure of the effective grid scale,

$$\Delta = (\Delta x \Delta y \Delta z)^{1/3}$$

The local Richardson number, Ri, is given by

$$Ri = g\frac{\partial}{\partial z}(\theta^* + \epsilon q_v - q_c - q_R) / DEF^2 \quad (17)$$

K_H is the eddy mixing coefficient for heat and a Prandtl (Pr) number of unity is assumed, therefore

$$\frac{\partial \tilde{\tau}_{\theta j}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\rho K_m \frac{\partial \theta^*}{\partial x_j}\right)$$

$$\frac{\partial \tilde{\tau}_{q_T j}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\rho K_m \frac{\partial q_T}{\partial x_j}\right)$$

etc

The parametrization of sub grid scale turbulence, on the assumption that it is three dimensional and isotropic, is valid only for grid intervals within the inertial sub range (Cotton 1975), Lilly (1979)). When used with grid intervals of the order of 1 km, as it will be, it can at best be regarded as a gross approximation (Klemp and Wilhelmson (1978)) or merely a numerical device for controlling non-linear instability (Pielke (1984) p 328). However Schlesinger (1978) and Clark (1979) have used it successfully in cumulonimbus integrations. Should it be felt necessary and warranted by any realism of the rest of the model's parametrizations the next stage up in realism is the 1½ order closure of Klemp and Wilhelmson (1978) (these parametrizations have not been fully verified, see Lilly (1979)).

3. Numerical Formulation

The analytic equations are approximated using second order accurate spatial differencing, while time derivatives are approximated by leapfrog differences for the advective terms, forward for the diffusive terms and implicit for the microphysical source/sink terms. Approximations to the microphysics mean that the time stepping is explicit (van den Berghe (1985a)). The model variables are held on a staggered grid.

This section describes the detailed numerical approximation of the above system ie spatial grid, space and time differencing, formulation of the diagnostic pressure equation and the construction of the initial balanced state.

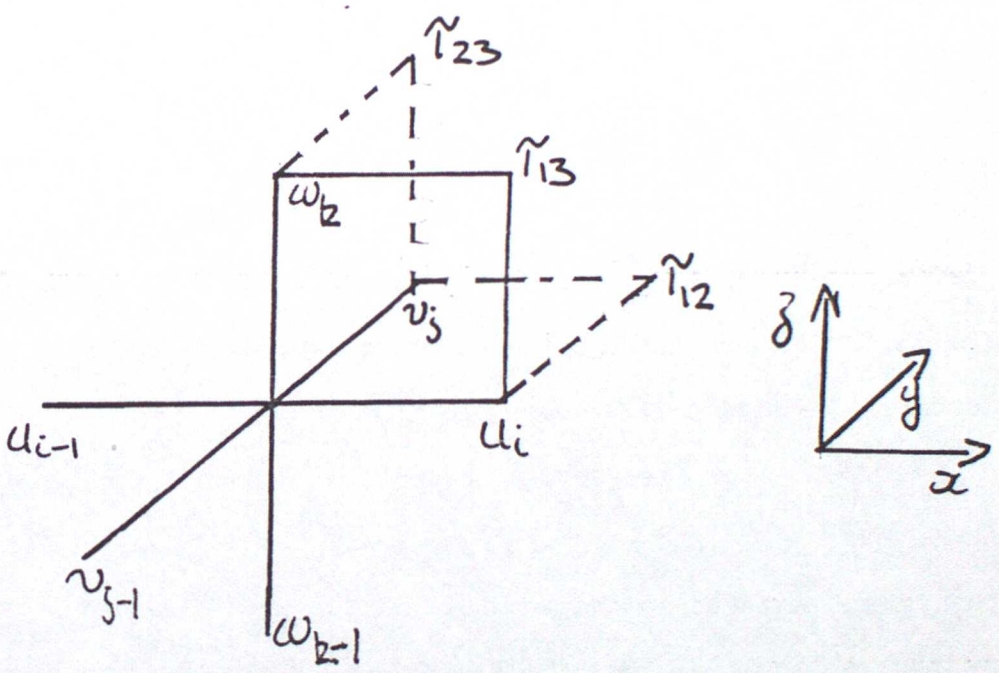


FIG 1 LOCATION OF VARIABLES AT A GRID POINT.

Θ^*, P'', q_v, q_c, q_R, $\tilde{\tau}_{11}$, $\tilde{\tau}_{22}$, $\tilde{\tau}_{33}$, K ARE HELD AT THE CENTRE POINT

3a. Model Domain and Grid

The variables are held on a grid that has constant increments, $\Delta x, \Delta y,$ in the horizontal directions and is variable in the vertical (rather than an explicit stretching of the vertical coordinate). The variables are held on the commonly used staggered grid (Arakawa 'C' grid, Fig 1 or see Haltiner and Williams (1980) p 226). Sun (1984) suggests that holding θ and w at the same point gives greater accuracy to the buoyancy term, equation (8c). This arrangement is used in the IBM model and may repay experimentation here.

The points holding w are termed 'z' points, those holding p, θ etc are termed 'zm'. Figures 1 and 2 show the vertical grid layout and define some terms.

The boundaries of the model are at $x=0, x=L_x, y=0, y=L_y, z=0, z=H$ The boundaries pass through the points at which the velocity normal to the boundary is held (see Figure 3). There are $II \times JJ$ points in the horizontal and $KK-2$ in the vertical direction. Computational points are used and so there are $(II+2)\times(JJ+2)\times KK$ points in the model. The points are numbered so that

$$p_{ijk} = p((i-\tfrac{1}{2})\Delta x, (j-\tfrac{1}{2})\Delta y, zm(k))$$
$$u_{ijk} = u(i\Delta x, (j-1/2)\Delta y, zm(k))$$
$$v_{ijk} = v((i-\tfrac{1}{2})\Delta x, j\Delta y, zm(k))$$
$$w_{ijk} = w((i-\tfrac{1}{2})\Delta x, (j-\tfrac{1}{2})\Delta y, z(k))$$

$i = 0, 1, 2, \ldots, II, II+1$
$j = 0, 1, 2, \ldots, JJ, JJ+1$
$k = 1, 2, \ldots, KK-1, KK.$

ALSO,

$$L_x = II\,\Delta x$$
$$L_y = JJ\,\Delta y$$
$$H = z(KK-1)$$

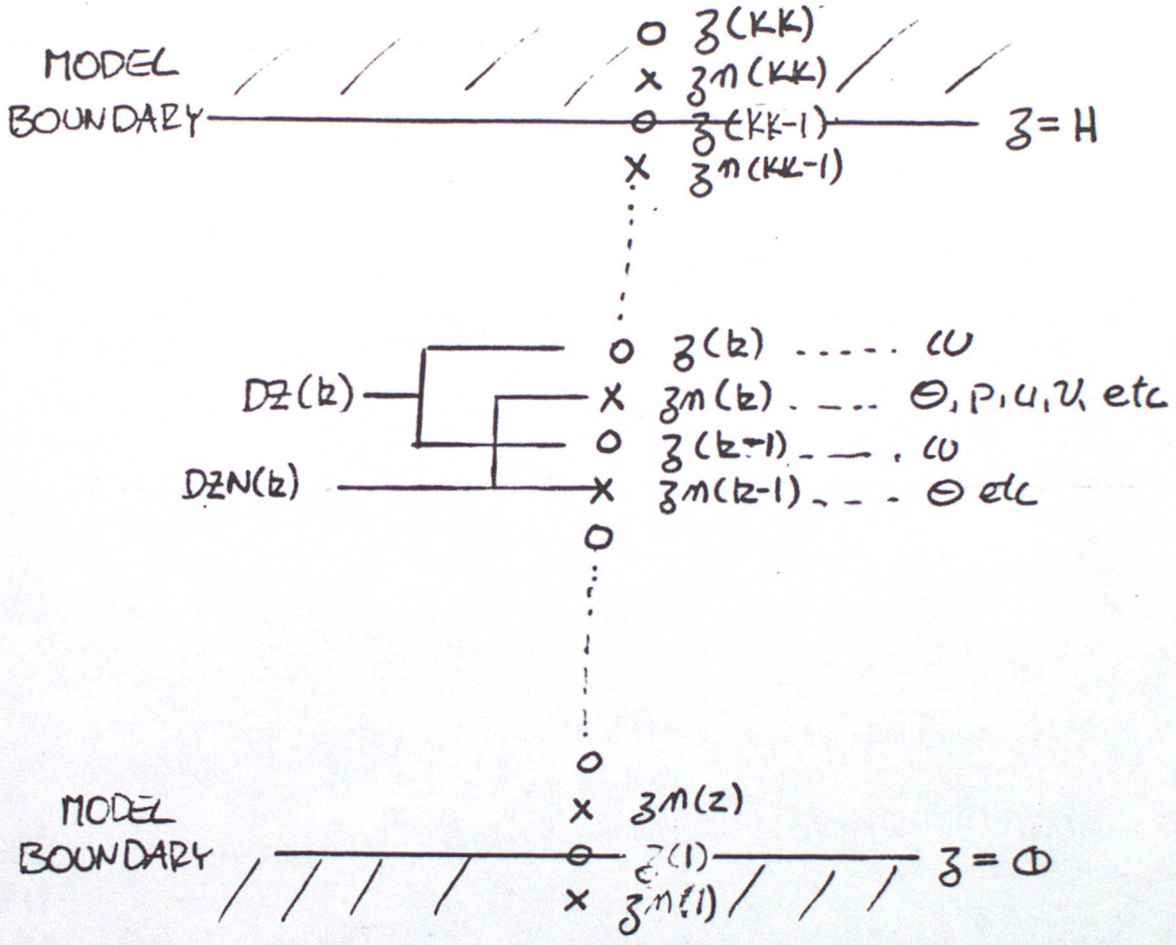


FIG 2 LAYOUT OF GRID IN THE VERTICAL

3b. Numerical Formulation

Equations (8) and (10) are written as

$$\delta_t \bar{u}^t = -\frac{1}{\rho}\delta_x p''^t + SLU^t \tag{18a}$$

$$\delta_t \bar{v}^t = -\frac{1}{\rho}\delta_y p''^t + SLV^t \tag{18b}$$

$$\delta_t \bar{w}^t = -\frac{1}{\rho}\delta_z p''^t - \frac{g}{\partial\tilde{\rho}} p''^t + SLW^t \tag{18c}$$

$$\delta_t \overline{\theta^*}^t = -ADV(\theta^*)^t + DIFF(\theta^*)^{t-1} - \frac{L}{c_p \Pi}\left(C^+_{VC} + C^+_{VR}\right) \tag{18d}$$

$$\delta_t \bar{q}_T^{\,t} = -ADV(q_T)^t + DIFF(q_T)^t - C^+_{VR} - C^*_{CR} \tag{18e}$$

$$\frac{q_R^{t+1} - q_R^t}{\Delta t} = -ADVP(q_R)^t - FALLP(q_R)^t + DIFF(q_R)^t + C^{t+1}_{VR} + C^{t+1}_{CR} \tag{18f}$$

Where $\delta_{mx}\phi = \dfrac{\phi_{i+m/2} - \phi_{i-m/2}}{m\Delta x}$, $\bar{\phi}^x = \frac{1}{2}\left(\phi_{i+1/2} + \phi_{i-1/2}\right)$

and $\tilde{\phi} = \underline{\phi}$ at a ζ point

$$SLU^t = -ADV(u)^t + DIFFM(u)^{t-1} + f\left(\bar{v}^{xy}{}^t + v_0\right) - \frac{1}{\tau_R}\left(u^t + u_0 - u_R\right) \tag{19a}$$

$$SLV^t = -ADV(v)^t + DIFFM(v)^{t-1} - f\left(\bar{u}^{xy}{}^t + u_0\right) - \frac{1}{\tau_R}\left(v^t + v_0 - v_R\right) \tag{19b}$$

$$SLW^t = -ADV(w)^t + DIFFM(w)^{t-1} + g\left\{\overline{\theta''}^{3}/\Theta + \epsilon\overline{q_v''}^{3} - \bar{q}_c^{3} - \bar{q}_R^{3}\right) - \left(\overline{1/\tau_R}^{3}\right) w \tag{19c}$$

and

$ADV(\phi)^t$ = the numerical approximation, at time t, for

$$\frac{\partial}{\partial x}u\phi + \frac{\partial}{\partial y}v\phi + \frac{1}{\rho}\frac{\partial}{\partial z}\rho w\phi \tag{20}$$

$ADVP(\phi)^t$ = positive definite approximation for equation (20)

$DIFF(\phi)^t$ = the numerical approximation, at time t, for

$$\frac{1}{\rho}\left\{\frac{\partial}{\partial x}\left(\rho K_m \frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho K_m \frac{\partial\phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(\rho K_m \frac{\partial\phi}{\partial z}\right)\right\}$$

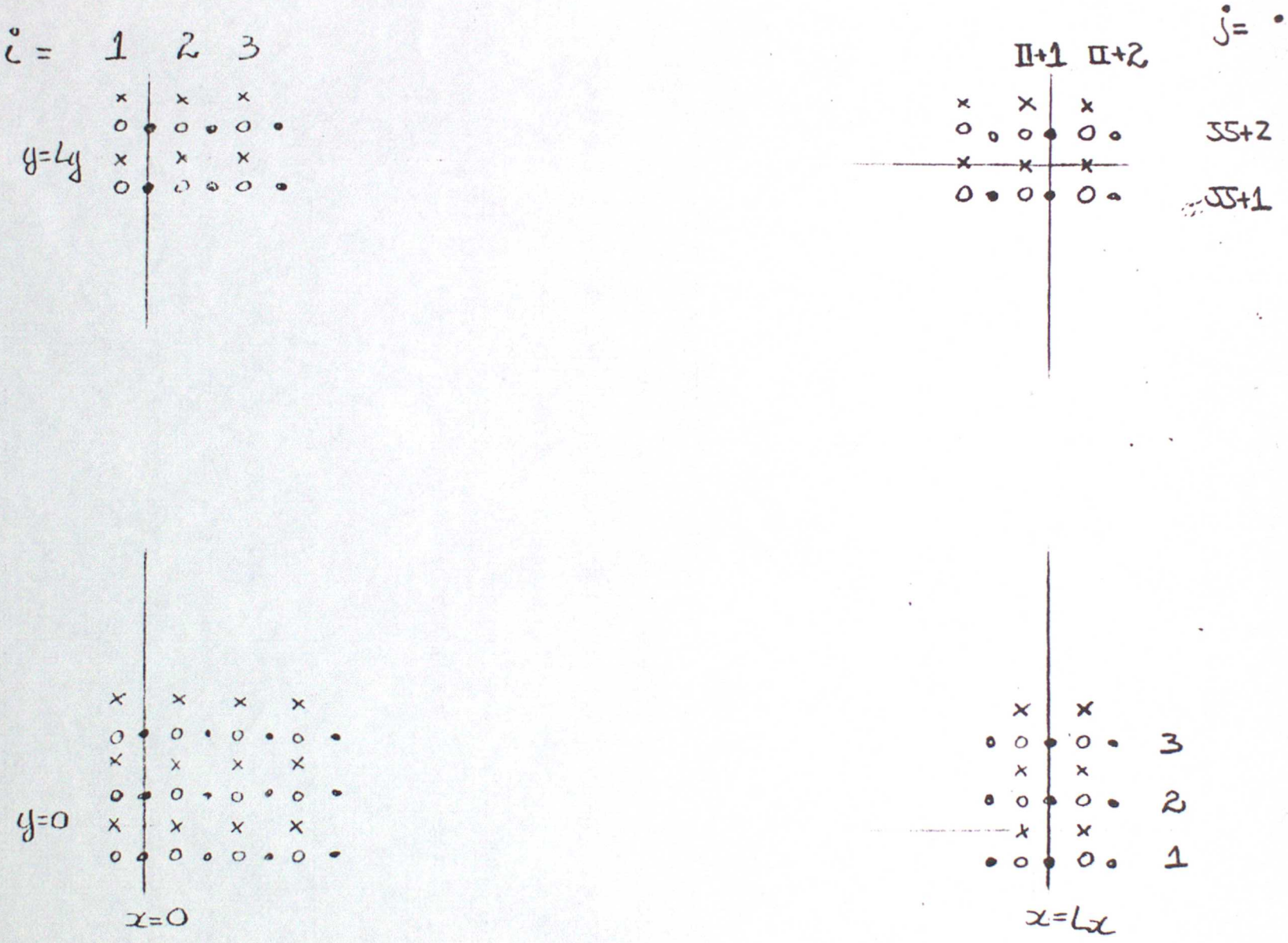


FIG 3 HORIZONTAL LAYOUT OF MODEL GRID

SOLID LINES ARE MODEL BOUNDARIES
• = U POINT, x = V POINT, o = P, Θ ew etc POINT

$DIFFM(u_i)^t$ = the numerical approximation, at time t, for

$$\frac{1}{\rho}\left\{\frac{\partial}{\partial x}\left(\rho K_M \frac{\partial D_{1i}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\rho K_M \frac{\partial D_{2i}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\rho K_M \frac{\partial D_{3i}}{\partial z}\right)\right\}$$

$FALLP(\phi)^t$ = the positive definite numerical approximation, at time t, for

$$\frac{\partial}{\partial z}\left(\rho \bar{V}_T \phi\right)$$

also $C_{ij}^{*} = \frac{1}{2}\left(C_{ij}^{t} + C_{ij}^{t+1}\right)$ $\qquad C_{ij} = C_{UC}, C_{UR}$ or C_{CR}

Equation (20) has been written in flux form by using the continuity equation ((9)).

Equations (18a) to (18c) differ from (8a) to (8c) in the addition of two terms;

(i) an initial, constant, velocity field (u_0, v_0, ϕ)

(ii) a damping term with time constant $\tilde{\tau}_R$.

(i) u_0, v_0 Most of the systems we shall be modelling move with a non zero horizontal velocity. We want to be able to keep the system motion as small as possible relative to the grid both for numerical purposes and to keep the system within the grid during the integration time. To do this we subtract off a system velocity that is constant in space and time $\{(u_0, v_0, \phi)\}$ and therefore only affects the integration through the coriolis term, boundary drag and the damping term.

(ii) $\tilde{\tau}_R$ The boundary conditions as set up (see section 4) allow the reflection of waves from the upper boundary. To minimize the effect of this on the integration we attempt to absorb the waves before they reach the upper boundary. A Rayleigh friction zone is set up in the upper layers of the model within which u, v and w are relaxed to (u_R, v_R, ϕ) with a time constant $\tilde{\tau}_R(z)$.

3c. Time Integration

The model is integrated forward in time using a step size Δt and an explicit scheme. The basic scheme is leapfrog ie

$$\phi^{t+1} = \phi^{t-1} + 2\Delta t f^t \qquad (21)$$

The diffusive parts are not stable for a leapfrog scheme and instead we must use a forward timestep over $2\Delta t$ ie

$$\phi^{t+1} = \phi^{t-1} + 2\Delta t f^{t-1} \tag{22}$$

Equation (18f) for q_R is approximated differently to enable the use of a spatial differencing scheme that is guaranteed to keep q_R positive. For further discussion of this point see van den Berghe (1985a). All schemes suited to this purpose need a forward time step.

The microphysical source/sink terms are formulated implicitly since the rate of condensation of water vapour to cloud water depends on the local values of q_v and T . There is also an implicit character to the formulation of the anelastic equations themselves (Ogura and Phillips (1962)) since the value of Θ depends on the release of latent heat which depends on the saturation vapour pressure which in turn depends on Θ . Both these links are in practice weak and can be approximated by an explicit calculation (see Clark (1979) and Wilhelmson and Ogura (1972); van den Berghe (1985a) gives the details of the approximation).

The different time differencing schemes in equations (18f) and (18d and e) mean that care must be taken to ensure that heat and water substance are conserved in the calculations (Clark (1973)). As an example consider the following coupled equations;

$$\frac{\partial \eta}{\partial t} = c \quad , \quad \frac{\partial \chi}{\partial t} = -c$$

approximate these equations as,

$$\eta^{t+1} - \eta^{t} = \Delta t\, c^{t+1} \quad , \quad \chi^{t+1} - \chi^{t-1} = -2\Delta t\, c^{*}$$

over two time steps

$$\eta^{t+1} - \eta^{t} = \eta^{t+1} - \eta^{t} + \eta^{t} - \eta^{t-1} = \Delta t (c^{t+1} + c^{t})$$

ie $$c^{*} = \frac{1}{2}(c^{t+1} + c^{t})$$

The leapfrog scheme with a **forward** step for diffusion may tend to time split the solution despite the recoupling by equation (18f) and the lateral boundary conditions. The model has two ways to recouple;

a. time smoothing $$\phi_R^{\epsilon+1} = \phi^{\epsilon+1}(1-\epsilon) + \epsilon\phi^{\epsilon}$$
$$\phi_R^{\epsilon} = \phi^{\epsilon}(1-\epsilon) + \epsilon\phi^{\epsilon+1}$$

(some of the boundary conditions are nonlinear and should be reapplied after smoothing. This is not done in the model, which is justified if changes are slow compared to Δt , a similar assumption is made in the boundary condition formulation).

b. Take an occasional forward step, ie replace both equation (21) and equation (22) by $$\phi^{\epsilon+1} = \phi^{\epsilon} + \Delta t f^{t}$$

(in the model code this is done by overwriting the $\epsilon-1$ fields by the ϵ fields and changing Δt, $1/\Delta t$ etc. A forward step is also used to start the model and to restart it. To ensure that a restart integration is the same as a continued integration it is essential to take a forward step at the same time as a restart dump is made).

3d. Spatial Integration

The spatial derivatives in the momentum equations are approximated using the Piacsek and Williams (1970) scheme, which conserves the square of the speed and so helps with the kinetic energy budget and in controlling non-linear instability. For θ^* and q_T the appropiate conservation is linear and so we use second order centred differences for their conservation equations.

THE TERMS IN SECTION 3b) BECOME (ALL AT TIME T)

$$ADV(u) = \frac{1}{2}\{\delta_x(\bar{u}^x\bar{u}^x) + \overline{u^x\delta_x u}^x\}$$
$$+\frac{1}{2}\{\delta_y(\bar{v}^x\bar{u}^y) + \overline{v^x\delta_y u}^x\}$$
$$+\frac{1}{2\rho}\{\delta_z(\overline{\rho w}^x\bar{u}^z) + \overline{\rho w}^x\delta_z u^z\}$$

$$ADV(v) = \frac{1}{2}\{\delta_x(\bar{u}^y\bar{v}^x) + \overline{u^y\delta_x v}^x\}$$
$$+\frac{1}{2}\{\delta_y(\bar{v}^y\bar{v}^y) + \overline{v^y\delta_y v}^y\}$$
$$+\frac{1}{2\rho}\{\delta_z(\overline{\rho w}^y\bar{v}^z) + \overline{\rho w}^y\delta_z v^z\}$$

$$ADV(w) = \frac{1}{2\rho}\{\delta_x(\overline{\rho u}^z\bar{w}^x) + \overline{\rho u^z}\delta_x w^z\}$$
$$+\frac{1}{2\rho}\{\delta_y(\overline{\rho v}^z\bar{w}^y) + \overline{\rho v}^z\delta_y w^z\}$$
$$+\frac{1}{2\rho}\{\delta_z(\overline{\rho w}^z\bar{w}^z) + \overline{\rho w}^z\delta_z w^z\}$$

$$ADV(\phi) = \delta_x(u\bar{\phi}^x) + \delta_y(v\bar{\phi}^y) + \frac{1}{\rho}\delta_z(w\overline{\rho\phi}^z)$$

WHERE $\phi = \theta^*$ OR q_T

THE EQUATION,

$$\frac{\partial}{\partial x}uq_R + \frac{\partial}{\partial y}vq_R + \frac{1}{\rho}\frac{\partial}{\partial z}(\rho(w+\bar{v}_T)q_R)$$
$$\approx ADVP(q_R) + FALLP(q_R)$$

is presently approximated using the flux form of upstream differencing eg

$$\frac{\partial}{\partial x}(u q_R) \approx \frac{F_i - F_{i-1}}{\Delta x}$$

where F_i is defined, at a u point, by

$$F_i = \begin{cases} (q_R)_i \, u_i & u_i \geq 0 \\ (q_R)_{i+1} u_i & u_i < 0 \end{cases}$$

The diffusive terms are written as

$$DIFFM(u) = \frac{1}{\rho}\left(\delta_x \tau_{11} + \delta_y \tau_{12} + \delta_z \tau_{13}\right)$$

$$DIFFM(v) = \frac{1}{\rho}\left(\delta_x \tau_{12} + \delta_y \tau_{22} + \delta_z \tau_{23}.\right)$$

$$DIFFM(w) = \frac{1}{\rho}\left(\delta_x \tau_{13} + \delta_y \tau_{23} + \delta_z \tau_{33}\right)$$

Where

$$\tau_{11} = \rho K_M D_{11}$$

$$\tau_{22} = \rho K_M D_{22}$$

$$\tau_{33} = \rho K_M D_{33}$$

$$\tau_{12} = \overline{\rho K_M}^{xy} D_{12}$$

$$\tau_{13} = \overline{\rho K_M}^{xz} D_{13}$$

$$\tau_{23} = \overline{\rho K_M}^{yz} D_{23}$$

$$D_{11} = \frac{4}{3}\delta_x u - \frac{2}{3}\delta_y v - \frac{2}{3}\delta_z w$$

$$D_{22} = \frac{4}{3}\delta_y v - \frac{2}{3}\delta_x u - \frac{2}{3}\delta_z w$$

$$D_{33} = \frac{4}{3}\delta_z w - \frac{2}{3}\delta_x u - \frac{2}{3}\delta_y v$$

$$D_{12} = \delta_x v + \delta_y u$$

$$D_{13} = \delta_x w + \delta_z u$$

$$D_{23} = \delta_y w + \delta_z v$$

$$K_M = \begin{cases} (c\Delta)^2 \, |DEF| \, (1 - R_i)^{1/2} & R_i \leq 1 \\ K_0 & R_i > 1 \end{cases}$$

$$R_i = g\,\delta_z(\bar{\theta}^{z} + \epsilon\bar{q}_v^{z} - \bar{q}_c^{z} - \bar{q}_R^{z}) \,/\, DEF^2$$

$$DEF^2 = \frac{1}{2}(D_{11}^2 + D_{22}^2 + D_{33}^2) + (\bar{D}_{12}^{xy})^2 + (\bar{D}_{13}^{xz})^2 + (\bar{D}_{23}^{yz})^2$$

also

$$DIFF(\phi) = \frac{1}{\rho}\{\delta_x(\overline{(\rho K_M)}^{x}\delta_x\phi) + \delta_y(\overline{(\rho K_M)}^{y}\delta_y\phi) + \delta_z(\overline{(\rho K_M)}^{z}\delta_z\phi)$$

The linear stability criteria for all the advection and diffusion terms reduce to the same as those for second order centred differences:

For advection : $$\Delta t\left\{\frac{|u|}{\Delta x} + \frac{|v|}{\Delta y} + \frac{|w|}{\Delta z}\right\} \leq 1$$

For diffusion : $$2\Delta t\left\{\frac{K_x}{\Delta x^2} + \frac{K_y}{\Delta y^2} + \frac{K_z}{\Delta z^2}\right\} \leq \frac{1}{2}$$

If we assume that $\Delta x = \Delta y = 2\Delta z = L$, $u = v = w = U$

$$K_x = K_y = K_z = K$$

then stability for advection requires

$$\Delta t \leq \frac{L}{4U} = 20 \text{ secs}$$

and diffusion

$$\Delta t \leq \frac{L^2}{24K} = 80 \text{ secs}$$

if $L = 10^3 m$, $U = 10 ms^{-1}$, $K = 500 m^2 s^{-1}$

That the model satisfies the stability criteria for advection and diffusion separately does not guarantee stability, for it is well known that the stability properties of the combined advective/diffusive scheme may be different. Cushman-Roisin (1984) has described a technique for analysing the stability of the combined scheme and this technique can be applied to our equations. An approximation to the stability criterion, which is sufficient but too restrictive, is that the combined scheme is stable if

$$K = 2\Delta t \left\{ \frac{K_x}{\Delta x^2} + \frac{K_y}{\Delta y^2} + \frac{K_z}{\Delta z^2} \right\} \leq \frac{1}{2}$$

and

$$\Delta t \left\{ \frac{|u|}{\Delta x} + \frac{|v|}{\Delta y} + \frac{|w|}{\Delta z} \right\} / (1 - 2K) \leq 1$$

In other words the stability of the combined scheme is still dominated by the stability with respect to advection, although reductions in the grid size will change this, eg with L = 500 m advective stability requires that $\Delta t < 10$, diffusive stability that $\Delta t < 20$ and the combined scheme that $\Delta t < 6.5$ secs.

3e. Diagnostic Pressure Equation

To advance the model in time we need to use equation (12) to diagnose the pressure. The model is in a correct balance only if equation (12) is obtained in a form that is consistent with the numerical formulation of the momentum equations. Taking $\frac{1}{4}\delta_x$ of equation (18a) $\frac{1}{4}\delta_y$ of (18b) and $\frac{1}{2}\delta_z$ of (18c) and adding we get

$$\frac{1}{2\Delta t}\left\{\nabla\cdot\rho\underline{u}^{t+1} - \nabla\cdot\rho\underline{u}^{t-1}\right\} = -\left\{\nabla^2 p + \frac{g}{\delta}\delta_z\left(\frac{\rho}{p}p''\right) + \nabla\cdot(\rho\underline{SL})\right\} \tag{23}$$

where

$$\nabla = \underline{i}\,\delta_x + \underline{j}\,\delta_y + \underline{k}\,\delta_z$$
$$\underline{u}^t = \underline{i}\,u^t + \underline{j}\,v^t + \underline{k}\,w^t$$
$$\underline{SL} = \underline{i}\,SLU + \underline{j}\,SLV + \underline{k}\,SLW$$

(ρ is a scalar)

The approximated form of the continuity equation is

$$\nabla\cdot\rho\underline{u}^t = 0 \tag{24}$$

We assume that the $t+1$ fields satisfy this condition but, being pragmatic, recognize that the numerical solution will not exactly satisfy equation (24). To provide a negative feedback on the resulting error growth (Harlow and Welch (1965)) we retain the $t-1$ term in equation (23). The pressure equation is then;

$$\nabla^2 p + \frac{g}{\delta}\delta_z\left(\frac{\rho}{p}p''\right) = \nabla\cdot\rho\left(\underline{SL} + \frac{\underline{u}^{t-1}}{2\Delta t}\right) \tag{25}$$

ie

$$\delta_x\delta_x p'' + \delta_y\delta_y p'' + \delta_z\delta_z p'' + \frac{g}{\delta}\delta_z\left(\frac{\rho}{p}p''\right) = \delta_x HU + \delta_y HV + \delta_z HW \tag{26}$$

where

$$HU = \rho\left(SLU + \frac{u^{t-1}}{2\Delta t}\right)$$
$$HV = \rho\left(SLV + \frac{v^{t-1}}{2\Delta t}\right)$$
$$HW = \rho\left(SLW + \frac{w^{t-1}}{2\Delta t}\right)$$

Equation (26) is solved using Fourier transforms in the horizontal directions and tridiagonal inversion in the vertical. The method is described fully in van den Berghe (1985b).

3f. Initial State

To avoid spurious forcing of the model from an out of balance initial state care must be taken to ensure that the $\underline{\phi}$ and ϕ' variables are set up in exact balance. This means both a consistent approximation and also balance at the machine accuracy at which the dynamical variables are held (32 bit and not the 64 bit of the main calculations). The model as coded satisfies this balance since initially still fields with no forcing remain still for all time.

EQUATIONS (5) ARE WRITTEN AS,

$$\underline{I} = \Theta\left(1 - \frac{z_m}{H_s}\right)$$

$$p = p_0\left(1 - \frac{z_m}{H_s}\right)^{1/k} \qquad \tilde{p} = p_0\left(1 - z/H_s\right)^{1/k}$$

$$\rho = \frac{p_0}{R\Theta}\left(1 - \frac{z_m}{H_s}\right)^{\frac{1}{k}-1} \qquad \tilde{\rho} = \frac{p_0}{R\Theta}\left(1 - z/H_s\right)^{1/k}$$

$$\rho' = \rho\left(\frac{p'}{\partial p} - \frac{\theta'}{\Theta} - \frac{R_v}{R_d} q_v'\right)$$

$$\tilde{\rho}' = \tilde{\rho}\left(\frac{\tilde{p}'}{\partial \tilde{p}} - \frac{\overline{\theta'}^z}{\Theta} - \frac{R_v}{R_d}\overline{q_v'}^z\right)$$

$$\frac{\partial p'}{\partial z} + \frac{g}{\partial R_d}\frac{p'}{I} = \rho g\left(\frac{\theta'}{\Theta} + \epsilon q_v'\right) \tag{27a}$$

$$\frac{\partial \tilde{p}'}{\partial z} + \frac{g}{\partial R_d}\frac{\tilde{p}'}{\underline{I}^z} = \tilde{\rho} g\left(\frac{\overline{\theta'}^z}{\Theta} + \epsilon \overline{q_v'}^z\right) \tag{27b}$$

$\sim$ indicates that the variable is held at a z point. Equations (27) are integrated numerically from the surface assuming that $\overline{p'}^z = 0$ at $z = 0$.

4. Boundary Conditions

The only real boundary of the model is the ground, all the other five boundaries are artifices introduced to allow adequate resolution with a realistic computer storage requirement. A major concern of limited area modelling (cumulonimbus modelling in particular) has been to minimize the effects of these artificial boundaries on the integration. The usual paradigm is that the effect of the

boundaries is minimized when any disturbance or energy radiated by the modelled system is not reflected back into the model domain. A simple and effective method is to absorb any incident radiation using an artificial zone near the boundary in which the model equations are changed to damp any disturbance (a Rayleigh friction zone). Since the model equations have been changed in the damping zone the solutions there are non physical and cannot be used. The damping zones can occupy 1/4 to 1/3 of the model domain and are therefore very wasteful of storage (eg Clark and Peltier (1977)). Over the past decade an alternative treatment of the horizontal boundaries has been developed (Orlanski (1976), Klemp and Wilhelmson (1978), Miller and Thorpe (1981) and Raymond and Kuo (1984)) which seeks to explicitly radiate any incident energy through the boundary. These "radiation" boundary conditions have proved effective and, since they have no non physical zone, storage efficient.

The horizontal boundary conditions of the model are the radiation conditions proposed by Miller and Thorpe (1981). The upper boundary is modelled as a rigid lid with a Rayleigh friction zone (see section 3a)). It is proposed to change the upper boundary to a radiative one based on the boundary condition given by Klemp and Durran (1983). The earth's surface is rigid and free slip with frictional drag through the stress tensor. Rain is assumed to drop through the bottom of the model with a fall speed determined by the amount of rain at $z=n(z)$. No other surface effects are considered explicitly at present.

At the top of the model domain, $z=H$, we have

$$w_{KL-1}=0$$

$$\delta_z(\rho u)=\delta_z(\rho v)=\bar{q}_v^{\,3}=\bar{q}_c^{\,3}=\bar{q}_r^{\,3}=\delta_z\theta^*=0$$

$$\tau_{13}=\tau_{23}=\delta_z\tau_{12}=\delta_z\tau_{11}=\delta_z\tau_{22}=\bar{\tau}_{33}^{\,3}=\tau_{\theta 3}=\tau_{q_v 3}=\tau_{q_r 3}=0$$

One further boundary condition is required to enable the calculation of SLW at the top of the model. We require conservation of momentum through the top of the model (Clark 1977) ie $\overline{(\rho w) w}^{\zeta} = 0$, at $\zeta = H$,

therefore $$\frac{\partial}{\partial \zeta}(\rho w w) = \frac{-1}{2DZN_{KL}} w^2_{KK-2} \tilde{\rho}_{KK-1}$$

The **lower** , $\zeta = 0$, boundary conditions are,

$$w_1 = 0$$

$$\frac{\partial}{\partial \zeta}(\rho w w)\Big|_{\zeta=0} = \frac{1}{2DZN_1} w_2^2 \tilde{\rho}_2$$

$$\delta_\zeta(\rho u) = \delta_\zeta(\rho v) = \delta_\zeta(q_v) = \delta_\zeta(q_c) = \delta_\zeta(q_R) = \delta_\zeta(\theta^+) = 0$$

$$\bar{\tau}_{11}^{\zeta} = \bar{\tau}_{22}^{\zeta} = \bar{\tau}_{33}^{\zeta} = \bar{\tau}_{12}^{\zeta} = \tilde{\tau}_{03} = \tilde{\tau}_{q_v 3} = \tilde{\tau}_{q_R 3} = 0$$

We model the exchange of momentum with the surface by a simple bulk drag law (eg Haltiner and Williams (1980) p 270).

$$\tilde{\tau}_{13} = \tilde{\rho}_1 C_D \left((u_0 + u_2)^2 + (v_0 + \bar{v}_2^{xy})^2 \right)^{1/2} (u_2 + u_0)$$

$$\tilde{\tau}_{23} = \tilde{\rho}_1 C_D \left((u_0 + \bar{u}_2^{xy})^2 + (v_0 + \bar{v}_2^{xy})^2 \right)^{1/2} (v_2 + v_0)$$

Where the surface tangential velocity is taken at $\zeta = \zeta_{N_2}$

The normal velocities at the lateral boundaries ($x=0$, $x=L_x$, $y=0$, $y=L_y$) are found by extrapolation using the following (radiating) replacement of the momentum equations;

$$\frac{\partial u_n}{\partial t} + c\frac{\partial u_n}{\partial x_n} = 0 \qquad (28)$$

n indicates the direction normal to the boundary, c the speed of the energy.

We shall describe two methods of solving equation (28) for u_n and, at the end of this section, describe some simple tests of the effectiveness of the boundary conditions.

Our choice of solution method is constrained by the pressure boundary condition for time t which assumes knowledge of u_n^{t+1} before the $t+1$ fields are found.

Miller and Thorpe (1981) have shown that accurate boundary conditions can be found by using a forward in time, and upstream space differencing scheme to solve equation (28) ie

$$u_6^{t+1} - u_6^t + c\left\{\frac{u_{6+1}^t - u_6^t}{\Delta x}\right\} = 0$$

(the formulae are given for the $x=0$ boundary, where 6 is the boundary point; extension to the other boundaries is trivial).

ie $$u_6^{t+1} = u_6^t(1+\hat{\tau}^t) - \hat{\tau}^t u_{6+1}^t \qquad (29)$$

$$\hat{\tau}^t = c\Delta t/\Delta x, \quad -1 \le \hat{\tau}^t \le 0$$

The methods to be described differ in their determination of $\hat{\tau}$.

Klemp and Wilhelmson (1978) assume that the energy to be radiated is carried solely by internal gravity waves. If this is so then these waves will be radiated if

$$c = u_6 + c^* \quad \text{ie} \quad \hat{\tau}^t = \frac{\Delta t}{\Delta x}(u_6 + c^*) \qquad (30)$$

where c^* is the phase velocity of the waves we want to radiate and is directed out of the domain. Klemp and Wilhelmson take c^* as a constant and show that energy will still be radiated if $|c^*|$ overestimates the phase speed of the actual waves. The fastest moving waves are horizontal with a large horizontal

wavelength and a vertical wavelength twice the model height, ie

$$c^* = \frac{NH}{\pi}$$

N = Brunt - Vaisala frequency. Miller and Thorpe (1981) follow Orlanski (1976) and determine the appropriate c from equation (28) by writing

$$c = -\frac{\partial u_n}{\partial t} \Big/ \frac{\partial u_n}{\partial x}$$

This has a non-trivial solution if referred to interior grid points at previous time steps (assuming that changes take place more slowly than Δt so that $\hat{r}^t = \hat{r}^{t-1}$), then

$$\hat{r}^t = (u^t_{i+1} - u^{t-1}_{i+1}) / (u^{t-1}_{i+2} - u^{t-1}_{i+1}) \qquad (31)$$

Equation (28) assumes that the disturbance to be radiated is one dimensional, which i a three dimensional integration is untrue. Raymond and Kuo (1984) show that the correct three dimensional version will improve the radiative properties of the boundary but the resulting boundary condition is too computationally expensive for a three dimensional integration.

The above provides boundary conditions for one variable per side, the remainder can be found by noting that Sundstrom and Elvius (1979) suggest that for our equation system a well posed set of boundary conditions will specify all variables on an inflow boundary but only specify one on outflow (the rest can be extrapolated). The extrapolation should be done using an appropriate form of equation (28) but experience has shown (Clark (1979), Klemp and Wilhelmson (1978), Raymond and Kuo (1984)) that an adequate approximation is to replace it by a simple extrapolation formula.

The lateral boundary conditions used are, for the $x = 0$ boundary;

(i) On u, equation (29) and (30) or equation (29) with $\hat{r}$ from (31) (if $\hat{r} > 0$, $\hat{r}^t = 0$, if $\hat{r} < -1$, $\hat{r}^t = -1$).

(ii) On ϕ $(\phi = \theta^*, q_v, q_c, q_R, v, w)$

if $\bar{u}_0 \geq 0$ $\quad \phi_0^{t+1} = \phi_0^t$

if $\bar{u}_0 < 0$ $\quad \phi_0^{t+1} = 2\phi_1^t - \phi_{i+2}^t$

where $\bar{u}_0$ is u_0 averaged to the point at which ϕ is held.

To calculate $SLU(0)$ we need $U(x=-\Delta x)$ which we extrapolate from U_0^t and U_{0+1}^t.

The corner values of all variables are found by averaging the values found by using either boundary.

The Reynolds stresses are given by

$$S_x\tilde{\tau}_{11} = S_x\tau_{22} = S_x\tau_{33} = S_x k_m = S_x\tau_{23} = 0$$

The pressure equation ((25)) was approximated in a manner consistent with the approximation of the momentum equations and so the boundary conditions must also be consistent. We rewrite equations (18a, b and c) with as the object.

$$\frac{\partial p''}{\partial x}\Big|_{x=0} = \rho\left(SLU_1 - \frac{1}{2\Delta t}\left(U_1^{t+1} - U_1^{t-1}\right)\right)$$

$$\frac{\partial p''}{\partial x}\Big|_{x=L_x} = \rho\left(SLU_{II+1} - \frac{1}{2\Delta t}\left(U_{II+1}^{t+1} - U_{II+1}^{t-1}\right)\right)$$

$$\frac{\partial p''}{\partial y}\Big|_{y=0} = \rho\left(SLV_1 - \frac{1}{2\Delta t}\left(V_1^{t+1} - V_1^{t-1}\right)\right)$$

$$\frac{\partial p''}{\partial y}\Big|_{y=L_y} = \rho\left(SLV_{JJ+1} - \frac{1}{2\Delta t}\left(V_{JJ+1}^{t+1} - V_{JJ+1}^{t-1}\right)\right)$$

$$\left(\frac{\partial p''}{\partial z} + \frac{g}{\delta}\frac{\rho}{\bar{\rho}}p''\right)\Big|_{z=0} = \tilde{\rho}_1\, SLW_1$$

$$\left(\frac{\partial p''}{\partial z} + \frac{g}{\delta}\frac{\rho}{\bar{\rho}}p''\right)\Big|_{z=H} = \hat{\rho}_{KK-1}\, SLW_{KK-1}$$

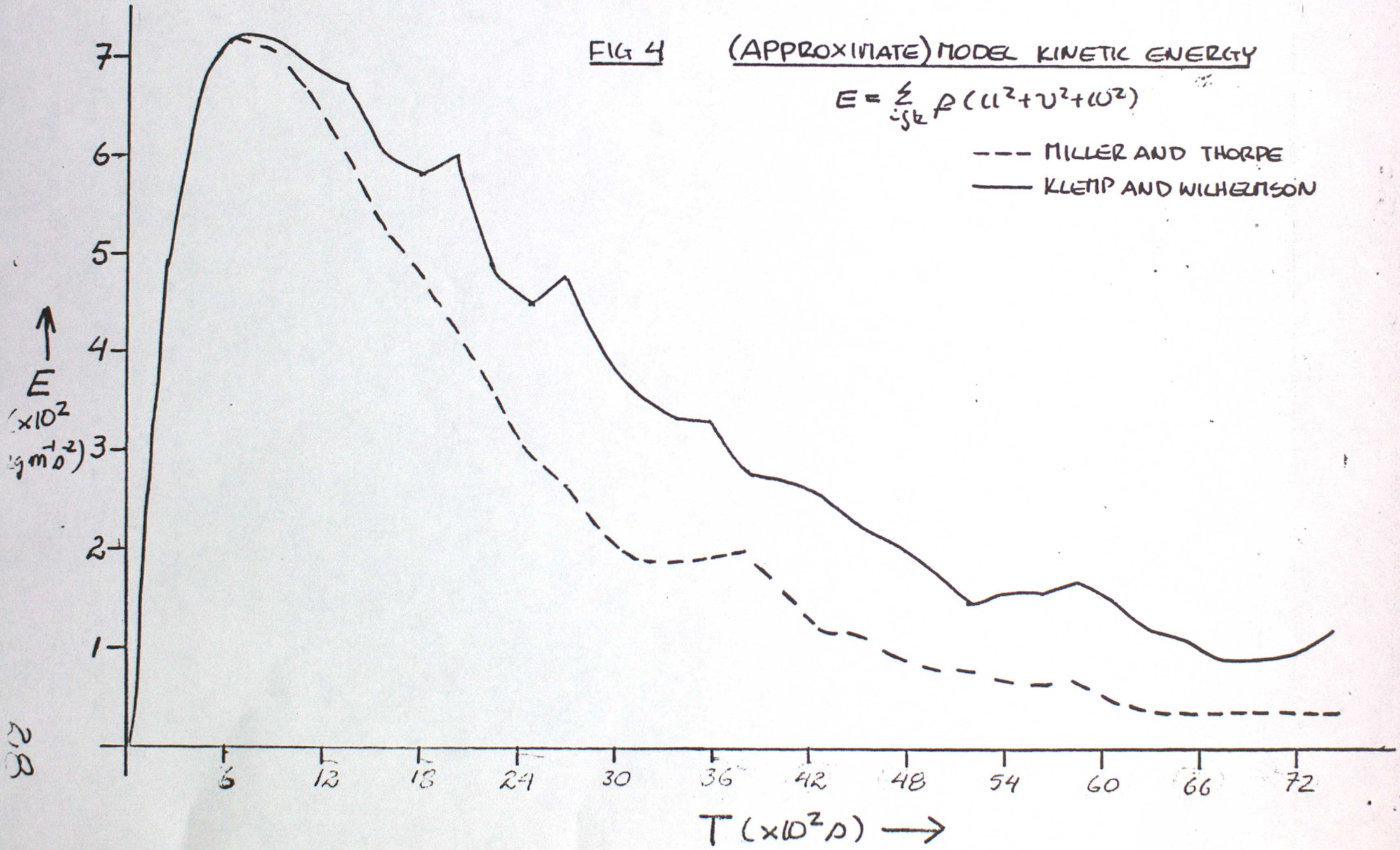
The boundary conditions have been formulated so that $SLW_{KK+1} = 0$ and the only contribution to SLW_1 is through the surface drag, which produces well posed upper and lower boundary conditions (Sundstrom and Elvius (1979)).

FIG 4 (APPROXIMATE) MODEL KINETIC ENERGY

$$E = \sum_{i,j,k} \rho (u^2 + v^2 + w^2)$$

--- MILLER AND THORPE

—— KLEMP AND WILHELMSON



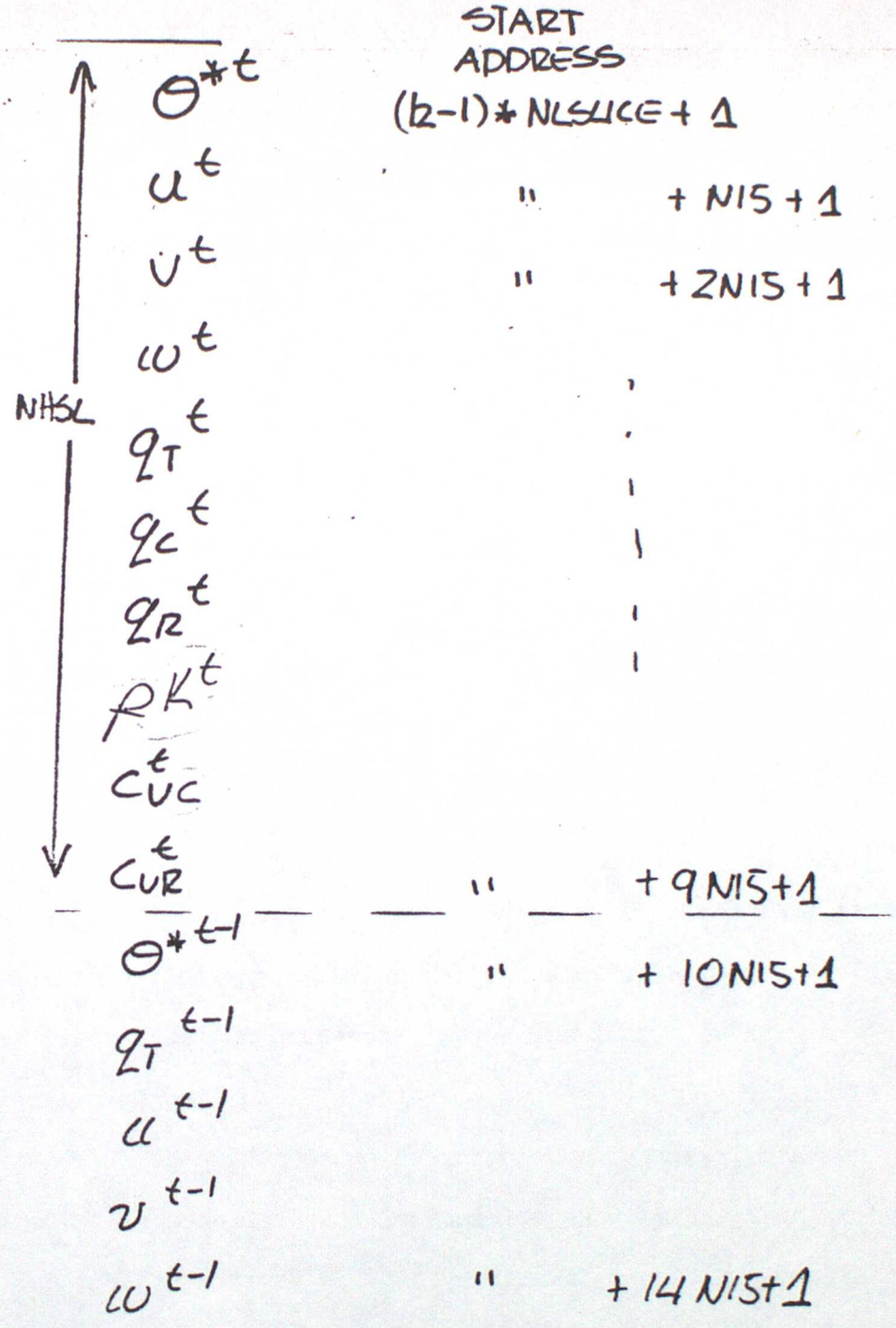
The model has been run in a dry $q_i' = 0$, quasi two dimensional ($L_x >> L_y$, initial field varying in x and z only) form to test the consistency of the boundary conditions and to compare the efficiency of the schemes of Miller and Thorpe (MT, equation (31)) and Klemp and Wilhelmson (KW, (30)). The test problem is similar to Orlanski (1976) with the initial θ profile stable except in a 4 km x 4 km square which is neutral. The collapse of this square generates gravity waves which should pass through the model boundaries without reflection. To isolate the effects of a boundary identical integrations were carried out on a small (32 km) and large (64 km) domain and compared. Qualitatively the results showed that the broad features were the same but the MT small domain integrations were closer in detail to the large than were the KW small. A clearer demonstration of the superiority of MT conditions is gained by plotting

$$E = \sum_{ijk} \rho (u^2 + v^2 + w^2)$$

against time for a MT and a KW small integration. E is a measure of the model kinetic energy. After the initial build up due to the collapse of the neutrally stable region we expect a monotonic decrease as the gravity waves pass out of the model domain. Figure 4 shows that the MT condition is the most effective since the decrease is almost monotonic and faster than the KW condition (which in the non-monotonic parts shows evidence of wave reflection). The superiority of the MT boundary conditions is also evident in 3D integrations but not so marked, possibly because the assumption of one dimensional radiation is less valid.

4. Model Structure

The model has been coded to run on the CYBER 205 with the calculations arranged so that the usual vector length is one horizontal slice ((II+2)*(JJ+2) points). The structure was originally based on the Met O 14 Large Eddy Model (Callen and Dickinson 1985)) but the overall control structure is new and the details have been substantially changed to accommodate the different equation set.



NLSLICE = length of one slice of horizontal fields = 15*N15

NHSL = 10*N15

length of store array = KL*N15 32 bit numbers

FIG 5 STORAGE OF FIELDS IN "STORE"

The field variables are held in a half precision (32 bit) array "STORE" and the calculations are performed in full precision (64 bit). Calculation proceeds in a vertical direction, at each step one horizontal slice of all the fields is expanded to 64 bit and an updated slice is contracted to 32 bit and stored. "STORE" is arranged in KK blocks, each block holding one horizontal slice of all the fields (in the order given in Fig. 5). The model reserves NIJ locations for each horizontal slice of every field of which (II+2)(JJ+2) are used (similarly there is space for NK slices).

To maximize the use of storage we calculate the divergence of the source terms (equation (25)),then solve the pressure equation and then recalculate the source terms for the advection equations. This approach is more costly in time but saves the use of 3 full field arrays as temporary storage for SLU, SLV and SLW. Figure 6 gives a schematic picture of the main stepping routine, ONESTEP, which uses this algorithm.

The model is restartable with a restart dump of one time level of the fields made every IRST steps (the restart is with a forward step). The dumping is done using concurrent I/O (Q7BUFIN/OUT) to minimize the likelihood of the calculations being I/O bound.

A different dump of fields is made for diagnostic purposes every IDIAGD steps. The aim is for the dumps to be transferred to the IBM for processing and storage on tape. Since one of the limiting factors on the size of the model is the amount of tape required to store the diagnostic dumps the dumped fields will have to be carefully chosen and stripped of any superfluous points.

The schematic structure of the dump/restart process is shown in figure 7. Most of this structure is found in the main program ("GECKO") but the input is performed in set up subroutines (SETCNTRL,SETFLDS), the dumps in ONESTEP and the monitoring output (zebra plots, simple field integrals etc) in PRNTDIAG, PRNTPRES and PRNTDAG2.

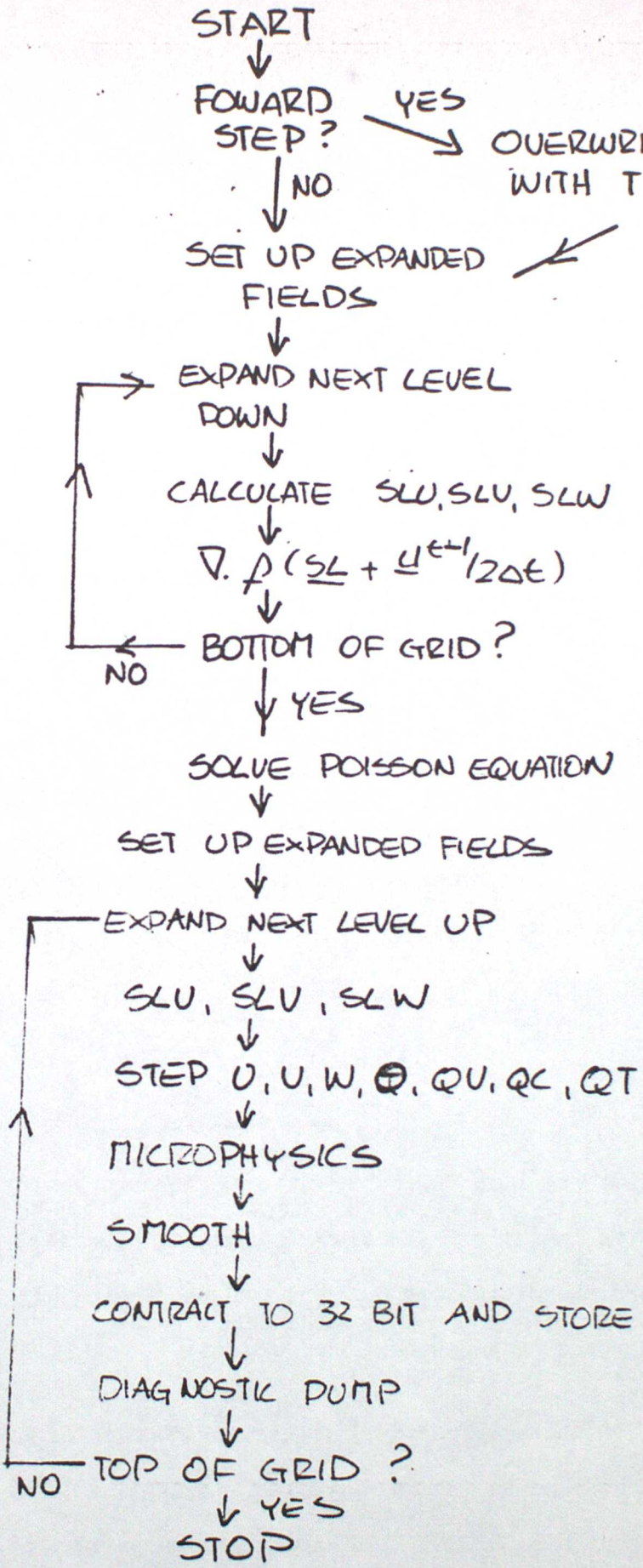


FIG 6 STRUCTURE OF STEPPING SUBROUTINE

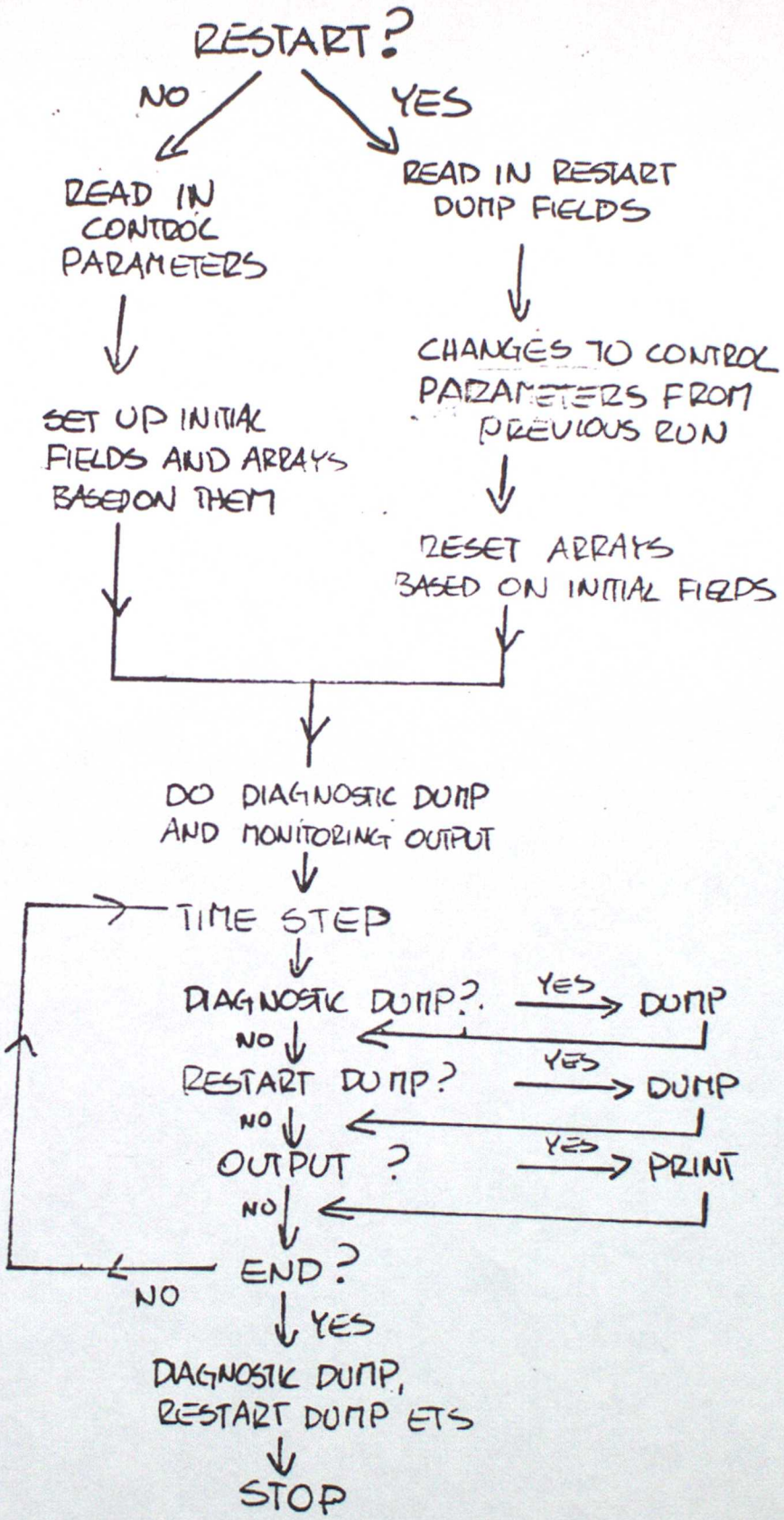


FIG 7 SCHEMATIC START, DUMP ETC STRUCTURE

References

BENNETTS, D A and RAWLINS, F, (1981)

Parametrization of the ice phase in a model of mid latitude cumulonimbus convection and its influence on the simulation of cloud development.

Q.J.R.M.S., 107, 477-501.

BENNETTS, D A and RYDER, P, (1984)

A study of mesoscale convective bands behind cold fronts. Part II: Cloud and microphysical structure.

Q.J.R.M.S., 110, 467-488.

CALLEN, N S and DICKINSON, P, (1985)

3-D Large eddy model. CYBER 205 version.

Met O 14 Internal Note.

CLARK, T L, (1973)

A study in cloud phase parametrization using the Gamma distribution.

J. ATMOS. SCI., 31, 142-155.

CLARK, T L, (1977)

A small scale dynamic model using a terrain-following coordinate transformation.

J. COMP. PHYS, 24, 186-215.

CLARK, T L, (1979)

Numerical simulations with a three-dimensional cloud model: Lateral boundary condition experiments and multicellular severe storm simulations.

J. ATMOS. SCI., 36, 2191-2215.

CLARK, T L and PELTIER, W R (1977)

On the evolution of finite-amplitude mountain waves.

J. ATMOS. SCI., 34, 1715-1730.

COTTON, W R, (1975)

Theoretical Cumulus Dynamics.

Rev. Geophys and Space Phys. 13, 419-448.

COTTON, W R, STEPHENS, M A, NEHRKORN, T and TRIPOLI, G J, (1982)
The Colorado State University three dimensional cloud/mesoscale model - 1982.
Part II: An ice phase parametrization.
J. RECH. ATMOS, 16, 295-320

CUSHMAN-ROISIN, B (1984)
Analytical, linear stability criteria for the Leapfrog, Dufort-Frankel method.
J. COMP. PHYS, 53, 227.

HALTINER, G J and WILLIAMS, R T, (1980)
Numerical prediction and dynamic meteorology. 2nd edition.
J Wiley and Sons Inc (New York).

HARLOW, F H and WELCH, J E, (1965)
Numerical calculation of time dependent viscous incompressible flow of fluid with free surface.
PHYS. FLUIDS, 8, 2182.

HAUF, T, HOLLER, H and SCHUMANN, U (1984)
Three dimensional convective cloud dynamics - a new integration scheme.
PROC 9th CLOUD PHYS. CONF, IAMAP, VOL II, 531-534.

KLEMP, J B and DURRAN, D R (1983)
An upper boundary condition permitting internal gravity wave radiation in numerical mesoscale models.
MON. WEA. REV, 111, 430-444.

KLEMP, J B and WILHELMSON, R B, (1978)
The simulation of three-dimensional convective storm dynamics.
J. ATMOS. SCI., 35, 1070-1096.

LILLY, D K, (1962)
On the numerical simulation of buoyant convection.
TELLUS, 14, 148-172.

LILLY, D K, (1979)

The dynamical structure and evolution of thunderstorms and squall lines.

Ann. Rev. Earth Sci, 7, 117-161.

MILLER, M J and MONCRIEFF, M W, (1983)

The dynamics and simulation of organized deep convection.

in "Mesoscale Meteorology - Theories, Observations and Models", D K LILLY and T GAL-CHEN (EDS), D REIDEL, 451-495.

MILLER, M J and PEARCE, R P (1974)

A three-dimensional primitive equation model of cumulonimbus convection.

Q.J.R.M.S., 100, 133-154.

MILLER, M J and THORPE, A J, (1981)

Radiation conditions for the lateral boundaries of limited-area numerical models.

Q.J.R.M.S., 107, 615-628.

ORLANSKI, I (1976)

A simple boundary condition for unbounded hyperbolic flows.

J. COMP. PHYS. 21, 251-269.

OGURA, Y and PHILLIPS, N A, (1962)

Scale analysis of deep and shallow convection in the atmosphere.

J. ATMOS. SCI., 19, 173-179

PASTUSHKOV, R S, (1975)

The effects of vertical shear on the evolution of convective clouds.

Q.J.R.M.S., 101, 281-291.

PIACSEK, S A, and WILLIAMS, G P (1970)

Conservation properties of convection difference schemes.

J. COMP. PHYS., 6, 392-405.

PIELKE, R A (1984)

Mesoscale meteorological modelling.

Academic Press, (Orlando, Fla)

RAYMOND, W H and KUO, H L, (1984)

A radiation boundary condition for multi-dimensional flows.

Q.J.R.M.S., 110, 535-553.

SCHLESINGER, R E, (1978)

A three dimensional numerical model of an isolated thunderstorm: Part I.

Comparative experiments for variable ambient wind shear.

J. ATMOS. SCI., 35, 690-713.

SMAGORINSKY, J, (1963)

General circulation experiments with the primitive equations:

I. The basic experiment.

MON. WEA. REV., 91, 99-164.

SUN, W-Y (1984)

Numerical analysis for hydrostatic and monohydrostatic equations of inertial-internal gravity waves

MONTH. WEA. REV., 112, 259-268.

SUNDSTROM, A and ELVIUS, T, (1979)

Computational problems related to limited-area modelling.

in "Numerical methods used in atmospheric models", GARP PUB. SERIES No. 17, ICSU/WMO.

TRIPOLI, G J and COTTON, W R, (1982)

The Colorado State University three-dimensional cloud/mesoscale model - 1982.

Part I: General theoretical framework and sensitivity experiments.

J. RECH. ATMOS., 16, 186-219.

VAN DEN BERGHE, C S, (1985a)

A three-dimensional cumulonimbus model: Warm microphysics.

Met O 15. Internal Report, in prep.

VAN DEN BERGHE, C S (1985b)

A direct Poisson solver with Neumann boundary conditions on a staggered grid.

Met O 15 Internal Report, in prep.

WILHELMSON, R B and OGURA, Y, (1972)

The pressure perturbation and the numerical modelling of a cloud.

J. ATMOS. SCI., 35, 1974-1986.