LONDON, METEOROLOGICAL OFFICE.

Met.O.19 Branch Memorandum No. 2.

Thickness retrievals using various sets of weighting functions and values of instrumental noise. By HUNT, R.D.

London, Met.Off., Met.O.19 Branch Mem.No.2, [1972], 31cm. Pp.7, pls.4.2 Refs.

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THICKNESS RETRIEVALS USING VARIOUS SETS OF WEIGHTING FUNCTIONS AND VALUES OF INSTRUMENTAL NOISE

by R D Hunt

INTRODUCTION

In order to retrieve temperature profiles of the atmosphere from satellite-borne radiometer measurements of radiances, detailed knowledge is required of the relationship between the measured radiation intensity and the vertical profile of the black-body intensity. Sets of normalised weighting functions describing this relationship need to be accurately determined. The accuracy of the inversion procedure is also dependant partly on the noise of the instrument. The effect of the accuracy of some retrievals of altering a) the weighting functions and b) the instrumental noise are described in this study.

PART 1 - Thickness retrievals using three sets of weighting functions

When the Tiros N satellite is launched, it will measure radiances at eleven frequencies within the 15 μm CO_2 band, the three channels which measure radiation emitted from the highest parts of the atmosphere being of particular importance to the High Atmosphere branch. In order to assess the effect on temperature retrievals of using various weighting functions for these three channels, three different sets of weighting functions were calculated in the form of L x M matrices, L being the total number of channels of the radiometer and M being the number of levels used to describe the temperature profile. In this case L = 11 and M = 50. In each set, the weighting functions for the lower eight channels (ie those channels measuring radiation emitted from the lower part of the atmosphere) were the same, but there were substantial differences between the weighting functions for the top three channels.

Large numbers of retrievals were then carried out with each set of weighting functions in turn using the 'maximum probability' method of inversion briefly described below. Comparisons were made between the standard of accuracy of retrievals for each set.

The three sets of weighting functions used are shown in figure 1, the weighting functions for the lower eight channels being based on those used for the American SIRS instruments. For the top three functions, set A was chosen to have the highest peak at about 50 km, near to the average stratopause height, while the weighting function for the second channel was taken to be the same as that for the top channel of the SCR instrument on Nimbus 4. Set B was chosen so that the peaks of the three functions were close together between about 30 and 45 km, while set C had the highest peak occurring at about 60 km (well above the usual height of the stratopause), with the second and third peaks spaced out evenly between this and the fourth peak. The high value of the top weighting function at level 50 is due to radiation emitted between this level and the satellite. It is difficult to give an exact figure for this value and only a rough estimate could be made, however the top peak was chosen to be about as high as possible without making this value unacceptably large.

Data used

Altogether, a total of 479 temperature profiles consisting of temperatures at 50 pressure levels were retrieved in this study. The profiles were a collection of interpolated rocket - and radio-sonde soundings as described in the High Atmosphere branch internal note HA/IG/1 (1) and were spread fairly well mainly over the northern hemisphere although with some in the southern hemisphere. The profiles were divided into seven groups according to latitude and season thus :-

tropical (0° - 30°)	64 profiles
30° - 50° summer	26 "
30° - 50° winter	25 "
50° - 70° summer	45 "
50° - 70° winter	207 "
70° - 90° summer	31 "
70° - 90° winter	25 "

There was one further group consisting of 56 profiles measured at West Gerinish during a stratospheric warming in December 1967.

Calculations were performed with each temperature profile to deduce the amount of radiation reaching the top of the atmosphere at each of the eleven frequenci at which the radiometer measures, and these values were used as 'measured' radiances for use in the inversion process. Deduced temperatures could then be compared directly with the original temperatures.

Retrieval method

The retrievals were performed using the 'maximum probability' method as described by Rodgers (2) for example, where details of the techniques are discussed. Basically the solution of the inversion equation found is that one of the infinite number which exist which is most probable and to find it necessitates solving the following equation :-

$$B - \overline{B} = CW^T \, (WCW^T + E)^{-1} \, (I - \overline{I}) \quad \ldots\ldots\ldots\ldots \quad (1)$$

If X (matrix of dimensions 50 x 1) is the temperature profile being retrieved, then

B(50 x 1) is a matrix of Planck functions at 50 levels for X, to be determined and which, once known, can be converted into a temperature profile,

$\overline{B}$ (50 x 1) is a mean Planck function profile for the particular latitude-season group to which X belongs,

C (50 x 50) is the covariance matrix of Planck functions for the sample,

W (11 x 50) is the weighting function matrix,

W^T (50 x 11) is the transpose of W,

E (11 x 11) is the covariance matrix of instrumental noise assessed to be diagonal, with values for the elements (91.00, 22.09, 10.00, 3.00. 0.30, 0.09, 0.06, 0.01, 0.01, 0.01, 0.01) based on figures for various instruments and, for the highest channels, much larger than the expected Tiros N values,

I (11 x 1) is a matrix of the 'measured' radiances from X, including the addition of a random noise term found by adding a number from a table of random numbers with mean zero and with standard deviation equal to the square root of the particular value in E and $\overline{I}$ (11 x 1) is the mean set of 'measured' radiances for the sample to which X belongs.

The derivation of the above equation can be found in the Rodgers' paper already mentioned. Computer programmes were written to find $\overline{B}$, $\overline{I}$ and C and then to solve the above equation for B. Each of the 479 profiles were retrieved in turn.

It should be noted that the profiles used in calculating $\overline{B}$, $\overline{T}$ and C were also those retrieved ie the retrieved profiles were not independant samples. Apart from finding temperature profiles and comparing them with the actual profiles, six standard thicknesses (1000 - 300 mb, 300 - 100 mb, 100 - 30 mb, 30 - 10 mb and 3 - 1 mb) were also calculated and compared with the same layer thicknesses from the original profiles. For each sample, the mean differences and the root-mean-square differences between the calculated and actual temperatures at each level, and the root-mean-square differences between the calculated and actual thicknesses were found.

Results

An example of the mean differences between calculated and actual temperatures at the 50 levels is shown in table 1. These figures are for the sample of 70° - 90° winter cases retrieved using sets A and C weighting functions. The main feature of these figures is the oscillatory form they take, with fairly large positive numbers around level 40 with set A for instance (ie a tendency for the retrieval procedure to give temperatures higher than actual) and a band of negative numbers near to the centre of the profile. The table for set C has a similar oscillatory form but with the positions of the positive and negative mean differences reversed. This implies that the shape of the weighting function controls the positions of the oscillations.

However, it is the results shown in table 2 which are more relevant to this report, showing the root-mean-square-errors (RMSEs) for the standard retrievals with the three sets of weighting functions. As may be expected, there are only very small differences between the three sets for the lower thicknesses - 1000 - 300 mb, 300 - 100 mb, 100 - 30 mb and, to a slightly lesser extent, 30 - 10 mb. (The large difference between set A and sets B and C for the 1000 - 300 mb thickness with the 30°- 50° winter sample has not been explained.) Differences of up to 2.65 decametres exist between the RMSEs of the highest two thicknesses, the actual amount seemingly depending on the relative positions of the oscillation peaks in the mean error for the temperature retrievals already described.

For instance, the figure in table 2 for the 3 - 1mb thickness RMSE using the Set B weighting functions with the 50° - 70° summer sample is much higher than for sets A and C. The 3 - 1 mb layer covers a region of quite large positive mean differences between derived and actual temperatures with set B, but with sets A and C, it covers regions with much smaller mean differences, negative in the case of set C and positive in set B. Other samples, especially the tropical group, show little to choose between the three sets even with the 3 - 1 mb thickness.

In general, it would seem that set A appears to give the best results using these six standard thicknesses as criteria , although there is only a quite small difference between them.

It should be pointed out that a situation which gives rise to smaller RMSEs for thickness retrievals is the case where the layer in question covers a region where the sign of the mean temperature difference changes, hence giving a cancelling-out effect for the thickness calculation. This situation does not arise frequently in the higher parts of the atmosphere where the oscillations in the mean temperature error profile have larger wavelength than in the lower part, (this apparently being due to the fact that the higher weighting functions are more spread out than the lower ones). It may be that there are other thicknesses which could be retrieved more successfully than those used in this study bearing in mind this effect, each set of weighting functions perhaps having its own optimum set of retrievable thicknesses although the total improvement would probably only be small.

PART 2 - Thickness retrievals using various values of instrumental noise

As described earlier, some values for instrumental noise need to be given for the matrix E in (1), and also to find the 'random noise' to add to the 'measured' radiances. To assess the effect on the thickness retrievals of altering the instrumental noise, some of the profiles were retrived using several different versions of E with the set A weighting functions.

Each version of E was diagonal, and the noise for the lower eight channels was held constant at 0.5 erg units for the highest and 0.15 erg units for the remainder. The six different values for the noise of the top three channels were as shown in table 3. Of these six sets, the first (set 1) was chosen to be very large in order to discover whether there was an upper limit for the RMSEs of the derived thicknesses and whether it coincided with the standard deviations of the actual thicknesses. The second version of E, set 2, was the same as that used in part 1, set 3 was chosen to have noise values approximately the same as the proposed values for the Tiros N instrument and the fourth version, set 4, was taken to be approximately half as noisy as set 3. Set 5 had no noise in the highest three channels. For the purpose of comparison, the programmes were also run with E being a zero matrix ie no instrumental noise in any of the eleven channels.

The summer 50° - 70° sample and the sample of profiles measured during the stratospheric warming at WestGerinish were retrieved using each version of E in turn, the latter sample being retrieved using values of $\overline{B}$, $\overline{T}$ and C in (1) based on the winter 50° - 70° sample, ie an independent sample.

Results

Table 3 shows the RMSE of the retrieved thicknesses with each set of instrumental noise. With both profile samples, it can be seen that altering the noise of the highest channels has little effect on the accuracy of retrieval of the 1000-300mb and 300-100mb thickness but generally makes a substantial difference to the remaining four standard thickness retievals. Of interest is the fact that the figures for set 1 are mostly only a little higher than those for set 2, implying that there may be a limit to the effect that increasing noise has on the retrieved thicknesses. The value of 8.75 decametres for the RMSE of the 3-1mb retrieved thicknesses with set 1 for the summer 50°-70° sample compares with a value of 25.10 decametres for the standard deviation of the actual 3-1mb thicknesses for the same sample.

Generally speaking, the smaller the noise the smaller the RMSEs, but the results for the case with no noise in any channels are significantly worse than for the case with noise in only the lowest eight channels. In fact, the RMSEs of the retrieved 1000-300mb thicknesses with no noise at all are up to twice as large as with any other set. This tends to highlight the ill-conditioned state of the matrix to be inverted in (1), $(WCW^T + E)$. Adding a small number to the diagonal terms of the matrix (which is the effect of having a non-zero E) makes it less ill-conditioned and, hence, better suited for inversion.

It can be seen from table 3 that the improvement in the RMSE of the 3-1mb thickness retrieval caused by reducing the noise in the top three channels by about half from somewhere near the expected Tiros N values is quite large - 16% with the summer 50°-70° sample and 33% with the stratospheric warming sample. The improvements in the 10-3mb thickness retrievals are slightly less - 12% and 29% respectively for the two samples.

References

1. Barwell, B R and Hoskin, G C. Vertical temperature profiles of the troposphere and stratosphere on punched cards. Met O 19 internal report HA/IG/1, 1972.

2. Rodgers, C D. Remote sounding of the atmospheric temperature profile in the presence of cloud. University of Oxford, Clarendon Laboratory Mem. 69. 3,1969.

Level	Mean difference °K set A	Mean difference °K set C	Level	Mean difference °K set A	Mean difference °K set C
1	0.05	0.04	26	-0.36	0.47
2	-0.11	-0.10	27	-0.29	0.48
3	-0.22	-0.19	28	-0.19	0.49
4	0.07	0.04	29	-0.21	0.32
5	0.27	0.25	30	-0.17	0.22
6	0.25	0.28	31	-0.04	-0.10
7	0.19	0.29	32	-0.02	-0.57
8	0.08	0.05	33	0.33	-0.77
9	0.03	-0.14	34	0.65	-0.99
10	-0.02	-0.18	35	0.96	-1.21
11	-0.07	-0.19	36	1.33	-1.40
12	-0.04	-0.13	37	1.87	-1.48
13	0.00	-0.07	38	2.40	-1.53
14	0.01	-0.07	39	2.79	-1.66
15	0.14	0.03	40	3.00	-1.34
16	0.26	0.12	41	3.07	-1.11
17	0.28	0.21	42	3.05	-0.89
18	0.27	0.28	43	2.74	-0.86
19	0.25	0.34	44	2.36	-0.81
20	0.26	0.46	45	2.00	-0.70
21	0.16	0.48	46	1.59	-0.63
22	0.07	0.53	47	1.13	-0.62
23	-0.03	0.57	48	0.77	-0.44
24	-0.30	0.45	49	0.24	-0.46
25	-0.35	0.44	50	-0.27	-0.72

Table 1 Mean difference between retrieved and actual temperatures at the 50 atmospheric levels. Sample : 70° - 90° (winter)

	layers (in pressure heights)	root-mean-square-error of thickness retrievals (in decametres) Set A	Set B	Set C
TROPICAL (0°-30°) 64 profiles	1000-300mb	0.65	0.64	0.64
	300-100	2.49	2.50	2.48
	100-30	3.90	3.82	3.87
	30-10	4.57	4.63	4.56
	10-3	7.39	7.40	7.34
	3-1	8.16	7.95	7.97
50°-50° SUMMER 26 profiles	1000-300	0.97	0.97	0.97
	300-100	2.99	3.04	3.05
	100-30	2.58	2.85	2.64
	30-10	5.50	5.53	5.61
	10-3	6.13	7.58	6.33
	3-1	10.57	12.34	10.37
30°-50° WINTER 25 profiles	1000-300	0.87	1.54	1.55
	300-100	4.04	4.33	4.37
	100-30	4.87	5.32	5.07
	30-10	7.33	7.27	7.45
	10-3	10.57	11.90	11.16
	3-1	13.22	14.73	13.97
50°-70° SUMMER 45 profiles	1000-300	0.92	0.93	0.95
	300-100	2.70	2.82	2.69
	100-30	3.74	4.22	3.71
	30-10	5.31	5.54	5.22
	10-3	10.01	10.99	10.25
	3.1	8.94	11.59	9.05
50°-70° WINTER 207 profiles	1000-300	1.00	1.00	1.00
	300-100	2.81	2.85	2.81
	100-30	5.17	5.20	5.11
	30-10	9.28	9.56	9.19
	10-3	16.31	16.52	16.07
	3-1	18.65	19.50	18.84
70°-90° SUMMER 31 profiles	1000-300	0.77	0.78	0.77
	300-100	2.02	1.99	2.03
	100-30	2.50	2.69	2.62
	30-10	4.58	4.61	4.65
	10-3	7.62	8.53	8.60
	3-1	10.90	12.33	11.20
70°-90° WINTER 25 profiles	1000-300	0.78	0.77	0.79
	300-100	3.56	3.49	3.62
	100-30	4.29	4.20	4.03
	30-10	10.25	10.03	10.56
	10-3	15.81	15.36	15.23
	3-1	20.46	21.53	21.44
STRATOSPHERIC WARMING DEC 1967 56 profiles	1000-300	0.76	0.76	0.80
	300-100	2.63	3.19	2.75
	100-30	5.26	7.15	6.13
	30-10	9.74	12.29	10.63
	10-3	20.23	22.57	24.27
	3-1	14.97	21.49	18.26

Table 2 Root-mean-square-errors of thickness retrievals with the three sets of weighting functions.

NOISE IN HIGHEST THREE CHANNELS (ERG UNITS)	ROOT-MEAN-SQUARE-ERROR OF DERIVED THICKNESSES (DECAMETRES) SUMMER 50°-70° SAMPLE 1000-300 mb	300-100 mb	100-30 mb	30-10 mb	10-3 mb	3-1 mb
31.6) 18.0) set 1 12.0)	0.93	1.93	3.03	4.54	7.89	8.75
9.5) 4.4) set 2 3.0)	0.91	1.66	2.90	4.54	7.65	8.11
0.5) 0.5) set 3 0.5	0.89	1.70	2.20	3.58	4.86	5.10
0.3) 0.2) set 4 0.2)	0.89	1.72	2.03	3.39	4.30	4.27
0.0) 0.0) set 5 0.0)	0.91	1.72	1.85	2.79	3.65	2.14
No noise in any channels	1.87	3.93	2.57	2.55	3.84	3.76

STRATOSPHERIC WARMING OVER WEST GERINISH SAMPLE

	1000-300 mb	300-100 mb	100-30 mb	30-10 mb	10-3 mb	3-1 mb
31.6) 18.0) set 1 12.0	1.04	1.75	6.27	11.82	20.04	24.53
9.5) 4.4) set 2 3.0)	1.02	1.68	5.66	10.15	18.15	20.26
0.5) 0.5) set 3 0.5)	1.11	1.86	3.89	6.62	14.69	12.22
0.3) 0.2) set 4 0.2)	1.06	1.68	3.13	6.55	10.40	8.21
0.0) 0.0) set 5 0.0)	1.07	1.67	2.79	6.30	9.56	6.31
No Noise in any channels	1.72	3.10	2.67	3.42	13.45	10.47

Table 3 Root-mean-square-errors of retrieved thicknesses with different values of instrumental noise.

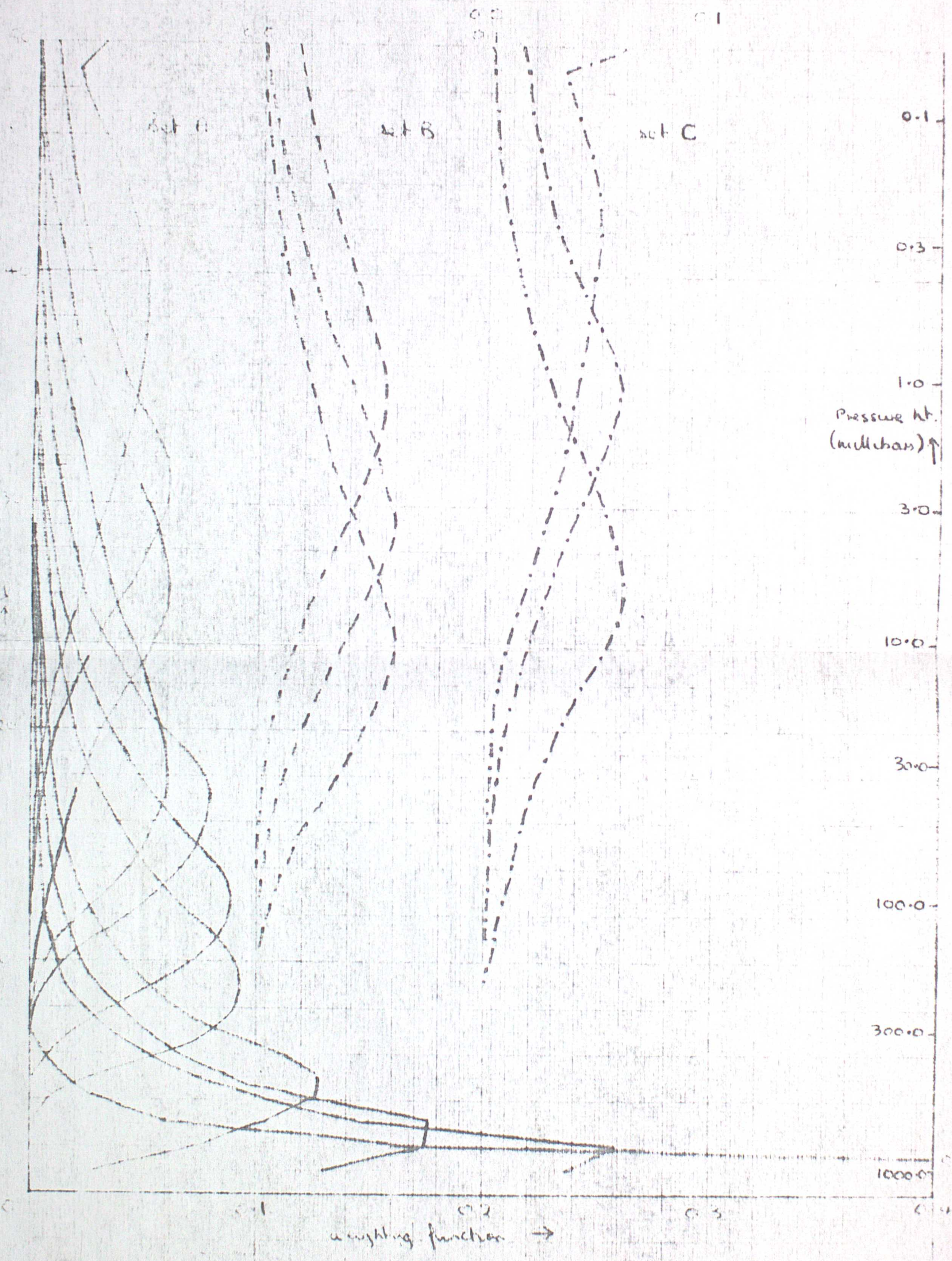
set A
set B
set C
0.1
0.3
1.0
Pressure ht. (millibars)
3.0
10.0
30.0
100.0
300.0
1000.0
0
0.1
0.2
0.3
0.4
weighting function →

Figure (1)