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Second Report arising from Charney and Eliassen's method of computing forecast 500 mb. contour charts

by

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1. Introduction

In M.R.P. 566 an account was given of a test made of the Charney and Eliassen (1) one-dimensional method of computing a forecast 500 mb. contour profile along a parallel of latitude. The method consists basically of assuming that the flow in a barotropic atmosphere would be equivalent to the flow in the actual baroclinic atmosphere at the 500 mb. level and that this flow can be considered as consisting of small perturbations superimposed upon a zonal west-east current constant with respect to time and longitude. There are some theoretical refinements to the basic assumption, but these have little practical significance. As such, the method ignores "development" in the sense used by Sutcliffe (2) and may therefore seem fundamentally inadequate even as a first step, but Charney's analysis purports to show that in spite of baroclinic development there is one level in the atmosphere at which there is "equivalent barotropic flow".

A test was made of the method in latitudes 45° North and 60° North, and the agreement between prediction and actuality seemed promising - better than could be expected if the method was fundamentally unsound. That was the conclusion of the first test and justified further work which is reported here.

2. The second test

Basic coefficients were calculated, so that Charney and Eliassen's formula could be used in latitudes 40°, 50°, and 60° North. These coefficients are given in the appendix. They permitted the computation of forecast contour profiles for 24 hours ahead and such profiles were computed for 34 days in June and July, 1950 from longitude 80° W to 20° E. Forecast charts were prepared by combining the profiles for latitudes 40°, 50° and 60° North. In estimating the zonal current, a 20° latitude band was used. In M.R.P. 566 it was stated that in certain synoptic situations the computed forecast value for the contour height would necessarily be unreliable, because some of the basic assumptions were unfulfilled. In drawing the forecast charts in the present experiment, the computed value was treated with circumspection if it occurred in a region dominated by one of those synoptic situations.

In order to test the success and value of the Charney and Eliassen method of forecasting the 500 mb. zonal profile, four different methods were adopted. These were as follows:-

(1) The forecast change of 500 mb. height was correlated with the actual change of the 500 mb. height and regression coefficients were calculated for the best fitting straight line giving the actual change (Y feet) in terms of the forecast change (X feet) in the form

$$Y = aX + b \qquad (1)$$

/ (2)

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(2) The average error (without regard to sign) and the root mean square error in the forecast 500 mb. heights were calculated.

(3) The forecast movements of the trough and ridge lines at 50° North were correlated with the actual movement and regression coefficients were calculated for the best fitting straight line giving the actual movement (N degrees of longitude) in terms of the forecast movement (M degrees of longitude) in the form

$$N = cM + d \qquad (2)$$

(4) The average error and the root mean square error in the forecast movement of trough and ridge lines at 50° North were calculated.

In order to provide a basis for the comparison of the success of Charney's forecasting method with that of other methods, the comparisons (1) (2) (3) and (4) were made in respect of some or all of the following methods of forecasting.

Forecast A Computation, using Charney's formula.

Forecast B From forecast charts, primarily based on the computed values obtained by forecast A.

Forecast C Movement forward of actual 500 mb. heights at the speed of the zonal current (constant with respect to longitude from 120° West to 40° East).

Forecast D From forecast charts prepared by conventional methods in the Forecast Research Division at Dunstable.

Forecast E Assuming no change.

The results of this statistical analysis are set out in Tables I - IV in which the various forecasts are identified by the letters given above.

Table I

Correlation of forecast change of 500 mb. height with actual change

r = correlation coefficient

a and b (in feet) = regression coefficients of equation (1)

Forecast compared with actuality	40° North			50° North			60° North		
	r	a	b	r	a	b	r	a	b
Forecast A	.35	.46	28	.54	.72	33	.24	.38	15
Forecast B	.28	.36	22	.52	.73	4	.19	.30	7
Forecast C	.34	.28	12	.55	.48	10	.27	.28	4
Forecast D				.70	.89	-16			

/ Table II

Table II

Average errors and root mean square errors (in feet) of the forecast 500 mb. height

	40° North		50° North		60° North	
	Average	Root Mean Square	Average	Root Mean Square	Average	Root Mean Square
Errors of forecast A	110	140	150	200	160	210
Errors of forecast B	110	140	150	200	150	200
Errors of forecast C	130	190	210	270	210	270
Errors of forecast D			130	170		
Errors of forecast E	120	150	190	250	160	230

TABLE III

Correlation of forecast movement of trough and ridge lines at 50° North with their actual movement.

r = correlation coefficient

c and d (in degrees of longitude) = regression coefficients of equation (2)

Forecast compared with actuality	r	c	d
Forecast A	.49	.54	2.9
Forecast B	.64	.79	1.3
Forecast C	.18	.61	0
Forecast D	.60	.69	2.3

Table IV

Average errors and root mean square errors (in degrees of longitude) of the forecast movement of troughs and ridges at 50° North.

	Average	Root mean square
Errors of forecast A	3.4	4.1
Errors of forecast B	2.5	3.2
Errors of forecast C	5.0	5.9
Errors of forecast D	3.1	3.6
Errors of forecast E	7.8	8.8

3. Interpretation of Results

Charney's formula gives a better result in latitude 50° North than in latitudes 40° and 60° North. This is mainly due to the zonal current in the latitude band about 50° North being stronger than in the latitude bands about 40° and 60° North. As Charney's formula is based on small perturbations on the zonal current, one would expect it to give better results when the zonal current is strong than when it is weak, for when the zonal current is strong the motion in the N - S direction will, in general, be relatively small.

Tables II and IV clearly show that forecasts A and B were better than assuming no change had occurred, but were generally not as good as the conventional methods of forecasting. However, forecast B predicts the movement of the trough and ridge lines more successfully than forecasts A and D. This is the only case in which forecast A or forecast B is not less successful than forecast D, and in which forecast B improved upon forecast A.

The change in the 500 mb. height forecast by Charney's method consists of two essentially independent parts; the first representing a displacement of the existing profile eastward with the zonal current and the second representing the contribution of various terms in which the variation of the Coriolis parameter with latitude plays an important part. The question arises as to whether the second set of terms do contribute significantly to the success of the method. In order to investigate this, corresponding forecasts were made on the basis of the first term only - viz. on the assumption that the 500 mb. profile moves eastward with the zonal current. If Tables II and IV are examined it will be seen that the Charney method does achieve greater success than a forecast which simply moves the 500 mb. profile eastward with the zonal current. This applies both to the forecast height of the 500 mb. surface and to the position of trough and ridge lines. On the other hand it will be seen from Table I that the correlation coefficients between the forecast and actual height changes are no less for the forecasts based on the zonal current alone than for those based on Charney's method. This implies that an empirical linear formula (the regression equation) involving the heights obtained from the zonal current method will give no greater a mean square error in the forecast 500 mb. height than any similar formula using the Charney forecasts. This fact might have some relevance if it was proposed to adopt either method in practice, but the regression equations themselves have no physical interpretation and it would be incorrect to argue from these correlation coefficients that moving the actual 500 mb. profile eastward with the zonal current gives as good a solution as the Charney formula.

4. Conclusions and proposed further work

These tests corroborate the conclusions reached in the first report. Although the method is based on treating the 500 mb. flow as if it consisted of small perturbations on a uniform zonal current, the method is more successful than simply moving the actual 500 mb. contour pattern at the speed of the zonal current. The method shows significant success, but the standard is rather less than that achieved by conventional methods. It is therefore not proposed to continue with the direct testing of the Charney method at present.

A report will shortly be available on a mathematical extension of the analysis to include some terms omitted by Charney and related with baroclinic development. It appears at present that it can be shown that these terms are just as important as those included by Charney, so that Charney's method is not, in general, a legitimate first approximation.

The object of introducing computational methods into synoptic forecasting must, however, be kept in mind, and a rather different approach is now being explored.

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5. References

1. Charney J.G. and Eliassen A. A numerical method for predicting the perturbations of the middle latitude westerlies. Tellus, Vol. 1, No. 2, 1949, p.38.

2. Sutcliffe R.C. A contribution to the problem of development. Q.J.R. Met. Soc. Vol. 73, 1947, p.370.

APPENDIX

Charney and Eliassen's one dimensional formula can be written

$$z(x+Ut,t) = z(x,0) + \int_{x-x_1}^{x+x_2} z(\alpha,0)\, I_{a^2}(x-\alpha,t)\, d\alpha \qquad (1)$$

$$\text{where} \quad I_{a^2}(x,t) = \frac{1}{2\pi}\sum_{-\infty}^{\infty}\left(\exp\frac{inbt}{n^2+a^2} - 1\right)\exp inx \qquad (2)$$

Values of $I_{18}(x,1)$ for latitudes 40°, 50° and 60° North have been interpolated from values given by Charney in Journal of Meteorology Vol. 6 No. 6 and are tabulated in Table I. Parameter b = 3.8, 5.9 and 8.2 for latitudes 40°, 50° and 60° North respectively.

Table I

x	40° North		50° North		60° North	
	$I_{18}(x,1)$	$I_{18}(-x,1)$	$I_{18}(x,1)$	$I_{18}(-x,1)$	$I_{18}(x,1)$	$I_{18}(-x,1)$
0	-4.82	2.13	-3.77	1.83	-2.61	1.36
10	-1.75	1.39	-1.27	1.08	-0.78	0.73
20	-0.55	0.90	-0.50	0.66	-0.42	0.42
30	-0.16	0.55	-0.17	0.38	-0.14	0.23
40	-0.03	0.34	-0.06	0.22	-0.06	0.14
50	-0.01	0.20	-0.02	0.13	-0.01	0.07
60	0.00	0.12	0.00	0.07	0.00	0.03
70	0.00	0.08	0.00	0.04	0.00	0.02
80	0.00	0.04	0.00	0.02	0.00	0.00
90	0.00	0.03	0.00	0.01	0.00	0.00
100	0.00	0.01	0.00	0.00	0.00	0.00
110	0.00	0.01	0.00	0.00	0.00	0.00
120	0.00	0.00	0.00	0.00	0.00	0.00
130	0.00	0.00	0.00	0.00	0.00	0.00
140	0.00	0.00	0.00	0.00	0.00	0.00
150	0.00	0.00	0.00	0.00	0.00	0.00
160	0.00	0.00	0.00	0.00	0.00	0.00
170	0.00	0.00	0.00	0.00	0.00	0.00
180	0.00	0.00	0.00	0.00	0.00	0.00

/ Now

Now $\int_{x-x_1}^{x+x_2} z(\alpha,0)\ I_{18}(x-\alpha,1)\ d\alpha$ can be evaluated by Simpson's rule, giving $\sum_{-x_2/10}^{x_1/10} A_N\ z(x+10N,0)$. For latitudes 40° and 50° North the limits of integration were $x_2 = 100°$, $x_1 = 40°$, but it was found that A_{-4}, A_8, A_9 and A_{10} were negligible in each case. For latitude 60° North, the limits of integration were $x_2 = 80°$, $x_1 = 40°$, and it was found that A_{-4}, A_6, A_7, and A_8 were negligible. Coefficients A_N are given in Table II.

Table II

N	A_N		
	40° North	50° North	60° North
-3	-.036	-.035	-.031
-2	-.068	-.056	-.047
-1	-.393	-.285	-.175
0	-.151	-.109	-.071
1	.312	.243	.164
2	.101	.074	.047
3	.124	.085	.052
4	.038	.025	.016
5	.045	.029	.016
6	.013	.008	-
7	.009	.009	-