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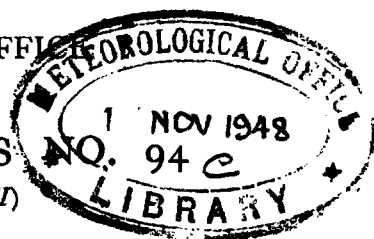
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VISIBILITY AND THE OPTICAL PROPERTIES OF THE ATMOSPHERE

By G. C. SIMPSON, K.C.B., F.R.S.



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VISIBILITY AND THE OPTICAL PROPERTIES OF THE ATMOSPHERE

By G. C. SIMPSON, K.C.B., F.R.S.

The measurement of visibility was introduced into meteorology in order to give practical information required by aviation. The aviator in planning his flights wishes to know not only the cloud and wind he is likely to meet, but also how far he can see, and he demanded that the weather reports should include this information. In order to meet this demand the meteorologist had no alternative at that time but to observe how far he could see. This was simplified by the fact that all the easily distinguishable features of a landscape appear and disappear more or less at the same distance, and he took as his definition of visibility the greatest distance at which the features of the landscape could be distinguished as such.

For practical reasons, both of observing and reporting (by code figures), the meteorologist chose a number of distances, and limited himself to observing and reporting whether features of the landscape at these distances could or could not be distinguished. The "features" which are to be distinguished have to be such that they are clearly visible in a clear atmosphere and under normal daylight illumination. Whether an object can be distinguished or not is a matter of whether its outlines can be seen sufficiently clearly against the background for one to be sure of what one is seeing, thus if the object is a tree the observer must be able to say that it is a tree and not a house.

The chosen distances were those of greatest practical importance. If one can see about 20 miles it does not matter much whether one can see 19 or 21 miles ; but if one can see only 2 miles it is not sufficient to be told one can see 1 mile or 3 miles. Thus the nearer objects should be closer together than distant ones. It was also necessary that the distances should be nice round numbers, and after international discussion the following distances, each of which is roughly twice the previous distance, were chosen :

Distance of visibility object	Code figure
0	0
50 m.	1
200 m.	2
500 m.	3
1 Km.	4
2 Km.	5
4 Km.	6
10 Km.	7
20 Km.	8
50 Km.	9

So far everything was empirical and practical and there was no reference to the optical properties of the atmosphere.

VISIBILITY IN DAYLIGHT

There have been a large number of papers published dealing with the physics of visibility in daylight. A bibliography of these, fairly complete up to 1939, will be found in the second edition of Middleton's book "Visibility in meteorology".* This book also contains a satisfactory summary of all the more important work on the subject, and should be consulted for details of the work discussed in this memorandum.

The theory of daylight visibility worked out by Koschmieder is the one which is now generally accepted, and is the only one which can be dealt with here.

Koschmieder's theory.—Koschmieder deals with the simplest form of visibility object, namely a perfectly black screen seen against the sky on the horizon.

The theory is very simple.

The cone of atmosphere extending from the eye of the observer to the contour of the screen is traversed by light from the sun, the sky and the ground. This light is scattered by the air molecules (Rayleigh scattering) and by the suspended matter. Some of the scattered light reaches the eye of the observer and appears to come from the screen so giving it an apparent surface brightness. This brightness depends on the amount of scattering between the eye and the screen and therefore on the distance of the screen; thus the surface brightness increases as the screen recedes from the observer. This surface brightness must obviously be less than the brightness of the sky adjacent to the screen, for the latter is made up of the light scattered by a cone practically similar to that in front of the screen, together with the light scattered in the extension of the cone to the limit of the atmosphere in the direction of the horizon.

Let H_1 and H be the apparent surface brightness of the sky and screen respectively, then as the screen recedes H_1 will remain constant and H will increase; the contrast between them is defined as $(H_1 - H)/H$ and it is a physiological law that the eye ceases to recognise this contrast when it falls below a small value ϵ , called the threshold of contrast. At this point the screen becomes indistinguishable from the sky and the visibility range is reached. It was Koschmieder's problem to relate this range with the physical characteristics of the atmosphere.

Factors and assumptions used by Koschmieder.—*Absorption and scattering.*—When a narrow pencil of light traverses the atmosphere its intensity decreases through scattering and absorption. Scattering is produced by material particles which in themselves are transparent but have a refractive index greater than that of free space. In the atmosphere these are the molecules of the air, the suspended water drops and a certain proportion of the nuclei. Nuclei may be divided into two classes: (a) hygroscopic nuclei and (b) dust. The former are, at all temperatures, in the form of solutions in water and therefore are transparent droplets which scatter the light without absorbing it.

Absorption is confined entirely to the non-hygroscopic nuclei. These are usually particles of soot and mineral matter, both of which absorb all or the

* MIDDLETON, W. E. K.; Visibility in meteorology. The theory and practice of the measurement of the visual range. 2nd edition. Toronto, 1941.

greater part of the light which falls upon them. There can be no doubt that in the atmosphere, except under the most highly polluted conditions such as occur in the neighbourhood of industrial regions or forest fires, the number of the non-hygroscopic nuclei is insignificant compared with the number of hygroscopic nuclei. Hence in unpolluted air the attenuation of a beam of light is entirely due to scattering and the absorption may be neglected.

This makes the solution of Koschmieder's problem relatively easy, for the production of scattered light is equal to the elimination of light from the direct ray. In other words the attenuation coefficient and the scatter coefficient are the same.

The attenuation or scatter coefficient, σ , is defined by the relationship

$$E = E_0 e^{-\sigma x}$$

in which E is the flux-density in a beam of light at a distance x which has the flux-density E_0 at $x = 0$.

It is very important to realise that the numerical value of σ depends on the unit of length used, and care must be taken that if distances are measured in feet the value of σ appropriate to feet is used.

Uniform atmosphere.—In order to make his equations tractable Koschmieder assumes that σ is constant throughout the cones of air he is investigating. This means that the physical condition of the air is uniform, at least along the line of sight to the horizon in the direction of the target. In the neighbourhood of towns this is often far from being the case, and therefore great care should be exercised if Koschmieder's results are to be applied to observations made in industrial regions. It is also clear for similar reasons that visibility observations made on the ground cannot be applied directly to visibility from aeroplanes, even of objects on the ground, for σ varies very much in the vertical direction.

Uniform illumination.—Koschmieder's analysis assumes that every element of the two cones of air receives the same illumination. This is true in two cases only: (a) with a clear sky so that the sun shines everywhere, and (b) with a completely overcast sky. As light also enters the cones from the ground, a uniform surface is also necessary; thus there should be no large expanses of water between the observer and the object, and if there is snow it should not be in patches. With a uniform surface it does not matter whether the surface is of land or water, of light or dark earth, or even of snow—uniformity is the criterion.

Threshold of contrast, ϵ .—The threshold of contrast varies from observer to observer and depends amongst other things on the general intensity of the light. As it is not practical to allow for these variations, ϵ is assumed to be constant and to have a numerical value of 0.02 (as ϵ is a ratio it is independent of the units of length). Experience has shown that in ordinary daylight conditions and with observers of normal eyesight the errors due to assuming ϵ to be 0.02 do not exceed those due to other unavoidable errors of observation.

Koschmieder's results.—With these assumptions and limitations Koschmieder's analysis shows that the visibility range V is given by

$$V = \frac{1}{\sigma} \log_e \frac{1}{\epsilon}$$

If the kilometre is taken as the unit for V and σ , and ϵ is taken to be 0.02 this reduces to

$$V = \frac{3.91}{\sigma} \text{ Km.}$$

Koschmieder also showed that this relationship is independent of the illumination so long as it is uniform in the sense described above. Also that it is independent of the position of the sun, so that the range of visibility of a black screen against the sky at the horizon is the same when looking towards the sun as in the direction away from the sun, although in the former case the brightness of the sky is much greater than in the latter case. He confirmed these results by actual experiments with a large black screen seen against the sky.

Thus if all visibility objects were black screens rising above the horizon the attenuation (scatter) coefficient of the atmosphere could be determined from the visibility observations. Visibility objects are, however, seldom black screens seen against the horizon and we must now examine the errors due to this fact.

Application of Koschmieder's results to ordinary visibility objects.—A discussion of the visibility of objects other than a black screen seen against the sky will be found in Middleton's book and a very useful summary has been given by Wright in Appendix I to his paper on "Atmospheric opacity".* The results derived in these discussions only will be given here, the analysis and data on which they are based must be consulted in the works mentioned.

Objects seen against the sky but not black.—If the object is not black some light is added to the scattered light in the cone between it and the observer; thus the object would become invisible at a shorter distance than if it were black. So long as the albedo of the object is not more than 0.25 no error larger than 3 per cent. is caused if the sky is overcast; but if the sun is shining on the object the error may be much larger. It is therefore desirable to choose objects as black as possible; a white house on the top of a cliff would be quite unsuitable as a visibility object, but a weathered building or a mass of dark trees would be satisfactory except when brightly illuminated by the sun.

Objects seen against a terrestrial background.—We have to differentiate between two cases: (a) the object and the background both at the same distance from the observer and (b) the background at some considerable distance beyond the object. In the former case even a black object seen against a background having an albedo of 0.25 (i.e. more than the albedo of most terrestrial backgrounds) would produce an error of 50 per cent. with an overcast sky and more than 70 per cent. if the sun were behind the object. Thus a tree on the edge of a wood would not be a suitable object. In the second case the object is not seen against the real background, but against the scattered light between the object and the distant background. If the background is not closer to the object than one-third the distance of the object from the observer the error would not be more than 10 per cent. assuming that the object is reasonably dark. As the objects used as visibility marks usually stand well in front of their background, only small errors are introduced.

Coloured objects.—As objects near the limit of visibility lose their colour, appearing grey, it does not matter what colour the object may be—it is not

* WRIGHT, H. L.; Atmospheric opacity: a study of visibility observations in the British Isles. *Quart. J. R. met. Soc., London*, 65, 1939, p. 411.

the colour, but whether the object appears light or dark which is the decisive factor.

Wright sums up his discussion of visibility objects in the following words "We may conclude that although the observed visibility is always less than the standard visibility, the differences for the kind of objects used at most meteorological stations are only likely to be serious in the somewhat special circumstances of an object viewed in bright sunshine when the solar bearing is near a critical azimuth".

Thus from visibility observations we obtain an approximate value of the attenuation coefficient σ , the accuracy of which is limited more by the fact that only a few objects are used than by the departure of the individual objects from the standard of a black screen seen against the sky.

The values of σ corresponding to the standard distances of the visibility objects adopted in international meteorology are shown in the second column of Table I.

TABLE. I

Distance of objects	Attenuation coefficient	Candle power of lights at the same distance as objects	Distance of lights of 100 c.p.
Km.	Km.	c.p.	Km.
0.05	78.2	0.05	0.14
0.2	19.5	0.7	0.39
0.5	7.8	4.3	0.78
1	3.91	17	1.3
2	1.95	70	2.1
4	0.98	184	3.3
10	0.39	1,740	5.9
20	0.19	6,960	8.0
50	0.08	43,500	11.0

VISIBILITY AT NIGHT

Owing to the absence of sunlight it is not possible to use the ordinary visibility objects at night. This difficulty can be overcome by the use of artificial lights, for in a given state of the atmosphere there are definite relationships between the brightness of a distant lamp and the daylight visibility. These relationships can be expressed in two ways:

(a) by the greatest distance at which a light of specified candle power can be seen, and

(b) by the candle power of the lamp which can just be seen at a specified distance.

These relationships will now be investigated.

General principles.—In daylight a distant object becomes invisible when the difference in the apparent brightness of the object and its background bears a certain ratio to the brightness of the background. At night a light becomes invisible for a totally different reason; namely, because the flux-density at the

eye due to the emitted light falls below a definite amount called the threshold of flux-density. This is a much more variable quantity than the ratio in the daylight observations ; it depends amongst other things on :

- (a) the condition of the eye, whether light adapted or dark adapted,
- (b) the brightness of the background, and
- (c) the presence of other lights in the field of view.

Fortzik has adopted 3.5×10^{-7} lux (3.5×10^{-11} lumens cm.⁻²) as the threshold of flux-density, while Middleton considers this to be on the high side and prefers 2×10^{-11} lumens cm.⁻². For practical purposes it is better to choose a high rather than a low value, and Fortzik's value will be used in this memorandum.

In discussing the daylight visibility it was assumed that the attenuation of the light was due to scatter and that there was little or no actual absorption of the energy. The same conditions will hold at night, hence the attenuation coefficient with a given state of the atmosphere will be the same by day and night. This is the link which makes it possible to connect the observations of lights at night with the visibility range in daylight.

Distance at which a specified light can be seen.—The illumination from a point source of light decreases with distance for two reasons :

- (a) the spreading out of the light, and
- (b) scatter.

Thus we have $E = I_0 x^{-2} e^{-\sigma x}$

in which

E = the flux-density

x = the distance of the light

I_0 = the intensity of the light in candle power

σ = the attenuation coefficient.

Let S be the distance in kilometres between the observer and the light when it is only just visible, then $E = 0.35$ lumens Km.⁻² and we have :

$$0.35 = I_0 S^{-2} e^{-\sigma S}$$

i.e.
$$S^2 e^{\sigma S} = \frac{I_0}{0.35}$$

This equation can best be solved by writing it in the form :

$$2 \log S + \sigma S \log e = \log I_0 - \log 0.35$$

and solving graphically.

The values of S for $I_0 = 100$ c.p. are entered in the last column of Table I against each of the daylight visibility ranges.

Candle power of a light which can be seen at a specified distance.—It is possible from the above equation to calculate the candle power of a light at a given distance which would just be visible with any given attenuation coefficient. This, however, has little interest except in the case of lights placed near to the visibility objects, when it would be useful to know what candle power these lights must have to disappear at night in the same state of the atmosphere as the objects disappear in daylight. In this case $S = V$.

As before we have :

$$I_0 = 0.35 S^2 e^{\sigma S}.$$

As $S = V$, $\sigma S = \sigma V = 3.91$,

thence $I_0 = 0.35 e^{3.91} S^2 = 17.5 S^2.$

This means that the light at a distance of 1 Km. would be 17.5 c.p. and all the other lights would increase as the square of the distance. It is interesting to note that if such a series of lights were set up in a perfectly clear atmosphere they would all appear equally bright, each giving at the observing post a flux-density of 17.5 lumens Km.⁻², i.e. 50 times the threshold flux-density; then as the air became hazy they would disappear one by one commencing at the most distant, and the last light visible would always be at the daylight range of visibility. The intensities of the lights for each of the international visibility ranges have been entered in the third column of Table I. It will be noticed that for distances greater than 10 miles the required candle power is too large to be practicable.

An interesting variation of the result in the last paragraph is seen if we consider a parallel beam of light, such as a searchlight beam with no spread. In this case there is no attenuation due to spreading and we have

$$E = E_0 e^{-\sigma x}$$

when $x = V$, $\sigma x = \sigma V = 3.91$

thence $E = E_0 e^{-3.91}$

i.e. $\frac{E}{E_0} = 0.02$

That is every parallel beam of light is reduced to 2 per cent. of its original intensity in a distance equivalent to the visual range in daylight.

Although the theory used in this memorandum for calculating the visibility of lights at night is now generally accepted, there is practically no experimental confirmation of the results. So far as I know no experiments have been published connecting day and night visibilities, and until that has been done there will be no final solution of the problems involved.

PHOTOMETRIC CONVERSION DIAGRAM

Purpose of the diagram.—The diagram has been prepared to represent graphically the relationship between the visibility range, the attenuation coefficient and the transmission coefficient in the atmosphere. It can be used for any of the following purposes :—

(a) To convert any one of the visual range, the attenuation coefficient or the transmission coefficient into either of the other two, e.g. visual range into transmission coefficient, transmission coefficient into attenuation coefficient, etc.

(b) To convert the unit of length from kilometre, mile or 1,000 yards. into either of the other two, e.g. attenuation coefficient expressed in miles into attenuation coefficient expressed in kilometres, etc.

(c) To combine (a) and (b) so that a given factor expressed in one unit may be converted into another factor expressed in a different unit, e.g. visual range in miles into transmission coefficient in kilometres.

Construction of the diagram.—*Attenuation coefficient*, σ .— σ is defined by

$$\frac{dE}{dx} = -E\sigma.$$

From which it follows that $E = E_0 e^{-\sigma x}$.

Let σ_1 and σ_2 be the attenuation coefficients, σ , expressed in units (1) and (2) respectively and x_1 and x_2 the distance x expressed in the same two units, then

$$E = E_0 e^{-\sigma_1 x_1} = E_0 e^{-\sigma_2 x_2}$$

hence

$$\sigma_1 x_1 = \sigma_2 x_2$$

and

$$\sigma_2 = \sigma_1 \frac{x_1}{x_2}. \quad \dots(1)$$

Koschmieder has shown that if V and σ_1 are the daylight visibility range and the attenuation coefficient expressed in kilometre units respectively

$$\sigma_1 = \frac{3.91}{V}. \quad \dots(2)$$

Substituting (2) in (1) we have

$$\sigma_2 = \frac{3.91}{V} \cdot \frac{x_1}{x_2}$$

or generally

$$\sigma_n = \frac{3.91}{V} \cdot \frac{x_1}{x_n}.$$

$$\text{Writing} \quad 3.91 \frac{x_1}{x_n} = K_n,$$

$$\text{we have} \quad \sigma_n = \frac{K_n}{V} \quad \dots(3)$$

$$\text{hence} \quad \log \sigma_n = \log K_n - \log V. \quad \dots(4)$$

The values of K for different units of σ are contained in Table II.

TABLE II—VALUES OF K FOR DIFFERENT UNITS OF σ

Units of σ		Values of $\frac{x_1}{x_n}$	Values of $K_n = 3.91 \frac{x_1}{x_n}$
Kilometres	(1)	$\frac{1}{1} = 1$	$K_1 = 3.91$
Miles	(2)	$\frac{1}{0.621} = 1.61$	$K_2 = 6.29$
1,000 yards	(3)	$\frac{1}{1.094} = 0.914$	$K_3 = 3.58$

Putting the appropriate values of K_n into equation (4) we have the three following equations giving the relationship between the attenuation coefficient, σ , and the daylight visual range, V .

$$\left. \begin{array}{l} \sigma \text{ in kilometres: } \log \sigma_1 = \log 3.91 - \log V \\ \sigma \text{ in miles: } \log \sigma_2 = \log 6.29 - \log V \\ \sigma \text{ in 1,000 yards: } \log \sigma_3 = \log 3.58 - \log V \end{array} \right\} \begin{array}{l} V \text{ in kilometres in all} \\ \text{three equations.} \end{array}$$

When plotted on double log paper these three equations give the three upper straight lines shown on the diagram facing this page.

Transmission coefficient, t .—The transmission coefficient is defined as the proportion of the incident flux transmitted through unit thickness of the absorbing (scattering) medium.

If σ_1 is the attenuation coefficient of the absorbing medium expressed in units (1) we have:

$$\text{Flux absorbed in unit distance} = E - Ee^{-\sigma_1} = E(1 - e^{-\sigma_1}).$$

$$\text{Flux transmitted through unit distance} = E - E(1 - e^{-\sigma_1}) = Ee^{-\sigma_1}.$$

$$\text{Hence} \quad t_1 = \frac{Ee^{-\sigma_1}}{E} = e^{-\sigma_1}. \quad \dots (5)$$

To express t in any other unit it is only necessary to express σ in that unit in equation (5), hence from equations (3) and (5) we have

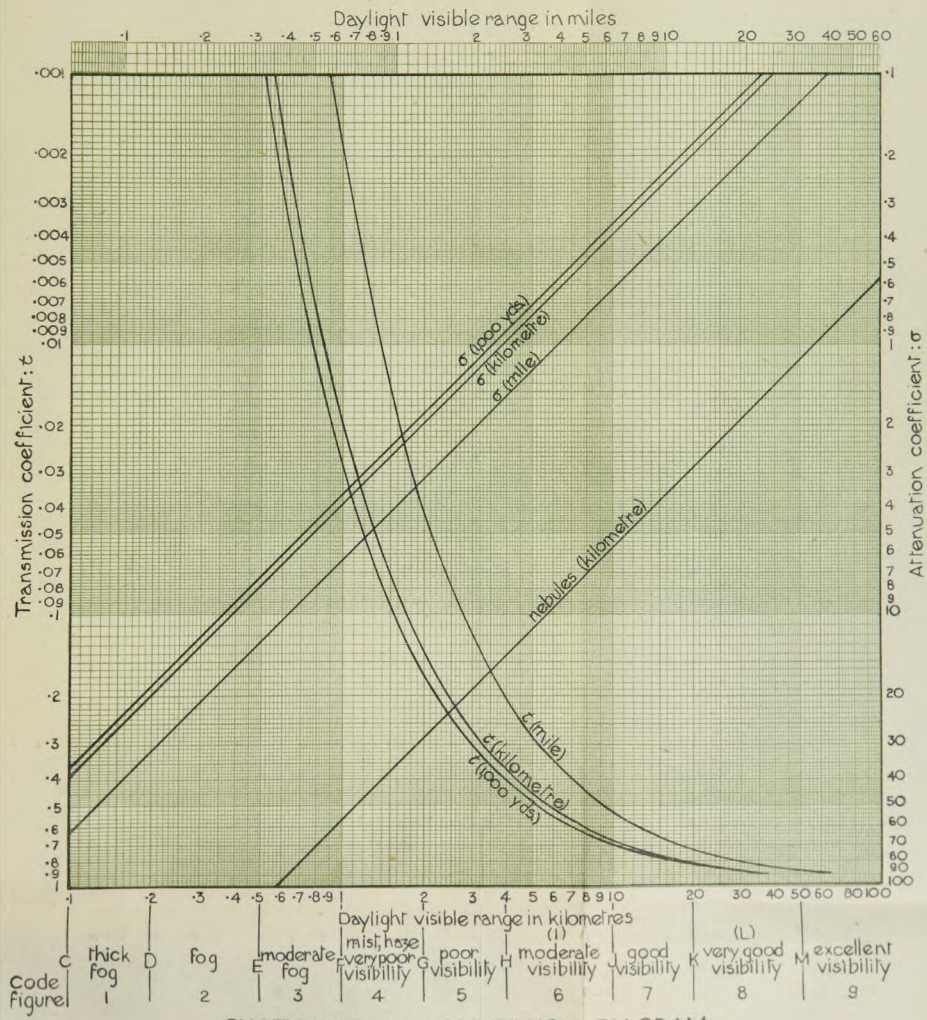
$$t_n = e^{-\sigma_n} = e^{-K_n/V}$$

$$\text{Taking logarithms} \quad \log_e t_n = -\frac{K_n}{V}$$

$$\begin{aligned} 2.30 \log_{10} \frac{1}{t_n} &= \frac{K_n}{V} \\ \log \frac{1}{t_n} &= \frac{K_n}{2.30V} \end{aligned} \quad \dots (6)$$

Putting the appropriate values of K_n from Table II in equation (6) we have the three following equations giving the relationship between the transmission coefficient, t , and the daylight visual range, V .

$$\left. \begin{array}{l} t \text{ in kilometres: } \log \frac{1}{t_1} = \frac{1.70}{V} \\ t \text{ in miles: } \log \frac{1}{t_2} = \frac{2.70}{V} \\ t \text{ in 1,000 yards: } \log \frac{1}{t_3} = \frac{1.56}{V} \end{array} \right\} \begin{array}{l} V \text{ in kilometres in all} \\ \text{three equations.} \end{array}$$



PHOTOMETRIC CONVERSION DIAGRAM

These three equations are represented by the three curved lines on the diagram.

Use of the diagram.—The daylight visibility range, V , is given in kilometres by the scale at the bottom and in miles by the scale at the top of the diagram.

The attenuation coefficient, σ , is given by the scale on the right.

The transmission coefficient, t , is given by the scale on the left.

The optical state of the atmosphere is characterised by a vertical line on the diagram. Where this line meets the scale at the bottom of the diagram we have the visibility range in kilometres and where it meets the scale at the top the corresponding visibility range in miles.

The same vertical line cuts the three parallel straight lines (the σ lines) in three points; the readings on the right-hand scale against each of these points give the values of the attenuation coefficient, σ , in the units entered against the line.

Similarly, the same vertical line cuts the three curved lines (the t curves) in three other points. The reading on the left-hand scale against each of these three points gives the value of the transmission coefficient, t , in the unit entered on the curve.

Thus for the optical state of the atmosphere represented by any vertical line of the diagram we can read off at once :

V (kilometre), V (mile),
 σ (1,000 yards), σ (kilometre), σ (mile),
 t (1,000 yards), t (kilometre), t (mile).

It is clear that conversion from one factor to another or from one unit to another can be effected at once by drawing the vertical line through the given value on its appropriate curve. It is then only necessary to read the scale value against the point where this line cuts the line or curve for the required factor and unit.

ADDENDUM

A line has been added to the diagram to represent nebules per kilometre.* A nebule is defined as a unit of obscuring power of such a magnitude that 100 nebules reduce the flux-density in a parallel beam to 1/1,000th of its original value. The relationship between nebules (kilometre) and σ (kilometre) is

$$n = 14.5 \sigma$$

The value of n for any point on the nebule line is to be read on the right-hand scale.

* See GOLD, E. ; A practical method of determining the visibility number V at night. *Quart. J.R. met. Soc., London*, **65**, 1939, p. 139.

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