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OBJECTIVE ANALYSIS IN THE NUMERICAL WEATHER FORECASTING SYSTEM

by

G. A. BULL

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Objective Analysis in the Numerical

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I. Definition of Objective Analysis

The equations of numerical forecasting permit the calculation of the future values of certain meteorological quantities at points within a rectangular area on the sides of which no change takes place through the period of the forecast. The calculation starts from an initial condition which is a statement of the values of the quantities concerned at all the grid points within and on the boundary of the area. A set of values of a particular quantity, say 500 mb height, at all the grid points is called the "field" of that quantity. The determination of the initial grid point values from the latest set of observations, and certain other data to be described, forms the objective analysis stage of numerical forecasting. It is vital to the success of the subsequent forecast that the initial values should be obtained with as high a degree of approximation to the true values as possible. Appreciable discordance in initial values at only one or two points, especially near the boundary, will lead to early failure of the subsequent forecast procedure. The analysis is a complex system of interpolation between the observations.

It should be mentioned here that before observations are used in the analysis system they have been inspected for hydrostatic consistency in the vertical, correct positions of ships, possible pressures in tropical latitudes etc. as described in Forecasting Techniques Branch Memorandum No. 4 (Ref. 4).

II. Outline of Principles of Analysis

The forecasting equations require a statement of the 1000, 600 and 200 mb heights and the 1000/600 and 600/200 mb thicknesses at all the grid points.

600 mb heights are not reported and the analysis is performed with the 1000, 500 and 200 mb heights. At the beginning of the forecast procedure the 600 mb height field is computed by applying at each grid point the formula:

$$H_{600} = 0.2H_{1000} + 0.8H_{500} \qquad (1)$$

The associated thickness fields are then computed. A justification of the use of the quoted formula is given by Bushby and Whitelam (Ref. 1).

First approximations to the grid point values of the fields are obtained by fitting a quadric (paraboloid) surface at each point of the grid for the height and thickness fields. It is emphasized that a separate quadric has to be determined at every grid point for every field concerned. In fact more than one quadric fitting operation is applied to each field. A further correction termed "plane-fitting" is applied to the 1000 mb field obtained from quadric fitting but not to the upper air fields.

The observations of 500 and 200 mb heights and 1000/500 mb thickness together with the observed winds and thermal wind corresponding to these fields reported by upper air observing stations are first analysed to give fields of these heights and thickness. This is called "upper air analysis".

Next an analysis using the complete set of observations of mean sea level pressure and temperature, and in the case of ships, wind, is made to determine the field of 1000 mb height. This is called the "surface analysis".

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The field of 1000 mb height given by the surface analysis is then compared with the field of that quantity provided by the upper air analysis and corrections are applied to the latter to bring the whole into agreement.

It is not possible to rely solely on the latest observations to obtain a representation of the current field. Over some parts of the area reporting stations are very sparse but the initial grid point values must be specified everywhere. So in addition to using observations from within a given radius, now six grid lengths, of the grid point for which the interpolation is being made a "background field" is taken into account. The background field is a set of grid point values of the quantity being analysed. In the upper air analyses the background field is <u>either</u> the field given by the previous numerical forecast for the time at which the analysis is being made <u>or</u> the climatological mean field. In the surface analysis the background field is the 1000 mb field derived from the upper air analysis. The climatological fields used are those for the nearest to the day of analysis of those for January, April, July, and October.

III. <u>Analysis of Upper Air Observations</u>

1. <u>Basic quadric fitting</u>

To make the description as concrete as possible we will describe the analysis used to produce a field of 500 mb heights. The same basic procedure is used for the analysis of 200 mb height and 1000/500 mb thickness.

If the grid point for which the interpolation is being made is the origin of rectangular Cartesian co-ordinates x, y in the stereographic plane of the chart the equation of the quadric surface specifying the 500 mb surface around the basic grid point is written:

$$H_{500} = ax^2 + by^2 + 2hxy + 2gx + 2fy + c. \qquad (2)$$

x and y are measured in units of grid length.

The coefficients a, b, h, g, f, c are determined by the least squares procedure. An expression E which gives a measure of the square of the difference between observed heights and the heights computed at observation points from formula (2) is minimized. E is a function of a, b, h, g, f, c. It is differentiated partially with respect to each of them to give six linear equations. It is only the value of c which is finally required.

E is defined by the formula:

$$E = \sum^{m} \left\{ p(H_{500} - H_o)^2 \right\} + T^2 \sum^{n} \left\{ p(\underline{V}_{500} - V_o)^2 \right\} + \sum^{l} \left\{ q(H_{500} - H_F)^2 \right\} \qquad (3)$$

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The symbols in this formula are defined as follows:

H_o = height of 500 mb surface in metres reported by an observing station.

H_{500} = height of 500 mb surface in metres given by the quadric expression (2) at the station reporting H_o.

p (not to be confused with pressure) = weighting factor given by the formula

$$p = \frac{1}{1 + 2^{-33} \times 10^8 r^4 \beta^{-4}} \qquad (4)$$

[$2^{-33} \times 10^8$ is entered in the programme as 0.01164115321].

r = distance in grid lengths of the reporting station from the basic grid point for which the interpolation is being made.

β = amplification factor $\frac{2}{1+\sin\varphi}$ from Earth to stereographic plane.

T^2 = a weighting factor of dimensions $[T]^2$ relating the fitting of wind observations and height observations. It has the value 16 seconds^2.

$\underline{V}_o$ = vector wind at 500 mb reported by an observing station; the observed winds are converted into components in units of km/hr by the programme before comparison with the geostrophic wind appropriate to the quadric.

$\underline{V}_{500}$ = vector geostrophic wind at the position of the station reporting $\underline{V}_o$ computed from the quadric surface of equation (2).

It has components $-\frac{\beta g}{fa}\frac{\partial H_{500}}{\partial y}$ and $\frac{\beta g}{fa}\frac{\partial H_{500}}{\partial x}$.

($f = 2\,\omega \sin\varphi$, a = grid length in kilometres at the North Pole). The factor $\frac{\beta g}{fa}$ is abbreviated to $\frac{K}{2}$ for convenience in writing the linear equations for the coefficients. One of these equations is written as a speciment at the end of this section.

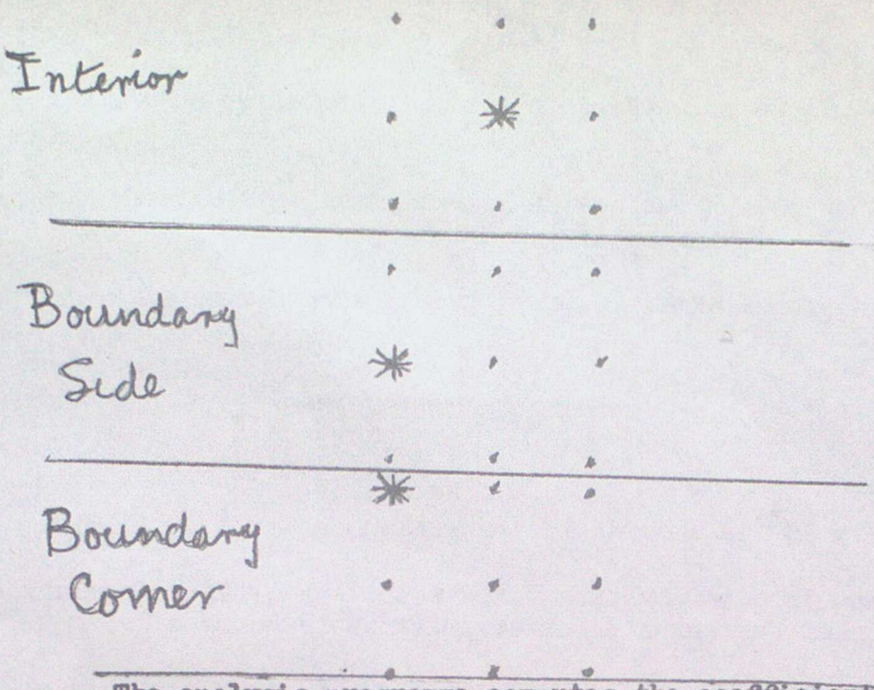
q = weighting factor equal to p/16, so that much less weight is given to the background field than to observations.

H_F = background field specified at grid points as described at the end of Part II.

H_{500} (in third bracket) = height of 500 mb surface given by the quadric (2) at the grid-point where the background field value is H_F.

The values of the weighting factors were determined empirically as described by Bushby and Huckle (Ref. 2).

The summations of the first and second brackets are taken over the nearest upper air observations, up to a maximum of six, within six grid lengths of the basic grid point. Observations outside the boundary of the analysis area but within six grid lengths of grid points in the analysis area or on its boundary are taken into account. The background field has values only at grid points. The method of summation of the third bracket will be clear from the following diagrams in which * is the basic grid point and . those used in the summation in addition to the basic point.



The analysis programme computes the coefficients of the unknowns in the linear equations formed by equating $\frac{\partial E}{\partial a}$, $\frac{\partial E}{\partial b}$, etc. to zero and solves them for the value of c which is the required value of the 500 mb height at the basic grid point concerned. This concludes the description of the basic system of quadric fitting.

The system analyses separately the 500 mb and 200 mb height fields and the 500/1000 mb thickness field. The thickness fitting requires, in principle, observed values of the 500/1000 mb thermal wind. It is clearly not possible to give an actual observed value for this - the 1000 mb level will often be underground - and the value used is obtained at each reporting station from the formula:

$$\underline{V}' = 1.1484375\left(\underline{V}_{500} - \underline{V}_{850}\right) \qquad (5)$$

It would be a useful exercise to form the linear equations for a etc. A sample one is

$$a\left\{\sum^{m} p x^4 + \sum^{\ell} q x^4 + T^2 \sum^{n} p K^2 x^2\right\} + b\left\{\sum^{m} p x^2 y^2 + \sum^{\ell} q x^2 y^2\right\}$$
$$+ h\left\{\sum^{m} 2p x^3 y + \sum^{\ell} 2q x^3 y + T^2 \sum^{m} p K^2 x y\right\}$$
$$+ g\left\{\sum^{m} 2p x^3 + \sum^{\ell} 2q x^3 + T^2 \sum^{m} p K^2 x\right\}$$
$$+ f\left\{\sum^{m} 2p x^2 y + \sum^{\ell} 2q x^2 y\right\} + c\left\{\sum^{m} p x^2 + \sum^{\ell} q x^2\right\}$$
$$- \sum^{m} p H_0 x^2 - \sum^{\ell} q H_F x^2 - T^2 \sum^{n} p K v x = 0$$

2. Comparison of Quadric Fitting with Observations

When a complete set of grid point values has been computed for the three upper air fields by quadric fitting a check is made by interpolating linearly between grid point values to find a value at each reporting station and then comparing this computed value with the observed value.

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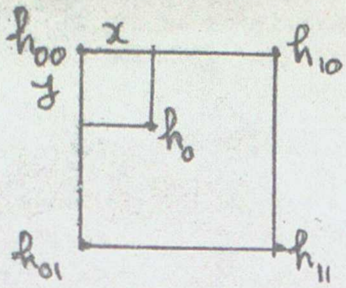
The interpolation formula used will be clear from the following iagram:



$$h_0 = (1-x)(1-y)\,h_{00} + x(1-y)\,h_{10} + (1-x)y\,h_{01} + xy\,h_{11} \quad (6)$$

The reporting station has co-ordinates x, y relative to the top left hand corner of the grid square in which it lies and h_{oo} etc. are the values computed by quadric fitting at the grid points.

Observations which fail to satisfy certain criteria on comparison with the computed value are rejected. These criteria are:

500 mb and thickness	Height within 16 dkm	Each component of the wind vector within 40 kt.
200 mb	Height within 12 dkm	

The rejected observations are printed-out on the line-printer.

The quadric fitting analysis is now repeated without the rejected observations and with a further modification termed "Curvature Correction" described in the next section.

3. Curvature Correction

It was found by Corby, as described in his paper, Ref. 3, that the quadric fitting analysis system described in section 2 systematically underestimates the depth of depressions. He attributed this to the comparison in the second term of E of observed winds with geostrophic winds. The observed wind speed in the upper air is nearer to the gradient wind speed than to the geostrophic wind speed. When contours are strongly curved as in deep depressions the gradient speed is substantially lower than the geostrophic speed. He, therefore, proposed the replacement of the observed wind speed by a new speed obtained by multiplication by a factor to bring it to the geostrophic value. This new value can be more fairly compared with the geostrophic wind appropriate to the quadric surface.

The relation between geostrophic and gradient speeds is easily seen to be:

$$V(\text{geos}) = V(\text{grad})\left(1 + \frac{V(\text{grad})}{Rf}\right) \quad (7)$$

In this formula R is strictly the radius of curvature of the trajectory which in the numerical forecast system is taken to be represented with sufficient accuracy by the contour.

The observed wind speeds V are multiplied by the factor $1 + \frac{V}{Rf}$, where R is the radius of curvature of the contour through the reporting station. The amended winds, which now approximate more closely to geostrophic winds, are substituted in the second term of E and a second quadric analysis is made.

The next few paragraphs describe how the curvature of the contours at reporting stations are computed and the checks made to ensure that absurdly large multiplying factors are not used.

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The radius of curvature R is computed at each grid-point from the field of the first quadric analysis by using the finite difference approximation shown in the following formula and diagram:

$$\begin{array}{ccc} 7 & 2 & 6 \\ 3 & 0 & 1 \\ 8 & 4 & 5 \end{array}$$

$$\frac{1}{R_0} = \frac{1}{l}\,\frac{2\left\{(h_1-h_3)^2(h_2+h_4-2h_0)+(h_2-h_4)^2(h_1+h_3-2h_0)\right\}-(h_1-h_3)(h_2-h_4)(h_8+h_6-h_7-h_5)}{\left\{(h_1-h_3)^2+(h_2-h_4)^2\right\}^{3/2}} \qquad (8)$$

R_0 is the radius of curvature at the grid point labelled 0 in the diagram, l is grid length and h_0, h_1 etc. are values at the labelled grid points.

This finite difference formula is derived from the usual expression,

$$\frac{1}{R} = -\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}.$$

by suitably differentiating the equation, $h = f(x, y)$, of the contour, and writing

$\dfrac{h_1-h_3}{2l}$ for $\left(\dfrac{\partial h}{\partial x}\right)_0$, $\dfrac{h_2-h_4}{2l}$ for $\left(\dfrac{\partial h}{\partial y}\right)_0$,

$\dfrac{h_1+h_3-2h_0}{l^2}$ for $\left(\dfrac{\partial^2 h}{\partial x^2}\right)_0$,

$\dfrac{(h_6-h_7)-(h_5-h_8)}{4l^2}$ for $\left(\dfrac{\partial^2 h}{\partial x \partial y}\right)_0$, and

similarly for the other differential coefficients.

The sign is chosen so as to obtain a positive value for the radius of curvature of cyclonic motion and negative for anticyclonic motion. The programme actually computes the reciprocal of R which is written k in the OC Analysis Programme Outline.

A value of k is determined for every point of the grid by using formula (8). It may happen that the grid point values are such as to give a very large value of k which would give an unrealistically large multiplying factor. To avoid this the value given by the formula is not accepted if $|lk| > 10$, i.e. if the radius of curvature is less than one tenth of the grid length. If the computed value of lk exceeds 10 it is taken as +10 if positive and -10 if negative. It is, however, at reporting stations that the values of k are required. They are found by linear interpolation between the four nearest grid point values in precisely the same way as computed values of height are found at reporting stations as described in section 2 of this Part.

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As a final precaution against the fortuitous occurrence of excessive differences between the observed wind and the "geostrophic" wind computed from it the multiplying factor $1 + \frac{KV\beta}{f}$ is allowed to have values only in the range $\frac{3}{4}$ to $1\frac{3}{4}$, being set to the nearer of these two values if outside the range. The factor ~~B~~ β allows for the variation with latitude of grid length on the Earth.

The last analysis of the upper air observations using these observations alone has now been made and we turn to the next stage, which is the analysis of the 1000 mb height field. This is termed the "surface analysis" in the OC Analysis Programme Outline. When the 1000 mb analysis is complete it is used, as will be seen, to produce a final analysis of the upper air fields.

IV. Quadric Analysis of Surface (1000 mb) Data

1. Basic System

The same basic principle of quadric fitting is applied to produce a first field of 1000 mb height, h_q. This first field is then compared with observations and corrected for curvature as in the upper air analysis. It then undergoes a further stage of amendment termed "plane fitting" which is not used in the upper air analysis.

The first step is to produce a value of 1000 mb height at every station reporting surface observations. The formula used is:

$$h_{1000} = 0.0029259(p - 1000)\, T_s \tag{9}$$

h_{1000} is the height of the 1000mb surface above sea level, $p - 1000$ is the difference between sea level pressure and 1000 in tenths of a millibar and T_s is temperature at the station in degrees absolute.

This formula follows from the hydrostatic relation in the form:

$$(dz)_{cm} = -\frac{2.8703 \times 10^6}{981} \frac{T\,dp}{p}$$

If dp and p are measured in tenths of millibars and p is taken to be 1000 mb then

$$(dz)_m = -\frac{2.8703 \times 10^4}{981} \frac{T_s\,dp}{10^4} \text{ (tenths mb)}$$

$$= -0.0029259\, T_s\, dp \text{ (tenths mb)}$$

The initial quadric fitting stage is done in the same way as for the upper air fields but the quantity E which is minimized is different. The differences are in the background field and in the wind comparison term.

E for the analysis of the 1000 mb field is defined by:

$$E = \sum^{m} \left\{ P(H_{1000} - H_0)^2 \right\} + T^2 \sum^{n} \left\{ \overset{P}{\cancel{q}}(V_{1000} - V_0)^2 \right\} + \sum^{\ell} \left\{ \cancel{p}\, q\, (H_{\cancel{s}\,1000} - H_u)^2 \right\} \tag{10}$$

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In this new E:

H_{1000} = height of 1000 mb surface computed from a quadric expression of the usual form.

$H_{1000} = ax^2 + by^2 + 2hxy + 2gx + 2fy + c$, referred to the grid point for which the interpolation is being made as origin.

H_o = height of 1000 mb surface at a reporting station.

$\underline{V}_o$ = vector wind reported by a ship within ~~ten~~ six grid lengths of the grid point for which the interpolation is being made. Ship observations alone are used.

$\underline{V}_{1000}$ = vector geostrophic wind at the position of a reporting ship computed from the quadric H_{1000}.

H_u = height of the 1000 mb surface computed at each grid point from the 500 mb and 1000/500 mb analyses described in Section III.

The summation (heights) of the first bracket is taken over the nearest stations including ships, up to a maximum of ten within six grid lengths of the basic grid point for which the interpolation is being made. The summation (winds) of the second bracket is taken over those ship observations, if any, which were used in the height summation. The summation of the final bracket is taken over the basic grid point and the eight surrounding ones as for the background field of the upper air analyses. No forecast field or mean field is used.

The values of p, q, and T are identical with those used in the upper air analyses.

A first analysis is made. From this values of 1000 mb height at reporting stations are deduced and compared with those reported. Observations which fail to satisfy the criteria; 1000 mb height within 10 dkm of computed value, each component of wind within 35 kt of computed value, are rejected and a second quadric analysis made without them. The rejected observations are printed out.

The next stage of 1000 mb height analysis is the curvature correction which is done in precisely the same way as for the upper air analysis except that the permitted range for the multiplying factor is enlarged to be from ½ to 2½. This produces a final quadric fitting the 1000 mb heights. The value is termed h_Q for convenience of reference in the next section.

3. <u>Plane-Fitting Correction</u>

The surface of the type of equation (2) used in quadric fitting is a paraboloid. Such a surface is rounded or bowl shaped near its vertex which is, of course the basic grid point. The slope of the paraboloid decreases too rapidly as the vertex is approached. This effect is particularly noticeable in intense depressions. One way of overcoming the difficulty would be to introduce terms involving the square of the height into the quadric expression. That however gives an excessively complex non-linear system of equations for the coefficients.

The curvature-correction system is one step towards the introduction of a more realistic correspondence between the true shape of the isobaric surface and the mathematical expression for it used in the analysis system.

A further stage is the "plane-fitting" correction which is used in the analysis of the 1000 mb surface.

The curvature and plane-fitting corrections were devised by

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Corby to whose paper (Ref. 3) reference should be made for a full discussion of the reasons for using them and the improvements which they give.

Plane-fitting is concerned with finding a plane which nearly represents the isobaric surface in the vicinity of the basic grid point and whose slope gives a geostrophic wind equal to the observed winds in the same regions. Suppose observations h_0 of the height of the isobaric surface and also of wind (components u, v corrected for curvature) are available at a ship at a position (x, y) relative to the basic grid point. Then extrapolation by means of the geostrophic equation gives for the height h at the grid-point the expression:

$$h = h_0 + \frac{f}{g\beta}(uy - vx) \tag{11}$$

x and y are measured on the stereographic plane and the factor β^{-1} is required to transform them into distances on the Earth.

From a number of such observations around a grid-point a weighted mean $h_{p'}$ of such values can be obtained where $h_{p'}$ is given by

$$h_{p'} = \frac{\sum p'\left(h_0 + \frac{f}{g\beta}(uy - vx)\right)}{\sum p'} \tag{12}$$

p' is a weighting factor of similar mathematical form to the quantity p used in quadric fitting but with a larger coefficient of r^4 so that the rate of decrease with distance from the basic grid point is more rapid than for p. The formula for p' is:

$$p' = \frac{1}{1 + 2^{-29} \times 10^8 r^4 \beta^{-4}} \tag{13}$$

[In the programme $2^{-29} \times 10^8$ is entered as 0.186]

The summation of $h_{p'}$ is taken over up to ten ships reporting both wind and pressure within six grid lengths of the basic grid point. The ship reports in question are taken out of those used in quadric fitting. In other words if the ship reports used in quadric fitting include some without both wind and pressure no search is made for others to replace them.

In computing $h_0 + \frac{f}{g\beta}(uy - vx)$

for any ship the full value is used when the direction to the ship from the basic grid-point is perpendicular to the wind vector reported by the ship. If these directions are not perpendicular the quantity in question is reduced by multiplying it by the quantity

$\dfrac{1}{1 + 2r^2\beta^{-2}\cos^2\gamma}$ where γ is the angle between the directions in question.

A value H between $h_{p'}$ and h_Q is now computed where

$$H = \frac{h_Q + K h_{p'}}{1 + K} \tag{14}$$

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K is a weighting factor defined originally by Corby by

$$K = \frac{\sum p'}{(\sum p')_{max} - \sum p'} \qquad (15)$$

Here the sum ($\sum p'$) is taken over the actual number of ship observations used on any given occasion and ($\sum p'$) max

is the maximum value $(\sum p')$ could assume if the maximum number of ship observations allowed for in the programme within a certain distance of the basic grid point. In the present working programme

$((\sum p')_{max} - (\sum p'))$ is set equal to 1.5 on all occasions.

If the difference between h_Q and $h_{p'}$ is large, as may occur in extrapolating across the centre of a depression, it is desirable to reduce the contribution of $h_{p'}$ in computing H. A check on the necessity for such reduction is made by computing the quantity,

$$\frac{150}{|h_Q - h_{p'}|} \text{ metres}^{-1}$$

If this quantity, symbolized by K', is less than unity at the grid point a new, smaller, value, value of $h_{p'}$ is obtained by multiplying the first value of $h_{p'}$ by K'. The new $h_{p'}$ is then used to form a final value of H. It is emphasized that only ships' wind observations are used in plane-fitting. The reported winds are corrected for curvature of the 1000 mb surface but are not otherwise modified.

This concludes the description of the analysis of the 1000 mb field.

V. Adjustment of Upper Air Analysis following Surface Analysis

The final stage is a modification of the fields of 500 mb height and 500/1000 mb thickness in the light of the 1000 mb analysis. The final h_{1000} field is compared with the original h_{1000} field (H_u of formula 10) and the difference,

$$\Delta h = \left[h_{1000}(\text{final}) - h_{1000}(\text{original}) \right]$$

formed at every grid-point. The 500 mb height of the Upper Air Analysis is then modified by adding $\frac{1}{2}\Delta h$

The 1000/500 mb thickness field of Upper Air Analysis is then also modified by adding $\frac{1}{2}\Delta h$. This procedure brings the 1000 mb, 500/1000 mb, and 500 mb fields into agreement. The 200 mb field derived from the Upper Air Analysis is not affected by the 1000 mb analysis and is not modified.

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