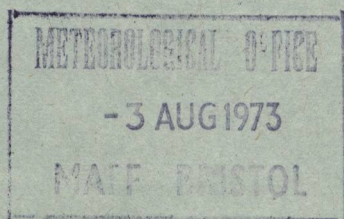


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THE PROBABILITY OF ENCOUNTERING RADAR WEATHER-ECHO NEAR GAN

By J. G. MOORE and R. P. W. LEWIS

Summary. A large number of photographs of the plan position indicator (PPI) display of a 10-cm radar located at Gan have been analysed in a way which provides assessments of the probability of encountering significant weather-echo at heights of 1 km and 5 km along tracks with lengths of from 25 nautical miles to 50 nautical miles; it is found that the probabilities follow a Modified Poisson, or negative binomial, law.

Regular measurements of the tops of weather-echo are used in conjunction with the photographic analysis in order to extend the range of applicability of the results and to investigate connections with routine climatological variables such as rainfall.

Introduction. A radar research project was set up on the island of Gan ($00^{\circ} 41' S$, $73^{\circ} 09' E$) during 1968 for the study of the structure of weather systems over the tropical oceans and the production of statistics of weather-echo; observations were made from August 1968 until July 1969. The observations described in the next section — only a small part of the whole — have been analysed to obtain frequencies of radar-echo occurrence along specified tracks, and these frequencies have then been compared with those derived from a simple Poisson distribution and a Modified Poisson distribution. Chi-square tests showed that the simple Poisson distribution did not give good agreement with the observations whereas the Modified Poisson produced a good fit. The Modified Poisson distribution was then used to derive estimates of the probability of an aircraft encountering significant weather-echo on tracks of specified lengths and heights near Gan. As the presence of radar-echo near the equator is a good indication of severe weather and of turbulence, the results may be of value for planning operations involving supersonic transport aircraft.

Some technical details of the radar used are given in Appendix I, and the formulae and mathematics used in the statistical arguments are collected together in Appendix II.

Data used. Photographs of the plan position indicator (PPI) display were taken at various elevations with a range setting of 60 nautical miles (n. mile) on occasions that were thought by the operator to be 'interesting'. Of these occasions 74 were selected with photographs at elevations of 12° which gave a fairly even coverage of the whole year and of various times of day, not more than one occasion being chosen on any one day. A larger sample of 190 occasions was chosen with photographs at elevations of 0° .

Some photographs taken at an elevation of 2° were also used. Additionally, the heights and positions of the highest echo-tops (up to a maximum of 10) within 60 n. mile were noted at hourly intervals by the observers at Gan during all shifts of a roster system designed to give adequate coverage of all seasons and times of day. The method used was to select likely echoes by visual examination of the PPI display and then to follow them up through the atmosphere by increasing the elevation angle of the aerial until they just disappeared; the heights of the tops were then read off from previously calculated tables which allowed for the curvature of the surface of the earth and the variation of refractive index with height. Observations at a total of 1466 separate hours were used, and these form a much more extensive and representative sample of data than do the PPI photographs. Virtually synchronous measurements of tops were available for all the 74 photographs at 12° elevation, and for 157 out of the 190 photographs at 0° elevation. These observations of echo-tops allowed extension of results obtained from photographs of 'interesting' occasions to the general statistical population of occasions of all types. Use was also made of the routine rainfall measurements.

Methods of analysis and measurement. The PPI photographs taken at an elevation of 12° were projected on a diagram containing 8 circles of circumference 25 n. mile and 4 straight lines of length 25 n. mile in positions corresponding to a height of about 5 km (see Figure 1); the circles were chosen to represent a plausible path for an aeroplane circling an airfield.

The photographs taken at 0° elevation were projected on another diagram (see Figure 2) containing 24 equally spaced radii with tracks of length 25 n. mile and 50 n. mile marked on them; the heights of the 25-n. mile tracks varied between 0.8 and 1.7 km, and of the 50-n. mile tracks between 0.2 and 1.7 km.

For all occasions chosen for analysis, the numbers of echoes intercepted by the various straight and circular tracks were counted. Additionally, results for diametrically opposite 50-n. mile tracks at 0° elevation were combined to give figures for tracks of length 100 n. mile.

Comparison of observations with Poisson and Modified Poisson distributions. The frequency distributions of numbers of echo-encounters made by straight tracks of lengths 25, 50 and 100 n. mile at about 1-km elevation, and by the straight and circular tracks of length 25 n. mile at about 5 km, are shown in Table I, which also shows the expected frequencies derived by fitting both Modified Poisson and simple Poisson distributions. The 'd' parameter for the Modified Poisson distribution is estimated from the sample variance (see Appendix II) and is a measure of the departure of the Modified Poisson from the simple Poisson distribution, being related to the correlation between numbers of encounters occurring simultaneously on two tracks of the same length. Now if the probability of encountering echo were always the same, i.e. if the weather situation and overall amount of convective activity near Gan never varied, one would expect the measured frequency distributions to conform to the simple Poisson form; however, as the overall activity does vary substantially from one day to another, the frequency distribution will be more complicated, and for reasons given in Appendix II will probably conform to the negative binomial, or Modified

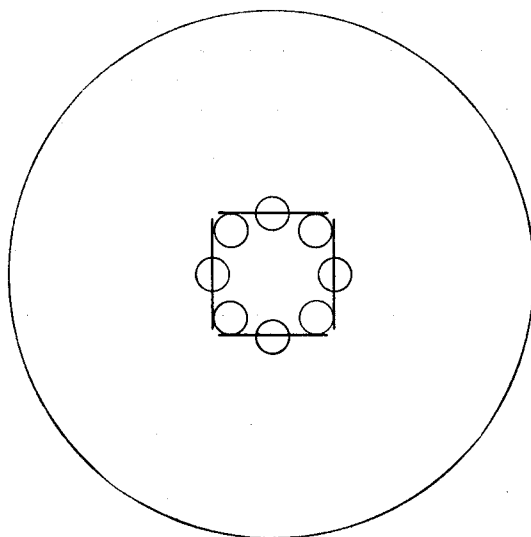


FIGURE 1—DIAGRAM OF TRACKS OF LENGTH 25 NAUTICAL MILES USED FOR MEASUREMENT ON PPI PHOTOGRAPHS AT AERIAL ELEVATION 12 DEGREES
Centres of small circles are 14 n. mile from centre of field; radius of large circle is 60 n. mile.

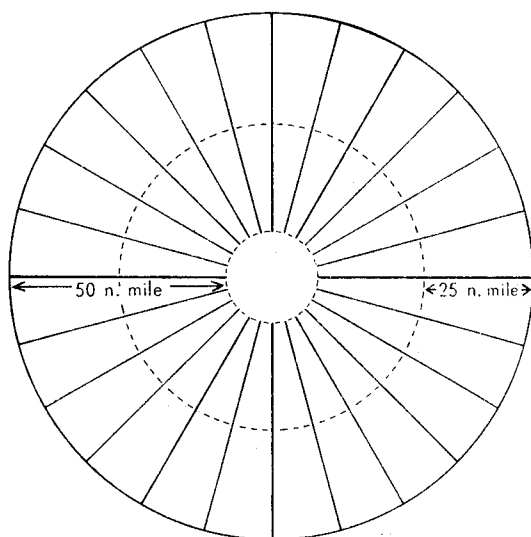


FIGURE 2—DIAGRAM OF TRACKS OF LENGTH 25 NAUTICAL MILES AND 50 NAUTICAL MILES USED FOR MEASUREMENT ON PPI PHOTOGRAPHS AT AERIAL ELEVATION ZERO DEGREES

Poisson, distribution. These theoretical expectations are confirmed by the figures of Table I: it is clear that the fit of observed and expected frequencies for the simple Poisson is bad, but that for the Modified Poisson the fit is reasonable except perhaps for the 100-n. mile tracks. Values of χ^2 for the fit of observed frequencies to those expected from the Modified Poisson distribution are, for the three lengths of track: 0.115 with two degrees of freedom (d.f.), 5.758 with four d.f., and 14.010 with five d.f.; these values of χ^2 have probabilities of being exceeded of 0.944, 0.218, and 0.016 respectively, which implies that the fit between observed and expected frequencies is likely to be as bad or worse on 94.4, 21.8, and 1.6 per cent of occasions and thus that the distribution fits the observed frequencies almost too well for the 25-n. mile tracks, well for the 50-n. mile, but very poorly for the 100-n. mile tracks.

A plot of m (mean number of encounters) and d against track-length for the tracks at about 1-km height (from photographs at 0° elevation) is shown in Figure 3; it is clear that the value of d for 100 n. mile is much smaller than would be expected from the theory of Appendix II, where it is shown

TABLE I(a)—FREQUENCY DISTRIBUTION OF NUMBERS OF ENCOUNTERS WITH RADAR-ECHO ON STRAIGHT TRACKS OF VARIOUS LENGTHS AT HEIGHTS OF ABOUT 1 KILOMETRE

Number of encounters	25-n. mile tracks			50-n. mile tracks			100-n. mile tracks		
	O	P	MP	O	P	MP	O	P	MP
0	3766	3703	3762	3297	3103	3275	1255	1056	1225
1	662	771	666	894	1194	931	572	813	622
2	111	80	109	275	230	254	262	313	264
3	18	6	17	69	29	68	131	80	105
4	3	0	3	22	3	18	41	15	40
5				2	0	5	15	2	15
6				1	0	1	1	0	6
7							3	0	2
8							0	0	1
	$m = 0.208$			$m = 0.385$			$m = 0.770$		
	$\sigma^2 = 0.245$			$\sigma^2 = 0.521$			$\sigma^2 = 1.168$		
	$d = 0.177$			$d = 0.355$			$d = 0.517$		

O is observed frequency; P is frequency from simple Poisson distribution; MP is frequency from Modified Poisson distribution.

m = mean number of echo-encounters; σ^2 = variance of the frequency distribution; d = second parameter of the Modified Poisson distribution.

TABLE I(b)—FREQUENCY DISTRIBUTION OF NUMBERS OF ENCOUNTERS WITH RADAR-ECHO ON STRAIGHT AND CIRCULAR TRACKS OF LENGTH 25 NAUTICAL MILES AT HEIGHTS OF ABOUT 5 KILOMETRES

Number of encounters	frequencies		
	O	P	MP
0	696	668	693
1	144	190	150
2	37	27	34
3	9	3	8
4	2	0	2
	$m = 0.285$		
	$\sigma^2 = 0.375$		
	$d = 0.316$		

Note: For explanation of symbols see foot of Table I(a).

that both m and d are proportional to the length of track; (values of m for 50 and 100 n. mile are constrained to lie on the same straight line through the origin by the method used for constructing the 100-n. mile data). The value of d for 100 n. mile estimated from the observed proportion of zeros is larger (0.63) than that estimated from the variance (0.52); it is still however appreciably less than twice the value of d for 50 n. mile.

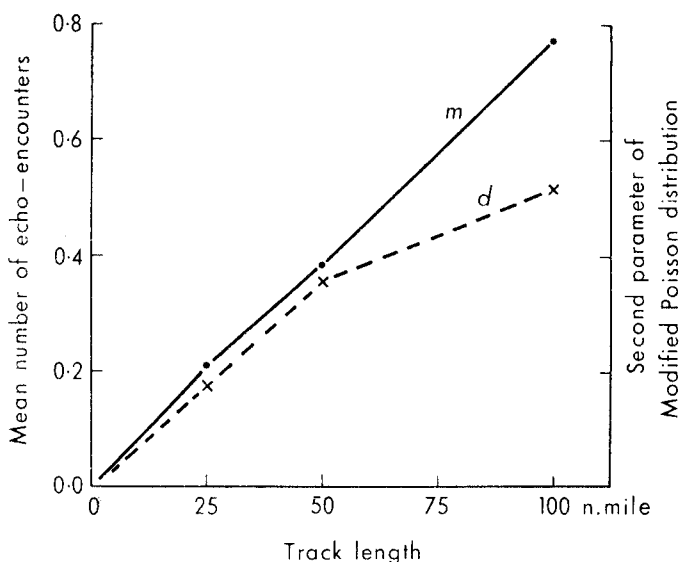


FIGURE 3—VARIATION WITH TRACK-LENGTH OF MEAN NUMBER OF ECHO-ENCOUNTERS (m) AND OF d (THE SECOND PARAMETER OF THE MODIFIED POISSON DISTRIBUTION)

The fact that the value of d for the 100-n. mile track is smaller than expected means that the total variance of the distribution is smaller than it would be if the distribution were of the Modified Poisson form. The total variance is composed of the variance of the parameters of the individual component distribution (i.e. $\text{var } \lambda$, also written σ_{λ}^2) plus the mean variance of the individual spatial distributions (equal to $\bar{\lambda}$ for individual Poisson distributions — see Appendix II), being the mean number of encounters over a large number of similar tracks at any one time. The most likely cause for the relative smallness of the total variance for the 100-n. mile tracks is a reduction of the mean variance of the individual spatial distributions below the Poisson value, and it may be argued that such a reduction is caused by, and is in fact indirect evidence of, negative correlation between the occurrences of radar-echo at distances of the order of the separation of the two halves of the 100-n. mile track, viz. 70 n. mile. (The correlation between the numbers of encounters on the two tracks as deduced from the present data is in fact positive, because these data are derived from a large number of occasions and thus extend over a spectrum of values of λ ; the suggestion is, however, that this correlation is not as large as it would be if the component distributions were simple Poisson.) The presumed existence of such a negative

correlation (the present data are inadequate to demonstrate it directly) is indicative of the mesoscale structure of systems producing weather-echo, and shows that such systems possibly have a characteristic size of from 50 to 100 n. mile, since the separation of the centres of the two component tracks is 70 n. mile. Other studies based on satellite observations¹ have shown that cloud-clusters in tropical regions tend to be of size 100 to 200 n. mile — figures in passable agreement with the present estimate, considering that the latter is indirect and that the former may be overestimates.¹

Relationship of numbers of echo-encounters to numbers of echo-tops. One would expect that a simple relationship — probably linear — would exist between the number of echo-encounters along tracks at a certain height H in the atmosphere and the number of echo-tops extending up to or above that height. The results of the investigation support this expectation as can be seen from Table II which shows for $H = 1$ km and $H = 5$ km the mean number m of echo-encounters for four different classes defined by the number of echo-tops equalling or exceeding H , namely: 1, 2 or 3 tops; 4, 5 or 6 tops; 7, 8 or 9 tops; 10 or more tops.

TABLE II—MEAN NUMBER OF ECHO-ENCOUNTERS (m) ON STRAIGHT TRACKS OF LENGTH 25 NAUTICAL MILES AT HEIGHT H FOR DIFFERENT NUMBERS OF ECHO-TOPS ABOVE H

Number of tops	$H = 1$ km	$H = 5$ km
	mean number of encounters	
≥ 10	0.46	0.44
7, 8, or 9	0.27	0.34
4, 5, or 6	0.14	0.18
1, 2, or 3	0.09	0.10

Extension of results to the general population. The results obtained so far are not necessarily typical of the general 'population' of occasions of encountering weather-echo since they are based on a special sample of 'interesting' cases when echo is likely to be more widespread and dense than the average. If we compare the statistics of frequency of occurrence of different numbers of echo-tops in the special samples and in the complete record — intended and designed to be representative (Table III) we see that at a height of 5 km there are indeed considerable differences, though at 1 km the differences are trivial.

TABLE III—PERCENTAGE FREQUENCIES OF DIFFERENT NUMBERS OF ECHO-TOPS AT HEIGHTS EQUAL TO OR EXCEEDING H IN SPECIAL SAMPLE AND OVER WHOLE YEAR

Number of tops	$H = 1$ km		$H = 5$ km	
	Sample	Whole year	Sample	Whole year
	percentage frequencies			
≥ 10	22	22	38	12
7, 8, or 9	19	19	16	18
4, 5, or 6	32	28	12	27
1, 2, or 3	27	31	34	43

It is possible to estimate values of the parameters of the Modified Poisson distribution appropriate to the general population by using relationships with echo-top data such as those described above.

The value of m (mean number of echo-encounters on a track) appropriate to the population, or m_p , is easily formed by weighting the class-values of m by the relative frequencies of occurrence of the various classes. ('Class' is defined with reference to the number of echo-tops occurring at or above the relevant height.) The class-values of m are taken from the column of Table II appropriate to height $H = 1$ km even when encounters at 5 km are being considered; this is because the two columns of Table II show no evidence of significant variation of class-values with height, and the figures for $H = 1$ km are based on a very much larger sample of data (3768 cases) than those for 5 km (74 cases) and are thus considered to be more reliable.

It is more difficult to estimate the population value of d , or d_p . In the notation of Appendix II, if the total variance of the observations of numbers of echo-encounters in a sample is σ^2 , then

$$\sigma^2 = \lambda^2 + \sigma_{\lambda}^2.$$

σ_{λ}^2 is the variance of λ over all component simple Poisson distributions and may be regarded as the variance of the class-means of λ (or $\sigma_{\lambda c}^2$) plus the mean variance of λ within classes (or $\overline{\Delta^2}$); that is

$$\sigma_{\lambda}^2 = \sigma_{\lambda c}^2 + \overline{\Delta^2}.$$

$$\text{Since } d = \sigma_{\lambda}^2 / \bar{\lambda},$$

$$d\bar{\lambda} = \sigma_{\lambda c}^2 + \overline{\Delta^2}. \quad \dots (1)$$

$\sigma_{\lambda c}^2$ is easily derived either for the special sample or for the general population by use of the appropriate class-frequencies and the class-means of λ . $\overline{\Delta^2}$ must be assumed constant from special sample to general population, and can be estimated by inserting the sample values of $\bar{\lambda}$ (or m), d , and $\sigma_{\lambda c}^2$ in equation (1); d_p is then found by applying equation (1) a second time using population values of $\bar{\lambda}$ (i.e. m_p), $\sigma_{\lambda c}^2$ and the value of $\overline{\Delta^2}$ just derived.

The actual working for the parameters of the Modified Poisson distribution for numbers of echo-encounters on 25-n. mile tracks at 5 km is as follows :

(a) Special sample

Classification by number of tops	Frequency (f_c)	Class-value of m ($= \lambda$)
≥ 10	28	0.46
7, 8, 9	12	0.27
4, 5, 6	9	0.14
1, 2, 3	25	0.09
Total	74	

These figures give values of $m = 0.265$ (to be compared with $m = 0.285$ from the actual counts given in Table I(b) and $\sigma_{\lambda c}^2 = 0.0266$).

Taking the values of m and d appropriate to all straight and circular tracks at 5 km as given in Table I(b), viz. $m = 0.285$ and $d = 0.316$, we find

$$\begin{aligned} \overline{\Delta^2} &= (0.285 \times 0.316) - 0.0266 \\ &= 0.0635. \end{aligned}$$

(b) Whole-year sample

Classification by number of tops	Frequency (f_c)	Class-value of m ($= \lambda$)
≥ 10	100	0.46
7, 8, 9	143	0.27
4, 5, 6	220	0.14
1, 2, 3	346	0.09
0	657	0.00
Total	1466	

These figures give $m_p = 0.100$, $\sigma_{\lambda c}^2 = 0.0164$

hence $d_p = (0.0164 + 0.0635)/(0.100) = 0.80$.

This calculation has assumed that $\overline{\Delta^2}$ is constant over *all* classes, including that of no echo-tops at or above 5 km. It is much more reasonable to assume however that only those classes with a non-zero number of echo-tops have any within-class variability, which means that $\overline{\Delta^2}$ for the general population should be multiplied by the ratio of the total frequency in non-zero classes to the total frequency in all classes or $809/1466$.

This reduces the value of $\overline{\Delta^2}$ from 0.0635 to 0.0350 and hence

$$d_p = (0.0164 + 0.0350)/(0.100) = 0.51.$$

For the calculation of m_p and d_p for encounters on tracks at 1 km, the close similarity of the frequencies of occurrence of different numbers of echo-tops in the special sample and in the general population (see Table III) enables us to apply a different method which does not depend on an attempt to assign a value to $\overline{\Delta^2}$. Assume that the frequency distributions for both special sample and general population are identical except for the frequency of occasions of no echo-encounter, and use equation (9) of Appendix II to show that

$$m + d = m_p + d_p.$$

If m_p is determined from the frequencies of numbers of echo-tops above 1 km given in Table III the value of d_p follows at once.

The working is as follows :

(a) Special sample : $m = 0.208$ $d = 0.177$

(b) Whole-year sample

Classification by number of tops	Frequency (f_c)	Class-value of m ($= \lambda$)
≥ 10	217	0.46
7, 8, 9	190	0.27
4, 5, 6	272	0.14
1, 2, 3	307	0.09
0	480	0.00
Total	1466	

These figures give $m_p = 0.148$, whence $d_p = 0.237$.

It is of interest that application of this latter method to the 5-km data yields a value of d_p for 25-n. mile tracks of 0.50 — very close to the value of 0.51 found above, even though the frequency distributions of numbers of echo-tops are markedly different for special sample and general population; this close agreement between estimates produced by two different methods encourages one to think that the extension of special results to the general population may be more reliable than it at first appears.

Extension of results to other levels. We may use the results of the routine measurements of echo-tops to make a rough estimate of the probability of encountering echo at levels in the atmosphere other than 1 km or 5 km. From Appendix II the probability P_1 of making at least one echo-encounter on a track is $(1 - (1 + d)^{-m/d})$. Assume that for 25-n. mile tracks the relationship between m (the mean number of encounters on a track) and numbers of echo-tops is that used above and that d is constant with height and equal to 0.50. (A better value of d could only be obtained from information on the magnitude of $\overline{\Delta^2}$ — the within-class variance — which we do not possess; also, the value of P_1 is not very sensitive to variations of d , those in m being far more important.) Values of m and d for 50-n. mile tracks may be obtained by doubling those for the 25-n. mile tracks, but in view of the results described earlier it was decided not to continue the process as far as 100 n. mile.

Table IV gives frequencies over the whole year of different numbers of highest echo-tops occurring at or above various levels from 3 km upwards. From these data, values of P_1 for 25-n. mile and 50-n. mile tracks as a function of height are derived, as shown in Figure 4.

It is hardly necessary to emphasize that the derivation of Figure 4 is crude; nevertheless it should give some idea of the probability of encountering echo at various levels near Gan.

Relationship of echo-encounter measurements to other meteorological variables. It is obvious that if a high correlation could be established between the frequency of radar-echo encounter as measured during the special short-term investigation and other meteorological variables (such as rainfall) which have good, consistent, long-term records, then it ought to be possible to make useful deductions about the seasonal and year-to-year variations of echo-activity which the radar records themselves would be quite inadequate

TABLE IV—FREQUENCIES OF NUMBERS OF ECHO-TOPS AT OR ABOVE VARIOUS LEVELS WITHIN 60 NAUTICAL MILES OF GAN

[illegible]

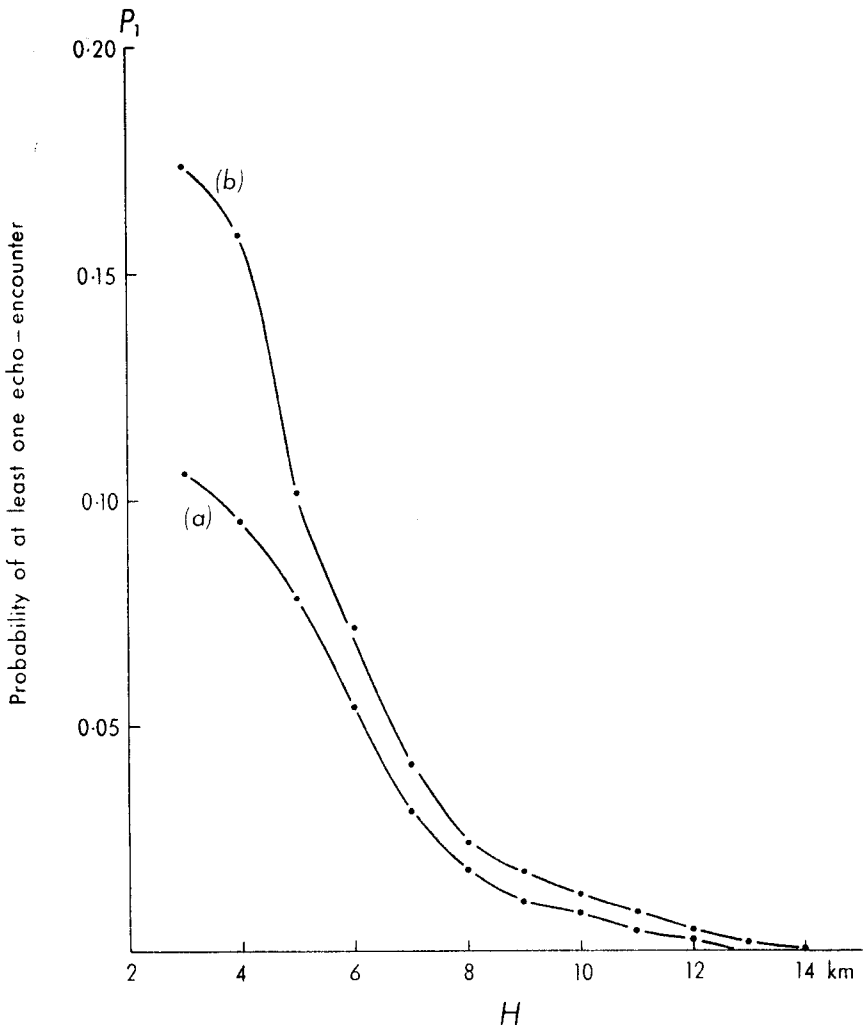


FIGURE 4—VARIATION WITH HEIGHT (H) OF THE PROBABILITY (P_1) OF AT LEAST ONE ECHO-ENCOUNTER ON TRACKS OF LENGTH 25 NAUTICAL MILES (a) AND 50 NAUTICAL MILES (b)

to reveal. Figure 5 shows the monthly rainfall amounts recorded at Gan from August 1968 to July 1969, and monthly values of the mean number m of echo-encounters expected on a 25-n. mile track at an altitude of 5 km; these values of m were calculated from the monthly frequencies of numbers of highest echo-tops at or above 5 km and the data of Table II. Values of both variables are plotted as time series and it is obvious that there is good agreement. The correlation coefficient between the two sets of figures is in fact 0.88 with 95 per cent confidence limits (assuming independence) of 0.62 and 0.97. The month with the greatest discrepancy between rainfall and m is November

1968; examination of the November data shows that during this month relatively more echoes were detected on the periphery of the scanned area than in the immediate vicinity of Gan.

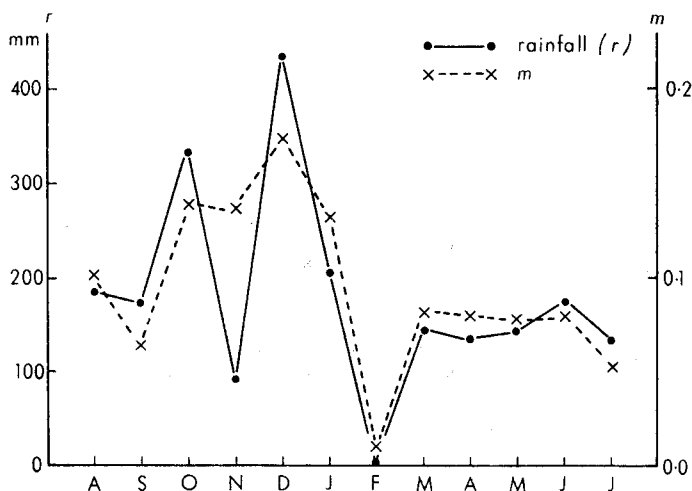


FIGURE 5—VARIATION FROM AUGUST 1968 TO JULY 1969 OF MONTHLY RAINFALL AT GAN (r) AND THE MEAN NUMBER OF ECHO-ENCOUNTERS (m) ON A TRACK OF LENGTH 25 NAUTICAL MILES AT A HEIGHT OF 5 KILOMETRES, ESTIMATED FROM ECHO-TOP DATA

Figure 6 shows a comparison of the monthly rainfall totals for the experimental period with those for the 11-year period 1961–71. Rainfall totals vary considerably from year to year and it would be difficult to describe many months of the experimental period as being particularly typical of the long-term pattern; there does appear, however, to be a tendency for smaller rainfall amounts to be recorded in February and March than in other months of the year.

Figure 7(a) shows expected frequencies of occurrence of one or more echo-encounters on 25-n. mile and 50-n. mile tracks at 5 km over the 12 months of the investigation calculated from the Modified Poisson distribution with $d = 0.50$ and m estimated from the monthly frequencies of numbers of echo-tops at or above 5 km (Table V) taken in conjunction with the data of Table II. Figure 7(b) shows corresponding estimates of monthly mean and extreme values of frequency of occurrence of echo-encounters for the 11-year period 1961–71, m being estimated by means of a regression equation on monthly rainfall derived from the data of Figure 5. (Frequencies for the 50-n. mile tracks were derived by using values of m and d equal to twice those used for the 25-n. mile tracks.) It may be noted that the frequencies of echo-encounters on both 25-n. mile and 50-n. mile tracks in February 1969 (Figure 7(a)) are appreciably lower than the February extremes shown in Figure 7(b) for the period 1961–71; this inconsistency is due to the regression on monthly rainfall used in the derivation of the frequencies shown in Figure 7(b).

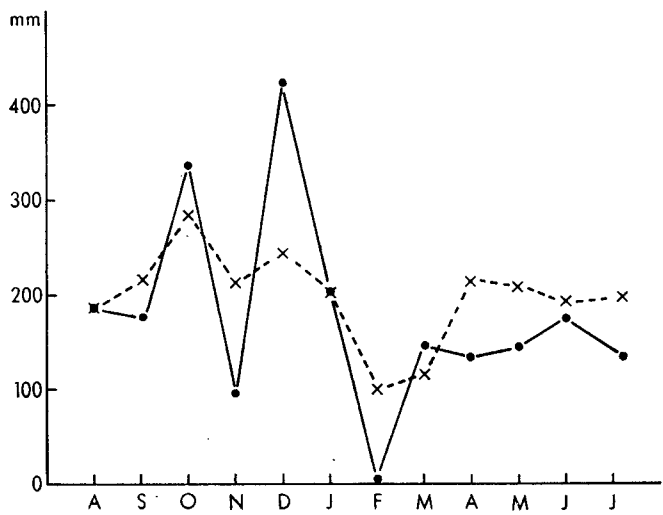


FIGURE 6—COMPARISON BETWEEN MONTHLY RAINFALL TOTALS AT GAN
· ———· from August 1968 to July 1969
x - - - x 11-year averages (1961-71)

TABLE V—MONTHLY FREQUENCIES OF NUMBERS OF ECHO-TOPS AT OR ABOVE 5 KILOMETRES

Number of tops	Aug. 1968	Sept. 1968	Oct. 1968	Nov. 1968	Dec. 1968	Jan. 1969	Feb. 1969	Mar. 1969	Apr. 1969	May 1969	June 1969	July 1969	All months
>10	7	2	17	12	27	14	1	1	6	7	5	1	100
9	0	1	13	6	12	9	0	1	4	1	4	2	53
8	2	1	5	5	13	7	0	2	4	1	4	2	46
7	1	1	4	12	9	6	0	2	0	2	5	2	44
6	4	4	9	4	12	8	0	4	3	6	5	7	66
5	4	11	9	9	3	9	0	4	6	10	4	1	70
4	5	7	6	12	13	9	0	5	4	9	9	5	84
3	5	2	8	12	6	7	0	12	8	8	8	10	86
2	6	11	7	11	14	6	0	13	15	12	9	5	109
1	4	4	7	16	14	7	1	19	30	14	13	22	151
0	34	55	52	38	41	53	64	30	63	73	72	83	657
	71	99	137	137	164	135	66	93	143	143	138	140	1466

The diagrams indicate that for most months a 25-n. mile track at 5 km will have at least one echo on about 8 per cent of occasions; in February the figure falls (on average) to about 5 per cent. Even in the wettest months the 25-n. mile tracks will have echo on no more than 14 to 15 per cent of occasions (8 to 9 per cent in February).

The other graphs in Figure 7 indicate that on average a 50-n. mile track at 5 km will have echo on about 14 per cent of occasions for most of the year, falling to about 9 per cent in February; the corresponding figures for the wettest years are 23 per cent and 14 per cent.

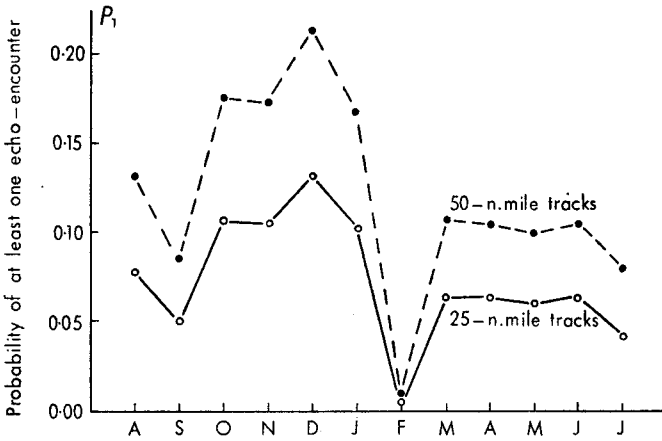


FIGURE 7(a)—VARIATION FROM AUGUST 1968 TO JULY 1969 OF PROBABILITY (P_1) OF OCCURRENCE OF AT LEAST ONE ECHO-ENCOUNTER ON TRACKS OF LENGTH 25 NAUTICAL MILES AND 50 NAUTICAL MILES AT A HEIGHT OF 5 KILOMETRES

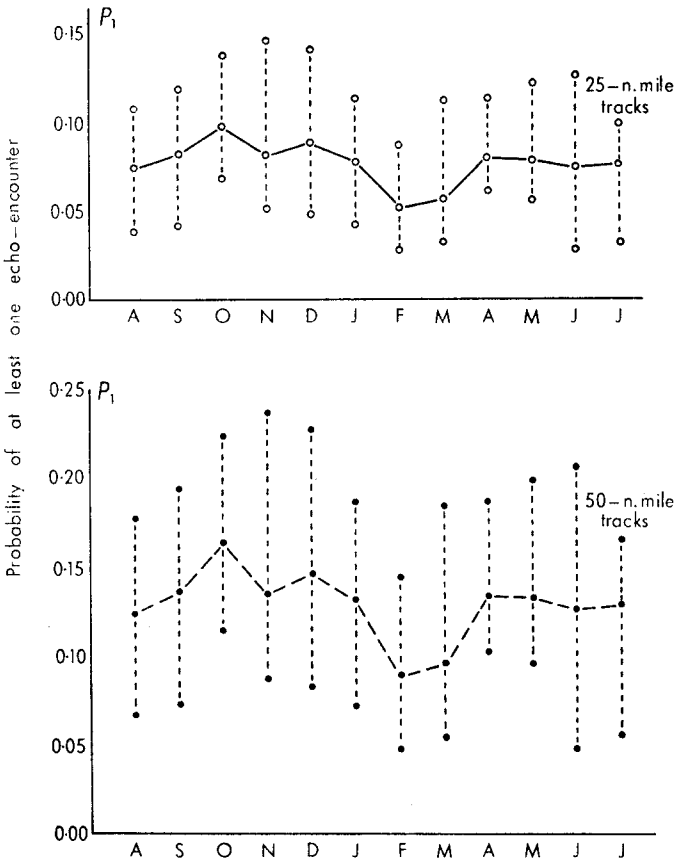


FIGURE 7(b)—CORRESPONDING MEAN AND EXTREME VALUES OF P_1 FOR THE PERIOD 1961-71 ESTIMATED FROM A REGRESSION ON MEAN AND EXTREME MONTHLY RAINFALL TOTALS

Length of echo intercepted by aircraft tracks. The frequency distribution of echo-lengths would be expected to be exponential in form if the echo-length were a non-negative random variable without memory.² The lengths of individual echoes intercepted by various tracks at 1 km and 5 km were measured, as were also the total lengths of echoes intercepted by the tracks. The resulting frequency distributions are illustrated in Figures 8 and 9 which show plots of the logarithm of the cumulative frequencies against length of echo. All points would lie on a straight line if the frequencies were exponential, but it is obvious that the frequencies of occurrence of large values are greater than would be expected on such an assumption; the obvious implication is that there is a considerable amount of 'memory' (or autocorrelation) involved in the radar-echo 'process' as would indeed be expected on physical and intuitive grounds.

The mean length of echo at 1 km is 3.6 n. mile and at 5 km, 3.2 n. mile.

It is interesting to apply the ideas contained in the discussion of the finite size of echo in Appendix II to the figures of Table I for the mean numbers of echo-encounters at 1 km on 25-n. mile and 50-n. mile tracks, each 25-n. mile track comprising one-half of a 50-n. mile track.

The total number of encounters on the 50-n. mile track is 1755. Using equation (11) of Appendix II we expect to have on the 25-n. mile track

$$\frac{1}{2} \times 1755 (1 + 3.6 / (25.0 + 3.6)) = 988 \text{ encounters.}$$

The number actually observed is 950; the discrepancy of 38 is within the range of normal statistical fluctuation. (If the effect of finite size of echo were ignored, one would expect only $\frac{1}{2} \times 1755$, or 877, encounters on the 25-n. mile track.)

Acknowledgements. The authors wish to acknowledge the hard and painstaking work of the meteorological and technical staff at Gan who made the observations upon which the present study has been based; they thank in particular Mr H. D. Westwood (the Meteorological Officer-in-charge) for his general support of the investigation, and Mr M. Morrish, who had local scientific control and who carried out the bulk of the photographic and other measurements.

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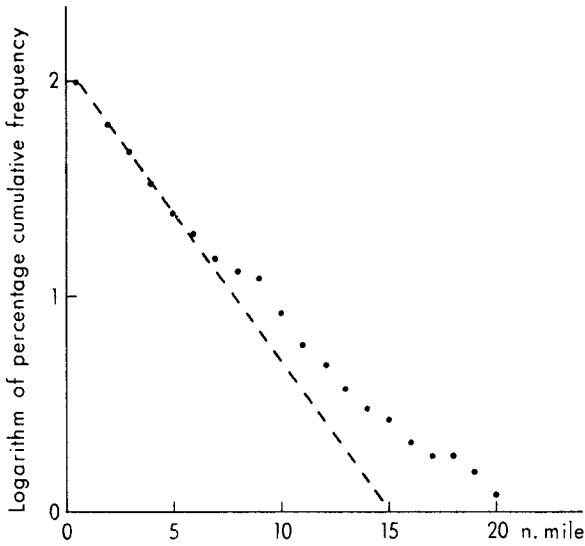


FIGURE 8(a)—LOGARITHM OF PERCENTAGE CUMULATIVE FREQUENCIES OF LENGTHS OF INDIVIDUAL ECHOES INTERCEPTED BY TRACKS OF LENGTH 25 NAUTICAL MILES AT A HEIGHT OF 1 KILOMETRE

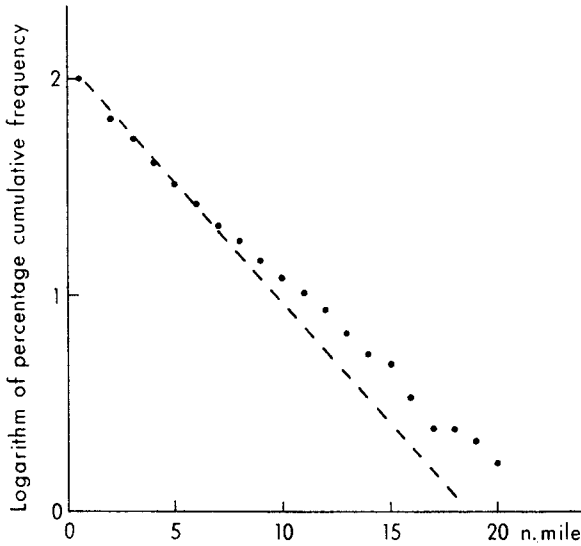


FIGURE 8(b)—LOGARITHM OF PERCENTAGE CUMULATIVE FREQUENCIES OF TOTAL LENGTH OF ECHO INTERCEPTED BY TRACKS OF LENGTH 25 NAUTICAL MILES AT A HEIGHT OF 1 KILOMETRE

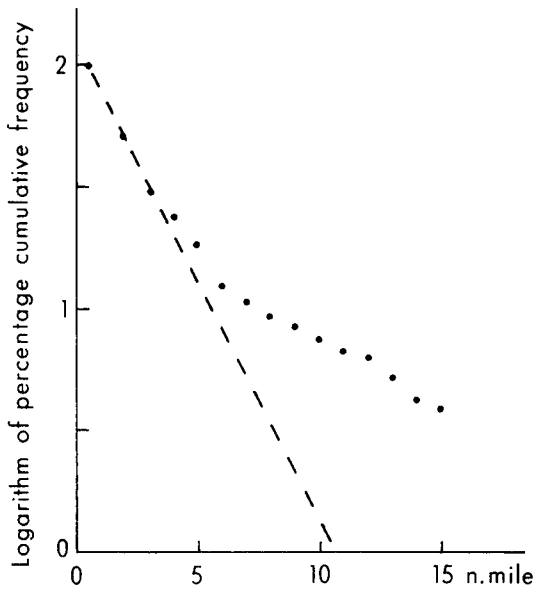


FIGURE 9(a)—LOGARITHM OF PERCENTAGE CUMULATIVE FREQUENCIES OF LENGTHS OF INDIVIDUAL ECHOES INTERCEPTED BY TRACKS OF LENGTH 25 NAUTICAL MILES AT A HEIGHT OF 5 KILOMETRES

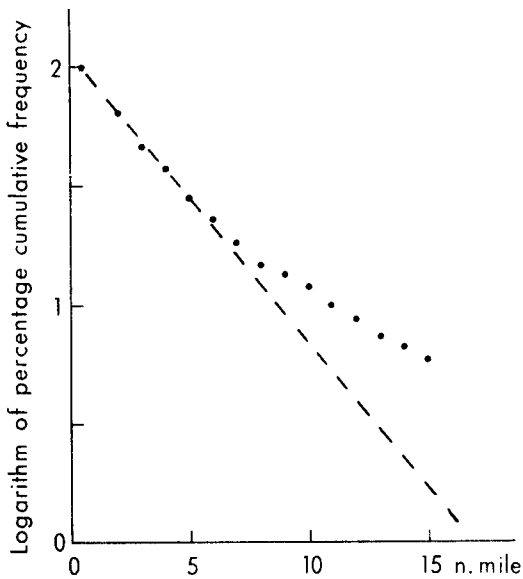


FIGURE 9(b)—LOGARITHM OF PERCENTAGE CUMULATIVE FREQUENCIES OF TOTAL LENGTH OF ECHO INTERCEPTED BY TRACKS OF LENGTH 25 NAUTICAL MILES AT A HEIGHT OF 5 KILOMETRES

APPENDIX I — CHARACTERISTICS OF THE RADAR

1. Make and type
Plessey Meteorological Radar, Type 43S
2. Aerial system
General : Parabolic dish, modified Cutler-type feed
Aperture : 12 ft (3.658 m) diameter
Polarization : Vertical
Gain : Not less than 37 dB with respect to an isotropic radiator
Beam width : $2^\circ \pm 0.25^\circ$ at -3 dB points
Sidelobes : Within $\pm 10^\circ$ — better than -27 dB
 Outside $\pm 10^\circ$ — better than -32 dB
Elevation limits : 0° to 35°
Modes of operation : (a) Manual setting
 (b) Continuous rotation at 10 rev/min or 20 rev/min
3. Transmitter/receiver
Transmitter characteristics :
 Frequency : 2700–2720 MHz
 Peak power : 675 kW (nominal)
 Pulse length : $2\mu\text{s} \pm 0.2\mu\text{s}$ (at 50 per cent peak amplitude)
 Mean power : 370 W (nominal)
 Pulse repetition frequency : 275 pulses per second ± 10 per cent
Receiver characteristics :
 Type : Linear/logarithmic
 Intermediate frequency : 30 MHz
 Video band width : 6 MHz (nominal)
 Noise factor : Better than 6 dB
Characteristic of sensitivity :
 Time control : Preset R^{-2} law to approx. 200 km
4. Calibrated intermediate-frequency attenuator
 Fine settings : 6 steps of 2.3 dB each
 Coarse settings : 3 steps of 16 dB each

APPENDIX II

The statistical distribution of the numbers of encounters with radar echo on tracks of various lengths

1. The simplest assumption to make about the distribution of significant radar-echo in space at any one time is that it is random in the following sense :

Consider an ensemble of lines, randomly distributed in space, all of the same length which is large compared to the dimensions of any single radar

echo. The passage of the line through any radar echo is regarded as a single encounter regardless of the length of echo intercepted. The number of encounters on a line is then assumed to have a Poisson distribution such that the probability p_k of k encounters is given by

$$p_k = e^{-\lambda} \lambda^k / k!, \quad \dots (1)$$

where λ is the usual Poisson parameter and is equal to the mean number of encounters on a track. (λ is proportional to the length L of the track so that $\lambda = L\lambda_0$ where λ_0 is the mean number of encounters per unit length.) Also assume that λ is the same for all tracks of the same length whatever their shape, so that the statistical distribution of numbers of encounters will be identical for both straight and circular tracks.

Although λ is assumed constant in space at any one time, it is certain to vary with time over periods greater than a few hours, and will itself have a frequency distribution — $f(\lambda)$, say.

The probability of k encounters over the track for all times will thus become

$$p_k = \int (e^{-\lambda} \lambda^k / k!) f(\lambda) d\lambda. \quad \dots (2)$$

The expected number of encounters at one time — which is equivalent to the mean number of encounters over a large number of similar tracks — is equal to λ . It follows that the expected number of encounters for all times, or m , is equal to the mean values of λ (or $\bar{\lambda}$) derived from the assumed frequency distribution $f(\lambda)$.

The variance of the number of encounters over all times, or σ^2 , is given by the sum of $\bar{\lambda}$ and the variance of λ , or σ_λ^2 .

Hence

$$\left. \begin{aligned} m &= \bar{\lambda} \\ \sigma^2 &= \bar{\lambda} + \sigma_\lambda^2 \end{aligned} \right\}, \quad \dots (3)$$

and these equations hold whatever the exact form of $f(\lambda)$ may be.

Assume that $0 \leq \lambda \leq \infty$ and that $f(\lambda)$ is markedly skew. If $f(\lambda)$ is of the Gamma (or Pearson Type III) form, i.e. if

$$f(\lambda) = \frac{1}{d^t (t-1)!} e^{-\lambda/d} \lambda^{t-1} \quad \dots (4)$$

then

$$\left. \begin{aligned} m &= \bar{\lambda} = td \\ \sigma^2 &= \bar{\lambda} + \sigma_\lambda^2 = td + td^2 = m(1+d) \end{aligned} \right\}, \quad \dots (5)$$

whence

$$\left. \begin{aligned} d &= (\sigma^2/m) - 1 = \sigma_\lambda^2/\bar{\lambda} \\ t &= m/d \end{aligned} \right\}, \quad \dots (6)$$

where d and t are parameters of the distribution. p_k is easily found as a negative binomial such that p_k for $k = 0, 1, 2, 3, \dots$ is given by

$$\left(\frac{1}{1+d}\right)^t \left\{ 1, \frac{td}{1+d}, \frac{t(t+1)}{2!} \left(\frac{d}{1+d}\right)^2, \frac{t(t+1)(t+2)}{3!} \left(\frac{d}{1+d}\right)^3, \dots \right\}$$

or

$$\left(\frac{1}{1+d}\right)^{m/d} \left\{ 1, \frac{m}{1+d}, \frac{m(m+d)}{2!(1+d)^2}, \frac{m(m+d)(m+2d)}{3!(1+d)^3}, \dots \right\}. \quad \dots (7)$$

If the modal value of λ is λm ,

$$\lambda m = (t-1)d = m - d. \quad \dots (8)$$

The parameter 'd' is a measure of the departure of the distribution of p_k from the simple Poisson; both m and d are proportional to the length of the track, i.e. they are linearly dependent on scale. The distribution (7) is referred to as the Modified Poisson.

Note that the expression $(m+d)$

$$\begin{aligned} &= m + (\sigma^2/m) - 1 = \{(m^2 + \sigma^2)/m\} - 1 \\ &= \{(\sum p_k k^2)/(\sum p_k k)\} - 1, \end{aligned} \quad \dots (9)$$

and is thus independent of p_0 , i.e. is independent of the number of occasions when no encounters were observed.

2. Any frequency table derived from practical observations or measurements is usually regarded as a particular sample drawn from a hypothetical infinite population, and we wish to use the statistical information contained in the sample to the best advantage in order to derive the most probable description of the frequency distribution of the infinite population.

The course generally adopted is, firstly, to assume the functional form of the frequency distribution — Gaussian, Poisson, negative binomial, etc. — and secondly, to assume that the numerical values of the parameters of the functional form (e.g. mean and standard deviation for the Gaussian) are such as to maximize the chances of the values associated with the particular sample occurring in a random set of observations. (The second assumption is the Principle of Maximum Likelihood.)

The two parameters associated with the Modified Poisson, or negative binomial, distribution are 'm' and 'd'. The Principle of Maximum Likelihood gives a simple expression for m — it is in fact the sample mean — but d is obtained only by solving a very complicated transcendental equation and it is therefore normally necessary in practice to use simpler methods.³

The two most common are estimation from the sample variance and equation (6) above, and from the observed proportion of zeros which by equation (7) is equal to $(1+d)^{-m/d}$.

The efficiency of these and other methods is discussed by Anscombe³ but since in the present case there is no real reason to believe that the parent population is *exactly* described by the Modified Poisson, great refinement of argument would be spurious.

A further reason for not being too worried over the theoretical accuracy of the methods employed is that frequencies calculated from equation (7) are not very sensitive to small variations in d for constant m . This is illustrated by Table VI which shows, for values of m of 0.2, 0.5 and 1.0, frequencies of occurrences of these events for a considerable range of values of d .

TABLE VI—PERCENTAGE FREQUENCIES OF OCCURRENCE OF THREE EVENTS DERIVED FROM THE MODIFIED POISSON DISTRIBUTION FOR VARYING VALUES OF THE PARAMETERS m AND d

m	d										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	<i>percentage frequencies</i>										
0.2	0.1	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.9	0.9	1.0
0.5	1.3	1.6	1.9	2.2	2.3	2.5	2.6	2.7	2.7	2.7	2.8
1.0	6.1	6.4	6.5	6.6	6.6	6.6	6.5	6.5	6.4	6.3	6.3

3. It was assumed above that the distribution of echo at any one time was purely random and hence describable by a Poisson formula. It is however quite likely on physical grounds that rather greater clustering takes place than would be expected on a purely random basis. Eggenberger and Pólya⁴ have shown that in such circumstances the basic probability distribution of numbers of encounters on any one occasion (over an ensemble of tracks of equal length) is itself of the negative binomial form, so that the final form of the overall probability distribution p_k will depend upon a convolution of a negative binomial with the frequency distributions of its parameters. An approximate analysis of variance shows that if the ' d parameter' of the spatial distribution (d_s) is constant in time, and if the resultant distribution is also of the negative binomial form, then the resultant d parameter is measured by an amount d_s over what it would have been if the primary spatial distribution were Poisson (with $d_s = 0$).

4. If for a mixture of simple Poisson distributions we have on each occasion *two* samples (i.e. two samples with the same parameter λ) then the overall mean number of events for all occasions of two samples will be $m_2 = 2m$. The variance of the number of events in the two samples combined we call σ_2^2 , so that

$$\sigma_2^2 = 2\sigma^2(1 + \rho),$$

where ρ is the correlation between the numbers of events in the two samples. If the Modified Poisson distribution for the combined number of events in the two samples has parameter d_2 , then we know that (because of the proportionality to scale) $d_2 = 2d$. Also,

$$d_2 = (\sigma_2^2/m_2) - 1,$$

whence

$$2d = \{2\sigma^2(1 + \rho)/2m\} - 1,$$

giving

$$d = \rho/(1 - \rho). \quad \dots (10)$$

This gives a further meaning to the parameter d . (For simultaneous coherent distributions, e.g. that of Eggenberger and Pólya, ρ is the correlation between numbers of events in the two halves of a straight track).

5. The discussion so far has ignored the effect of the finite size of echoes.

Consider a straight track ABC made up of two halves AB and BC each of length L . Consider an echo of length e , where $e \ll L$, occurring at random

anywhere on AC. The probability that the echo straddles the point of division B, so that it is recorded as occurring both in AB and BC, is $(2e/2(L + e))$ or $e/(L + e)$. If there are $n(e)$ echoes of length e , and N echoes altogether, then the number straddling B is

$$\begin{aligned}\Sigma n(e) \cdot e/(L + e) &= N \overline{e/(L + e)} \\ &\approx N \cdot \overline{e}/(L + \overline{e}),\end{aligned}$$

where the overbar indicates a mean value.

Thus if N echoes are observed on a track of length $2L$, we may expect to observe, on average,

$$\begin{aligned}\frac{1}{2}\{N - N\overline{e}(L + \overline{e})^{-1}\} + N\overline{e}(L + \overline{e})^{-1} \\ = \frac{1}{2}N\{1 + \overline{e}(L + \overline{e})^{-1}\} \quad \dots (11)\end{aligned}$$

echoes in either half of the track where \overline{e} is the average length of echo.

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A COMPARISON OF GEOSTROPHIC AND ROCKET WINDS AT STRATOSPHERIC LEVELS, MEASURED FROM A SMALL NETWORK OF ROCKET SOUNDING STATIONS

By G. C. BRIDGE

Summary. The temporary formation during January and February 1971, of a network of three stations over north-west Europe (West Geirinish and Aberporth in the United Kingdom, and Kiruna in Sweden) recording near-simultaneous observations of temperatures and winds at stratospheric levels, provided an opportunity to compare theoretical geostrophic winds, calculated from contour-height differences between these stations, and the actual winds measured during the descent stage of the skua rocket payload. Analysis of the results showed that provided changes of temperature and wind direction in the stratosphere were slow, values of geostrophic wind were calculated which compared favourably with the actual measured wind.

Introduction. During the latter part of January and early February 1971, the British Meteorological Office programme of skua rocketsonde observations of temperature and wind from West Geirinish in the Outer Hebrides was supplemented by similar observations from the European Space Research Organization (ESRO) rocket range (ESRANGE) at Kiruna, situated in the far north of Sweden (see Figure 1). These additional observations were obtained for a research project carried out by University College, London, in conjunction with ESRO, to clarify more fully the mechanism and structure of stratospheric warmings which occur over north-west Europe during the winter period.

The occasion was considered suitable for operating a third rocketsonde station in order to obtain sets of data suitable for synoptic interpretation. The only additional site available was at Aberporth, on the Cardiganshire coast, which had previously been found suitable for the firing of the short version of the skua rocket. However, as a result of the close proximity of populated land areas and somewhat restricted sea and air boundaries to the range, success of the firing programme was largely governed by the frequency of weather conditions giving generally light winds at most levels up to 100 mb.

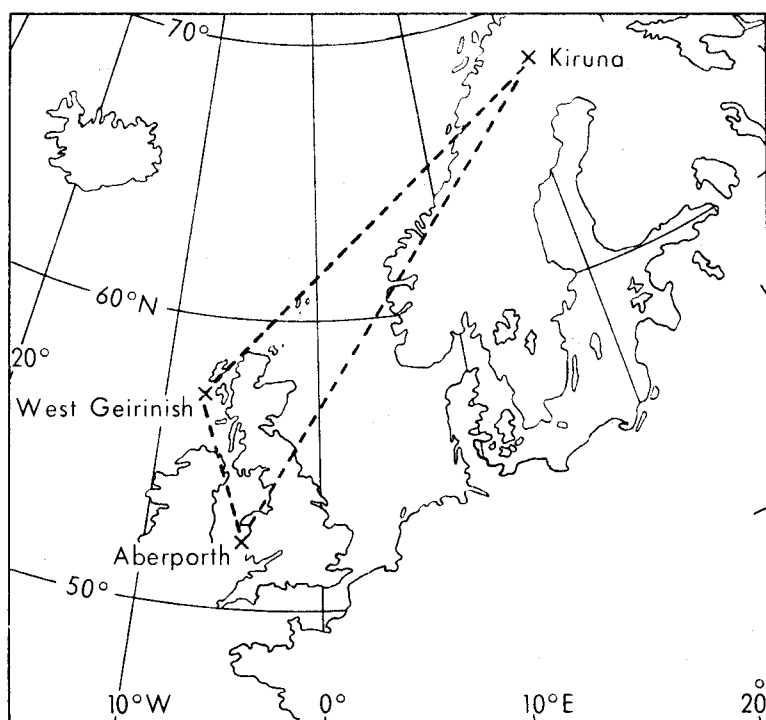


FIGURE 1—LOCATION OF THE THREE ROCKETSONDE STATIONS

It was intended to fire rockets from the three stations simultaneously; the extent to which inclement weather, unserviceability of equipment, etc. affected the success of the venture can be seen from Table I.

Contour height and wind calculation. As a first and necessary step towards the calculation of geostrophic wind, the geopotential heights of selected pressure surfaces were calculated. Firstly, integration of the equation

$$d \log P = \frac{-g}{RT} dz$$

gave the height z in geometric metres of the selected levels, where

P is the pressure in millibars,

T is the temperature in kelvins,

$g = 9.8166 \{1 - (3.143 \times 10^{-7}z)\} \text{ m s}^{-2}$

$R = 287.05 \text{ J/(kg K)}$.

Since all rocketsonde descents terminated at around 20 km, 50 mb was taken as the standard lower boundary for the integration. Values of the contour height (converted to geometric metres) and temperature at this level were found by interpolation from 50-mb synoptic charts based on midnight radiosonde measurements. The mean difference between temperature measured by the rocketsonde at 50 mb and the chart value was found to

TABLE I—PROGRAMME OF ROCKET LAUNCHES FROM WEST GEIRINISH, ABERPORTH AND KIRUNA DURING THE PERIOD 19 JANUARY–11 FEBRUARY 1971

	West Geirinish (57° 21' N 7° 22' W)		Aberporth (52° 08' N 4° 34' W)		Kiruna (67° 49' N 22° 00' E)	
	Rocket No.	Useful data range km	Rocket No.	Useful data range km	Rocket No.	Useful data range km
19 Jan.			M320A	nil	004	20–60
20 Jan.	M325	20–60				
21 Jan.			M322A	nil	005	20–58
22 Jan.	M327	20–60 ←	M324A	20–60	006	nil
23 Jan.	M329	19–55				
24 Jan.						
25 Jan.	M331	20–60 ←	M326A	19–61	007	20–58
26 Jan.						
27 Jan.					008	20–35
28 Jan.	M333	20–47 ←	M328A	20–57	009	20–60
29 Jan.						
30 Jan.					010	20–37
31 Jan.						
1 Feb.	M335	19–58				
2 Feb.						
3 Feb.	M336	19–58 ←	M330A	19–62	011	20–55
4 Feb.						
5 Feb.	M337	19–58				
6 Feb.						
7 Feb.						
8 Feb.			M332A	20–62	012	20–44
9 Feb.					013	20–50
10 Feb.	M338	19–60 ←	M334A	19–62	014	20–55
11 Feb.						

Arrowed lines joining Aberporth to West Geirinish and Kiruna show how comparisons were made to arrive at data presented in Table II.

be about 1 degC, which led to an estimated mean difference of 20 metres in contour height. The only corrections applied to the sonde temperature determinations were those for dynamic heating (the fall-speeds at 60 km and 20 km being 120 m/s and 10 m/s respectively), and for cooling by infra-red radiation loss. Solar-radiation error was avoided by launching in darkness (usually soon after sunset at 75 km).

The integration was performed from the 50-mb level upwards by the use of interpolated temperature data over steps of one kilometre. The height values of the selected pressure levels for each of the three stations were obtained and converted to geopotential metres. In this exercise, 5, 2, 1, 0.5, 0.3 and 0.2 mb were selected and the relevant contour heights entered on the triangular grid ABC formed by the three launching sites (see Figure 2).

Low contour height prevailed over the region to the north and east of Norway, hence values at points C and A were always the smallest and the greatest respectively. A contour height equal in value to that at point B was always located somewhere along AC. Its position (labelled D in Figure 2) was calculated on the assumption that uniform contour gradients existed along each of the sides of ABC. BD was then drawn and its angle with a true-north direction through the centroid O of the grid triangle represented the direction of the wind, on the assumption that there was no ageostrophic motion over the area of the grid.

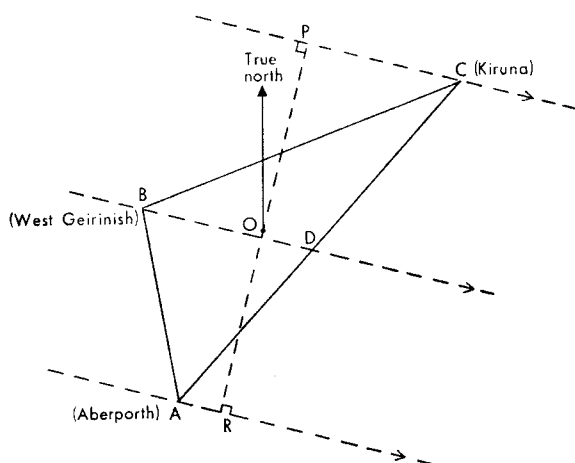


FIGURE 2—THREE-STATION GRID DIAGRAM AND CONSTRUCTION LINES

Geostrophic wind velocity V_g was then evaluated by means of the equation

$$V_g = \frac{g}{f} \times \frac{dh}{dn},$$

where g is the local acceleration due to gravity,
 f is the Coriolis parameter, which varies with latitude, and
 $\frac{dh}{dn}$ is the gradient of geopotential height.

One contour having already been constructed through BD, two others were then drawn parallel to it through points A and C respectively. Perpendiculars from these to point O were then drawn at R and P and this process yielded two values for dn . Contour-height differences A-B and B-C gave two values of dh ; hence, by deducing values for f at the mid points of PO and RO, two values for V_g were calculated. The mean of these two values was assumed to be equal to the mean wind over the grid ABC.

Comparison between calculated and observed winds. The next step was to make the comparison between the theoretical winds thus computed and the rocketsonde wind observations from the three stations. Rocket winds were calculated by tracking the falling sonde-parachute by radar, the parachute being constructed with silvered reflecting panels. The parachute was assumed to take up the environmental wind at a height of about 0.1 mb or 65 km, hence, from a series of readings of azimuth, elevation and slant range, together with time elapsed, wind-height profiles could be plotted and the value of the wind be calculated at each kilometre level, meaned over a 2-kilometre layer centred on the level in question.

Since it was only possible to calculate a mean geostrophic wind over the grid, winds observed at points A, B and C had to be averaged before any comparison could be made. This was achieved by taking the means of the zonal and meridional components of the observed winds and converting

the final result back to represent a grid rocketsonde wind. A limitation in the method employed is the assumption of the existence of a constant wind gradient along AB, BC and CA. In practice this would rarely occur, and deviations would be accentuated by the large distance of point C from both A and B. Reference to Table I shows how the data were assembled from the various comparisons available. The results of the comparisons are presented in Table II.

TABLE II—COMPARISON OF THE MEAN THEORETICAL GEOSTROPHIC WIND AND MEAN OBSERVED ROCKET WIND OVER THE THREE-STATION GRID AT VARIOUS PRESSURE LEVELS

Height	Wind type	22 Jan. 1971	25 Jan. 1971	28 Jan. 1971	03 Feb. 1971	10 Feb. 1971
0.2 mb	Rocket Theory	280° 130 kt 255° 150 kt	265° 170 kt 255° 160 kt			
0.3 mb	Rocket Theory	275° 130 kt 245° 160 kt	265° 150 kt 260° 130 kt		270° 175 kt 255° 200 kt	265° 200 kt 240° 230 kt
0.5 mb	Rocket Theory	280° 130 kt 245° 160 kt	260° 130 kt 265° 110 kt		265° 170 kt 260° 200 kt	270° 180 kt 270° 200 kt
1.0 mb	Rocket Theory	300° 75 kt 240° 110 kt	260° 70 kt 275° 65 kt	255° 95 kt 240° 115 kt	265° 110 kt 260° 140 kt	270° 140 kt 255° 190 kt
2.0 mb	Rocket Theory	335° 65 kt 255° 55 kt	290° 30 kt 305° 40 kt	280° 45 kt 250° 55 kt	275° 75 kt 290° 65 kt	265° 90 kt 265° 120 kt
5.0 mb	Rocket Theory	340° 80 kt 290° 50 kt	345° 40 kt 335° 50 kt	355° 40 kt 345° 40 kt	305° 45 kt 305° 55 kt	285° 50 kt 285° 55 kt

Note: Wind direction is reported to the nearest 5°. Wind speed is reported to the nearest 5 kt up to 100 kt, and to the nearest 10 kt over 100 kt.

It was obvious that another limitation on the method was that on no occasion were three simultaneous observations obtained. Consequently data for the day preceding or following the date in question had to be used. At Kiruna, however, on three occasions the means of both were available to yield a more acceptable value. Since, as can be seen from Figure 3, no stratospheric warming occurred during the observational period, day-to-day changes at these high levels were considered to be gradual and quite small, and this inspired confidence in the use of data from adjacent dates.

A convenient method of expressing the overall error between the two sets of winds is to take the root-mean-square (r.m.s.) error derived from their component values. Thus

$$\text{r.m.s. wind error} = \sqrt{\left\{ \frac{1}{n} \sum (u_g - u_r)^2 + \frac{1}{n} \sum (v_g - v_r)^2 \right\}},$$

where u_g and v_g , u_r and v_r are the geostrophic and rocket wind components respectively, and n is the number of observations. The daily r.m.s. errors obtained by summing all heights are

Date	22 Jan.	25 Jan.	28 Jan.	3 Feb.	10 Feb.
r.m.s. wind error	79 kt	21 kt	25 kt	34 kt	50 kt.

Combination of all days, and tabulation against height levels gives

Height level	0.3 mb	0.5 mb	1.0 mb	2.0 mb	5.0 mb
r.m.s. wind error	70 kt	51 kt	55 kt	41 kt	29 kt.

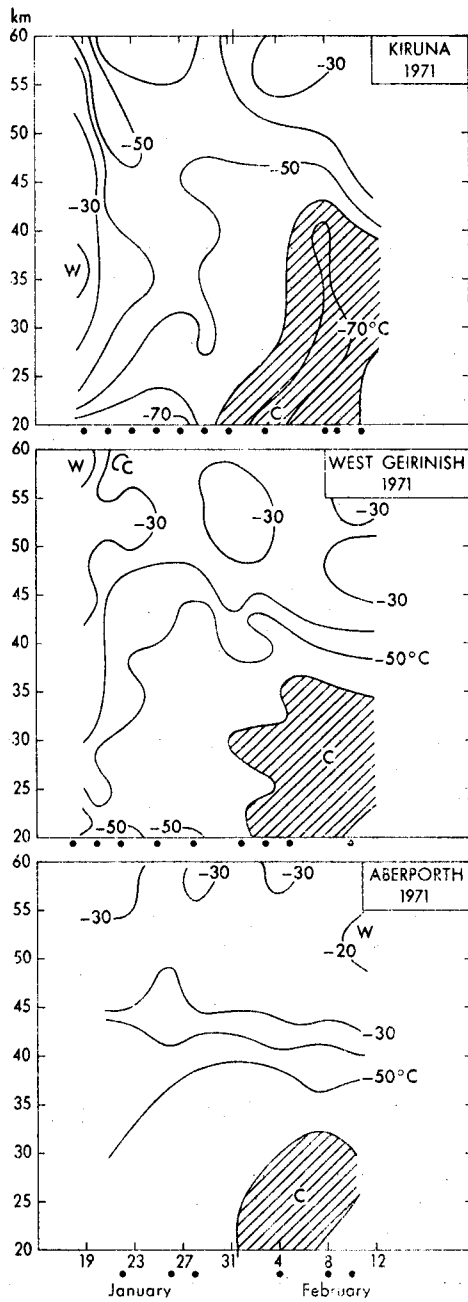


FIGURE 3—HEIGHT-TIME CROSS-SECTIONS OF TEMPERATURE DURING THE PERIOD OF DATA COMPARISON

C denotes cold; W denotes warm; ● indicates a rocket sounding; hatched areas represent temperature below -60°C (a small area has inadvertently been omitted from the top diagram).

However, at Kiruna and West Geirinish, fluctuations of the temperature field with time were present initially and may well account for the large wind-direction discrepancies which occurred on 22 January 1971. If data for this date are ignored, the following results are obtained :

Height level	0.3 mb	0.5 mb	1.0 mb	2.0 mb	5.0 mb
r.m.s. wind error	66 kt	27 kt	41 kt	24 kt	9 kt.

The apparent increase of error with height is most probably associated with the gradual increase in actual wind speed with height. If the ratio of the r.m.s. wind error to the scalar mean rocket wind speed is computed for the selected height levels, the following values are obtained :

Height level	0.3 mb	0.5 mb	1.0 mb	2.0 mb	5.0 mb
Ratio $\frac{\text{r.m.s. wind error}}{\text{scalar mean}}$	0.38	0.17	0.39	0.40	0.21.

Similarly, comparison of the mean vector difference between the observed rocket winds and the theoretical geostrophic winds with the scalar mean rocket wind speed yields

Height level	0.3 mb	0.5 mb	1.0 mb	2.0 mb	5.0 mb
Ratio $\frac{\text{mean vector error}}{\text{scalar mean}}$	0.33	0.16	0.36	0.38	0.20.

The absence of any gradual changes in magnitude of the error with height implied by these ratios would suggest that inaccuracies produced by the upward integration are small, once a stable temperature field has been established.

The results of Table II, and the subsequent tabulated values, imply that, bearing in mind the various assumptions made during the computations, it would seem that the determination of stratospheric geostrophic winds by contour-height comparison over a small grid yields fairly acceptable values. Much improvement would be realized by reducing the grid size or by using data from stations forming square or rectangular grids. A network of interconnecting grids is the next logical step, the analysis of data from which would give stratospheric winds at many levels over vast areas. Such a network made up from rocketsonde stations would however be quite impracticable for logistic and financial reasons. On the other hand satellites are producing radiance data from the stratosphere with coverage on a global scale. By using overlapping sets of radiance data from various levels over one position, a mean temperature profile up to stratospheric heights can be obtained. The contour data at say 50, 100 or even 200 mb having been established from radiosonde observations, and the temperature profile above being known, then contour heights at many levels over large areas could easily be computed and would yield sequences of stratospheric upper-air charts. Data produced at these levels would enable atmospheric models to include regions at present largely ignored, and would be of significant importance in the prediction of conditions likely to affect the performance of high-level supersonic transports.

THE PERFORMANCE OF WET-BULB THERMOMETERS IN THE LARGE THERMOMETER SCREEN

By H. E. PAINTER

Summary. By comparing temperatures from a fully aspirated psychrometer with simultaneous readings of mercury-in-glass and platinum resistance thermometers in a Large Thermometer Screen it is shown that, by using the accepted psychrometric coefficients, the glass thermometer with a standard muslin cap gave wet-bulb depressions on the average about 7 per cent too small, while a platinum resistance thermometer with sleeving covering its entire length gave wet-bulb depressions about 2 per cent too great. There was also some variation with wind speed.

Introduction. From the beginning of 1969, a recording aspirated resistance psychrometer¹ became the official instrument at Kew for measuring dry-bulb and wet-bulb temperatures. It was considered desirable to have comparisons of these temperatures with temperatures recorded simultaneously in a Large Thermometer Screen. Two platinum resistance thermometers were therefore set up, in the screen, in addition to the standard mercury-in-glass thermometers. The temperatures from these resistance thermometers were recorded by the same recorder as was used for the aspirated psychrometer. In the course of checking the self-consistency of these adjacent glass and resistance thermometers, it was found that the temperature of the two wet-bulb thermometers in the screen deviated more and more as the wet-bulb depression increased. The same two thermometers when used as dry bulbs gave entirely consistent readings over a wide range of temperatures. The glass thermometer always gave a smaller wet-bulb depression than the resistance thermometer and this suggested that more heat was being conducted down its stem to the bulb. It is possible that the psychrometric coefficient used for the screen measurements had been selected to suit such a glass thermometer with muslin cap and that this coefficient was not necessarily applicable to readings from the resistance thermometer. To investigate this problem a series of simultaneous readings from the glass and resistance thermometers in the screen and from the aspirated psychrometer was taken.

Site and observations. The Large Thermometer Screen was situated 5 metres to the east of the aspirated psychrometer, and contained the standard four glass thermometers, thermograph and hygrograph. The wet-bulb thermometer was covered with a standard circular muslin cap.² Standard Mk 2 Meteorological Office platinum resistance thermometers were fixed horizontally in the screen so that the ends with the resistance elements were 5 millimetres from the corresponding glass thermometers. A close-fitting woven sleeve was fitted to the wet-bulb resistance thermometer and this sleeve extended 10 centimetres along its length; the other end of the sleeving passed into the water reservoir. The temperature recorder registered the temperatures of the resistance thermometers at 10-second intervals, the sequence being: aspirated dry-bulb, aspirated wet-bulb, screen dry-bulb and screen wet-bulb. The time between successive measurements from the same thermometer was two minutes. Each glass thermometer was read at exactly the same time as the recorder was registering the temperature of the associated screen resistance thermometer. The mean of six consecutive readings from each of the six thermometers provided the simultaneous data

to be compared and any shorter-period variations of temperature were smoothed out. All index and recorder corrections were applied as necessary. From March to October 1970 about 140 sets of such observations were obtained.

Treatment of the data. In addition to the two wet-bulb temperatures obtained from the screen, another wet-bulb temperature, which has been used as a reference, was deduced from the dew-point derived from the aspirated psychrometer and the screen dry-bulb temperature. This procedure is considered to be justified since, under the conditions of the investigation, the dew-point is conservative, particularly with respect to radiation which affects dry-bulb and wet-bulb temperatures in a screen. The dew-points were derived from tables for aspirated psychrometers based on a psychrometric coefficient of $0.667 \times 10^{-3} \text{ degC}^{-1}$; the reference wet-bulb temperatures were derived from tables for the screen instruments based on a psychrometric coefficient of $0.799 \times 10^{-3} \text{ degC}^{-1}$. In both tables the atmospheric pressure is assumed to be 1000 mb. Differences of the temperature of each wet-bulb thermometer from the reference wet-bulb temperature have been plotted in Figure 1 against the depression of the latter below the screen dry-bulb temperature. Since the ventilation in the screen varies with wind speed, the differences have been plotted using three different symbols, depending upon the wind speed given by the Observatory pressure-tube anemometer. Although this was some considerable distance from the thermometers, and about 19 metres higher it has been assumed to give a measure of the ventilation in the screen. Subsequent measurements have shown that the wind speed at a height of 2 metres at a site near the thermometer screen is, in general, about one-half of that given by the pressure-tube anemometer, but it can be significantly different from this on occasions. The three ranges of wind speed into which the readings were divided were: 0–5 knots, 6–10 knots and 11 knots or more. The greatest wind speed during these observations was 19 knots. The best-fitting straight lines through the origin have been evaluated and drawn for each combination of thermometer and wind speed. It can be seen that the slopes of the lines vary according to the wind speed. It can also be seen that the difference between the mercury-in-glass wet-bulb temperature and that from the resistance thermometer is about 8–9 per cent of the wet-bulb depression irrespective of the wind speed. A mean of all readings shows that the glass thermometer gives wet-bulb depressions that are about 7 per cent too small whilst the resistance thermometer gives wet-bulb depressions about 2 per cent too large.

The suggested inadequate covering of the glass thermometer with the muslin cap was further investigated on four occasions by setting up a second wet-bulb thermometer which was covered with a woven sleeve extending 2 centimetres above its bulb and with threads tied above and below its bulb so that the sleeve fitted the bulb of the thermometer closely. Table I gives the results of simultaneous readings, meaned over a period of 10 minutes, from each of the glass wet-bulb thermometers and the resistance thermometer. The reference wet-bulb temperature and its depression below the screen dry-bulb temperature are also included together with the wind speed from the pressure-tube anemometer.

In the case of standard glass thermometers the difference in temperature with different coverings on the wet bulbs can be clearly seen in this table.

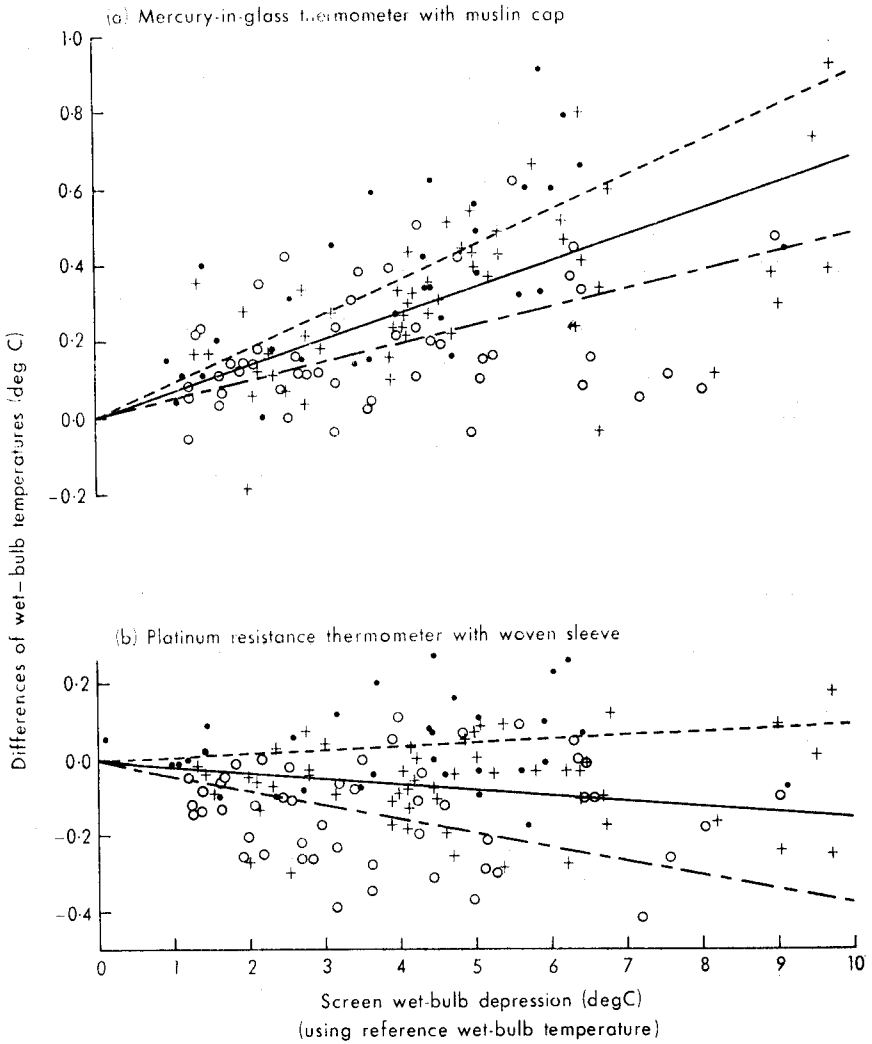


FIGURE 1—DIFFERENCES BETWEEN WET-BULB TEMPERATURES FROM THERMOMETERS EXPOSED IN A LARGE THERMOMETER SCREEN AND REFERENCE WET-BULB TEMPERATURES, PLOTTED AGAINST THE WET-BULB DEPRESSION

- (a) for a standard mercury-in-glass thermometer with muslin cap.
- (b) for a standard Mk 2 Meteorological Office platinum resistance thermometer with 10 cm of the sheath covered with sleeving.
- — — — for anemograph wind speeds 0-5 knots
- + — — — for anemograph wind speeds 6-10 knots
- o — — — for anemograph wind speeds ≥ 11 knots

It can also be seen that the glass thermometer with a sleeve is in closer agreement with the reference and resistance wet-bulb readings than a glass thermometer with a muslin cap.

TABLE I—WET-BULB TEMPERATURES MEASURED IN A LARGE THERMOMETER SCREEN BY DIFFERENT THERMOMETERS WITH SPECIFIED WET-BULB COVERINGS

Glass thermometer with sleeve	Glass thermometer with muslin cap °C	Resistance thermometer with sleeve °C	Reference wet-bulb temperature °C	Wet-bulb depression degC	Wind speed kt
16.5	16.8	16.3	16.35	6.3	14
13.9	14.0	13.8	13.75	3.0	12
16.2	16.5	16.3	16.35	5.4	7
14.3	15.1	14.4	14.25	6.95	2

Conclusions. The standard mercury-in-glass thermometer under the weather conditions experienced at Kew appears to need a wet covering over about 2 cm of its stem in order to be compatible with Meteorological Office humidity tables for screen psychrometers. The resistance thermometer requires a sleeve somewhat shorter than 10 cm along its length in order to be compatible with the same tables.

REFERENCES

1. PAINTER, H. E.; A recording resistance psychrometer. *Met Mag, London*, 99, 1970, pp. 68–75.
2. London, Meteorological Office. Observer's handbook. London, HMSO, 1969, p. 117.

OBITUARY

It is with regret that we have to record the death of Mr E. G. Cowler, Senior Scientific Officer, H.Q. Strike Command, Royal Air Force, on 23 March 1973.

PUBLICATIONS RECEIVED

The following have been received from the Meteorological Institute of the University of Thessaloniki :

Meteorologika 18: *Evaporation in Thessaloniki—Greece*. By G. C. Livadas and P. Chr. Machairas. 1972.

Meteorologika 19: *Contribution to the study of warm invasions in Greece*. By A. A. Flocas. 1972.

Meteorologika 20: *Contribution to the study of atmospheric refraction index and photogrammetric refraction in the area of Greece by means of meteorological data*. By E. N. Patmios. 1972.

Meteorologika 21: *The cooling power in Thessaloniki—Greece (III)*. By Chr. J. Balafoutis and G. C. Livadas. 1972.

Meteorologika 22: *Earth surface temperature*. Part I. Bare-soil surface. By G. C. Livadas and Y. A. Goutsidou. 1972.

NOTES AND NEWS

Retirement of Mr. J. Harding, O.B.E.

Mr John Harding, who has been Assistant Director, Agriculture and Hydrometeorology since 1966, retired from the Meteorological Office on 30 April 1973. He entered the Office early in 1936 after graduating in physics and taking his Master's Degree at Trinity College, Dublin. He trained for a few months at Croydon and then until the beginning of the War forecast for the operation of long-distance flights of flying boats from Foynes and Hythe. During the period 1939 to 1943 he served at the Central Forecasting Office and also at Prestwick, Gloucester and Plymouth before returning in 1944 to CFO at Dunstable, where he remained until 1949 and where he gained his reputation as an outstanding forecaster. He was promoted to Principal Scientific Officer in 1948. From 1949 to 1955 he was deputy to the Chief Meteorological Officer, Middle East, at Ismailia and then was in charge of the Meteorological Office at RAF Wyton until 1960, when he was promoted to Senior Principal Scientific Officer and made Assistant Director, General Services. For the next six years he was responsible for services to the general public and to agriculture and it was during this period that Weather Centres were opened in Manchester, Glasgow and Southampton and that the volume and type of services available to the public were greatly expanded.

With the reorganization of some branches of the Office in 1966 Mr Harding was made responsible for hydrometeorology and agriculture and he remained in this post until his retirement, building a high reputation among hydrologists at home and abroad. He was appointed an Officer of the Order of the British Empire in 1972.

John Harding's considerate, sympathetic and kindly approach to every task or problem was widely appreciated and not least by the Staff General Purposes Committee, which co-ordinates the social and sporting activities of the Office, and of which he was Chairman from 1967 to 1973.

We wish him and Mrs Harding many years of happy retirement.

J. K. BANNON



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NOTICES

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