The Procedures for Objective Analysis of Height and Humidity Data for the 10 level model

1. Introduction

The initial data required by the 10 level model consists of grid point values of contour height and wind components for the ten levels spaced at 100mb intervals from 1000mb to 100mb, and grid point values of humidity mixing ratio representing the vertical mean over each of the seven lowest layers of the model. The model assumes that the humidity mixing ratio is zero for the top two layers.

This note describes the procedures used for the objective analysis of contour height and relative humidity for both the rectangle (fine mesh) and the octagon (coarse mesh) versions of the model. The analysis area for the rectangle has dimensions 66x50, and that for the octagon contains 3209 points, having 63 points across the diameter. The wind is derived from the height fields by solving the balance and omega equations; the humidity mixing ratio is obtained from the relative humidity and the partial thicknesses derived from the analysed height fields. These processes take place during the initialisation. Relative humidity is analysed rather than humidity mixing ratio as it is a more meaningful quantity in relation to synoptic features such as depressions, anticyclones and fronts.

The objective analysis procedure uses upper air and surface observations of heights winds and relative humidities and first guess grid point values for all fields being analysed. The observations undergo a number of quality controls such as the hydrostatic check before being used in the analysis programmes.

2. Initial Background Fields

The initial first guess (background) fields are normally modified 12hr forecast fields, which are derived by programmes separate from the analysis programme. For both the octagon and the rectangle the modified 12hr forecast field is assumed to be an adequate first guess in the interior of the field, but in both cases boundary effects make the outer few rows of grid points unreliable. In the case of the octagon, the boundary values remain fixed throughout the forecast period and, despite a region of high diffusion extending to the outer three rows of grid points, the resultant inaccuracies can be expected to penetrate some distance into the field. The octagon forecast is at present computed first, and the rectangle boundary values are modified during the forecast by using hourly tendencies derived from the octagon forecast. However, this does not ensure that the resultant boundary values will be accurate, and there is once again a region of high diffusion to limit the extent of these inaccuracies. The field over which the analysis is performed for both the octagon and rectangle contains an extra row of grid points round the perimeter of the forecast area and not even a boundary value of a forecast field is available at these points. These additional grid points have to be present in the background field, and therefore have to be generated in some way.

To create the initial background field of heights for the octagon, the outer two rings of grid points of the forecast field are removed. These are then replaced, one ring of grid points at a time by extrapolation from the interior of the field, first in the x direction, then in the y direction, using climatological gradients at each point; where a value may be obtained by both x and y extrapolation, the mean of the two resultant values is taken. The extra ring of grid points, present only in the analysis area, is obtained by extrapolation as above, but using half the climatological gradient. Finally the completed field is smoothed by (1-2-1) smoothing in both the x and y directions.

The procedure to obtain the initial background field to the rectangle analysis is as follows. First the rectangle 12hr forecast field is smoothed by (1-2-1) smoothing in both the x and y direction and then grid point values are computed from the octagon 12hr forecast field at the outer eight rings of the rectangle analysis area by linear interpolation from the octagon grid. The outer eight rows of the rectangle forecast field are then merged with the interpolated octagon field according to the following formula.

$$h_m = \beta h_o + (1-\beta) h_R$$

h_m = merged background height

h_o = interpolated octagon forecast height

h_R = smoothed rectangle forecast height

β varies with row and level according to the following table

Row	1	2	3	4	5	6	7	8
100mb	1.0	1.0	1.0	0.84	0.665	0.49	0.32	0.16
All other levels	1.0	1.0	1.0	0.80	0.62	0.47	0.32	0.16

Row number refers to the analysis area of the rectangle and is (G+1), where G is the number of grid lengths from the nearest edge.

The forecast produces grid point values of humidity mixing ratio at the seven lowest layers of the model. Using the forecast partial thicknesses, the forecast relative humidity field is derived. To produce background fields for the octagon the forecast area of the field is left untouched.

Values at the grid points outside the forecast area are taken to be the same as that of the nearest grid point on the boundary of the forecast area. If two of these points are equi-distant from a point outside, then a mean is taken.

For the rectangle, the relative humidity background field is derived from the octagon and rectangle forecasts in exactly the same way as the height field.

3. Objective analysis of height fields

(1) General

The height analysis procedure uses upper air observations at 1000mb, 850mb, 700mb, 500mb, 400mb, 300mb, 200mb, 100mb; surface observations and background height fields. The procedures for both the octagon and rectangle are similar to that described by Bull (1966), but there are some differences in detail, and there are also small differences between the scheme for the rectangle and octagon. In addition it is possible to introduce artificial observations at the surface, 1000, 500, 300 and 100mb levels and to delete or correct actual observations, should it appear necessary.

At every level there are several "scans" across the whole field before the final analysis is produced. The procedure is the same for each scan, a quadric surface being fitted to the data and background field around each grid point independently.

The levels are analysed in four groups in the following order:-

(1) 100mb

(2) 300, 500, 1000mb (upper air data only)

(3) 200, 400, 700, 850mb

(4) 1000mb using surface data

(2) Quadric Fitting

To obtain the value of the height field at any level at a grid point, the field in the neighbourhood of that point is represented by

$$H_A = ax^2 + by^2 + 2hxy + 2gx + 2fy + c$$

where (x, y) are coordinates in number of grid lengths (dimensionless) in the stereographic plane with origin at the grid point. The following expression for E is then minimized with respect to variation in a, b, h, g, f, c

$$E = \sum \{p(H_A - H_o)^2\} + \sum \{q(H_A - H_B)^2\} + \sum \{p(\underline{V}_A - \underline{V}_o)^2 S^2\} \qquad (1)$$

where H_o = observed height at observation point in geopotential metres

H_B = background field height in geopotential metres

$\underline{V}_o$ = observed wind at observation point in metres sec^{-1}

$\underline{V}_A$ = geostrophic wind at observation point in metres sec^{-1} derived from H_A

$$\underline{V}_A = \left(\beta g / af \left[-\frac{\partial H_A}{\partial y}, \frac{\partial H_A}{\partial x} \right] \right)$$

f = Coriolis parameter

β = map magnification factor

a = grid length at North Pole

$p = 1/(1+p'r^n)$ = distance weighting factor for observations

r = distance of observation from grid point measured in number of grid lengths

q = weight of background heights

$S = T \sin\phi (1+\sin\phi)$ (ϕ = angle of latitude)

T = weighting factor of winds relative to heights (units of time)

H_A = height at observation point calculated from the quadric.

p', n, q, T^2 vary with scan and level and the values are given in the Appendix. T is scaled by $\sin\phi(1+\sin\phi)$ so that gradients of the height field, instead of winds are effectively being fitted. This is because $\beta g/af$ can become large near the equator.

The summations in the first and third terms on the right hand side of equation (1) are taken over the nearest six (ten at the surface) stations, with height or wind data at the level being analysed, within G grid lengths of the grid point. These stations are drawn from the "married list" of observations which is formed from the following three sets of data:

(1) a predetermined list of up to twelve of the nearest land stations within twelve octagon (thirty-six rectangle) grid lengths of the grid point.

(2) All ship data within G grid lengths of the grid point.

(3) Airep data. These are wind only data and are used only for the 300mb analysis where the observation is made between 260 and 340mb, within 3hrs of the synoptic hour and within G grid lengths of the grid point.

These three sets of data are merged together to form the married list of the nearest observations (up to twelve in number) within G grid lengths of the grid point. Since aireps are wind data only, they are not permitted to replace the nearest three upper air observations in the list, so that there is likely to be observed height data available for the quadric fitting procedure. The married list contains up to twelve observations so that it is probable that at any level six (ten at the surface) will be found with data. G varies with level and scan and is tabulated in the Appendix.

The summation in the second term on the right hand side of equation (1) is taken over nine grid points of the background field. When the background field for the octagon is read into the analysis programme, it is expanded into a set of grid point values over a 63x 63 square by extending any row shorter than 63 grid points along the x direction, copying the boundary value into each of the extra points created. Then if the grid point is not on the edge of the 63x63 square the nine points are as indicated in Fig. 1(a), and if it is see Fig.1(b). This means that a grid point lying on an edge of the octagon analysis area which slopes at 45° relative to the coordinate axes, will use three of the created background points from outside the analysis area.

For the rectangle, nine background points are still used, but they are on a grid of the octagon grid length. See Fig. 1(c). Similarly Fig. 1(b) applies in the same way as (a) if the point under consideration is within three rectangle grid lengths of the edge of the grid and (d) applies if the point is within three grid lengths of two of the edges simultaneously.

(Fig. 1(d) is the case where the point is at the top left hand corner of the grid.)

Minimising E with respect to variations in a, b, c, h, g, f

ie $$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = \frac{\partial E}{\partial c} = \frac{\partial E}{\partial h} = \frac{\partial E}{\partial g} = \frac{\partial E}{\partial f} = 0$$

yields six linear simultaneous equations in these six unknowns, which are then solved by the method of pivotal condensation. This gives a value for H_A at the origin, which is the grid point being analysed.

Where no height observations are found within G grid lengths of the grid point, the height is set equal to the background field value at that grid point.

It has been found that with only one observation of height, the surface fitted tends to be an unreliable estimate of the field in the region of the grid point and so in this case the following formula is used.

$$H_A = \frac{pH_0 + qH_B}{p+q} \qquad (1a)$$

where p is the distance weighting factor for the observation

H_o is the observed height

q is the weight of the background field at the grid point being analysed

H_B is the value of the background field at the grid point being analysed.

(3) First and Second Scans including Data Check

The first scan is used to produce a preliminary analysis for checking the horizontal consistency of the data.
After this scan has been made observations are compared with the analysis and where a difference exceeds a criterion dependent upon the level being analysed (see Appendix) the observation of height or wind is rejected. There are also differences in the height criteria between ships and land stations which are in sparse data areas, and stations in dense data areas. Observations from the former are allowed to differ from the first scan by a larger amount than observations from the latter. This is to avoid the rejection of a correct piece of data which may be the first indication of a new synoptic feature not present in the forecast field. Also a less strict criterion is allowed for artificial upper air height observations.
In order to compare an observation with the analysis produced by the first scan the heights and geostrophic winds are interpolated linearly from the four surrounding grid points. If O is an observation (see Fig.2) surrounded by four heights h_5, h_6, h_9, h_8 and R and S are the distances from h_5 as shown, then

$$h_o = (1-R)(1-S)\,h_5 + R(1-S)h_6 + (1-R)S\,h_8 + RS\,h_9 \qquad (2)$$

$$u_o = -\beta g/af \frac{\partial h}{\partial y}$$
$$= -\beta g/af\left\{R(h_6 - h_9) + (1-R)(h_5 - h_8)\right\} \qquad (3)$$

$$v_o = \beta g/af \frac{\partial h}{\partial x}$$
$$= \beta g/af\left\{S(h_9 - h_8) + (1-S)(h_6 - h_5)\right\} \qquad (4)$$

In general if a height or wind observation is rejected for a particular level it is removed from the data for all subsequent scans for that level. However in order to improve the vertical consistency of the final analysis a height observation rejected at any of the levels 300, 500, 1000mb, causes the observation to be rejected at all levels except 100mb.

Having removed data rejected by the first scan a second scan is made using the same set of constants and background field as in the first scan.

(4) Third Scan including Curvature Correction

In the quadric fitting procedure the winds which are fitted are assumed to be geostrophic. In the first two scans the observed winds are used, but these lead to some errors caused by the geostrophic approximation. After the second scan a correction is made to the observed winds by assuming that they are gradient winds and replacing them in the quadric fitting procedure by the equivalent geostrophic wind using the formula

$$V_{geos} = V_{grad} \left(1 + \frac{V_{grad}}{R f} \right)$$

where R is the radius of curvature of the contour which is an approximation to the radius of curvature of the trajectory at the observation. To derive R at the observation points, a field of R is generated at the grid points using the second scan of the height analysis in the following way.

If $h(x,y)$ is a field of heights, then the radius of curvature of a contour $h(x,y) =$ constant is given by

$$\frac{1}{R} = - \frac{d^2y/dx^2}{\left(1 + (dy/dx)^2\right)^{3/2}} \qquad (5)$$

($h(x,y)$ = constant implies $y = g(x)$)

Using the finite difference form for the derivatives (see Fig. 2).

$$\left(\frac{\partial h}{\partial x}\right)_5 = \frac{h_6 - h_4}{2\ell}, \quad \left(\frac{\partial h}{\partial y}\right)_5 = \frac{h_2 - h_8}{2\ell} \qquad \text{etc.}$$

where $l = \alpha/\beta$ = grid length in metres, yields the formula

$$\left(\frac{l}{R}\right)_5 = \Big[2\{(h_6-h_4)^2(h_2+h_8-2h_5)+(h_2-h_8)^2(h_4+h_6-2h_5)\} - (h_6-h_4)(h_2-h_8)(h_7+h_3-h_1-h_9)\Big] \Big/ \{(h_6-h_4)^2+(h_2-h_8)^2\}^{3/2} \qquad (6)$$

(h_i refers to the height at grid point i in Fig. 2)

If $|l/R| > 10$ (i.e. if the radius of curvature is less than one tenth of a grid length) then l/R is set equal to ± 10 depending on whether l/R is positive or negative. The value of R at an observation is determined from the grid point field of R by linear interpolation from the four nearest grid points. The method of interpolation is identical to that used for heights in the data check. For the upper air levels, to prevent excessive corrections to the observed wind the factor

$$(1 + V_{grad}/Rf)$$

is only allowed to have values between 0.75 and 1.75; if it is outside this range it is set to the nearer of these two limits. At the surface these limits are 0.5 and 2.5 respectively, since air trajectories at the surface tend to have larger curvature.

A third scan is then made through the field using the grid point field resulting from the second scan as a background field. The purpose of the second scan was to obtain the correct general height level, and the purpose of the third scan is to fit the winds more closely and for the rectangle particularly to obtain a smooth field. The latter are accomplished by the use of a distance weighting factor which decreases more quickly with distance from the observation than that used in the second scan. This means that the quadric is effectively being fitted over a smaller area surrounding the grid point and since a height field only approximates to a quadric over a limited area the data near the grid point will be fitted more closely, giving a better approximation to the true height at the grid point. This also means that the gradient between adjacent grid points will be nearer the true value so that the resulting analysis fits the winds more closely. Also on the fine mesh it avoids roughnesses in the field caused by different shaped surfaces being badly fitted to slightly different sets of data around adjacent grid points.

On the octagon at 100mb the distance weighting factor is the same for each scan as this has been found to give a satisfactory analysis in operational use.

(5) Details related to specific groups of levels

(a) 100mb

The 100mb analysis presents problems specific to that level; the height observations are subject to quite substantial random and systematic errors which cause them to be inconsistent with the wind observations. However, the field tends to change more slowly than at other levels so that the 12hr forecast field can be treated as quite a good first guess to the analysis.

To take account of the systematic errors in the height observations, all ascents are corrected using the British radio sonde as standard. Proportions of these systematic errors are also applied to the levels 500, 400, 300, 200mb.

After the sonde correction, there are three scans at 100mb as at all other groups of levels. Then, because of the inconsistencies between the height and wind data, there is a fourth scan at 100mb using wind data only, before the final analysis is produced.

After the final (fourth) scan all the height observations within the analysis area are compared with the analysis field and the differences are presumed to represent the random errors. If an error exceeds 40 decametres the height observation is rejected at all subsequent groups of upper air levels. Otherwise the ascent is adjusted using the following constants, down to 500mb

$$h'_l = h_l + \beta_l R_E \qquad (7)$$

h_l = height at level l β_l = constant for level l

h'_l = adjusted height at level l R_E = 100mb random error

Level	200mb	300mb	400mb	500mb
β_l	0.71	0.45	0.31	0.17

NB the same constants are used to make the systematic corrections.

(b) 1000mb (upper air), 500mb 300mb

These three levels are analysed together with the usual three scans. If there is no 1000mb wind present on the ascent, it is computed from the following formula:-

$$\underline{v}_{1000} = \underline{v}_{500} - 1{\cdot}1484(\underline{v}_{500} - \underline{v}_{850}) \qquad (8)$$

The background field point corresponding to the grid point under consideration is given eight times the weight of the other eight background points in the second term in E. This is to try and fit detail which is only contained in the forecast field.

(c) 200, 400, 700, 850mb

There is no 12hr forecast field available at 850mb to be used as background field for the first two scans, so this is produced by taking the mean of the 800 and 900mb forecast fields at each grid point. Then the twelve hour forecast fields at these levels are adjusted according to the following formulae.

Let h^B_n = 12 hour forecast

h'^B_n = adjusted 12 hour forecast

h^A_n = analysed height fields

$$h'^B_{200} = h^B_{200} + \tfrac{1}{2}\left\{(h^A_{100} - h^B_{100}) + (h^A_{300} - h^B_{300})\right\} \qquad (9)\ (a)$$

$$h'^B_{400} = h^B_{400} + \tfrac{1}{2}\left\{(h^A_{300} - h^B_{300}) + (h^A_{500} - h^B_{500})\right\} \qquad (9)\ (b)$$

$$h'^B_{700} = h^B_{700} + \tfrac{3}{5}(h^A_{500} - h^B_{500}) + \tfrac{2}{5}(h^A_{1000} - h^B_{1000}) \qquad (9)\ (c)$$

$$h'^B_{850} = h^B_{850} + \tfrac{7}{10}(h^A_{1000} - h^B_{1000}) + \tfrac{3}{10}(h^A_{500} - h^B_{500}) \qquad (9)\ (d)$$

These adjustments to the initial background fields are made for two reasons;

(a) to improve the vertical consistency between the eight analysed levels of the model;

(b) to ensure that artificial data, which is only inserted at the four monitored levels of the model - viz 1000mb, 500mb, 300mb, 100mb - has some effect at the remaining four levels.

There are three scans at this group of levels, and the background field point corresponding to the grid point under consideration is given sixteen times the weight of the remaining eight background points, so that the modifications to the initial background field will be reflected to some extent in the final analysis.

(d) Surface Analysis

Finally the 1000mb field is analysed using surface reports, with the 1000mb upper air analysis as initial background field. The surface pressure reports are converted to 1000mb heights using the formula

$$h_{1000} = 0.0029259\,(p - 10000)\,T_S \qquad (10)$$

T_S = surface temperature in degrees absolute

p = surface pressure in tenths of a millibar

Wind reports are used from ships only, since those from land stations are assumed not to give an accurate indication of the gradient.

There are five scans altogether at the surface of which the first three are the usual quadric fitting procedure; ten observations with data instead of six, are used in the summations in E (see equation (1)), to overcome the reduction in data caused by the neglect of wind observations from land stations. The background point corresponding to the grid point under consideration is given a weight of eight times that of the other eight points.

The last two scans are "plane-fitting" instead of quadric fitting, as it has been found that the quadric fit procedure tends to underestimate the depth of surface lows quite considerably.

The procedure in the plane fitting scans is as follows; given an observation of h (height), u, v (x, y components of wind) at (x, y) relative to a grid point, then extrapolating the observed gradient to the grid point yields a value of h_G

$$h_G = h + {af}/{\beta g}\,(uy - vx) \qquad (11)$$

h_G = height at grid point β = map magnification factor

a = grid length at North Pole

f = coriolis parameter

From the collection of observations in the married list a weighted mean h'_G is obtained where

$$h'_G = \sum p\, h_G / \sum p \qquad (12)$$

For the octagon:- $p = (1/(1+\beta^{-n} p' r^{n}))(1/(1+2(\underline{r}.\underline{v})^2/\beta^2 v^2))$

For the rectangle:- $p = (1/(1+p'\beta^{-n} r^{n}))(1/(1+2(\underline{r}.\underline{v})^2/9\beta^2 v^2))$

$\underline{r}$ is the position vector of the observation relative to the grid point on the stereographic plane $\underline{v}$ is the wind vector at the observation point. p' is as tabulated in the Appendix. The background field for each of the plane-fitting scans is the analysis from the previous scan. Writing h_B as the background field value at a grid point

$$h_A = \frac{K h'_G + h_B}{1+K} \qquad (13)$$

where

$$K = \sum p / 1{\cdot}5$$

if $|h_B - h'_G| > 150$, then K is multiplied by

$$\frac{150}{|h_B - h'_G|}$$ in the above formula.

If an observation in the married list has no height it is ignored; if it has no wind, then the value of h_G given by that observation is

$$h_G = h_B + (h_O - h_{INT}) \qquad (14)$$

where h_{INT} = height interpolated from the analysis field (see data check)

h_O = height at the observation

h_B = background height

The weighting factor

$$p = 1/(1 + p' r^n \beta^{-n})$$

At the end of the second "plane-fitting" scan at the surface, there are now height fields at

1000(surface and upper air), 850, 700, 500, 400, 300, 200, 100mb.

To obtain the fields at 900, 800, 600mb the following interpolation formulae are used

$$h_{900} = 0.3714\, h^{(u)}_{1000} + 0.6286\, h_{850} + 30.65 \qquad (15)\ (a)$$

$$h_{800} = 0.7051\, h_{850} + 0.2949\, h_{700} + 34.615 \qquad (15)\ (b)$$

$$h_{600} = 0.566\, h_{700} + 0.434\, h_{500} + 83.0 \qquad (15)\ (c)$$

Then the difference between the 1000mb (surface analysis) and the 1000mb (upper air analysis) is calculated at every grid point.

Finally the upper levels are corrected according to the following formula

$$h'_n = h_n + \frac{(n-1)}{9}\left(h^{(s)}_{1000} - h^{(u)}_{1000}\right) \qquad (16)$$

h_n = original value of height at n x 100millibars

h'_n = corrected value of height at n x 100millibars

$h^{(u)}_{1000}$ = Upper air 1000mb analysis

$h^{(s)}_{1000}$ = Surface 1000mb analysis

These corrected fields are the final output of the analysis programme at the ten levels.

4. Objective Analysis of the Fields of Relative Humidity

(1) General

An analysis of relative humidity is required for the seven 100mb layers centred at 950, 850,........350mb. The procedure is the same for both the rectangle and octagon versions of the 10 level model.

The analysis makes use of upper air and surface relative humidity data together with 12hour forecast fields of relative humidity. The forecast gives fields at the seven layers required but the only upper air relative humidity observations which are used are at the standard levels 1000, 850, 700, 500, 400, 300mb. Rather than analyse data at the standard levels and interpolate the forecast fields, the layers required are analysed directly and the upper air observations are interpolated in the vertical from the standard levels to the layers required by the model before the horizontal analysis takes place. The following formulae are used:-

$R_{950} = \frac{2}{3} R_{1000} + \frac{1}{3} R_{850}$ or $R_{950} = R_{1000}$ if R_{1000} available and R_{850} missing

$R_{850} = R_{850}$

$R_{750} = \frac{1}{3} R_{850} + \frac{2}{3} R_{700}$ or $R_{750} = R_{700}$ if R_{700} available and R_{850} missing

$R_{650} = \frac{3}{4} R_{700} + \frac{1}{4} R_{500}$ or $R_{650} = R_{700}$ if R_{700} available and R_{500} missing (17)

$R_{550} = \frac{1}{4} R_{700} + \frac{3}{4} R_{500}$ or $R_{550} = R_{500}$ if R_{500} available and R_{700} missing

$R_{450} = \frac{1}{2} R_{500} + \frac{1}{2} R_{400}$ or $R_{450} = R_{500}$ if R_{500} available and R_{400} missing

or $R_{450} = R_{400}$ if R_{400} available and R_{500} missing

$R_{350} = \frac{1}{2} R_{400} + \frac{1}{2} R_{300}$ or $R_{350} = R_{400}$ if R_{400} if R_{400} available and R_{300} missing

or $R_{350} = R_{300}$ if R_{300} available
and R_{400} missing

or $R_{350} = R_{500}$ if R_{500} available and
both R_{400} and R_{300} missing

In all other cases the interpolated relative humidity is set as missing data.

Each layer is analysed independently and like the height analysis the analysis of a given layer consists of a number of scans through the field in which values are obtained for the relative humidity at each grid point from the observations surrounding that grid point together with a background field. All the layers are first analysed using upper air data only. Surface data is used in the 950mb analysis at a later stage. Two scans are made through the field for each layer. In the first scan the background field is the modified 12hour forecast and in the second scan the analysed field resulting from the first scan. is used.

(2) Method of analysis for each grid point

The value of the relative humidity at each point is found by taking a weighted mean of the surrounding observations together with the value of the background field at the grid point concerned. The following formula is used:-

$$R_g = \left(\sum_i p_i R_i + q R_{bg}\right) \Big/ \left(\sum_i p_i + q\right) \qquad (18)$$

where the sum is taken over the six nearest observations within G grid lengths. The value of G (see Appendix) depends on the scan number and the grid length.

R_g is the analysed value at the grid point

R_{bg} is the value of the background field at the grid point being analysed

q is the weight of the background field

R_i is the i^{th} observation of relative humidity

p_i is the weighting factor for the i^{th} observation.

$$p_i = \frac{1}{1 + p' |\underline{r}_i|^n + 1000\,((\underline{\nabla} R_b)_i \cdot \underline{r}_i)^2}$$

where

$\underline{r}_i = (x_i, y_i)$ is the position vector of the i^{th} observation relative to the grid point being analysed in units of grid length.

$(\underline{\nabla} R_b)_i$ is the interpolated value at the i^{th} observation point of the gradient of the background field.

$$(\underline{\nabla} R_b)_i \cdot \underline{r}_i = x_i \left(\frac{\partial R_b}{\partial x}\right)_i + y_i \left(\frac{\partial R_b}{\partial y}\right)_i$$

where

$$\left(\frac{\partial R_b}{\partial x}\right)_0 = S\,(R_{b_9} - R_{b_8}) + (1-S)(R_{b_6} - R_{b_5})$$

$$\left(\frac{\partial R_b}{\partial y}\right)_0 = R\,(R_{b_6} - R_{b_9}) + (1-R)(R_{b_5} - R_{b_8})$$

$R_{b_5}, R_{b_6}, R_{b_9}, R_{b_8}$ are the values of the background field at the four grid points surrounding the observation (see Fig. 2) R, S are the distances in the x and negative y directions from the point R_{b_5} 0 is the observation.

The effect of the term $p' |\underline{r}_i|^n$ in the weighting factor is to weight nearer observations more heavily than farther ones. n varies with the scan number being equal to 4 in the first scan and 8 in the second. This results in the weighting factor decreasing more quickly with distance in the second scan. The first scan is intended to give the general level of relaive humidity and the second to fit the detail. The value of p' depends on the scan number and grid length and is given in the Appendix. The gradient term $1000\,(\underline{\nabla} R_b)_i \cdot \underline{r}_i$ has the effect of weighting an observation more heavily in a direction parallel to the isopleths of the background field than in a direction perpendicular to them, and as the term depends on the magnitude as well as the direction of the gradient, this effect is more pronounced when the isopleths are closely packed. In this way the shapes of features in the forecast field such as fronts which show up as a narrow band of wet air may be preserved in the final analysis. Even if the forecast front is slightly displaced, as long as its orientation is correct this procedure will help to analyse the front in the correct position if there is at least one observation in the frontal zone.

As already stated this procedure of finding a weighted mean of the observations surrounding a grid point is executed for all the points of the field to form a scan. Two scans are made through the field for all layers simultaneously using upper air relative humidity data only, the background field for the second scan being the result of the first.

(3) <u>950mb Layer</u>

Two further scans are then made over the 950mb layer using surface relative humidity data derived from the surface temperature and dew point. The procedure is the same as that for the upper air data. The background field for the first surface scan is the 950mb upper air analysis, and that for the second scan is the field resulting from the first surface scan. No adjustment based on the final 950mb analysis is made to the upper layers as the vertical structure of relative humidity is so variable. The final analysis therefore consists of the upper air analysis for the layers from 850mb to 350mb and the 950mb analysis based on surface data.

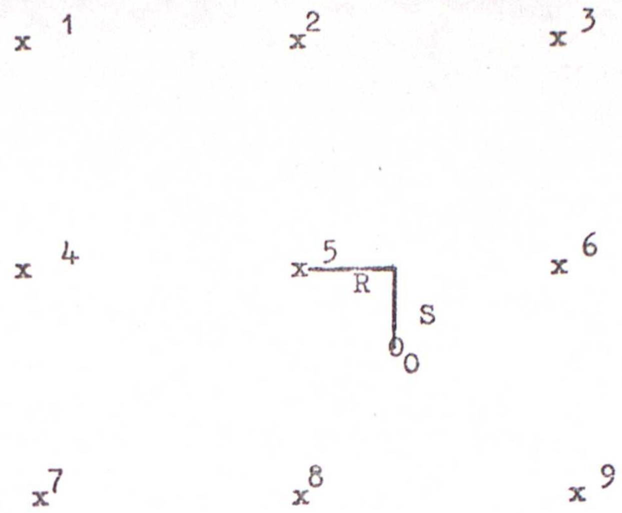

Fig. 2

The points numbered 1 to 9 are any set of 9 grid points in the field. O is an observation point with coordinates (R,S) in the x and negative y direction relative to point 5.

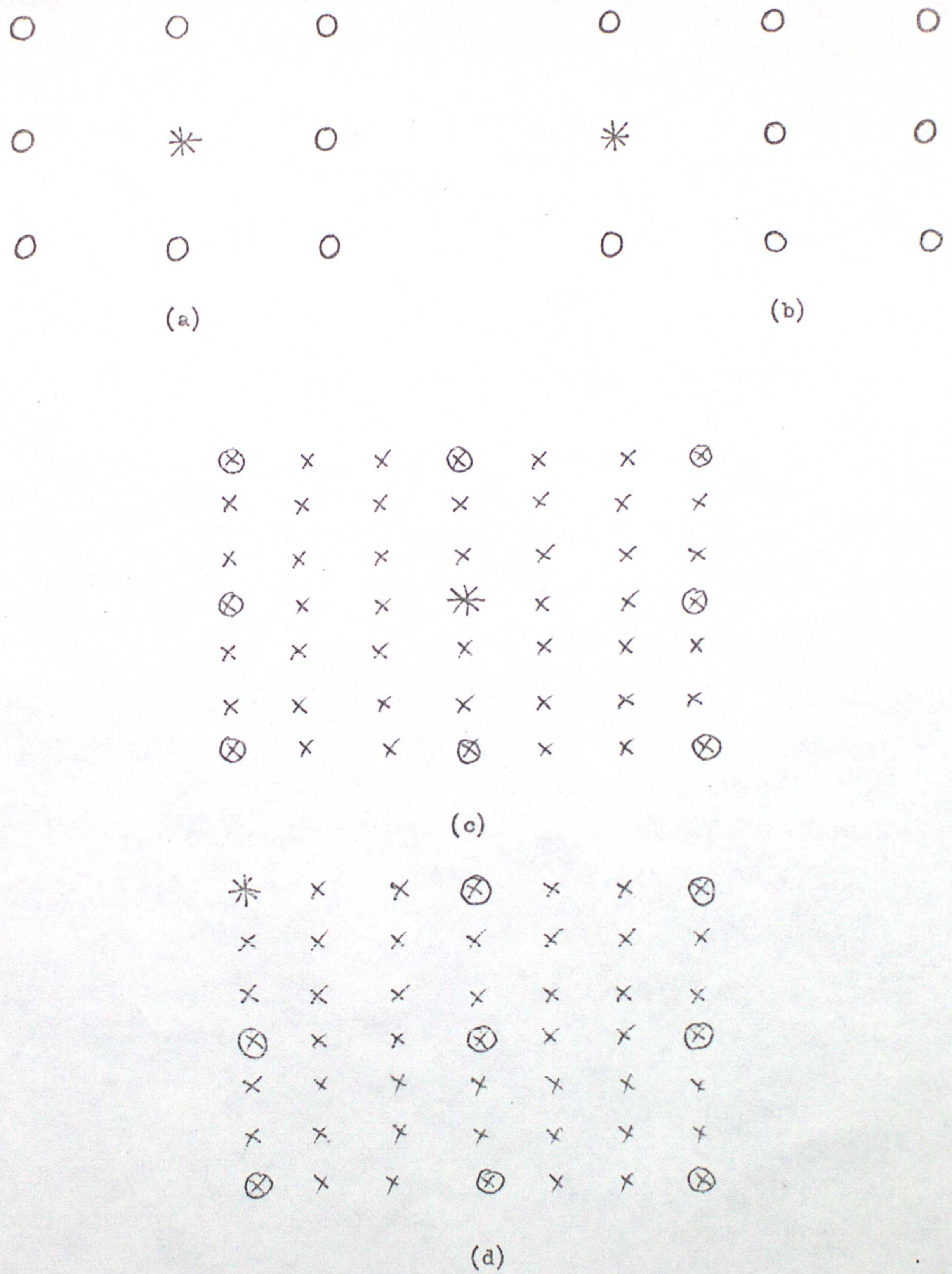

Fig. 1

✳ = Grid point being analysed

O = Background points used

× = Rectangle background points

APPENDIX

Octagon 100mb

Scan	1	2	3	4
p'	0.0116411532	0.0116411532	0.0116411532	0.0116411532
q	0.125	0.125	0.125	0.125
T^2	16	16	16	16
n	4	4	4	4
G	12	12	6	6
Background	12hr F/C	12hr F/C	2nd Scan	2nd Scan

Remaining U/A levels

Scan	1	2	3
p'	0.001	0.001	0.1
q	0.0625	0.0625	0.0625
T^2	16	16	64
n	8	8	8
G	12	12	6
Background	12hr F/C	12hr F/C	2nd Scan

G is in grid lengths

Surface

Scan	1	2	3	4	5
p'	0.001	0.001	0.1	0.186	1.86
q	0.0625	0.0625	0.0625	---	--
T^2	16	16	64	---	--
n	8	8	8	4	4
G	12	12	6	6	6
Background	1000mbU/A	1000mbU/A	2ndScan	3rdScan	4thScan

Rejection Criteria

Heights

Observation rejection if $\left| \text{Height (observed)} - \text{Height Calculated} \right| >$ X metres

	X
Land data (U/A except 100mb)	60
Land data 100mb	70
Sparse data areas U/A	120
Surface	100
Artificial observations U/A	240

Winds

Upper Air — Observation rejected

for $V_{obs} < 40\ \text{msec}^{-1}$ if $(V_{cal} - V_{obs})^2 > 400\text{m}^2\text{sec}^{-2}$

$40\ \text{msec}^{-1} \leqslant V_{obs} \leqslant 80\ \text{msec}^{-1}$ $(V_{cal} - V_{obs})^2 > \frac{1}{4}V^2_{obs}$

$V_{obs} > 80\text{msec}^{-1}$ $(V_{cal} - V_{obs})^2 > 1600\text{m}^2\text{sec}^{-2}$

Surface $(V_{cal} - V_{obs})^2 > 225\text{m}^2\text{sec}^{-2}$

Rectangle

100mb

Scan	1	2	3	4
p'	1.234567×10^{-3}	1.234567×10^{-3}	1.524158×10^{-7}	1.524158×10^{-5}
q	0.125	0.125	0.125	0.125
T^2	16	16	16	64
n	4	4	8	8
G	36	36	36	36
Background	12hr F/C	12hr F/C	2nd Scan	3rd Scan

Remaining U/A levels

Scan	1	2	3
p'	1.524158×10^{-7}	1.524158×10^{-7}	1.524158×10^{-5}
q	0.0625	0.0625	0.0625
T^2	16	16	64
n	8	8	8
G	36	36	36
Background	12hr F/C	12hr F/C	2nd Scan

G is in grid lengths

Surface

Scan	1	2	3	4	5
p'	1.524158×10^{-7}	1.524158×10^{-7}	1.524158×10^{-5}	2.296×10^{-3}	2.296×10^{-2}
q	0.0625	0.0625	0.0625	--	--
T^2	16	16	64	--	--
n	8	8	8	4	4
G	36	36	36	36	36
Background	1000mbU/A	1000mbU/A	2nd Scan	3rd Scan	4th Scan

Rejection Criteria

Heights

Observation rejected if $\left| \text{Height (observed)} - \text{Height (calculated)} \right| > X$ metres

	X
Land data (U/A levels except 100mb)	60
Land data (100mb)	70
Sparse data (U/A levels except 100mb)	120
Sparse data (100mb)	90
Surface	70
Artificial observations U/A	240

Winds

Upper air — Observation rejected

for $V_{obs} < 40 \text{ msec}^{-1}$ if $(V_{cal} - V_{obs})^2 > 400 \text{ m}^2\text{sec}^{-2}$

$40 \leqslant V_{obs} \leqslant 80 \text{ msec}^{-1}$ $(V_{cal} - V_{obs})^2 > \frac{1}{4} V_{obs}^2$

$V_{obs} > 80\text{msec}^{-1}$ $(V_{cal} - V_{obs})^2 > 1600 \text{ m}^2\text{sec}^{-2}$

Surface

$$(V_{cal} - V_{obs})^2 > 225\text{m}^2\text{sec}^{-2}$$

Humidities

Scan	Rectangle 1	Rectangle 2	Octagon 1	Octagon 2
p'	0.01	0.01	0.81	65.61
q	0.2	0.2	0.2	0.2
n	4	8	4	8
G	9	3	3	1
Background field U/A	12hr F/C	1st Scan	12hr F/C	1st Scan
Background field surface	950mb U/A	1st Scan surface	950mb U/A	1st Scan surface

CONSTANTS

R (earth's radius)	=	6367.3 km	
a (grid length at pole)	=	326,683.635 m	(Octagon)
	=	108,894.545 m	(Rectangle)
g	=	$9.8\ \text{msec}^{-2}$	
π	=	3.14159	
$\frac{a^2}{8R^2}$	=	$10^{-4} \times 3.29043$	(Octagon)
	=	$10^{-4} \times 0.365604$	(Rectangle)
$\frac{\beta g}{af}$	=	$\frac{0.412508}{\sin \phi(1+\sin\phi)}\ \sec^{-1}$	(Octagon)
	=	$\frac{1.237525}{\sin\phi(1+\sin\phi)}\ \sec^{-1}$	(Rectangle)

where β = map magnification factor

$$= \frac{2}{1+\sin\phi}$$

ϕ = latitude

1 knot = 1.9425 metres $\sec^{-1}$

Useful Formulae

1) Latitude ϕ and longitude θ in terms of x, y of grid

$$(1+\sin\phi)^{-1} = 0.5 + \frac{a^2(x^2+y^2)}{8R^2}$$

$$\tan(\theta - 55°) = \frac{y}{x}$$

2) Wind components u , v interms of speed V and direction α

$$u = -V\sin(\alpha - (\theta + 35))$$

$$v = -V\cos(\alpha - (\theta + 35))$$

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