

Library

MET.O.14

METEOROLOGICAL OFFICE
BOUNDARY LAYER RESEARCH BRANCH
TURBULENCE & DIFFUSION NOTE



T.D.N. No. 63

0118885

BRIEF NOTES ON THE SPECIFICATION OF THE BOUNDARY LAYER IN TERMS
OF EXTERNAL READILY ASSESSED PARAMETERS.

by

F.B.Smith

March 1975

Please note: Permission to quote from this unpublished note should be
obtained from the Head of Met.O.14, Bracknell, Berks., U.K.

BRIEF NOTE ON THE SPECIFICATION OF THE BOUNDARY LAYER
IN TERMS OF EXTERNAL READILY ASSESSED PARAMETERS.

by F.B. Smith Met O 14

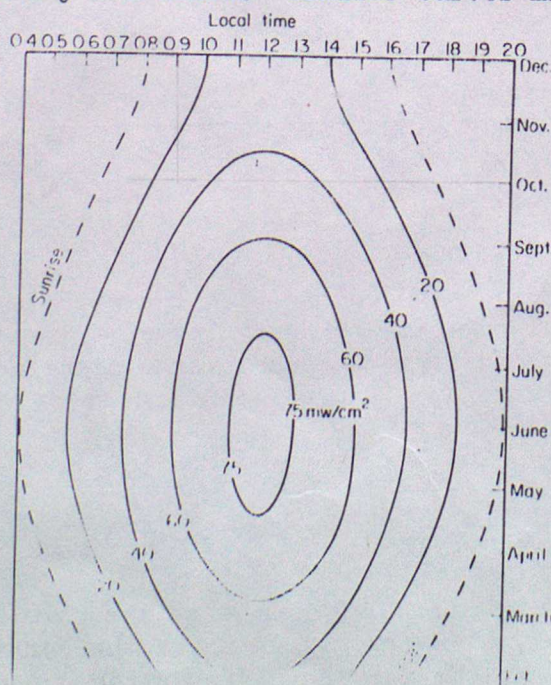
The following notes inevitably simplify the true complexity of the problem of specifying the structure of the boundary layer solely in terms of readily assessed "external" parameters. However they do provide a means of providing an informed estimate of the most important aspects of the structure when more elaborate methods are inconvenient or impossible, and apply to land surfaces. An example is worked through to provide estimates of structure and diffusion properties.

The external parameters we consider are

- (i) solar elevation expressed if required in terms of month and time of day.
- (ii) cloud cover.
- (iii) the albedo of the surface and the available moisture (in rather broad terms).
- (iv) the roughness characterised by the type of surface and its orography.
- (v) the pressure gradient (or geostrophic wind $G \equiv \frac{1}{\rho f} \frac{dp}{dy}$)
- (vi) the coriolis paramter $f = 2 \Omega \sin \phi$ (Ω = Earth's angular vel., ϕ = latitude : $f = 10^{-4}$ at $50^\circ N$)

1. Specification of the surface head flux

The first figure shows the incoming solar radiation (for a latitude of $52^\circ N$ in mw/cm^2 reaching the ground, assuming relatively unpolluted air, in terms of time of day and month. Similar curves may be drawn for other latitudes.



Reduction factors for cloud in eighths to the incoming solar radiation

Cloud amount	Multiply I.S.R. by:
0	1.00
1	0.89
2	0.81
3	0.76
4	0.72
5	0.67
6	0.59
7	0.45
8	0.23

The table above gives a very broad idea of the expected modification arising from cloud.

Contours of the incoming solar radiation in mw/cm^2 in clear sky conditions at latitude $52^\circ N$ as a function of time of day and month.

^{incoming?}
This increasing energy is subdivided into

- (i) back long wave radiation
- (ii) warming the soil (a soil heat flux)
- (iii) sensible heat flux into the atmosphere
- (iv) latent heat flux (evaporation of available water).

The last flux is very variable, being dominant in very damp ground conditions. It is also likely to be greater in early morning and in the growing season when the plant stomata are fully open.

The next Table gives some guidelines as to the likely magnitude of the sensible heat flux. More work is required on this particular aspect and it is hoped measurements specially made at Cardington will be able to help in clarifying the picture still further in the near future.

Table of typical values of sensible heat flux as a fraction of incoming solar radiation over various ground surfaces. The nett radiation (i.e. the effective incoming radiation) is less than the actual incoming solar radiation since some fraction of the latter will be reflected back from the surface. Snow, being white, is an effective reflector and has the smallest fraction in the above column. The nett radiation is absorbed by the surface layer; some of this energy is used in warming the ground, some in evaporation of water and some appears as a flux of sensible heat into the atmosphere above.

Ground type	Nett radiation Incoming solar radiation	Sensible heat flux		
		Incoming solar radiation		
		Daytime average	After rain	Dry conditions (no rain for 10 days)
Ocean	0.95	0.1	—	—
Tropical forest	0.90	0.2	0.1	0.4
Mixed agriculture	0.85	0.33	0.2	0.5
Grassland	0.80	0.4	0.2	0.6
Deserts	0.70	0.6	0.3	0.9
Snow	0.35	0.3	—	—

2. Specification of surface roughness.

Terrain is rarely homogeneous and when dealing with the structure of the atmosphere as deep as the usual day-time boundary layer it is necessary to average the drag effect of the underlying ground over a significant area. The averaging should be done in terms of an effective drag coefficient defined as

$$C_D = \frac{\tau}{\rho G^2} = \left(\frac{ku}{G \ln z/z_0} \right)^2$$

where τ is the surface shearing stress, G is the geostrophic wind and u is measured at some height z a few metres above the surface. averaging the respective values of C_D for the different components of the "landscape", weighted according to their respective areas. This has been done for the UK for 10 x 10 km squares.

The following Table gives typical values of z_0 and drag coefficient for various surfaces.

Table gives typical values of z_0 and C_D over normal countryside.

Table 1. Values of z_0 and the geostrophic drag coefficient C_D .

Type	z_0	C_D
Short grass	0.5 cm	1×10^{-3}
Long grass	3	1.3×10^{-3}
Root crops	10	1.6×10^{-3}
Large open fields (200-500 m)	10	1.6×10^{-3}
Smaller fields with hedges and trees	25	2×10^{-3}
Parkland with trees	30	2.1×10^{-3}
Suburban or village woodland	80	2.3×10^{-3}

The above assumes level, or nearly level, countryside. Hills and mountains are a difficult problem and the drag has to be assessed over large areas where the full effect of the form drag can be felt.

The following formula has been used:

$$z_0 = 0.2 \frac{\overline{\Delta h}^2}{d}$$

where $\overline{\Delta h}$ = average height range between peaks and valleys in the area
 d = average distance between peaks separated by valleys, or between successive ridges.

For mountainous areas an effective drag has to be associated with an area much larger than that of an individual mountain. A typical value for the Welsh hills is $C_D = 3.2 \times 10^{-3}$ which may be compared with an average value for English plains (using (b) above) of 1.9×10^{-3} .

3. Specification of the wind profile

(i) In neutral conditions

Smith's neutral wind profile model apparently gives satisfactory results. It deduces from the momentum equations that near the top of the boundary layer h :

$$\begin{aligned} \text{shearing stress} &= \tau(z) \propto (h-z)^2 \\ \sin \alpha &\propto (h-z) \end{aligned}$$

where α is the turning of the wind from the geostrophic direction. If u_* is the friction velocity and G the geostrophic wind, S_0 is the surface value of the $\sin \alpha$ and k is von Karman's constant (~ 0.4) then:

$$S_0^2 = \frac{1}{k} \frac{u_*}{G} \quad \text{where} \quad u_*^2 = \frac{\tau(0)}{\rho} \quad \text{by definition}$$

$$h = 2k \frac{u_*}{f} \left[1 + \frac{Gk}{u_*} \right]^{-1/2}$$

$$\text{and } \ln \frac{G}{fz_0} = k \frac{G}{u_*} + 1 - \ln \left[2k \frac{u_*}{G} \left(1 + k \frac{G}{u_*} \right)^{-1/2} \right]$$

from which we can deduce S_0 , h and $\frac{u_*}{G}$ in terms of G/fz_0 .

log ₁₀ $\frac{G}{fz_0}$	NEUTRAL				$H/G^2 = 0.2$											
	u_*/G	h/G	fh/u_*	α_0	0.2		0.4		2		4		20			
9	.025	55	.20	16°	.027	15°	.029	12°	.034	7°	.037	5°	.045	3°		
8	.030	72	.22	17°	.033	16°	.035	13°	.041	10°	.043	8°	.051	4°		
7	.035	90	.23	18°	.039	18°	.042	16°	.047	13°	.051	10°	.058	7°		
6	.042	120	.25	21°	.048	20°	.051	19°	.056	17°	.061	15°	.071	12°		
5	.054	167	.28	25°	.061	25°	.066	22°	.074	20°	.078	19°	.09	16°		

The wind speed above $z = 5z_0$ is given by

$$u = \frac{u_*}{k} \left[\ln \frac{z}{z_0} - \frac{z}{h} \right]$$

the turning by:

$$\sin \alpha = S_0 \left(1 - \frac{z}{h} \right)$$

and the stress components parallel and at right angles to the isobars are

$$\tau_x = \frac{1}{2} \rho f G S_0 h \left(1 - \frac{z}{h} \right)^2$$

$$\tau_y = \frac{1}{2} \rho f \frac{u_*}{k} h \left[1 - \left(\frac{z}{h} \right)^2 + 2 \frac{z}{h} \ln \frac{z}{h} \right]$$

Specifying the
full PROFILES OF
WIND & STRESS

(ii) In UNSTABLE conditions, u_*/G and S_0 can still be specified reasonably well in terms of G/fz_0 and the additional parameter H/G^2 (related to $\mu = ku_*/fL$ where $L = -\frac{\rho C_p T u_*^3}{kgH}$ using a very similar MODEL.

L is called the Monin-Obukhov length scale and is a measure of the height where free convection begins to dominate over forced convection.

These results may be presented in a different and basically easier form (see the next Figure).

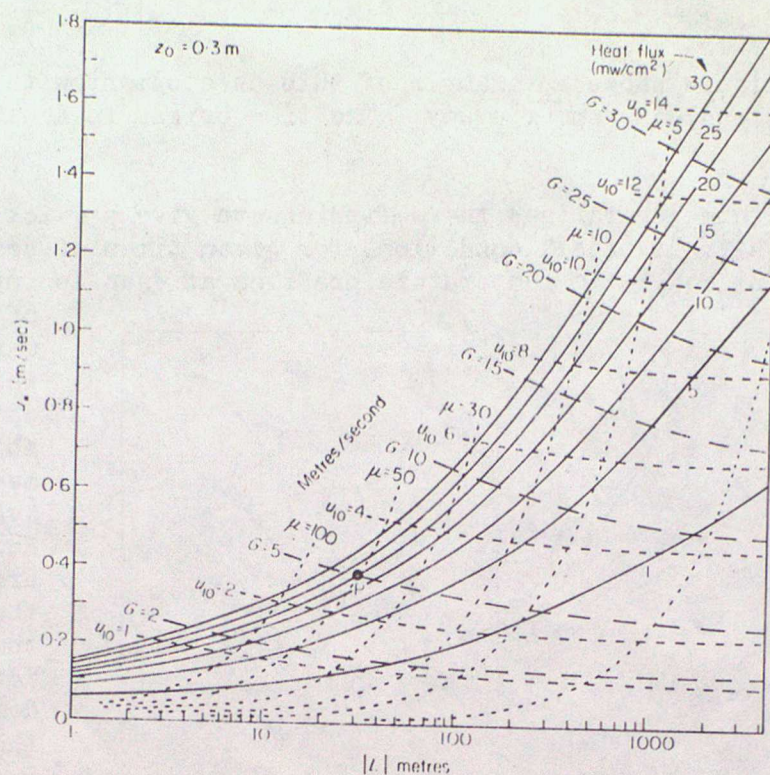
This Figure shows an example of a whole set of similar diagrams. Each is for a specific z_0 . Given H and G it is possible to deduce

$$u_*, L, \mu, \text{ and } u(10m)$$

The last parameter can be deduced using Businger's form

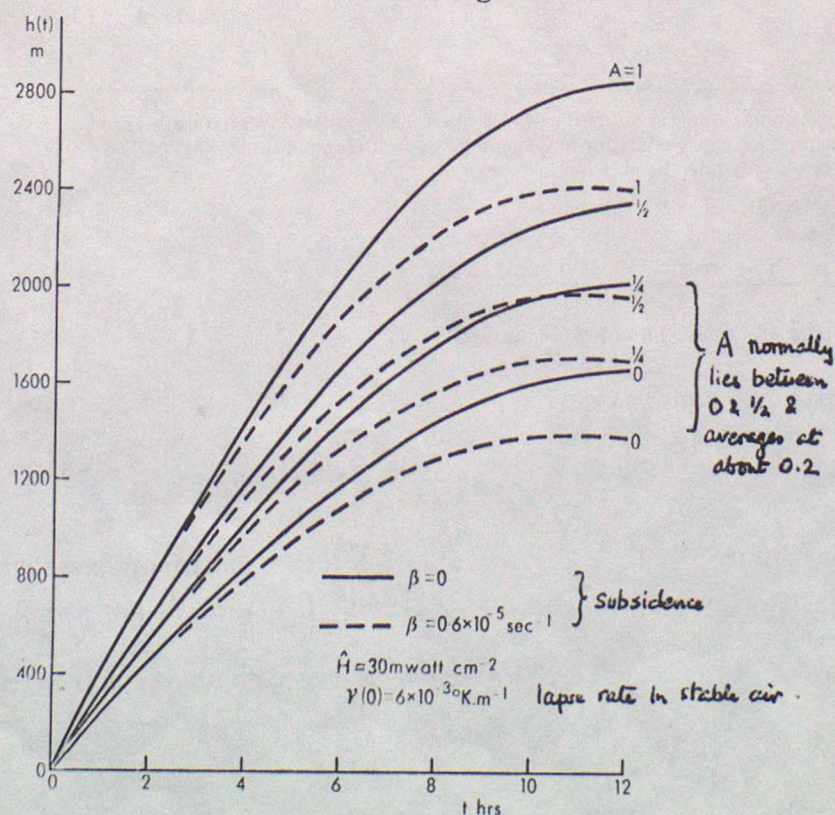
$$\frac{du}{dz} = \frac{u_*}{kz} \left(1 + 16 \frac{z}{|L|} \right)^{-1/4}$$

which can be readily integrated.



The inter-relationship between the heat flux in mw/cm^2 , the geostrophic wind speed G , the wind speed (m/sec) at 10 m, the surface friction velocity u_* (m/sec), the Monin-Obukhov length-scale L (metres) and stability parameter μ . As an example, consider the point P where $H = 20 \text{ mw/cm}^2$ and $G = 5 \text{ m/sec}$. The appropriate values of the other variables at P can be immediately obtained: $\mu = 50$, $u_{10} = 2.8 \text{ m/sec}$, $u_* = 0.38 \text{ m/sec}$, $L = -32 \text{ m}$.

The parameter which cannot be estimated accurately using the model is the boundary layer depth h . This is because h depends on the HISTORIC heat input and the initial lapse rate configuration



The development of the depth of the convectively unstable boundary layer, $h(t)$, for various degrees of interfacial mixing, for typical values of $\gamma(0)$ and H in the cases of no subsidence and typical subsidence. Sinusoidal heat flux model.

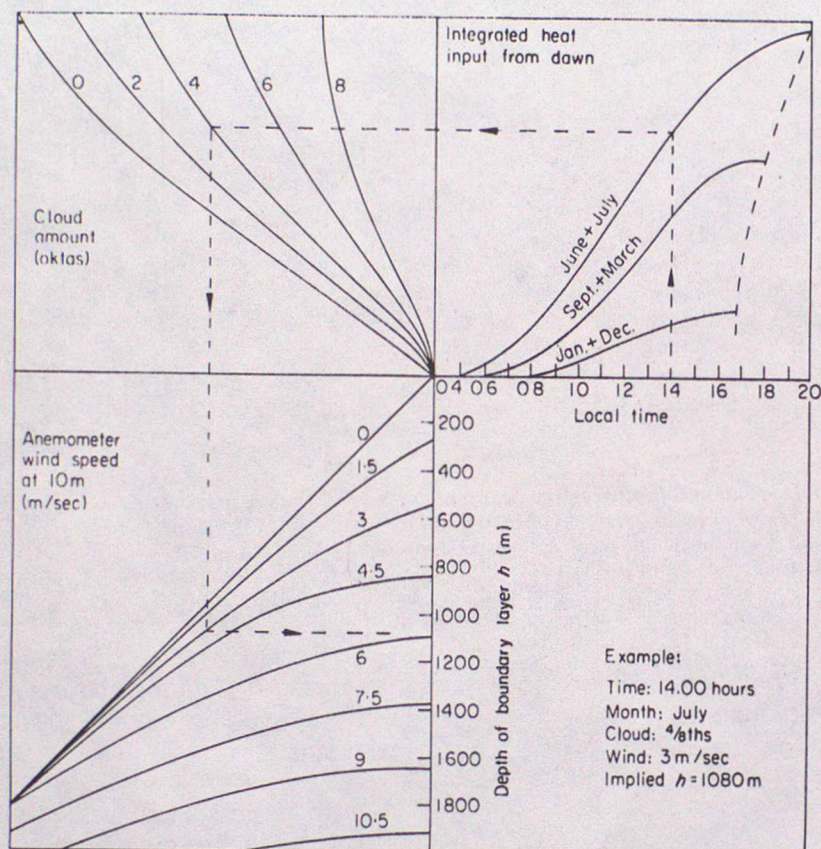
The above figure shows an example of this development with time of day on a typical cloudless summer's day. The time origin is about 1 hour after sunrise.

The next figure generalises these findings to give a quick means of estimating h in "typical" conditions for given times of year and cloud amount. It allows for realistic temperature profiles at dawn as implied by an analysis

of Balthum ascents at Cardington. On the whole the diagram's estimates agree reasonably well with such measured results as are available.

However some discrepancies are inevitable because of the simplifying assumptions that have been made.

Note the example which demonstrates how to use the figure.



A nomogram for estimating the depth of the boundary layer in the absence of marked advective effects or basic changes in weather conditions. The marked example shows how the diagram is to be used.

4. The temperature profile

In neutral conditions θ is const.

In unstable conditions:

$$(i) \quad \text{for } \frac{z}{|L|} \leq 1: \quad \frac{d\theta}{dz} = \frac{\theta_*}{kz} \psi_h \left(\frac{z}{|L|} \right) \quad \text{where } \theta_* \equiv \frac{H}{\rho_c u_*}$$

$$\text{where } \hat{k} \text{ (the Businger constant)} \approx 0.53$$

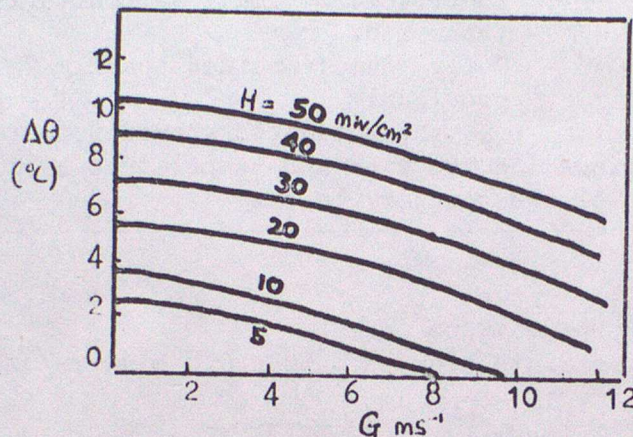
$$\text{and } \psi_h \left(\frac{z}{|L|} \right) \approx \left(1 + 9 \frac{z}{|L|} \right)^{1/2}$$

$$(ii) \quad \text{for } \frac{z}{|L|} > 1 \text{ we enter a free convection regime where}$$

$$\frac{d\theta}{dz} = -C \left(\frac{H}{\rho c_p} \right)^{1/3} \left(\frac{g}{T} \right)^{-1/3} z^{-4/3}$$

and C is close to 1 in magnitude.

(different estimates range from 0.84 to 1.4)



This diagram is deliberately rough to emphasise the present uncertainty in the exact values. The numbers are deduced using Clarke's value for C in the above formula (namely $C = 0.84$).

Note how insensitive $\Delta\theta$ is to the wind speed. Remember also that normally large wind speeds are associated with cloud, damp soil and correspondingly low values of heat flux H.

5. Conclusion

In both numerical forecasting and in making rough but informed estimates of the nature of the boundary layer, only the external parameters are available to determine whatever is required concerning the internal structure. We have shown that given :

- f: the Coriolis parameter
- Time of day, month and cloud amount
- z_0 and the amount of available moisture (in rough terms) at the surface
- the soil type
- the pressure gradient

we can deduce the sensible heat flux, the wind profile, the temperature profile and the shearing stress profile.

These estimates are most likely to be in error near marked discontinuities of underlying surface (sea to land, mountains, marked topography etc). Much more work still has to be done; many interesting research projects await attention. This is only a very interim Report.

Example

Given the following information:

Day: March 15th
Time: 11^h
Weather: 2/8th cloud, $G = 6$ m/sec
Potential temperature at top of boundary layer obtained from earlier radiosonde ascent = 6°C
Countryside: small agricultural fields, trees etc.
Soil: damp; rained heavily 2 days previously.

find the surface friction velocity u_* , the estimated ground temperature, the turning of the wind at the surface and the wind and temperature at 10 metres; and finally the expected depth of the boundary layer.

Solution

From page 1: in clear skies $R_0 = 50$ mw/cm²
with the specified 2/8th cloud $R = 50 \times 0.8 = 40$ mw/cm²

From page 2: $H/R \approx 0.25$ i.e. $H = 10$ mw/cm².

From page 3: $Z_0 = 25$ cm.

From page 4: $\frac{H}{G^2} = \frac{10}{6^2} \approx 0.3$ $\frac{G}{fz_0} = \frac{6}{10^{-4} \times 0.25} = 2.4 \times 10^5$

$\frac{u_*}{G} \approx 0.058$ i.e. $u_* \approx 0.35$ m/sec

also $\alpha_0 \approx 23^{\circ}$

From page 5: We can use Fig. 1 to determine L in terms of our values of H and u_* . $L \approx -40$ metres.
(note this Figure is for a different Z_0 , but this does not affect this answer).

To find u_{10} we integrate $\frac{du}{dz} = \frac{u_*}{kz} \left(1 + 16 \frac{z}{|L|}\right)^{1/4}$

If $y = \left(1 + 16 \frac{z}{|L|}\right)^{1/4}$ then $k \frac{u}{u_*} = \ln \frac{y-1}{y+1} + 2 \tan^{-1} y + \left[\ln \frac{4|L|}{9z_0} - \frac{\pi}{2} \right]$

Inserting $z = 10$, $|L| = 40$, $k = 0.4$, $u_* = 0.35$ and $z_0 = 0.25$

gives (approximately): $u_{10} = 3.5$ m/sec

From page 6: The depth of the boundary layer may be assessed using Fig. 6.
Using the Figure as described:

$h = 720$ metres

From page 7: the total $\Delta\theta$ is given by $G = 6 \text{ m/sec}$ and $H = 10 \text{ mw/cm}^2$
 $\Delta\theta = 2.2^\circ\text{C}$

$$\text{Hence estimated ground temperature} = 6 + 2.2 \\ = 8.2^\circ\text{C}$$

To find the temperature at 10 metres we return to the equation on page 6 for $\frac{z}{|L|} < 1$, and integrate it from $z = z_0$ to $z = 10$.

$$\text{If } y = (1 + \frac{qz}{|L|})^{1/2} \text{ then } \Delta\theta = \frac{\theta_*}{k} \left[\ln \frac{4|L|}{9z_c} \frac{y-1}{y+1} \right]$$

$$\text{Putting } \theta_* = \frac{H}{\rho c_p u_*} = \frac{10}{100 \times 0.35}, \quad k = 0.53, \quad |L| = 40, \quad z_c = 0.25$$

$$\Delta\theta = 1.6^\circ$$

$$\text{i.e. } \theta \text{ at 10 metres} = 8.2 - 1.6 = 6.6^\circ\text{C}$$

$$\theta_{10} = 6.6^\circ\text{C}$$

6. Estimating vertical spread of a passive gas released from a ground-level source

This method may be used to give estimates of vertical spread when direct measurements of the vertical fluctuations of the wind are unobtainable. Solutions of the diffusion equation using reasonable profiles of wind and diffusivity $K(z)$ suggested by actual measurements in unstable, neutral and stable conditions. The solutions have been checked against actual dispersion data wherever possible.

Note the vertical spread is represented by σ_z where

$$\sigma_z^2 = \frac{\int_0^\infty z^2 C(z) dz}{\int_0^\infty C(z) dz}$$

and $C(z)$ is the concentration. The scheme only applies within the boundary layer and once the "plume" begins to interact with any inversion marking the top of the layer the scheme becomes inaccurate and a "box-model" solution is approached

$$\text{i.e. } C \approx Q \bar{u} h$$

for an across-wind line source.

(i) Deduce the P-stability. Using Fig. 2 the numerical value of P is expressed in terms of heat flux H and wind speed u at 10 metres. The values of P for night-time stable conditions are rather more uncertain and are based on estimates of wind speed and cloud amounts.

(ii) Using Fig. 3 read σ_z for neutral $P = 3.6$ and a supposed $z_c = 10 \text{ cms}$, at the distance x required.

(iii) Multiply this value of σ_z by the correction factor for stability given in Fig. 4.

(iv) Multiply this corrected value of σ_z by a further correction factor for terrain roughness using Fig. 5.

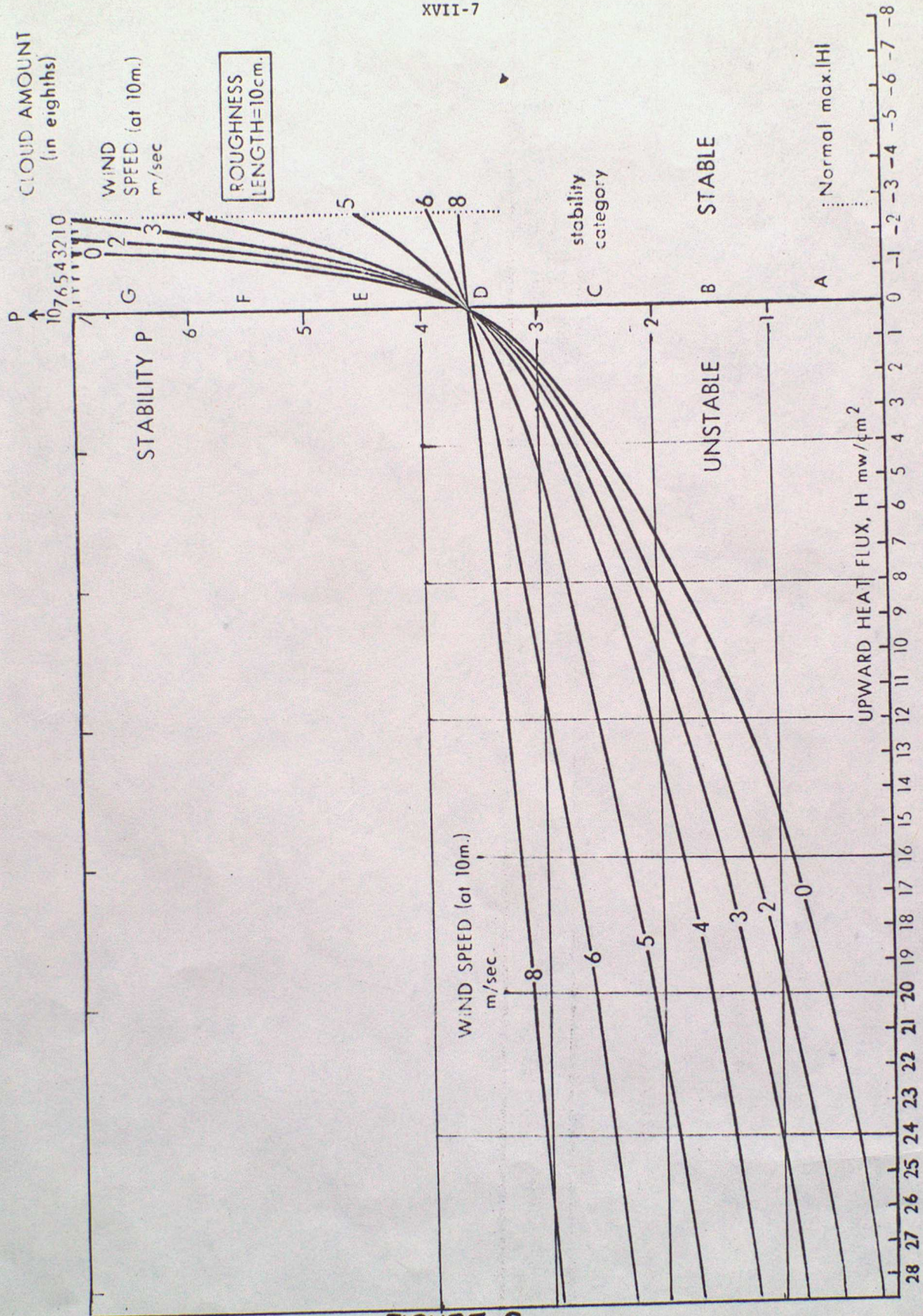


FIGURE 2.

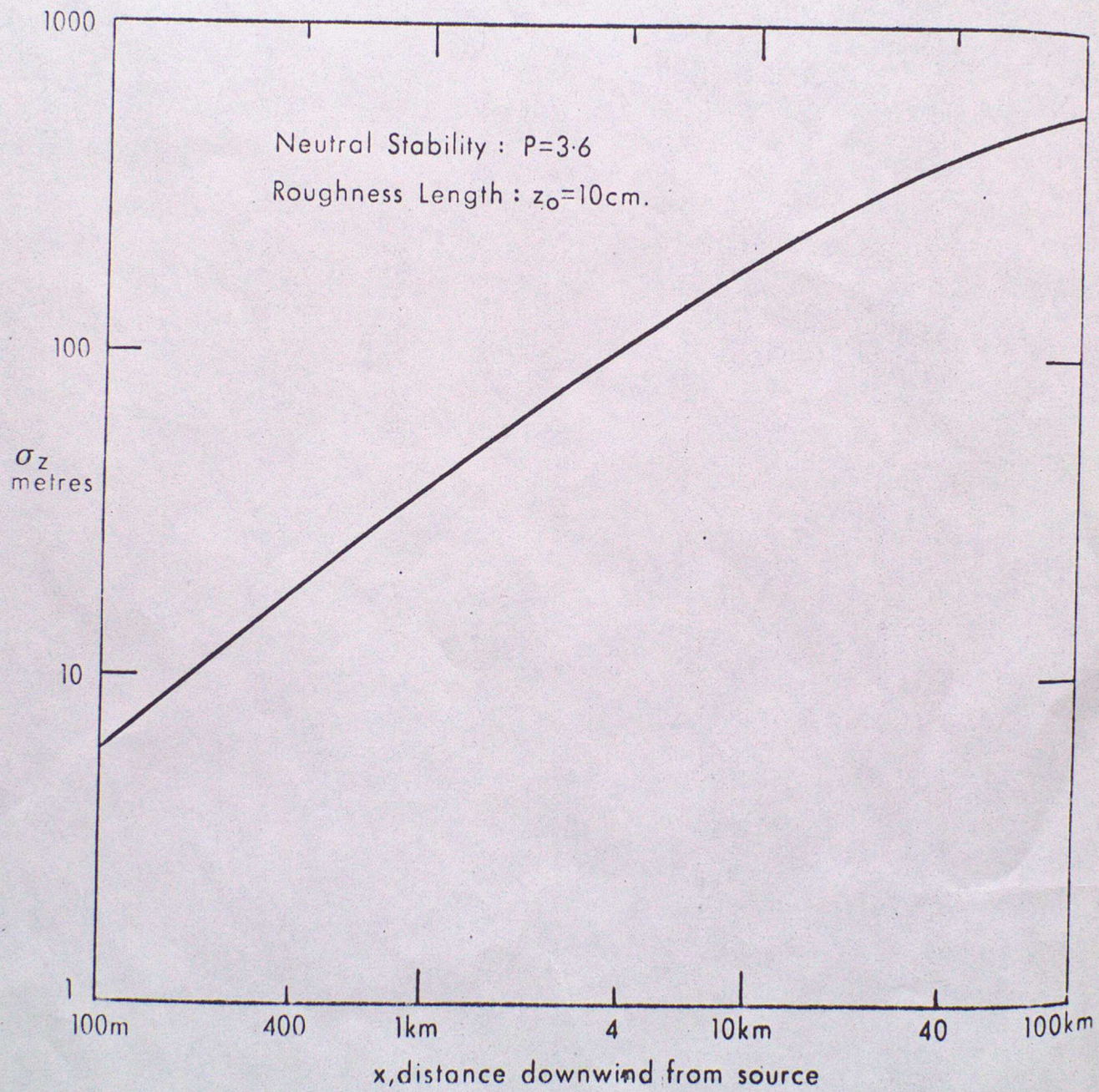


FIGURE 3.

9(b)

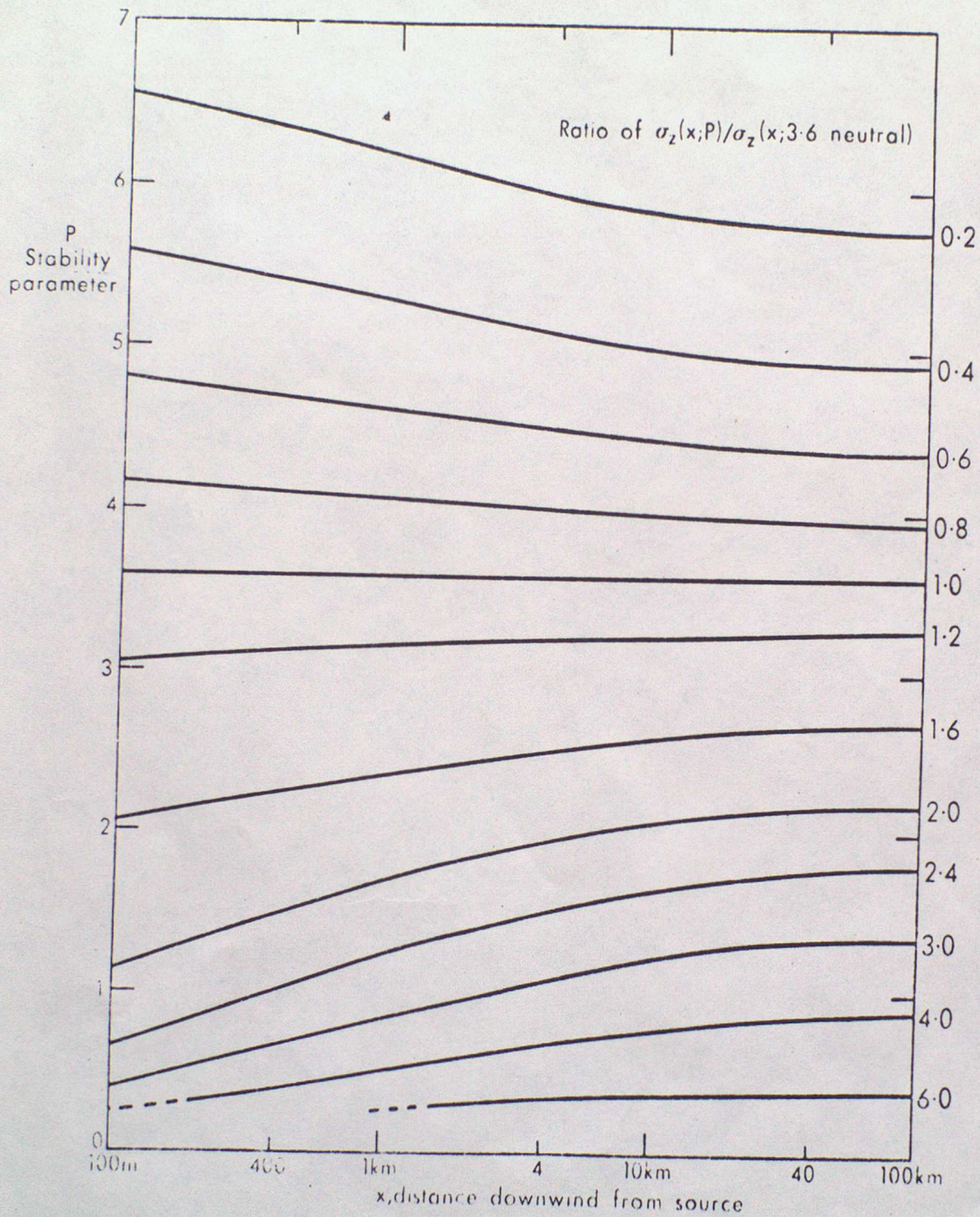
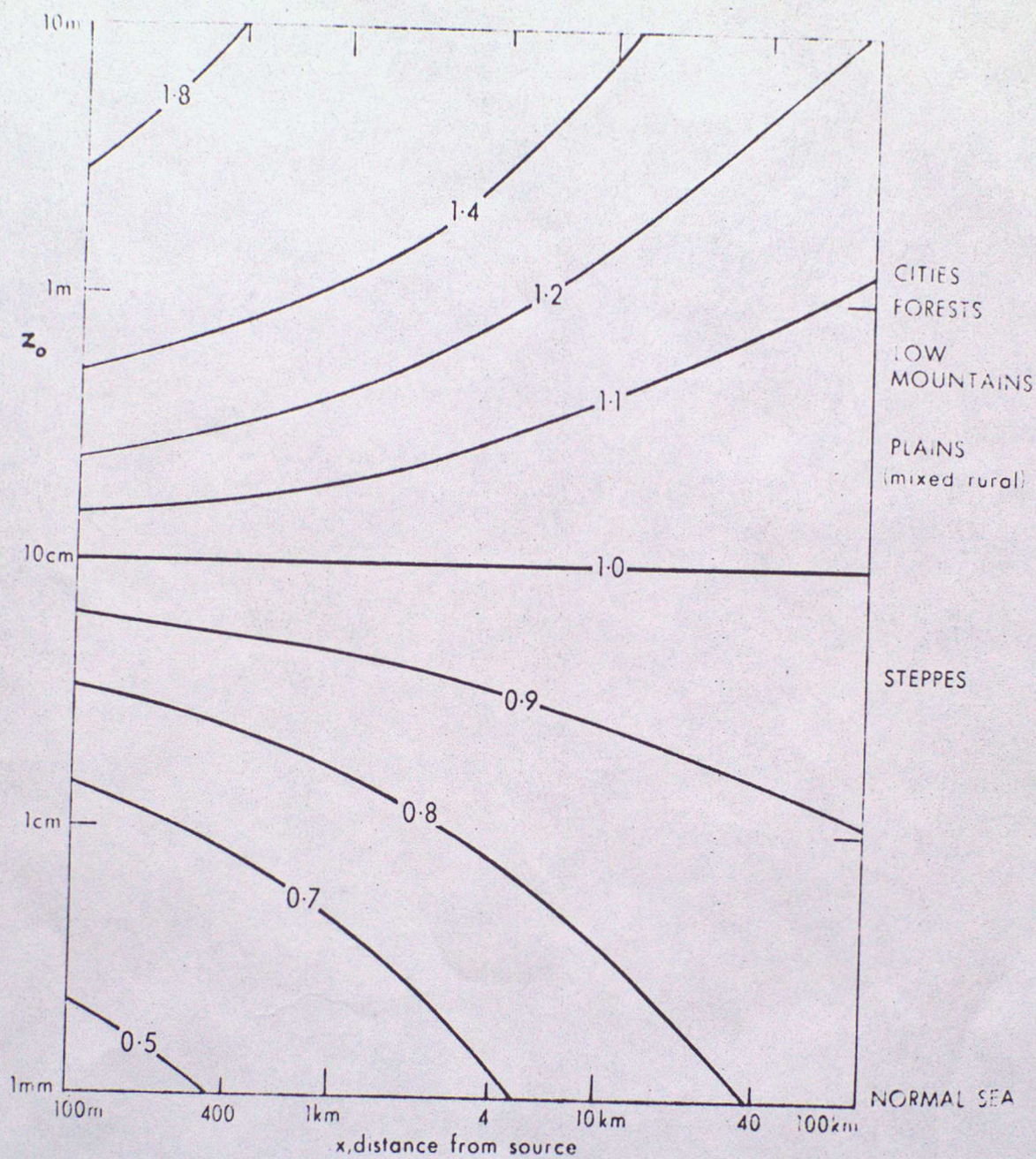


FIGURE 4.



CONTOURS OF

$$\frac{\alpha_z(x; z_0)}{\alpha_z(x; z_0 = 0.1m)}$$

These curves are virtually
independent of heat flux

FIGURE 5.

Example

Given the same conditions as in the previous example deduce σ_z for $x = 10$ km.
Has the plume interacted with the top of the boundary layer?

Solution

Using Fig. 2 with $H = 10$ mm/cm² and $u_{10} = 3.5$ m/sec
we find $P = 2.2$

Using Fig. 3 (for $P = 3.6$ and $z_0 = 10$ cms) at $x = 10$ kms
 $\sigma_z \approx 200$ metres.

Correcting for P using Fig. 4 a factor of 1.8 is implied ($x = 10$ km, $P = 2.2$)

$$\sigma_z = 200 \times 1.8 = 360 \text{ m.}$$

Finally correcting for z_0 using Fig. 5 a factor of 1.08 comes from

$$z_0 = 25 \text{ cms, } x = 10 \text{ km}$$

$$\text{Thus } \sigma_z = 360 \times 1.08 = 390 \text{ m}$$

$$\text{i.e. } \underline{\sigma_z = 390 \text{ m}}$$

The top of the plume (approx $2\sigma_z$) will probably have just reached the top of the boundary layer but the effect will be quite negligible on the ground-level concentration.

7. Estimating lateral spread

This has proved a much more difficult problem than estimating vertical spread since horizontal dispersion varies significantly with sampling time, meso-scale meteorological processes (large "eddies" on the scale 10-100 km) and so on.

The total angular width θ of the plume is very approximately given by the following table:

For short releases:

P	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7
$x = 100$ m	60°	45°	30°	20°	15°	10°	5°
$x = 100$ km	20°	15°	10°	10°	5°	5°	2°

$$\sigma_y \text{ is given by: } \sigma_y = \frac{\theta}{57} \cdot \frac{x}{4} \quad \text{approximately}$$

At large distances $x \gg 100$ km a rough rule-of-thumb is that σ_y increases by about 1 km for every hour of travel.

8. Plume rise

If the plume is hot and comes out of a stack at height h_s then it will rise and act as though it were emitted from a higher virtual source. In near neutral conditions the plume rise is given by

$$\Delta h = 1.6 \frac{(3.7 \times 10^{-5} Q_H)^{1/3} x^{2/3}}{\bar{u}} \quad (x \leq 10h_s)$$

\bar{u} cal s⁻¹ coll/sec

where Q_H is the output in Mw (typically 10^6 for a power station)

x is in metres

\bar{u} is the mean wind at stack height in m/sec.

Beyond $x = 10h_s$ the plume effectively no longer rises

In stable conditions

$$\Delta h_{\text{final}} \approx 2.9 \left(\frac{3.7 \times 10^{-5} Q_H}{\bar{u} S} \right)^{1/3}$$

where S = stability parameter = $\frac{g}{T} \frac{d\theta}{dz}$

9. Concentration estimates

In general, provided there is no low-level inversion the plume from a point source is approximately Gaussian; and the ground-level concentration is therefore:

$$C = \frac{Q}{\pi u \sigma_y \sigma_z} \exp \left\{ - \left(\frac{y^2}{2\sigma_y^2} + \frac{h^2}{2\sigma_z^2} \right) \right\}$$

where Q is the source strength of the gas

h is $h_s + \Delta h$

The maximum concentration depends on the behaviour of σ_y and σ_z .
In LIGHT WINDS during the day, vertical motions are dominated by convection and

$$\sigma_z \approx A \frac{x}{u}$$

$$\sigma_y = Bx$$

defining A and B . A and B may be determined using the schemes outlined earlier.

Then

$$C_{\text{max}} = \frac{2QA}{\pi e B h^2 u^2} \quad \text{at} \quad x = \frac{uh}{\sqrt{2}A}$$

In STRONG WINDS vertical motions are dominated by dynamically generated turbulence of a generally smaller scale

Now $\sigma_z \sim A \sqrt{ux}$

Whilst $\sigma_y = Bx$

again defining A and B

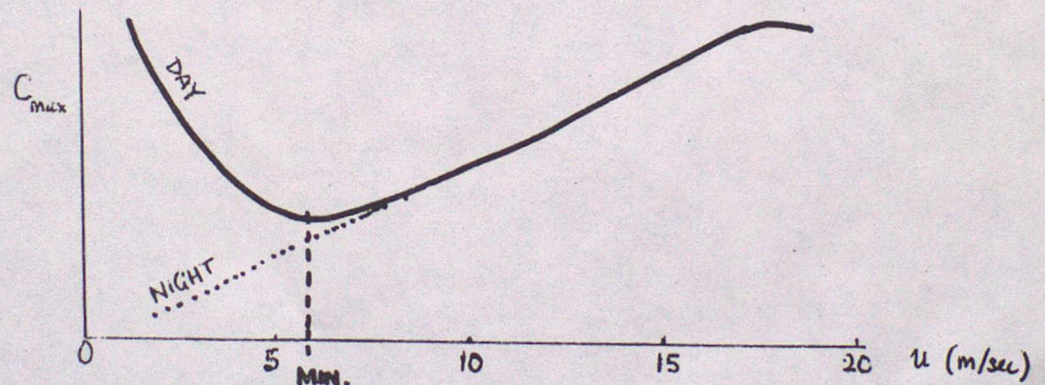
Now
$$C_{max} = \frac{3\sqrt{3} Q A^2}{\pi e^{3/2} B h^3}$$

A general interpolation formula for all u is

$$C_{max} = Q \left[\frac{L}{h^3} + \frac{K}{h^2 u^4} \right]$$

where typically $L \sim 8K$. Since h itself depends on wind speed, because $\Delta h \propto u^{-1}$, the maximum concentration during the day tends to show a minimum at a wind speed near 6 m/sec.

Schematically:



The following Figures (due to Dr B. Turner, E.P.A., N.Carolina) indicate typical solutions as a function effective emission height, stability, distance and boundary layer depth.

Example

For the same conditions as before, and for a power station chimney 100 m high emitting SO_2 at a rate of 1 kg/sec and a heat output of 10^6 Mw :
determine: the effective stack height
the maximum ground level concentration
the concentration 30 km downwind.

Solution

The plume rise is, from above,

$$\Delta h = \frac{1.6 (3.7 \times 10^{-5} \times Q_H)^{1/3} (10h_s)^{1/3}}{\bar{u}} = \frac{1.6 (37)^{1/3} \cdot 100}{6} \approx 90 \text{ m.}$$

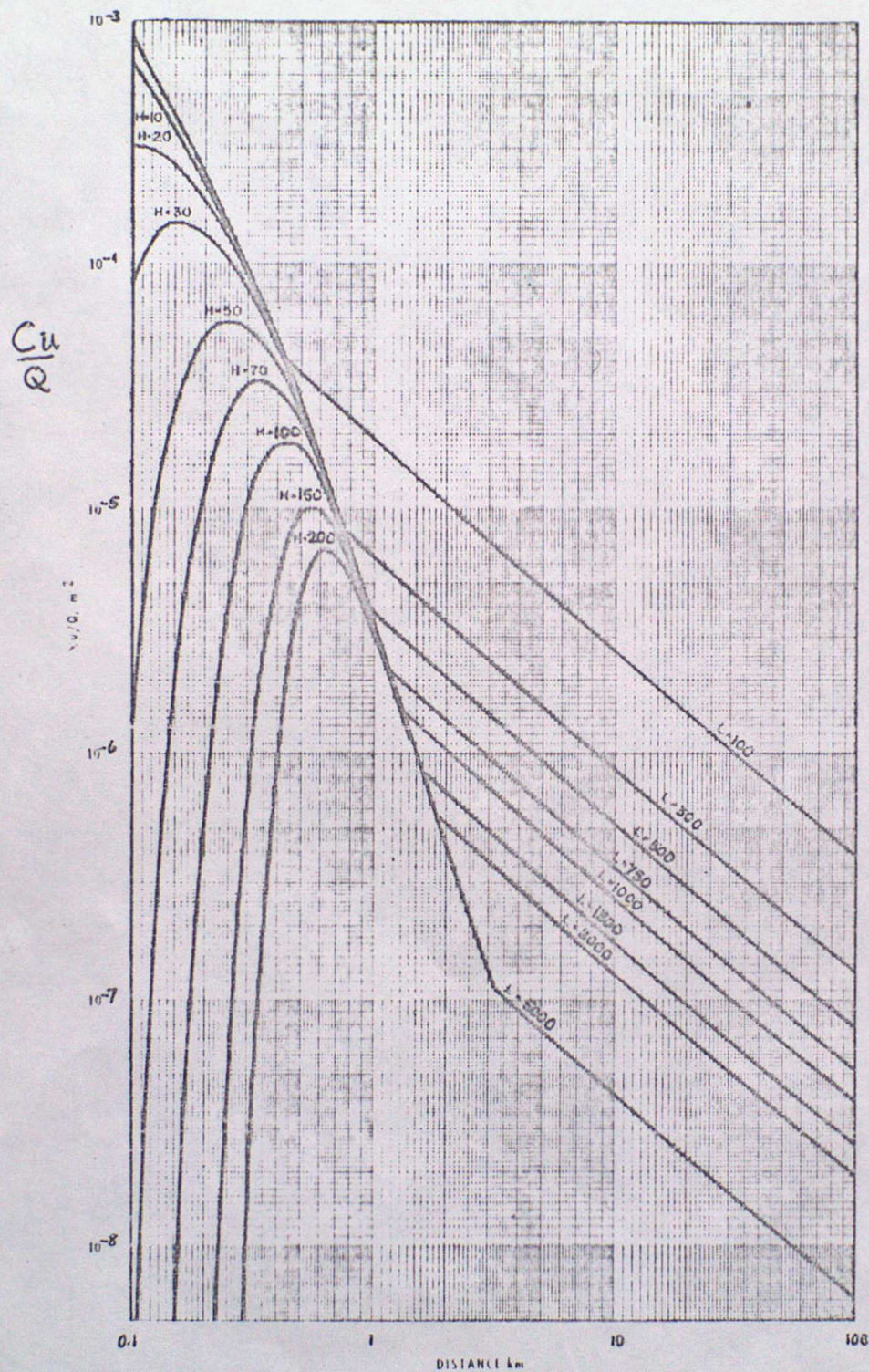


Figure 3.5A. x_u/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), A stability.

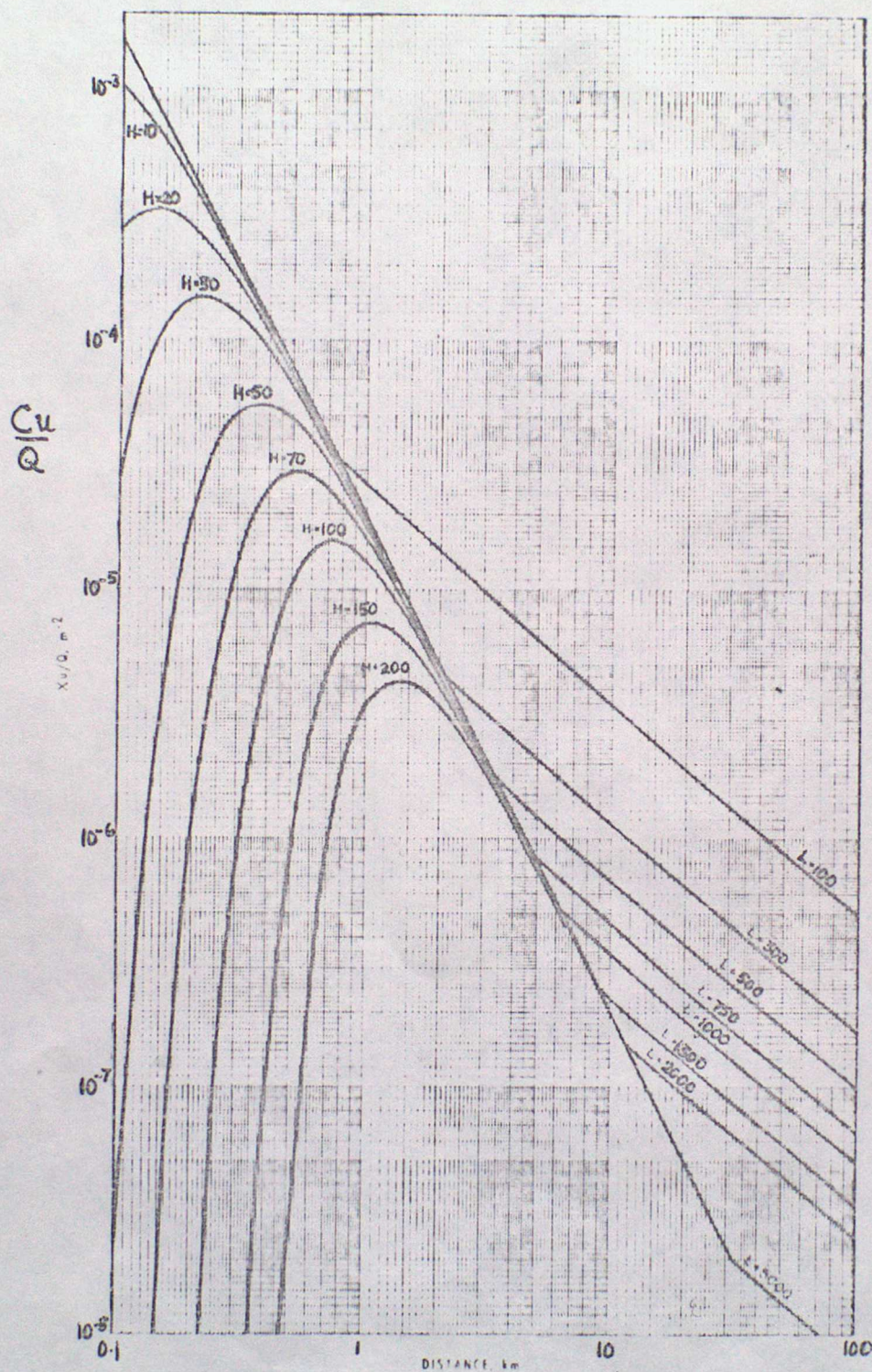


Figure 3.5B. C_u/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), B stability.

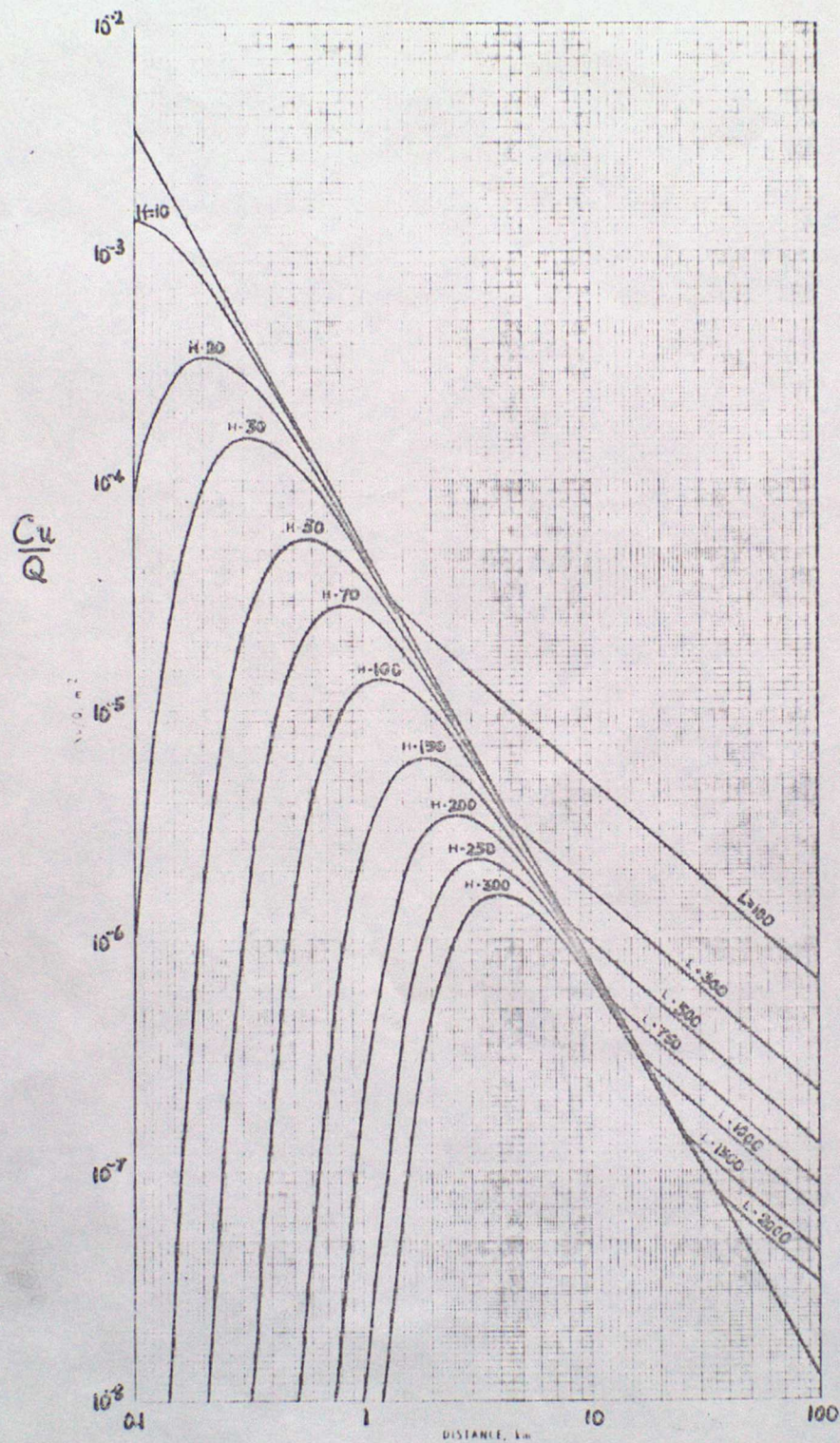


Figure 3-5C. Q/C_u with distance for various heights of emission (H) and limits to vertical dispersion (L), C stability.

Estimates

12(c)

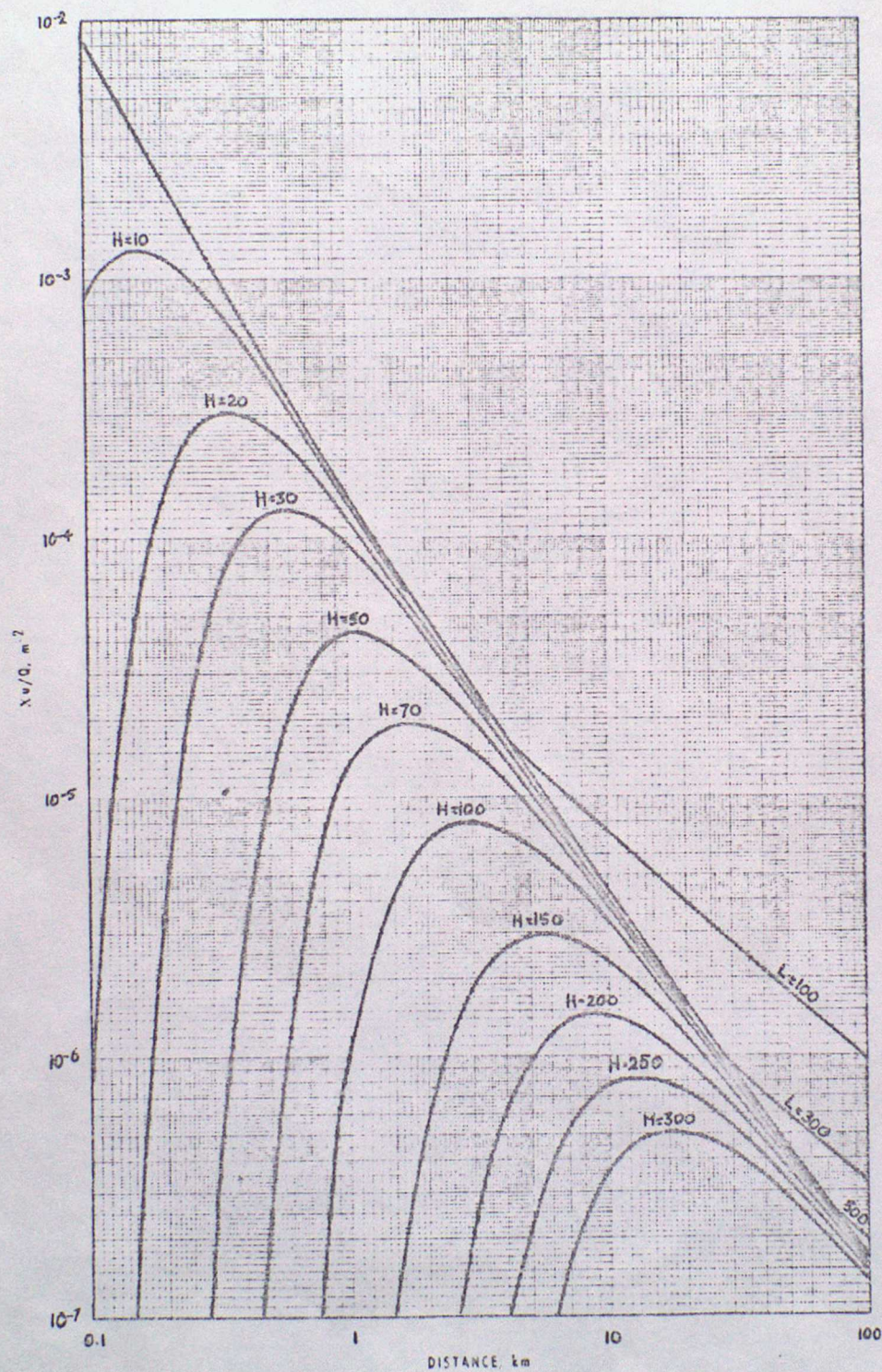


Figure 3-5D. $x u / Q$ with distance for various heights of emission (H) and limits to vertical dispersion (L), D stability.

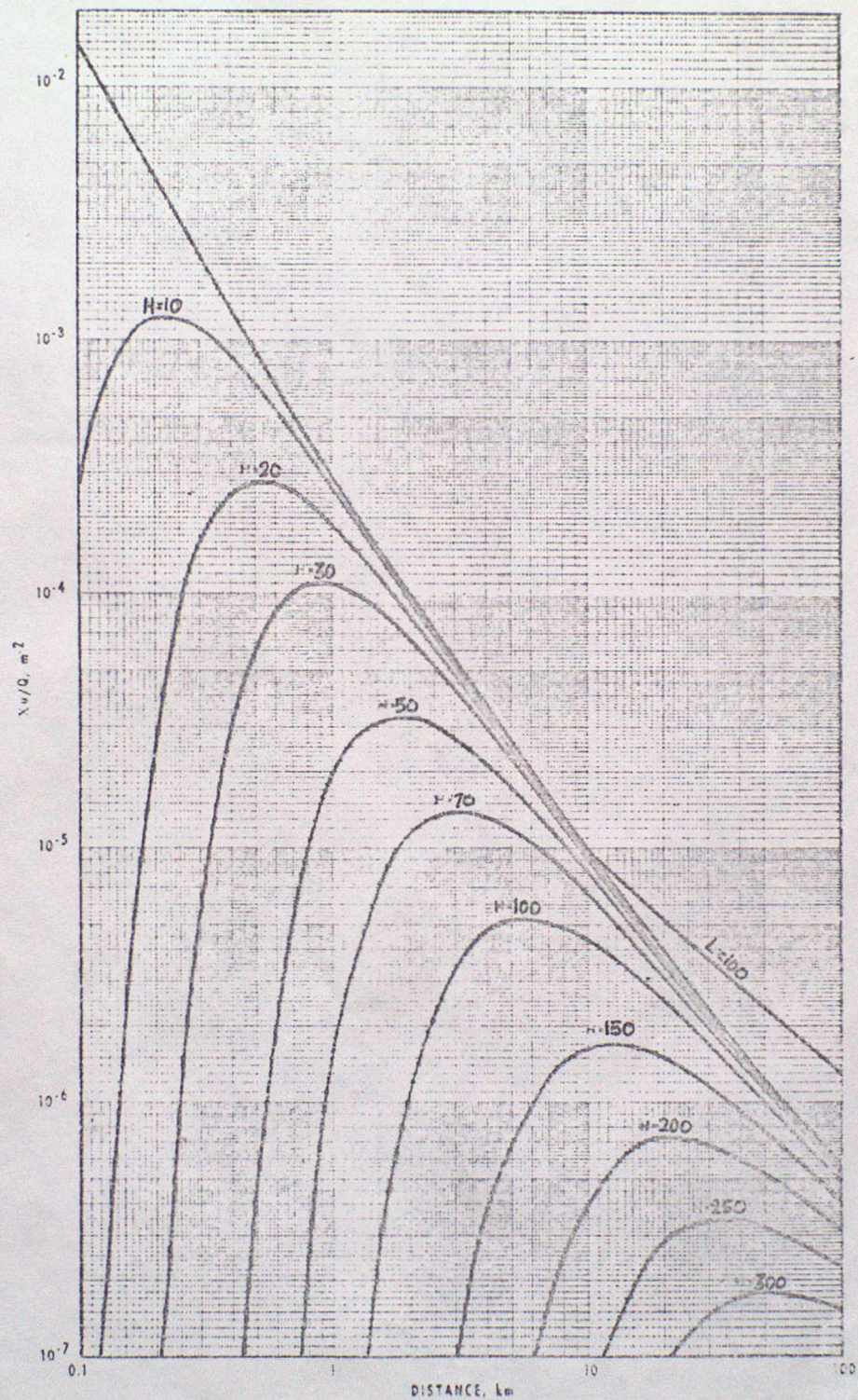


Figure 3-5E. x_u/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), E stability.

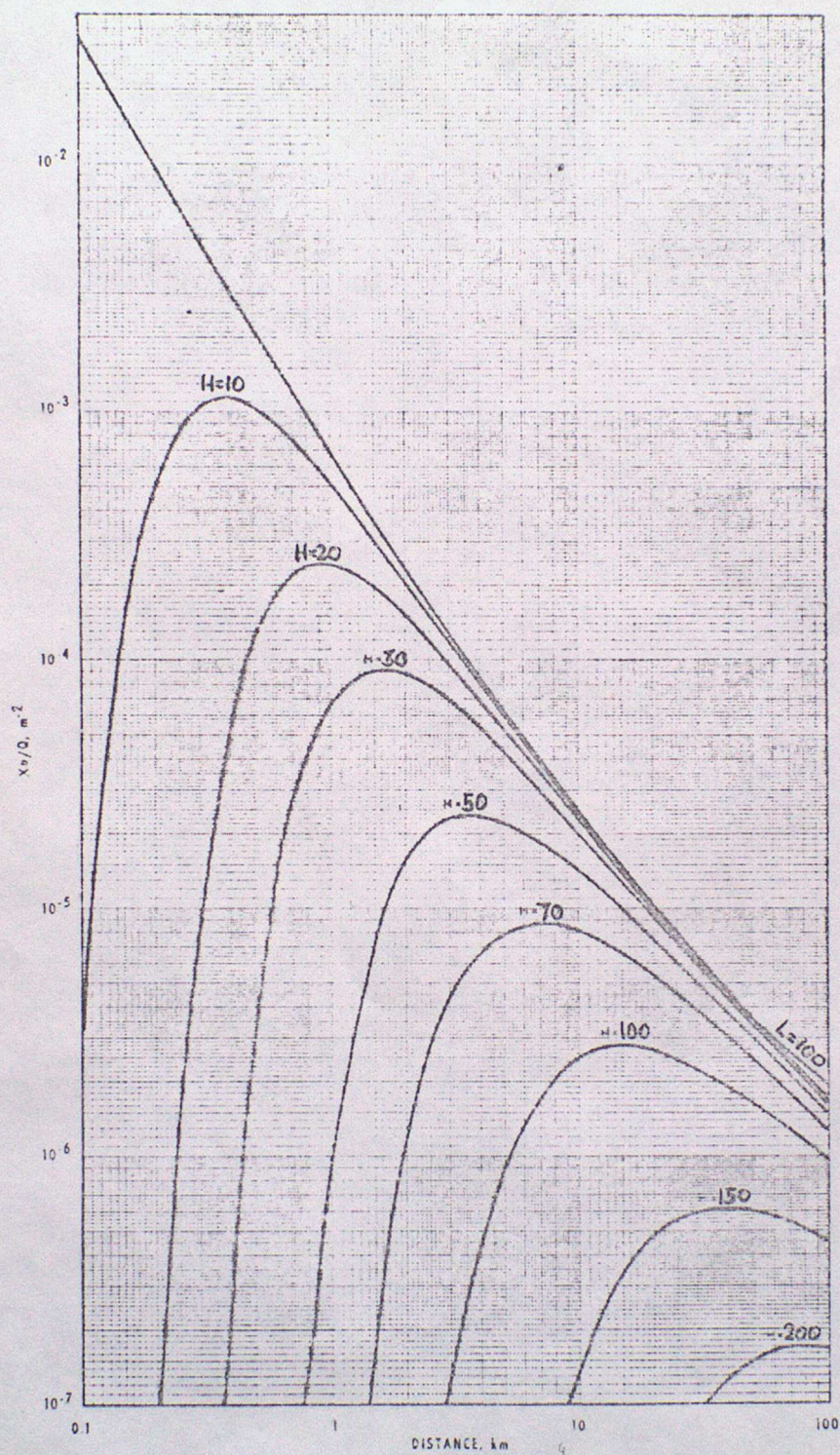


Figure 3-5F. xu/Q with distance for various heights of emission (H) and limits to vertical dispersion (L), F stability.

Effective stack height is therefore 190 m.

Since $P = 2.2$, Figure 3.5 C is appropriate. This shows that, approximately,

$$\frac{Cu}{Q} = 4 \times 10^{-6} \quad \text{at about 2.4 km downwind.}$$

$$C_{\max} \approx \frac{4 \times 10^{-6} \times 10^9}{6} \mu\text{g/m}^3$$

$$\text{i.e. } \underline{C_{\max} = 670 \mu\text{g/m}^3}$$

At 30 km the plume has been affected by the top of the boundary layer (estimated earlier at 720 m.). The figure estimates the concentration to be given by

$$\frac{Cu}{Q} = 2.7 \times 10^{-7}$$

and this agrees well with assuming vertical uniformity in C below 720 m and a lateral $\theta = 20^\circ$ from the Table.

$$C = 2.7 \times 10^{-7} \times \frac{10^9}{6} \mu\text{g/m}^3$$

$$\text{i.e. } \underline{C = 45 \mu\text{g/m}^3}$$

due to the power station source, and on the plume axis at $Z = 0$.