Numerical Weather Prediction

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Forecasting Research Technical Report No. 507

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Document History

Date	Version	Action/Comments	Approval
03-12-07	1.0	Approved for release.	Andrew Brown

Abstract

The Met Office Unified Model is modified to include diffusion based on the Smagorinsky-Lilly sub-grid model and tested for a case of convective boundary layer turbulence at 50 m resolution. Comparison with the Met Office Large-eddy model reveals that, by using a smaller mixing length to compensate for damping from interpolation, this semi-Lagrangian model can produce a good convective turbulence simulation.

1 Introduction

With the arrival of more powerful supercomputers, operational weather centres can now run limited area models at horizontal grid lengths as small as a few kilometres. For example, the 4 km resolution Met Office Unified Model (UM) provides high resolution forecasts of, among other things, severe convective precipitation and fog for domains covering the UK. In order to improve this capability, there is increasing need for new critical tests. One approach is to perform reruns of real case studies (May *et al.*, 2004). A complementary approach is to prescribe idealised (but still realistic) forcings and initial conditions and compare with analytical or well established scaling laws. Moreover, it is crucial that the model converges to the correct limit at even higher resolutions.

With these motivations in mind, this paper describes tests of the UM at 50 m resolution. A simple case of a dry convective boundary layer (CBL) case is adopted. The use of these idealised forcings and removing moisture from the problem enables an isolation of the role of the dynamical core and the diffusion. It is intended to build on this with moist convective cases in the future. The dry convective boundary layer has also been much studied using independent Large-eddy models (Nieuwstadt *et al.*, 1992; Mason and Brown, 1999). One of these models, the Met Office Large-eddy model (LEM), is used here as a reference.

Large-eddy simulation (LES) is appropriate for a high Reynolds flow such as a dry CBL as it simulates the resolved scale and parametrizes the diffusion associated with the unresolved turbulence. Some of this diffusion is provided by the sub-grid model. However, there is also diffusion implicit in the advection scheme and this needs to be balanced against that provided by the sub-grid model. In fact, there is a body of literature which argues that the implicit diffusion associated with the advection scheme can perform the same role as the sub-grid model (Drikakis, 2003). The interpolation in semi-Lagrangian advection provides an implied diffusion which may need to be taken into consideration when configuring the sub-grid model. The authors are not aware of a dry CBL LES being performed using a semi-Lagrangian model before.

2 Method

The case used is one of free convection over a homogeneous land surface. This involves prescribing a weak wind and a large surface sensible heat flux. The initial potential temperature is a mixed layer up to an inversion height (z_{i0}), and then an overlying stratification above:

$$\begin{aligned}
\theta &= \theta_0 \quad z < z_{i0} \\
&= \theta_0 + \Gamma(z - z_{i0}) \quad z > z_{i0} \\
\theta_0 &= 293K \ ; \quad z_{i0} = 1000m \ ; \quad \Gamma = 0.003Km^{-1}
\end{aligned} \tag{1}$$

where Γ is the overlying vertical temperature gradient. The Coriolis parameter (f_0) is 0.0001 s^{-1}. The initial wind in the x direction is 2 ms^{-1} and zero in the y direction. A random perturbation of amplitude 0.1 K is applied below 250 m to initiate turbulence. Roughness lengths of 0.1 m for momentum and 0.01 m for heat are used, typical of a rural land surface. The surface sensible heat flux is constant at 300 Wm^{-2}. A domain of 5 km x 5 km x 5 km is used to allow reasonable room for convective eddies, with 50 m resolution in the horizontal. The simulation is run for 10 hours, typically generating a mixed layer which deepens to 3.5 km, and warming of the mixed layer by about 6 K.

Table 1 compares the UM and LEM model configurations. The LEM uses the Smagorinsky-Lilly model (Smagorinsky, 1963; Lilly, 1967) as described in Mason and Derbyshire (1990). A very similar sub-grid model was implemented the UM, with an identical formulation for 3D sub-grid eddy viscosity (ν) and thermal diffusion (ν_h):

$$\nu = \lambda^2 S f_m(Ri) \ ; \quad \nu_h = \lambda^2 S f_h(Ri) \ ; \quad \frac{1}{\lambda^2} = \frac{1}{[\kappa(z+z_0)]^2} + \frac{1}{\lambda_0^2} \ ; \quad \lambda_0 = C_s \Delta \tag{2}$$

where λ, Δ, S, Ri, κ, z_0, $f_m(Ri)$ and $f_h(Ri)$ are the mixing length, horizontal grid length, shear tensor magnitude, Richardson number, von Karman constant, momentum roughness length, momentum and heat stability functions respectively. λ_0 is the neutral mixing length and, following Lilly (1967), scales with the grid length multiplied by the Smagorinsky constant (C_s). Assuming an isotropic inertial sub-range and sharp cut-off sub-filter, Lilly (1967) calculated a value of C_s of 0.17. In practical LES, the turbulence is non-isotropic and contains sources of diffusion beyond the sub-grid model, notably the advection scheme. Thus, the Smagorinsky constant is often adjusted in the region of 0.17 for different applications. The value used in the LEM is 0.23. Following Mason and Callen (1986), this value is sufficiently large to dampen grid scale numerical noise (from finite difference errors for example), whilst not too large to waste the resolution used.

The UM implementation of the sub-grid model differs slightly from Lilly (1967) in that it only multiplies the diffusion by the diagonal components of the shear tensor for calculating the sub-grid stress. The existing explicitly time-stepped horizontal viscosity and thermal diffusion (which is normally set constant in the horizontal) is modified to vary in the horizontal and set to the horizontal components of ν and ν_h (ν^{hor}, ν_h^{hor}). Since the horizontal diffusion is explicitly time-stepped, its value is limited to prevent numerical instability:

$$\nu^{hor}, \nu_h^{hor} \leq \frac{\Delta^2}{16\Delta t} \tag{3}$$

where Δt is the UM timestep. The existing boundary layer vertical diffusion is extended to all model levels and set to the vertical components of ν and ν_h. Since the time-stepping for the boundary layer diffusion is implicit, no limiting is applied to it. Identical forms of the stability functions $f_m(Ri)$ and $f_h(Ri)$ are used in the LEM and UM.

Model aspect	control UM	LEM
Dynamical formulation	Compressible; non-hydrostatic	Anelastic; non-hydrostatic
Coordinate system	Cartesian	Cartesian
Advection scheme	semi-Lagrangian; cubic interpolation; monotone θ	centred differences for momentum; monotone θ
Time step	5 s	variable (0.8 s average)
Time stepping	semi-implicit	explicit
Pressure solver	GCR iterative Helmholtz	FFT Poisson solver
Vertical staggering	Charney-Phillips	Lorenz
Horizontal staggering	C-grid	C-grid
Smag. constant ($\lambda_0/\Delta x$)	0.23	0.23

Table 1: A comparison of the model configurations used in the LEM and control UM simulations. Smag. stands for Smagorinsky. λ_0 and Δ are the maximum mixing length and the horizontal grid length respectively. The LEM timestep varies during the simulation so that the sum of the viscous and velocity Courant numbers is 0.4.

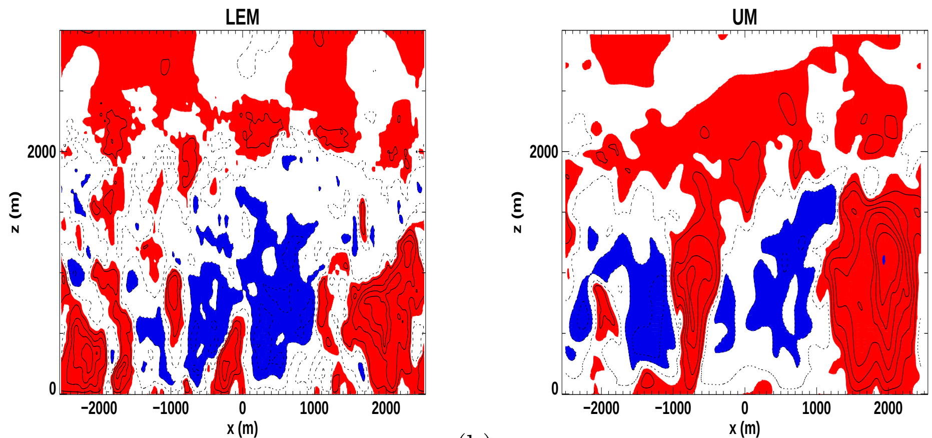
Both the LEM and UM use identical horizontal grid lengths. Whilst the LEM uses a uniform vertical grid length of 50 m, it became apparent that this was not ideal for the UM dynamics. The control UM run uses a quadratically varying vertical grid (to give good accuracy of the semi-Lagrangian scheme) with a vertical grid length near the surface of 1 m, increasing to 99 m at height 5 km, giving a similar vertical resolution to the LEM in the interior of the convective boundary layer (about height of 1 km). The additional near surface vertical resolution in the UM was required to give a good representation of the super-adiabatic surface layer in the dry CBL, since the UM semi-Lagrangian dynamics applies an isentropic assumption to potential temperature between the first level and the surface.

3 Results

Vertical cross sections of the vertical velocity 3 hours into the simulations are shown in Fig. 1. These both show gravity wave activity above the inversion (height 1300 m), entrainment near the inversion and convective plumes within the boundary layer. Although both simulations show convective mixing over similar depths, the entrainment, convective plume and gravity wave fields are qualitatively smoother in the UM than the LEM, despite using the same mixing length in the sub-grid model.

A useful summary measure of the turbulence is the time-area average velocity variance and third moments. Figure 2 compares vertical profiles of the mean vertical velocity variance and third moments for the 4-5 hour averaging period. Both the variances and the mean transports (given by the third moment profiles) have a similar shape and span very similar depths; the UM has slightly greater variance and transport in the interior of the boundary layer compared with the LEM.

A significant source of dissipation for large-eddy simulations is the sub-grid model. Both the LEM and the control UM are configured with the same filter scale (via the Smagorinsky constant (C_s) of 0.23). Figure 3 demonstrates how the sub-grid model can be adjusted in the UM to give different smoothness of vertical velocity fields. Decreasing C_s to 0.115 (Fig. 3a) gives more smaller scale structure, closer to the LEM simulation. Conversely, increasing C_s to 0.46 gives smoother fields.



(a) (b)

Figure 1: Vertical cross-sections of instantaneous vertical velocity through the domain centre at time 3 hours for: (a) the LEM, (b) the UM. Contour interval 1 ms^{-1}, dotted values negative. Values greater than 1.5 ms^{-1} shaded red, values less that -1.5 ms^{-1} shaded blue.

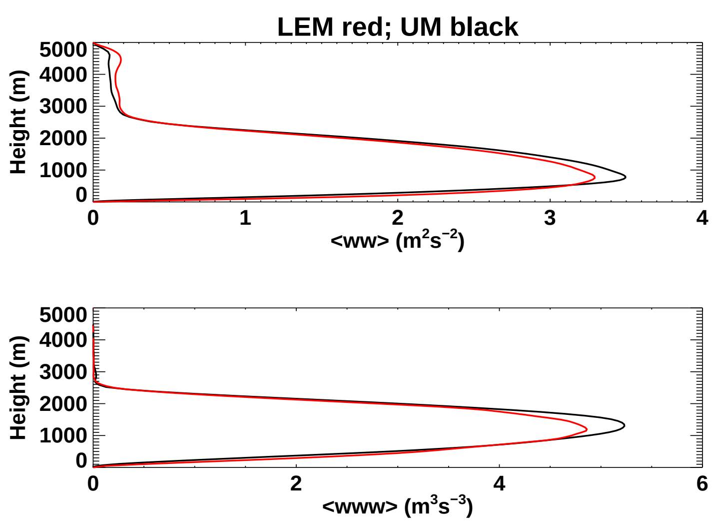


Figure 2: Time-area averaged vertical velocity variance (top) and third moment (bottom) profiles for LEM (red) and UM (black) valid averaging from 4 to 5 hours.

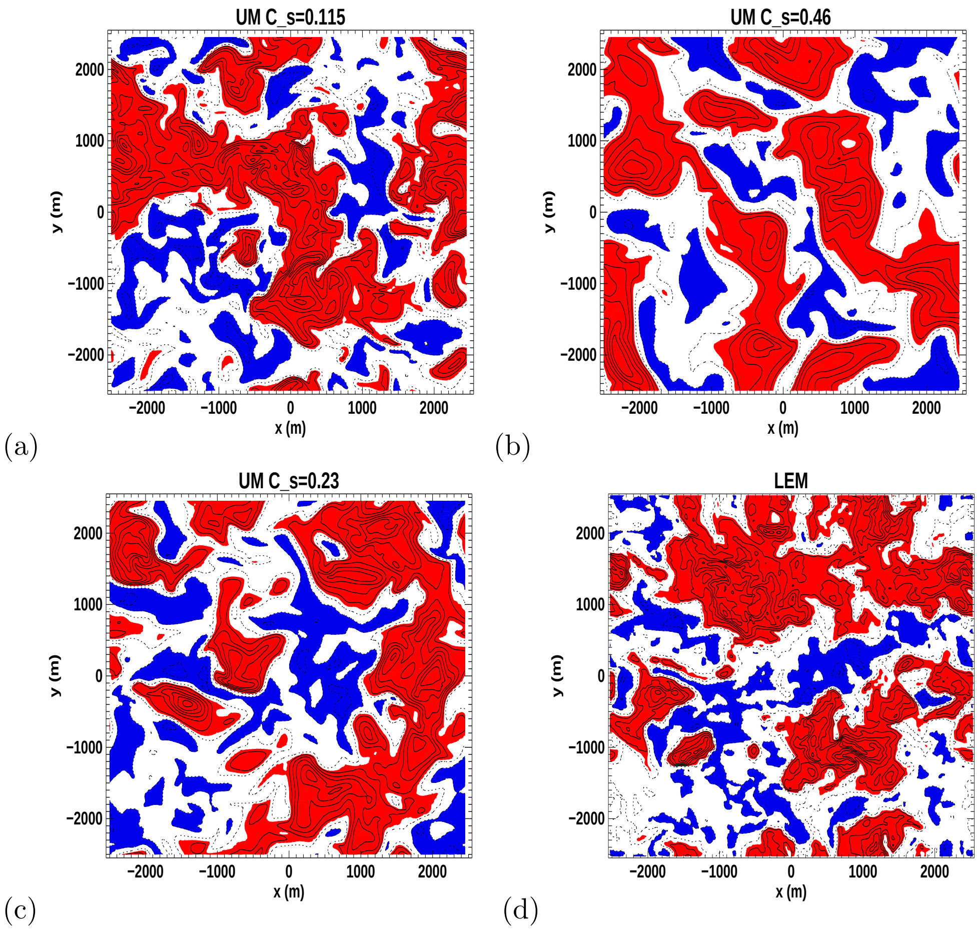


Figure 3: Horizontal cross-sections of instantaneous vertical velocity at height 1km at time 3 hours for (a) C_s=0.115, (b) C_s=0.46, (c) C_s=0.23, (d) The LEM with C_s=0.23. Contour interval 1 ms^{-1}, dotted values negative. Values greater than 1.5 ms^{-1} shaded red, values less that -1.5 ms^{-1} shaded blue.

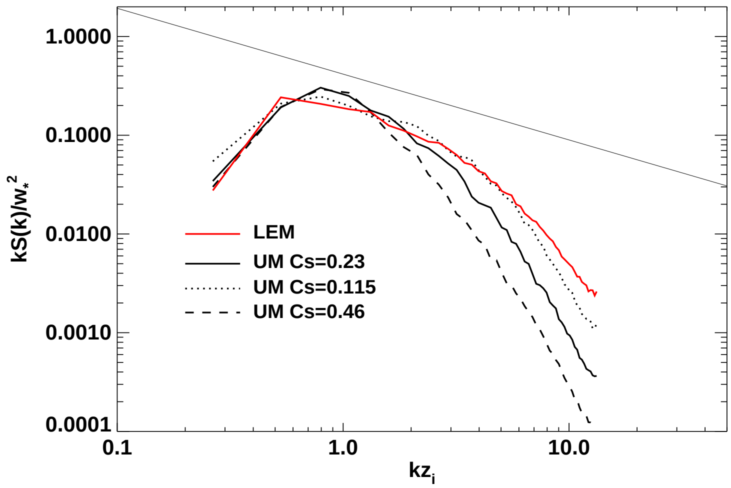


Figure 4: Normalised vertical velocity power spectra (averaged between hour 1 and 2) at height 1 km for 50 m horizontal resolution dry convective boundary layer runs. Thin line is the inertial subrange -2/3 power law. The wavenumber (k) is defined as 1/wavelength. $z_i = 1325m$ is the inversion height of the control simulation and w_* is the convective boundary layer velocity scale. $kz_i = 2$ corresponds to a wavelength of 13Δ, where Δ is the horizontal grid length.

A complementary assessment of the sensitivity to sub-grid model is provided by the vertical velocity power spectra within the convective boundary layer. Figure 4 shows spectra for different UM sub-grid model configurations compared with the LEM. The LEM spectrum has a maximum near $kz_i \sim 1$, a wavelength close to the depth of the mixed layer. It follows the inertial subrange up to wavenumbers $kz_i \sim 3$, and then falls off smoothly with increasing wavenumber. It was found by Mason and Brown (1999) that such a fall off was close to much higher resolution results, and thus can be considered a robust reference for this study.

The control UM spectrum ($C_s = 0.23$, Fig. 4) decays more rapidly with increasing wavenumber than the LEM simulation, despite the use of the same mixing length. The spectrum of the control UM simulation decays from the LEM simulation at values above $kz_i \sim 2$, wavelengths of 13Δ. The UM simulation with $C_s = 0.46$ falls off even more severely, from $kz_i \sim 1$, wavelengths of 26Δ, clearly a configuration of the UM which is too dissipative. The best match to the LEM is achieved with $C_s = 0.115$ with only a slight fall off from the LEM spectrum, starting at $kz_i \sim 6$, a wavelength of 4Δ. So, the UM requires approximately half the mixing length to compensate for other sources of dissipation in the semi-Lagrangian advection scheme.

Similar spectra were calculated (not shown here) for sensitivity to decreasing the UM timestep from 5 s to 1 s and also making the implicit timestepping weights less off centred; both changes had little effect on the spectrum. The UM and LEM simulations consumed similar amounts of computational time.

4 Conclusions

This paper described a comparison between the Met Office Unified Model and Large-eddy model at a turbulence resolving resolution of 50 m. For a dry convective boundary layer case with prescribed surface heat flux, it was demonstrated that a close match between the two could be achieved by using a smaller mixing length in the UM. The UM run with a Smagorinsky constant of 0.115 produced the best match in terms of spectra. Presumably the smaller mixing length compensates for the extra dissipation provided by the semi-Lagrangian cubic interpolation. This study has confirmed in the context of the UM the need to consider the numerical dissipation from the both the advection scheme and sub-grid model when configuring it as a Large-eddy model (Brown *et al.*, 2000). It is pleasing that the UM can produce a good convective boundary layer turbulence simulation.

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