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HANDBOOK

OF

WEATHER FORECASTING

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PREFACE

The Handbook of Weather Forecasting was written mainly for distribution within the Meteorological Office to provide forecasters with a comprehensive and up-to-date reference book on techniques of forecasting and closely related aspects of meteorology. The work, which appeared originally as twenty separate chapters, is now re-issued in three volumes in loose-leaf form to facilitate revision.

Certain amendments of an essential nature have been incorporated in this edition but, in some chapters, temperature values still appear in degrees Fahrenheit. These will be changed to degrees Celsius when the chapters concerned are completely revised.

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19.1. INTRODUCTION

When an aircraft leaves a visible trail behind it, both position and heading of the aircraft may be revealed to observers on the ground. Knowledge concerning the conditions in which trails are likely to occur, and if and how they can be avoided, is clearly important for military purposes so long as visual observations continue to supplement radar in any system of aircraft detection. The physics of aircraft trails have been extensively studied from about 1941 onwards. There are two types of trail, which may be referred to as "aerodynamic" and "exhaust" trails respectively. Aerodynamic trails are formed by adiabatic reduction of temperature to below dew-point in the vortices which are shed from the tips of the propellers and wings (Richards[1*], Parker[2]). They are infrequent, faint and ephemeral and usually occur in moist air; they have been observed not only in flight but even before the aircraft has left the ground. As they do not present any operational or forecasting problem, it will be unnecessary to make any further reference to them in this chapter. Exhaust trails commonly form in the upper troposphere or stratosphere and are often long and persistent. They arise from condensation of water vapour in the exhaust gases after mixing with ambient air, but no completely satisfactory theory of their formation and persistence has yet been published and current methods of forecasting their occurrence are largely empirical. A summary of theoretical work on exhaust trails is given below, followed by an account of forecasting methods.

19.2. CRITICAL TEMPERATURES FOR SATURATION OF THE TRAIL

19.2.1. Physical theory

Combustion of hydrocarbon fuel in the engine results in heat and water vapour passing out with the exhaust gases and mixing with the ambient air in the wake of the aircraft. The added water vapour tends to increase the relative humidity in the wake above the ambient value, the heat tends to decrease it, which effect is dominant depending on the characteristics of the aircraft and engine as well as on the atmospheric conditions. It is evident that a necessary condition for a cloud trail to form behind the aircraft is that the mixture of ambient air and exhaust gas in the wake should attain saturation at the enhanced temperature. Consideration of the conditions in which this is just possible leads at any level to a critical temperature above which trail formation cannot take place, subject to the assumptions made.

Atmospheric conditions will be specified by the conventional symbols. In addition, certain aircraft and engine characteristics and some other variables will be denoted by the symbols listed below. Units are in the c.g.s. system unless otherwise stated, the unit of heat being the calorie.

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*The superscript figures refer to the bibliography at the end of this chapter.

A cross-sectional area of the trail

C heat produced by combustion of one gm. fuel

F rate of consumption of fuel (gm. sec.$^{-1}$)

m mass ratio of ambient air to exhaust gases

m' mass ratio of ambient air to combustion products in the exhaust gases

M mass flow of air through engine (gm. sec.$^{-1}$)

P proportion of heat of combustion of fuel which passes into the trail

V airspeed of the aircraft

W mass of water vapour produced by combustion of one gram of fuel

X rate at which vapour is condensed in the trail (gm. sec.$^{-1}$)

It is assumed that the heat and water vapour from the exhaust are mixed uniformly with the ambient air throughout the cross-section of the wake at a short distance behind the engine. The heating effect on the air in the wake due to the combustion of F gm. fuel together with condensation of X gm. water is given by

$$C_p \rho A V \delta T = P C F + L X \quad \ldots (1)$$

where δT is the increase of temperature, C_p is the local specific heat at constant pressure, L is the latent heat of condensation to water or sublimation to ice, whichever is relevant. Since the composition of the wake consists predominantly of ambient air and since at heights important in the contrail problem the water content of the ambient air is small, C_p for the wake may be taken to have the value for dry air (0.24 cal. gm.$^{-1}$ °C^{-1}). If the ambient air has relative humidity U per cent, then the conservation of water is expressed by:-

$$WF + \frac{E e U \rho A V}{100 p} = X + E \frac{(e + \delta e) \rho A V}{p} \quad \ldots (2)$$

where the left-hand side includes the water vapour introduced per second by the exhaust gases and entrained ambient air, and the right-hand side includes the condensed vapour together with the vapour contained in the now saturated wake at its enhanced temperature; e, $e + \delta e$ are the saturation vapour pressures at temperatures T, $T + \delta T$ respectively; E is the ratio of the molecular weights of water and of dry air. If entrainment of ambient air has proceeded far enough for δT and therefore δe to be regarded as small compared with T and e respectively, then these two equations together yield an expression for de/dT which can be eliminated by means of the Clausius-Clapeyron equation,

$$\frac{de}{dT} = \frac{J e E L}{R T^2} \quad \ldots (3)$$

where J is the mechanical equivalent of heat and R the gas constant.

Thence is obtained an expression for the rate of condensation of vapour per second in the form

$$\left(1 + \frac{E^2 e J L^2}{R p C_p T^2}\right) X = W F - \frac{E e}{R T}\left(1 - \frac{U}{100}\right) A V - \frac{E^2 e J L P C F}{R p C_p T^2} \quad \ldots (4)$$

For a condensation trail to be possible, X must exceed zero and the right-hand side of (4) must be positive. The most favourable case for this occurs when the ambient air is just saturated (the possibility of supersaturation being excluded for the present) so that the critical temperature T_c for trail formation is given by the limiting condition

$$W = \frac{E^2 e J L P C}{R p C_p T_c^{\,2}}$$

$$\text{whence} \qquad T_c = \left(\frac{E^2 J}{R C_p}\right)^{\frac{1}{2}} \left(\frac{P C}{W}\right)^{\frac{1}{2}} \left(\frac{e L}{p}\right)^{\frac{1}{2}} \quad \ldots (5)$$

in which the first factor contains only physical constants, the second depends on characteristics of the aircraft and fuel, and the third on meteorological conditions. By differentiation of equation (4) it may be shown that if the wake is saturated then dX/dT is negative, so that if the ambient temperature exceeds T_c, there can be no condensed water and no cloud trail. The temperature T_c was at one time called the "immunity" temperature; it is the basis of the "Mintra" line shown on the Meteorological Office tephigram forms 2810 and 2810A (1956 Edition).

The above treatment follows that given in M.O.479[3] and by Briggs[4]; it leads to a simple formula for computing the critical temperature, since the latter refers in practice to a part of the trail where considerable mixing has taken place. If however it is required to discuss conditions nearer the engine where temperature may be much in excess of the ambient temperature, then some other method must be used (see for example sections 19.3.4., 19.3.5.).

The critical temperature so defined is independent of the cross-section of the trail, of the fuel consumption and of the speed of the aircraft; it varies, on the other hand, with the type of fuel (through C and W), with the design of the aircraft (through P), with the ambient pressure and temperature (the latter through the saturation vapour pressure), and with the saturation process according as this refers to a surface of ice or water. If the known values of the physical constants (E = 0.621, $J = 4.18 \times 10^7$ erg.cal.$^{-1}$, $R = 2.87 \times 10^6$ erg.gm.$^{-1}$ °K^{-1}; $C_p = 0.24$ cal.gm.$^{-1}$ °K^{-1}) are inserted, then (5) becomes

$$T_c = 4{\cdot}84 \left(\frac{P C}{W}\right)^{\frac{1}{2}} \left(\frac{e L}{p}\right)^{\frac{1}{2}} \quad \ldots (6)$$

19.2.2. Aircraft and fuel characteristics

For the petrol ordinarily used for piston-engined aircraft, W is about 1.4 and C about 10,570 cal.gm.$^{-1}$. The proportion (P) of the heat of combustion which finds its way, directly or indirectly, into the trail, varies to a large extent with the type of aircraft. It is stated[3] that in a single-engined propeller aircraft with radiator, engine and exhaust in line, the value of P is between about 100 and 75 per cent; at most only some 25 per cent of the fuel energy goes into the propeller and even some of this mechanical energy is transformed back into heat and passes into the same trail as the exhaust. If the radiator were greatly offset laterally from the exhaust it is conceivable that conditions might approximate to P = 50 per cent, or even 25 per cent, through much of the heat going off on a different track from the exhaust gas. The published (1956) Mintra line is based on the piston-engined Spitfire III aircraft, for which P is given as 77 per cent at 20,000 ft. and 80 per cent at 40,000 ft. (Goldie[5] or M.O.479[3]). It is further assumed that the combustion of fuel is complete; on examination, the effect on the Mintra or critical temperature of incomplete combustion appears to be negligible, except momentarily from sudden variation in mixture control. A consideration of various types shows that the differences between conventional piston-engined aircraft do not affect the critical temperatures by more than about 2°C.

For the kerosene fuel used in jet aircraft, W is given[3] as 1.3 and C as 10,280 cal. $gm.^{-1}$, whence C/W is 7,910. Bannon[6] gives W as 1.45 and C as 9,800, whence C/W is 6,760; also for the Meteor IV jet he quotes the value of P as 87 per cent at 20,000 ft., 86 per cent at 30,000 ft., 84 per cent at 40,000 ft. and (by extrapolation) 81.5 per cent at 50,000 ft. The critical temperatures given for the Canberra, and by Bannon for the Meteor IV, tend to be slightly lower than those for the Spitfires but the difference is at most about 1°C.

In theoretical work, the value of P for jet aircraft is commonly taken as 100 per cent, and C as 10^4 cal. $gm.^{-1}$, but there is no essential difficulty in changing to other values if required.

19.2.3. Condensation processes

The critical temperature given by equation (6) differs according to the nature of the condensation of vapour in the trail. Water vapour either changes phase directly to ice in a trail maintained saturated with respect to ice, or it condenses to water in a trail saturated with respect to water. Since the saturation vapour pressure over water is substantially greater than that over ice at low temperatures, the critical temperature is lower for water saturation than for ice saturation, the difference increasing from 2°C. at low levels to 4°C. at high levels (Table 19.1). Deposition to ice has been thought to require the presence of deposition nuclei, but it is now doubtful whether these exist. In recent tests[7] with a jet engine, no such nuclei were found in the exhaust gases, although it is suggested that there may have been some present which were rendered ineffective by other constituents. Evidence for the existence of water drops in the early stages of a trail comes from observations of ice accretion on parts of an aircraft immersed in its own condensation trail, and from observations of coronae in trails (Dobson[8], Aanensen[9]). Experiments[10] on non-persistent trails produced in a laboratory indicate that they often do not glaciate in these conditions, even at temperatures several degrees below -40°C. Persistent trails, on the other hand, probably consist of ice crystals in most if not in all cases, the presence of which has been indicated by reports of mock suns in the trails (Hancock[11], Botley[12]). Even when ice cyrstals are present, it is nevertheless considered that the initial change of phase is one of condensation to water drops, and that freezing takes place subsequently. The critical temperature for initiation of trails should therefore be related to the initial attainment of saturation with respect to water in the mixture of exhaust gases and ambient air.

19.2.4. Computed critical temperatures

Table 19.1 gives values of critical temperatures computed from equation (6) for the pressure levels shown in column 1.

TABLE 19.1 CRITICAL TEMPERATURES (°K.)

Pressure	Piston aircraft		Jet aircraft	
(mb)	Ice	Water	Ice	Water
(1)	(2)	(3)	(4)	(5)
1000	250	248	246	244
900	248	247	244	242
800	247	245	243	241
700	245	243	242	239
600	243	241	240	238
500	241	239	238	236
400	239	236	236	233
300	236	233	233	230
200	232	228	230	226
150	229	225	227	223
100	225	(221)	223	(219)
50	219	(215)	217	(213)

(Figures in brackets are extrapolated)

The figures in columns 2 and 3 are derived from the data given in M.O.479[3] for a piston-engined aircraft (Spitfire III): P being about 80 per cent but varying slightly with height, and C/W has the value 7550 cal. $gm.^{-1}$. The figures in columns 4 and 5 are appropriate for a jet aircraft and are based on a value of 100 per cent for P and 7690 cal. $gm.^{-1}$ for C/W. For columns 2 and 4 the trail is assumed to form on reaching saturation with respect to ice, while for columns 3 and 5 saturation with respect to water is required. Values of saturation vapour pressure and latent heat were taken from the Smithsonian Meteorological Tables[13]. The figures in column 2 are in close agreement with the temperatures indicated by the Mintra line on the tephigram (Form 2810, 1956).

19.2.5. Note on terminology

The Mintra temperature as defined above is one above which trail formation is unlikely at the level concerned. Subsequent work has shown the need for revision of the values depicted by the Mintra line on the 1956 tephigram or as given in Table 19.1, column 2, but in order to avoid confusion, any revised values such as those given in columns 3 to 5 of Table 19.1 will be referred to in this chapter as "critical" values, the use of word Mintra being restricted to the original values. The term "critical" will also be used on occasions to include the Mintra values and at times also the "Drytra" values, the temperatures below which trail formation necessarily occurs.

19.3. THE INITIATION OF VISIBLE TRAILS

19.3.1. Inadequacy of Mintra temperatures

The Mintra temperature, at any level as defined above, is such that condensation trails cannot ordinarily form at any higher ambient temperature. The converse, that contrails should form whenever the air temperature is less than Mintra at the same level, is not true; experience shows that the initiation of cloud trails is delayed until the temperature is reduced by several degrees below the Mintra value. The main problem regarding the initiation of trails is to predict the temperature at which they will necessarily form in any given atmospheric conditions, but complete success in this has not yet been achieved. The various difficulties and relevant theories are reviewed in the following sections.

19.3.2. Conditions for a visible trail

The attainment of saturation in the trail, formed of exhaust gas mixed with ambient air, is not of itself sufficient for the trail to become visible; for this to be the case, condensed water or ice particles must be present in a sufficient concentration. What the number of particles must be in relation to their size has not been clearly demonstrated, but it must depend on the conditions of viewing, including illumination, background contrast, distance from the observer, acuity of the observer's eyes and the constitution of the particles.

In most work the limiting condition for visibility is given simply as the mass of condensed water or ice per unit volume. Goldie estimates that a trail about 20 feet in diameter is just visible with a concentration of 0.02 gm. m.$^{-3}$; Appleman[3] assumes arbitrarily 0.004 gm. m.$^{-3}$, for a "faint" trail and 0.01 gm. m.$^{-3}$ for a "distinct" trail. Jones[14] thinks these values are much too low and suggests that a reasonable figure would be of the order five to ten times greater than Appleman's. Pilié and Jiusto[10] find 0.004 gm. m.$^{-3}$ barely sufficient for visibility of a water drop trail in laboratory conditions and suggest 0.055 gm. m.$^{-3}$ as a more realistic minimum value. Further, suppose water droplets are formed initially but in insufficient numbers for a visible trail; on freezing they begin to grow by deposition of ice and continue to do so until the vapour pressure is reduced to saturation with respect to ice, so that in this way the ice content may perhaps reach or exceed the visibility threshold value. It follows that in certain circumstances the formation of a visible trail is delayed until after the freezing of the droplets has taken place, but the process is further complicated by continued entrainment of ambient air in the meanwhile. Appleman, in fact, finds that his minimum ice concentration of 0.01 gm. m.$^{-3}$ is quantitatively satisfied if water saturation is first attained and if the excess vapour, above that required for ice saturation, subsequently condenses and freezes.

When other conditions such as illumination are satisfied, the visibility of a trail depends on scattered reflection of light from the constituent particles. If the concentration is small enough, all the particles, in a tube along the line of sight, will contribute to the scattered light which reaches the observer. A visibility criterion based on the total number of particles has been derived by Frost[15] and Briggs[4] in their work on the application of diffusion theory to condensation trails. From consideration of the brightness of a water drop at 30,000 ft. under zenith sun, and its contrast with the brightness of the sky, Frost obtains the formula

$$N = 8.5 \times 10^{-4} \times r^{-2} \qquad \ldots (7)$$

for the minimum number of droplets of uniform radius r, in a tube of unit cross-section, which will make the trail visible. For a trail of diameter 6 m. and drops of radius 10^{-3}cm. this gives 1.4 drops per cubic centimetre. On the other hand, Goldie's estimate, already mentioned, indicates that for the same size of drops, a trail 20 feet in diameter is just visible when there are five drops per cubic centimetre. Part, at least, of the difference between these two results can probably be attributed to Frost's assumption of the most favourable conditions for visibility.

It is not possible, with present knowledge, to make a firm recommendation regarding the choice of a visibility criterion. When more definite information becomes available it should be possible to determine approximately the depression of ambient temperature below Mintra at which a visible trail should occur, given reasonable conditions of illumination. Meanwhile, observations of the ambient temperature at which trails first form may be used to assess the concentration of the liquid or solid particles in the trail (Jones[14]).

19.3.3. Critical temperatures for water saturation

The Mintra line printed on the tephigram Form 2810 (1956 Ed.) is based on deposition to ice in the trail. It has been seen that if condensation in the trail is delayed until saturation with respect to water is attained, then the visibility condition is likely to be satisfied as soon as the droplets freeze. If the critical temperature is referred to saturation with respect to water, there is a reduction of about 3°C. compared with the ice Mintra values (Table 19.1, columns 2 and 3; see also Fig. 19.5); a further reduction of about 3°C. is obtained by taking P, the proportion of the heat of combustion going into the trail, as unity for a jet aircraft, although this appears to be more a matter of theoretical convenience than correspondence with the facts. In this way, for any pressure level, a critical temperature is obtained which is about 6°C. below the originally computed value and which is in closer, but not yet complete, accord with observations. These critical temperatures for water saturation are given in column 5 of Table 19.1. Several sets of comparisons with observations are available. Appleman[16,17] obtains the same critical temperatures by a graphical method; comparison with observations[18] of trails made by United States jet aircraft showed that of 1,784 occurrences, only four per cent occurred at temperatures above the critical values; these observations are admittedly subject to some uncertainty since they were made by crews of operational aircraft on routine flights. Frost[15] gives a diagram in which the occurrences of the first appearance of contrails are plotted against height and temperature, using a series of observations made by the Meteorological Research Flight in 1951-53 with piston-engined aircraft; only one observation out of 52 falls on the wrong side of the critical line, which agrees with Appleman's figures and with Table 19.1, column 5. Another series of observations[19] made by the Meteorological Research Flight with jet aircraft is plotted in Figure 19.1 (see Section 19.6.1 for fuller description). The critical line now under discussion corresponds with the line 5½°C. below Mintra, and it is seen that only one trail (out of 180 occurrences) falls on the wrong side of this line. Nevertheless, with all such sets of observations the great majority of trails do not begin to form until the ambient temperature is reduced below this critical value by a further 3° to 6°C.

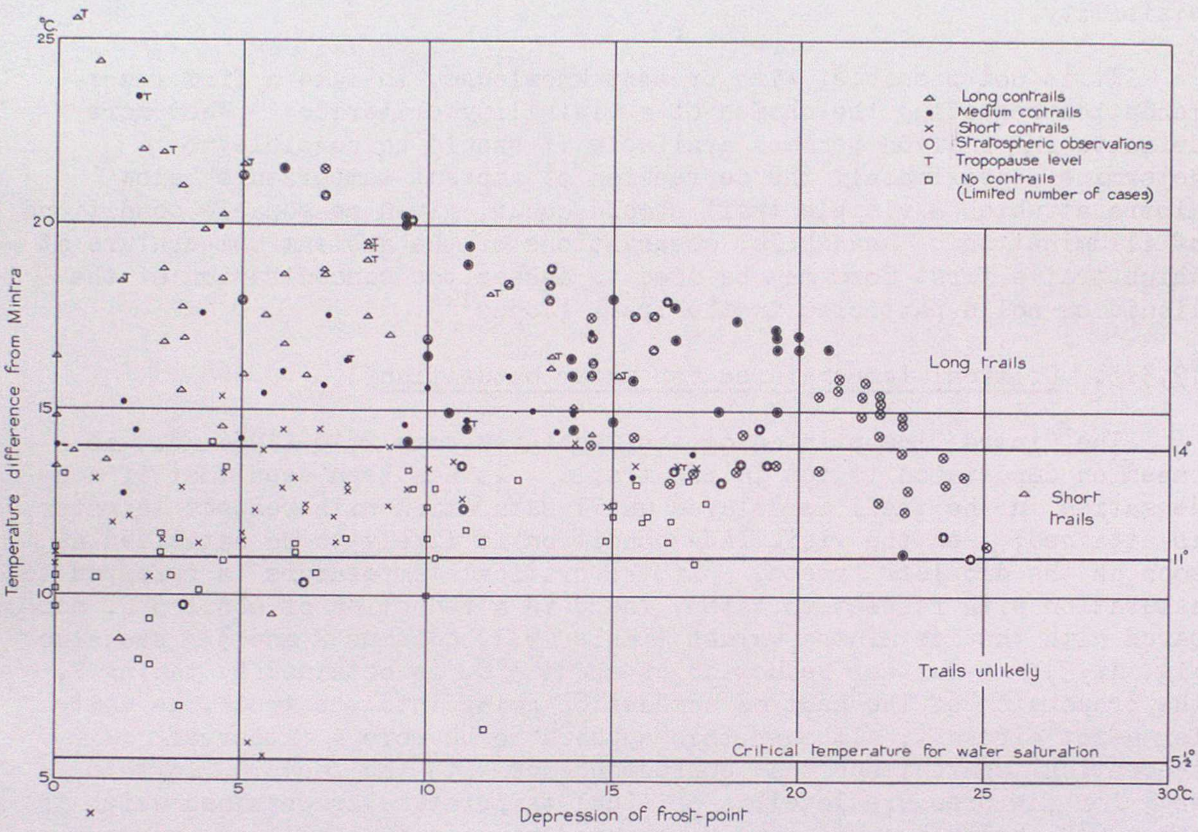

FIGURE 19.1 LENGTH OF CANBERRA CONTRAILS FOR VARYING TEMPERATURE AND HUMIDITY (HELLIWELL AND MACKENZIE[19], REVISED)

If the droplets which form on reaching saturation do not freeze immediately, as supposed by Appleman and others, then the visibility condition will not be satisfied at once; if the trail is to become visible while the water is still liquid, then a certain minimum concentration must be attained. Osterle[20] computed that Appleman's requirement of 0.01 gm. m.$^{-3}$ results in lowering the critical temperature for saturation by 3 to 4°C. at 30,000 ft. to 40,000 ft. A reduction of temperature of this amount in a wind tunnel did not always result in a visible trail, but lighting conditions had a pronounced effect. By daylight, the water concentration was increased by a factor of nearly four without necessarily producing a visible trail. From the observations of Helliwell and MacKenzie (Section 19.6.1.) it is seen that trails appear only when the temperature is reduced to about 5°C. or 6°C. below the critical value for saturation. This result would therefore be explicable if the trails first become visible while still in the liquid state, and when the water concentration reaches a certain minimum value which is probably substantially greater than 0.01 gm. m.$^{-3}$. The transition of the trail particles from water to ice is discussed in section 19.4.

19.3.4. Further theory of jet exhaust trails

Theoretical considerations have been carried a stage further by Jones[14] who assumes knowledge of the physical state of the exhaust gases as they leave the jet pipe. If ambient air of humidity mixing ratio x gm. kg.$^{-1}$ passes through the engine at the rate of M gm. sec.$^{-1}$, the humidity mixing ratio of the exhaust air about to leave the jet pipe is shown to be given by

$$x_e = \frac{W\,F\,(1000 + x) + M\,x}{M - F\,(W - 1)\,(1000 + x)\,/\,1000}\ \text{gm.kg.}^{-1}$$

where F and W are as defined in Section 19.2.1. If the exhaust gas is at temperature T_1 and pressure p_1 then on emerging it immediately expands adiabatically to the ambient pressure p and is cooled to a temperature T_1' given by

$$T_1' = T_1\left(\frac{p}{p_1}\right)^{(\gamma - 1)/\gamma}$$

where γ is the ratio of the specific heats of the exhaust gas; the humidity mixing ratio meanwhile remains unchanged.

The gases leave the exhaust pipe at a high speed v (of the order of 5×10^4 cm. sec.$^{-1}$) relative to the aircraft, and undergo considerable mixing with the ambient air during which both the thermal and kinetic energy of the exhaust gas contribute heat to the trail. If one gram of exhaust gas moving at speed v_1 cm. sec.$^{-1}$ relative to the ambient air $(v_1 = v - V)$ mixes with m gm. of ambient air, then from equations expressing the conservation of momentum and the loss of kinetic energy as the speed of the exhaust gas is reduced to that of the ambient air, it is found that the temperature of the mixture of air and exhaust gas in the trail is raised by an amount

$$\delta T = \frac{m\,v_1^2}{2\,J\,(1 + m)^2\,C_p}$$

wherein C_p for the trail is taken to be the same as that for dry air since both burnt fuel and water vapour form only a small proportion of the mixture. The result of mixing one gram of exhaust gas at temperature T_1' with m gm. of ambient air at temperature T, both being at ambient pressure, is to give a temperature $(T_1' + m\,T)\,/\,(1 + m)$ for the mixture. Together with the contribution from the kinetic energy, this gives the trail temperature in degrees Kelvin as

$$T_m = \left(\frac{T_1' + m\,T}{1 + m}\right) + \left(\frac{v_1^2}{2\,J\,C_p}\right) \cdot \frac{m}{(1 + m)^2} \qquad \ldots (8)$$

After some reduction and approximation, the humidity mixing ratio of the mixture is shown to be

$$x_m = \frac{1000\, W\, F}{(1 + m)\,(M + F) - W\, F} + x \qquad \ldots (9)$$

in which the first term on the right depends (apart from m) only on engine and fuel characteristics, and the second only on the ambient air.

The expressions (8) and (9) give the temperature and mixing ratio in the trail so long as the trail remains unsaturated. Since T_m defines the saturation mixing ratio (with respect to water or ice) at the same temperature, two sets of curves can be drawn on the same diagram which show (i) the variation with m of saturation mixing ratio in the trail, and (ii) the values of x_m for any selected value of ambient pressure, temperature and humidity mixing ratio. The points of intersection of the two sets of curves give the value of m (that is, the degree of entrainment) at which the trail becomes saturated and the corresponding saturation mixing ratio within the trail. In particular, if the ambient air is saturated, a critical ambient temperature will be defined, above which saturation of the trail cannot be achieved; this would define the critical temperature for the pressure level concerned.

As an example, Jones assumes for a hypothetical jet engine, (in addition to the values already given above), $T_1' = 600°K$, $v = 2.5 \times 10^4$ cm. sec.$^{-1}$ relative to the ambient air, F = 230 gm. sec.$^{-1}$, M = 1.48×10^4 gm. sec.$^{-1}$, and he gives a diagram, reproduced as Fig. 19.2, showing the sets of curves mentioned for 200 millibars. From a series of such diagrams for different pressure levels, critical temperatures can be obtained corresponding with initial saturation of the environment with respect to ice or water, but the results have not yet been published.

Jones' equations do not include ab initio the effect of latent heat, but he discusses this separately and an approximate allowance for it is discussed; it appears that the trail temperature might be increased by about 0.5°C. above the value previously calculated.

A table given by Jones for 200 millibars shows that, with the values quoted above, the trail temperature exceeds the ambient temperature by about 40°C. for an entrainment ratio m = 10, and by about 10°C. for m = 40. Also from equation (8) with m = 10, it is seen that the kinetic effect contributes about 3°C. to the trail temperature. On leaving the engine, the exhaust gases immediately start to mix rapidly with the ambient air, m reaching a value of 10 within a few feet of the jet outlet, and a value of 20 to 30 within about 100 yards, but thereafter the mixing increases more slowly. When the ambient temperature is near the critical value, a large amount of mixing is required before a trail can form and the trail temperature at that point is not much above the ambient temperature. This explains why the method of Section 19.2 gives results which agree with those of other methods in which the assumption of a small excess temperature in the trail is not made. Similarly, Jones' method is likely to lead to closely similar values for critical temperatures (apart from a possible change in the visibility criterion). As ambient temperature becomes more favourable, so trail formation takes place with a smaller amount of mixing, in conformity with the observation that dense trails can form within a few feet of the exhaust pipe.

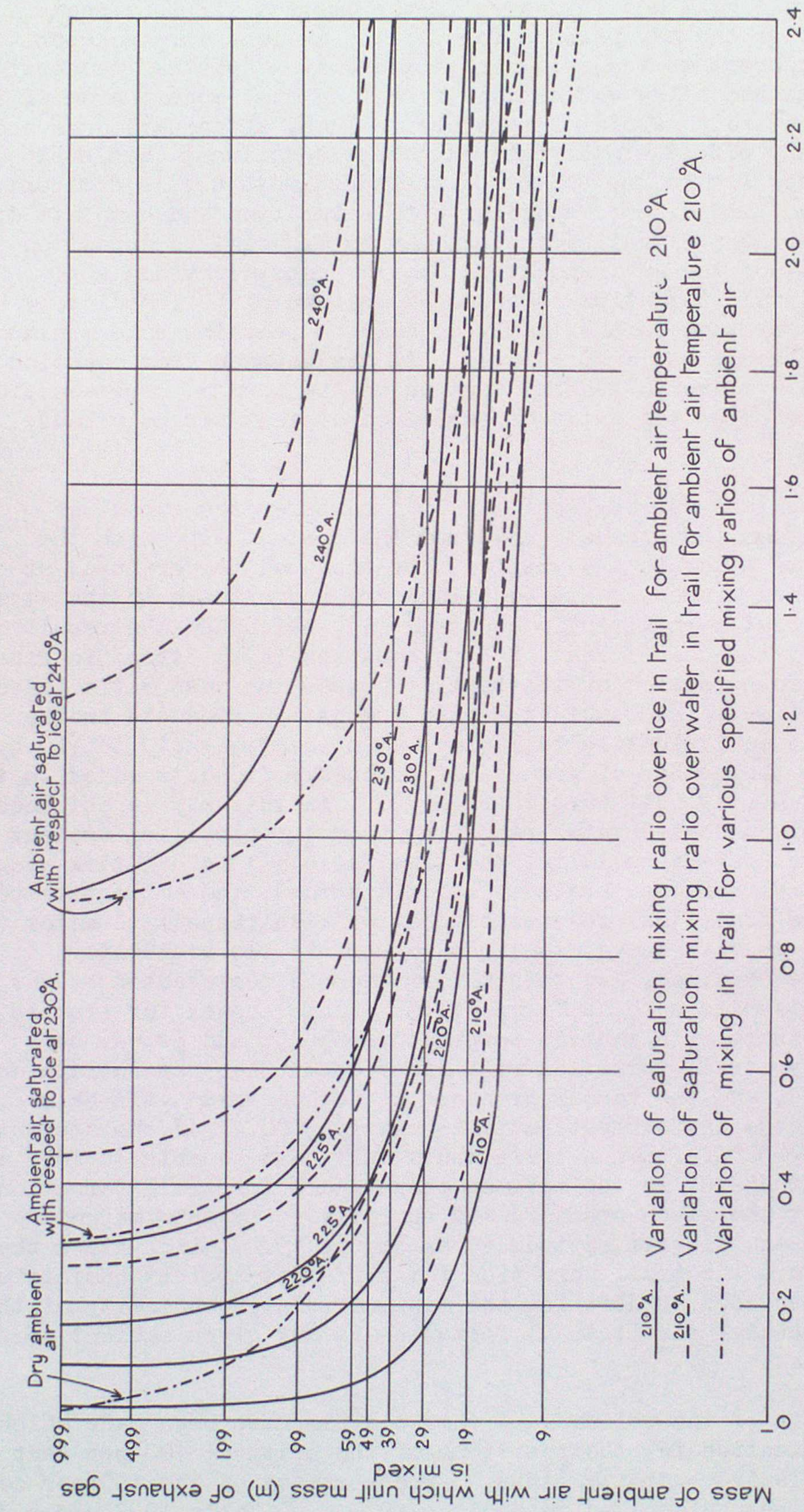

FIGURE 19.2 CONDITION IN EXHAUST TRAIL AT 200 mb.
(R.F. JONES[14])

19.3.5. Effect of ambient humidity

The Mintra temperature as originally determined is that temperature above which a trail will not form even if the ambient air is already saturated (although one may perhaps form if the ambient air is supersaturated with respect to ice). In a similar way a "Drytra" temperature may be defined as one below which trail formation must occur, even if the ambient air is entirely devoid of moisture. Thus, at temperatures above Mintra the heating effect of the exhaust gases is dominant, while at temperatures below Drytra the effect of the added moisture is dominant. Between these two limits, corresponding with saturation and complete dryness of the environment, trail formation may be expected to depend on the relative humidity of the environment. Similar considerations apply whether visible trail formation is supposed to take place immediately on attainment of saturation in the trail, or only on reaching some minimum concentration of condensation products. As may be seen from equation (4), the critical temperature in general depends on the area of cross-section (A) of the trail, or on the ratio of entrained ambient air to exhaust gases.

Using a graphical method, Appleman[16,17] compares the amount of water available from the exhaust, and from the ambient air, with the amount required to saturate the trail at its enhanced temperature, or to produce a specified excess. His equations are approximate in that they omit the latent heat term given in equation (1), which has the result of reducing to unity the coefficient of X in equation (4). Also, in place of the area of cross-section of the trail, he uses the mass ratio (here denoted by m') of entrained ambient air to combustion products in the exhaust; this is equivalent to replacing ρAV in equations (1) to (4) by m'fF, where f is the number of grams of combustion products added to the wake for each gram of fuel burned (f = 12). In this way is obtained the net amount of condensed water resulting from any specified ambient pressure, temperature and humidity, and mass ratio m'; a negative amount means that the trail is unsaturated. From diagrams, the ambient temperature at which the trail just reaches saturation with respect to water (a condition which, as seen above, is taken to satisfy the visibility criterion) can be obtained, and this information is represented by the solid lines in diagrams such as Figure 19.3. This shows, for example, that at 200 millibars with ambient temperature -58°C. and 60 per cent relative humidity, trail formation will first occur when the entrainment ratio reaches 175, and the vapour pressure is not reduced again below saturation until the entrainment ratio is nearly 3000. If the ambient temperature were -52°C., then a trail would only just be able to form at m' = 500, so that -52°C. is the maximum temperature for trail formation to be possible at the given pressure and humidity. By this method Appleman obtains the diagram reproduced as Figure 19.4, which gives the maximum temperature for trail formation for different ambient humidities; it can also be regarded as defining the minimum relative humidity of the environment which will permit trail formation at any given ambient temperature and pressure.

Several tests of the validity of this diagram have been made which give some justification for the positions of the critical 100 per cent and zero per cent relative humidity lines. The accuracy of the 100 per cent line has already been discussed in Section 19.3.3. Table 19.2 now gives a summary of an analysis of trail observations made by the Air Weather Service, Washington[18].

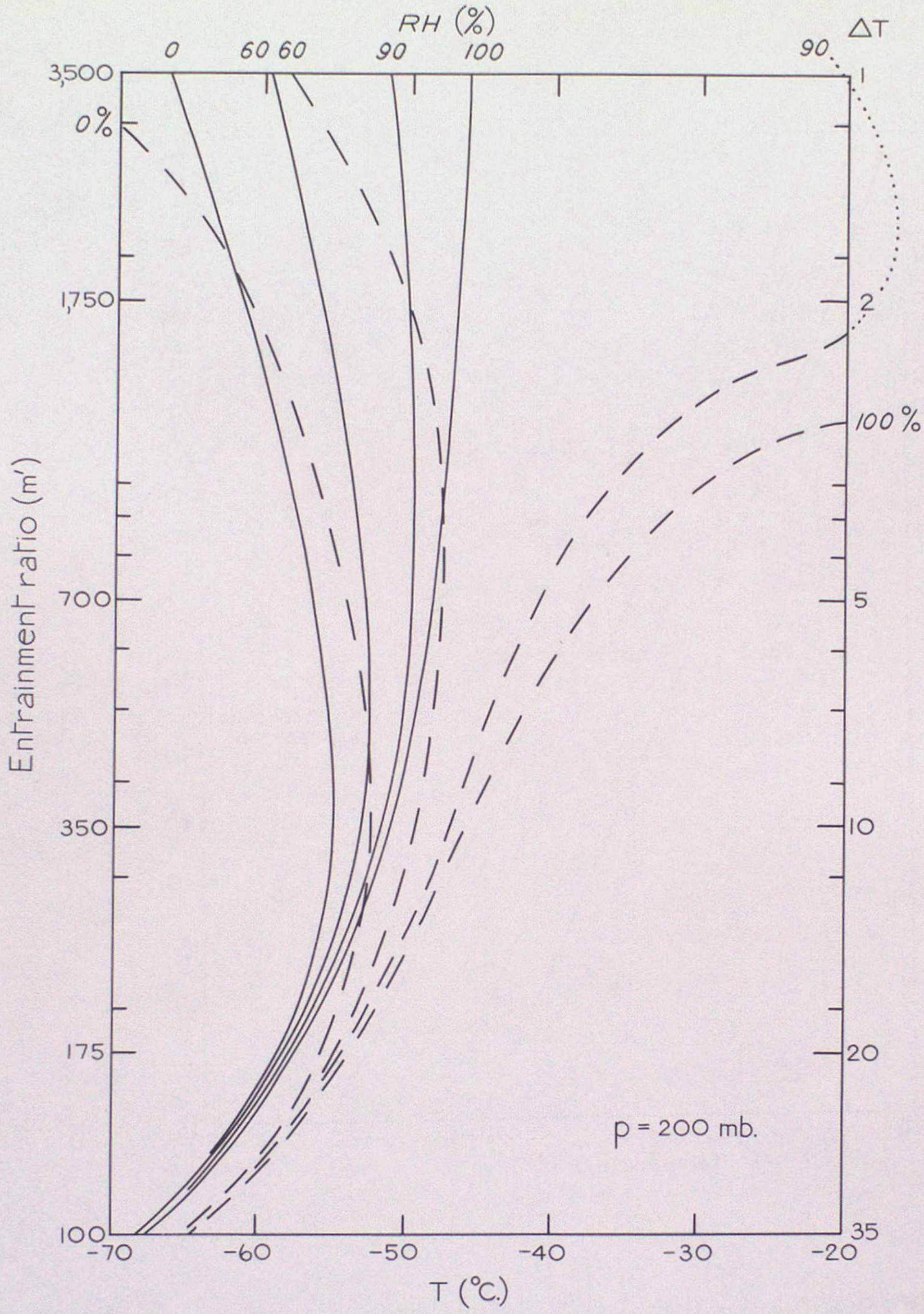

FIGURE 19.3 VALUES OF ENTRAINMENT RATIO (m') REQUIRED FOR FORMATION OF A WAKE SATURATED WITH RESPECT TO WATER (SOLID CURVES), AND SATURATED WITH RESPECT TO ICE PLUS AN ICE-CRYSTAL CONTENT OF 0.01 gm. m.$^{-3}$ (DASHED CURVES) (AFTER APPLEMAN[18,17])

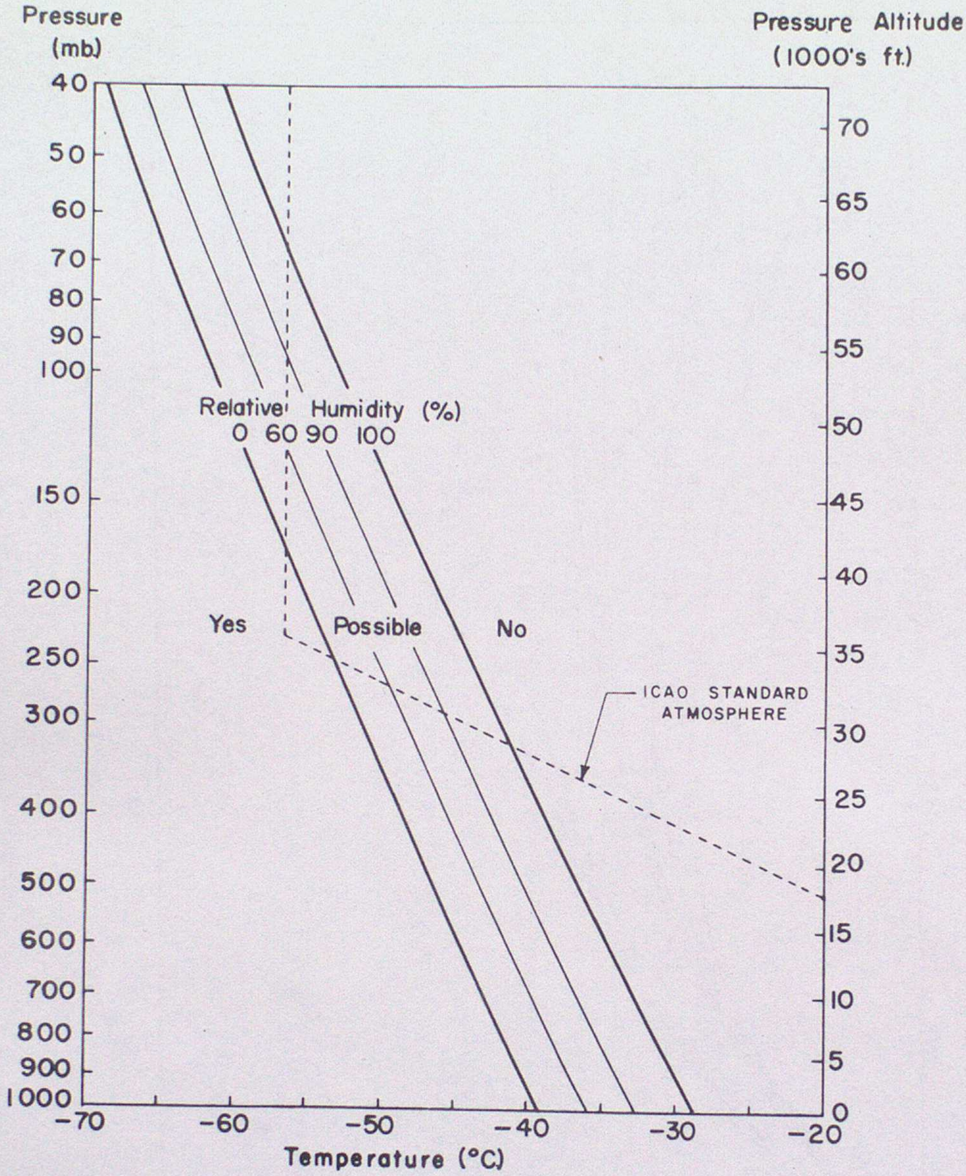

FIGURE 19.4 APPLEMAN'S[16,17] FORECASTING DIAGRAM. A GRAPH OF THE RELATIVE HUMIDITY REQUIRED FOR JET-AIRCRAFT CONTRAIL FORMATION AS A FUNCTION OF PRESSURE AND TEMPERATURE OF THE ENVIRONMENT

TABLE 19.2 PERCENTAGE FREQUENCY OF 3271 OBSERVATIONS WITH AND WITHOUT TRAILS IN RELATION TO APPLEMAN'S DIAGRAM

Temperature	Trails	No Trails	Total
Above 100% value	26	4	30
Intermediate	26	25	51
Below 0% value	2	17	19
Total	54	46	100

All the observations were made by the crews of operational jet aircraft. The table shows that the critical temperatures fulfil expectations for all but a small proportion of the observations. When the ambient temperature fell between these two values, a trail was as likely to form as not; unfortunately no humidity observations were available with which to check the theoretical expectations in this range.

In an analysis of observations made by the Meteorological Research Flight with piston-engined aircraft, Bannon[21] found only a "very rough" correspondence between trail formation and ambient humidity as measured by a frost-point hygrometer, and concluded that the formation of trails and their persistence must be governed by factors other than humidity. From a later series of ascents by the Meteorological Research Flight, Frost[15] confirmed that the first appearance of a trail bears no relationship to the humidity of the environment; this again refers to piston-engined aircraft. For jet aircraft, Helliwell and MacKenzie[19] conclude that relative humidity has only secondary effects on trail production. An expression due to Jones for the humidity mixing ratio in an unsaturated trail has been given above as equation (9). With the values there quoted, this equation reduces approximately to

$$x_m = \frac{20}{1 + m} + x$$

At temperatures of, say, $-50°$C. or less, and pressure about 200 millibars, x is at most 0.20 gm. kg.$^{-1}$ (corresponding with saturation over water) and is usually considerably less. The ambient contribution to the mixing ratio of the trail is therefore small in comparison with the contribution $20/(1 + m)$ from the exhaust unless m is large; moreover the relative effect of the ambient contribution continues to decrease with the ambient temperature. Observations and theory therefore agree in showing that the ambient humidity is generally of secondary importance for the initial formation of trails.

19.4. PERSISTENCE AND LENGTH OF EXHAUST TRAILS

In the previous section, the conditions in which a visible exhaust trail can form have been discussed. It is now necessary to consider the further history of a condensation trail, in particular its length and persistence in relation to the state of the atmosphere. The main factors which control persistence are: (i) the constitution of the trail - water drops or ice particles, (ii) the temperature and humidity of the environment, (iii) the rate of evaporation of the trail particles, (iv) the degree of mixing between the exhaust gases and the environment and (v) diffusion of the trail particles.

Persistence is a somewhat arbitrary term which has been variously defined. If a trail terminates within, say, a minute it would probably be described as ephemeral, or evanescent or at least non-persistent; if it lasts several minutes or longer, it is likely to be regarded as persistent.

Bannon[6], in his analysis of ascents made by the Meteorological Research Flight with piston-engined aircraft, remarks that the trails usually dissipated within a few seconds, and he defines a persistent trail as one which endures for at least two to three minutes. He found that persistent trails occur with widely varying ambient humidities, although there is some tendency for them to form in conditions moister than average. He suggested that the degree of turbulence present in the ambient atmosphere might be important in determining persistence, but records of bumpiness measured on the investigating aircraft did not support this idea. Some theoretical work on the turbulent diffusion of the trail particles has since been carried out and is described in Section 19.5. Frost[15] on the other hand suggests, also in relation to piston-engined aircraft, a duration in excess of about seven minutes for persistent trails.

As already discussed in Section 19.2.3 it appears that condensation in a trail begins only when the vapour pressure has reached saturation with respect to water, and that water drops are formed in the first instance. If these remain liquid they will evaporate more or less rapidly according to the humidity and rate of entrainment of ambient air, which is always unsaturated with respect to water except in a water-drop cloud. Such trails will be generally non-persistent in character, although their actual duration in time will vary to some extent with the circumstances. If, on the other hand, the droplets freeze before they can evaporate, then the ice particles grow further by deposition until the vapour pressure in the trail is reduced to saturation with respect to ice, the trail at the same time becoming optically denser. If the ambient air is unsaturated with respect to ice, the ice particles will tend to evaporate with continued mixing; the duration of the trail will be limited although greater than that for a purely water-drop trail, if only on account of the greater density of the ice trail. If the ambient air is supersaturated with respect to ice, then the trail persists indefinitely and the ice particles continue to grow until they become large enough to fall out as precipitation.

Whether the water drops freeze soon after their formation is therefore of first importance for the further life of the trail, and it has in fact been generally assumed in the literature that a persistent trail is composed of ice crystals. However, even a non-persistent trail made by a fast aircraft can be long enough to be operationally important. Thus, a trail of length one mile at 50,000 ft. would subtend an angle of six degrees at the ground, although for a fast aircraft any one part of the trail will have a duration of less than 10 seconds. For operational and hence for forecasting purposes the length of the trail is surely of more importance than its persistence, and when the duration is brief, the change to ice crystals may not take place.

Appleman's diagrams, one of which is reproduced as Figure 19.3, illustrate how, on his theory, the length of trail is related to the degree of mixing between exhaust gas and environment. For example, if the temperature and relative humidity at 200 millibars are -55°C. and 60 per cent respectively, a trail first forms where the mass ratio (m') of ambient air to exhaust gas is 250. If freezing takes place, the trail remains visible until the mass ratio reaches nearly 3000, given by the point of intersection of the -55°C. ordinate with the broken curve for 60 per cent relative humidity, corresponding with an ice content of 0.01 gm. $m.^{-3}$. Diagrams of this type give at present only a qualitative idea of the length or persistence of trails, since the relationship between entrainment ratio and distance from the aircraft has not been determined.

Jones'[14] theory, like Appleman's, yields values of entrainment ratio (m) between which a trail (or visible trail) will exist. (His m is the mass ratio of entrained air to total exhaust, and so is smaller than Appleman's). Thus, Figure 19.2, reproduced from Jones, shows, for example, that saturation of the trail with respect to water is achieved with completely dry ambient air at a temperature of 220°K. and entrainment ratio 25, and that the trail persists until the water content is again reduced to saturation at an entrainment ratio of m = 71, or, if freezing has taken place, until m = 166. With a suitable modification, values of m can be found which mark the beginning and end of a visible trail. Here, again, the result remains only qualitative as regards the length of trail until this can be related to the entrainment ratio.

The relationship between trail diameter and entrainment is also discussed by Jones. From Figure 19.2 it is seen that saturation of a trail (at 200 millibars) cannot be achieved until m has attained a value of about 10, and the diameter of the trail is then about three times the diameter of the jet orifice. The trail diameter increases only slowly as mixing with ambient air proceeds, as shown by the following figures obtained theoretically:

Trail diameter (D_m) and entrainment ratio (m)
(ambient temperature 200°K.)

(Jet pipe diameter taken as unity)

m	D_m
10	2.61
20	3.47
30	4.15
40	4.74
50	5.26
60	5.73
80	6.58
100	7.32
1,000	22.8

19.5 APPLICATIONS OF THEORIES OF TURBULENT DIFFUSION

The passage of an aircraft, through the atmosphere, leaves immediately behind it a wake in which the principal eddies are those set up by the aircraft itself. The initial mixing of exhaust gas with ambient air takes place in this wake and may lead to the formation of a visible trail. Once the disturbance due to the aircraft has died down, the subsequent history of the trail elements is governed, according to the theories about to be described, mainly by the inherent turbulence of the free atmosphere. This causes the particles to diffuse further away from the axis of the trail until with their reduced concentration the trail is no longer visible, or until the particles have evaporated. There is, however, no really suitable theory of turbulent diffusion in the wake or in the free atmosphere, hence existing work has proceeded by applying formulae which have been developed to describe the effects of diffusion in the shear layer near the ground. This procedure is unsatisfactory; but it is worthwhile indicating very briefly the attempts so far made, if only to bring out the kind of results that may be expected from this line of attack.

In an application to the region in which turbulence is set up by the passage of a piston-engined aircraft, Frost[15] regards the condensation elements in the trail as if they were diffused from a continuous point source moving with the speed V of the aircraft, and takes the result to be the same as that of diffusion from a fixed source into a turbulent medium moving with mean speed V. By assuming further that a certain minimum number of droplets in a tube of unit cross-section along the line of sight is required to render the trail visible (see Section 19.3.2), he obtains an equation for the visible outline of a trail in the form

$$(1 + m) \; z^{(1+m)} = C^2 \, x \, \log_e \frac{l}{x} \qquad \ldots (10)$$

where x is distance behind the aircraft, z is distance from the axis of the trail, l is the value of x for which the trail vanishes, C is a diffusion constant and m a stability parameter (m = 1/7 corresponding with neutral stability). It follows that the result of plotting $z^{(1+m)}/x$ against $\log_e x$ should be a straight line of slope $C^2/(1 + m)$ from which C could be determined in terms of m without any knowledge of the number or size of the trail particles; this expectation was confirmed by observations of a trail made by a Mosquito aircraft of the Meteorological Research Flight, and the value found for C was in reasonable agreement with that derived from certain observations of smoke puffs. By differentiation of (10) with respect to x, the trail width is found to be a maximum when x = 1/2.7, that is, at about 3/8 of its length, which is in fair agreement with observations of short non-persistent trails. The length of the turbulent wake to which the preceding theory applies is of the order of 250 metres, and if the turbulence in the wake is sufficient to dissipate the trail before this length is attained, then only non-persistent trails can be formed. This leads to a necessary condition for persistent trails, namely, $X/V > 0.005$ gm. cm.$^{-1}$, the notation being that given in Section 19.2.1; in other words, the rate of condensation per unit length of trail must exceed a certain value. In an appendix to that paper, certain formulae are derived for trails formed by a jet aircraft.

In a paper by Briggs[4], it is assumed that for jet aircraft the initial mixing behind the aircraft is rapidly completed and that thereafter the trail particles are subject only to undisturbed atmospheric diffusion; because of the high aircraft speed it is suggested that the problem should be treated as one of diffusion of droplets produced instantaneously along a line. After the application of a visibility criterion on the lines of Frost, an equation for the visible outline of a trail is obtained in the form

$$(1 + m) \, z^2 = C^2 \left(\frac{\bar{u}\, x}{V}\right)^{2/(1+m)} \log_e \frac{l}{x} \qquad \ldots (11)$$

which introduces the mean wind speed $\bar{u}$. Briggs also deduces a formula for the critical temperature for visible trails to persist for at least 10 secs.; this formula is dependent on wind speed and it is deduced that trails can persist for 10 secs., only if the mean wind is less than about 75 knots. This result has not been confirmed; it would seem to require that the mean wind speed should be a measure of the vertical shear which in turn is related to the intensity of turbulence.

The formula for diffusion from an instantaneous line source has also been applied by Clodman[22] to the dissipation of trails; in addition to the usual parameters, consideration is given to the height of tropopause, lapse rate of temperature, wind shear and curvature of the stream-lines. This emphasizes the complexity of the problem. As with the work described above on the diffusion aspect, there remains some doubt regarding the applicability of the basic formulae and, on the whole, little confidence can be based on the theoretical results unless, and until, they have been fully substantiated by comparison with observations.

19.6. FORECASTING CONDENSATION TRAILS

The theoretical work discussed above has had one or more of the following objectives: (i) to determine at any level a critical (Mintra) temperature, such that if the ambient temperature exceeds this value then trails will not be expected; (ii) to determine at any level a critical (Drytra) temperature, such that if the ambient temperature is less than this value then trails will certainly be expected whatever the ambient humidity; (iii) when the ambient temperature lies between the two critical values, to determine the minimum ambient relative humidity which will permit formation of trails; (iv) to determine the length and persistence of trails in any given ambient conditions.

There has been little theoretical or practical success with objective (iii); on the contrary, both observations and theory indicate that the humidity in this range is generally of secondary importance. Further, the need for objective (iv) has become less as the speed of aircraft has increased. The main forecasting need is therefore to specify the conditions in which initiation of trails may or may not take place, that is, to specify the critical temperatures at any height. For ambient temperatures between these limits, trail formation is more problematical. While the effect of humidity is apparently small in this range, it should perhaps be given some weight, particularly when the ambient temperature is near one of the critical values. Although direct observations of humidity are not available as a routine at the heights usually concerned, some indirect information is often available, and it is advisable to estimate the humidity as best one can and to weight the forecast accordingly.

The forecasting of contrails depends then on (i) forecasting the ambient temperature for the path followed by the aircraft, including any variations of height en route and if necessary the initial climb and final descent; (ii) forecasting the ambient humidity for the path of the aircraft; (iii) assessing the likelihood of trails at any stage of the flight in terms of the relation of ambient temperature to the critical temperatures, and in terms of the humidity. Forecasting the temperature is fully explained in Chapter 14 and need not be discussed here. Humidity forecasts in the lower and middle troposphere are discussed in Chapter 15: forecasts of humidity in the upper troposphere are difficult because of the absence of routine observations at those levels, hence it is necessary to fall back on indirect information (see Chapter 15, Section 15.6.3) or on statistical methods (Chapter 15, Section 15.5.5.3). There are in use at present two basically different methods for forecasting condensation trails, one by the British and the other by the United States meteorological services.

19.6.1. Empirical forecasting

Helliwell and MacKenzie[19] discussed observations made by a Canberra jet aircraft of the Meteorological Research Flight. Meteorological observations, which included measurements with the Dobson-Brewer frost-point hygrometer, were made by an observer in the aircraft, and by means of a camera mounted in the tail, photographs of the trails were taken. Further information was obtained by use of a second aircraft for observing the trails made by the first. In general it was not possible to derive the lengths of the trails at all precisely, so that for purposes of analysis they were roughly grouped into three categories - short, medium and long. For all available photographic observations, the depression of ambient temperature below the Mintra was plotted against the frost-point depression, and the estimated length of trail was noted; similarly the plots of the observations were suitably labelled when they referred to the tropopause or the stratosphere. The observations were found to be grouped into fairly well defined zones according to the length of trail, and the diagram was made the basis of an empirical method of forecasting.

Helliwell's and MacKenzie's diagram has since been revised and some additional observations included with the result as shown in Figure 19.1. The original classification of the trails into three groups was thought to be unrealistic because of the lack of precise measurements of their length. The medium category was therefore merged with the other two, so that the zoning now separates, necessarily somewhat roughly, areas of long, short and no trails respectively. There is, moreover, little suggestion in this diagram of dependence on ambient humidity. While there are a few occurrences of trails at high relative humidity (frost-point depression 6°C. or less) in association with temperatures within 10°C. of the Mintra value, it should be realised that there must have been a large number of flights in this region when no trails were formed but nil reports were not recorded; the positive occurrences must represent only a small proportion of all flights in this part of the diagram. The following rules for empirical forecasts are based on the revised diagram:

(a) Temperature less than 11°C. below Mintra, trails in general not expected.

(b) Temperature 11°C. to 14°C. below Mintra, short (non-persistent) trails should be forecast.

(c) Temperature more than 14°C. below Mintra, long (persistent) trails should be forecast.

While these rules take no specific account of humidity, it is considered that (when the information is available) due weight should be given to humidity when the temperature is near to one of the limiting values. Moreover, it has already been suggested (Section 19.4) that even if the initiation of trails shows little relation to ambient humidity, persistence is likely to be affected by it. Further, it should be remembered that trails have occurred occasionally at temperatures above the Mintra value.

This scheme of forecasting avoids a defined Drytra temperature, but it is seen that (Figure 19.5) the temperature 14°C. below Mintra, below which long trails are to be forecast, is practically identical with Appleman's zero per cent line or Drytra, the difference at any level being at most 1°C. The critical temperatures on this scheme, immediately below which short trails are to be forecast, are about 6°C. below Appleman's 100 per cent humidity line.

19.6.2. <u>Forecasting by Appleman's method</u>

Appleman's forecasting diagram is shown in Figure 19.4. It may be regarded as showing the minimum ambient relative humidity with respect to water which will permit trail formation by a jet aircraft at any given pressure and temperature. The diagram divides into three separate areas: (1) if the temperature at the given pressure lies to the left of the zero per cent relative humidity line, trails should form whatever the humidity; this line therefore defines the Drytra temperatures; (2) if the temperature falls to the right of the 100 per cent humidity line, trails should never form; (3) if the temperature falls between the zero and 100 per cent lines, trails should form only if the ambient humidity is at least equal to the value indicated at the corresponding point of the diagram. For practical purposes it seems best to regard it as an empirical diagram. When the zero and 100 per cent humidity lines are entered on the tephigram, they have an immediate application to forecasting (see Figure 19.5). Some tests of the validity of the diagram have been quoted in Section 19.3.5,

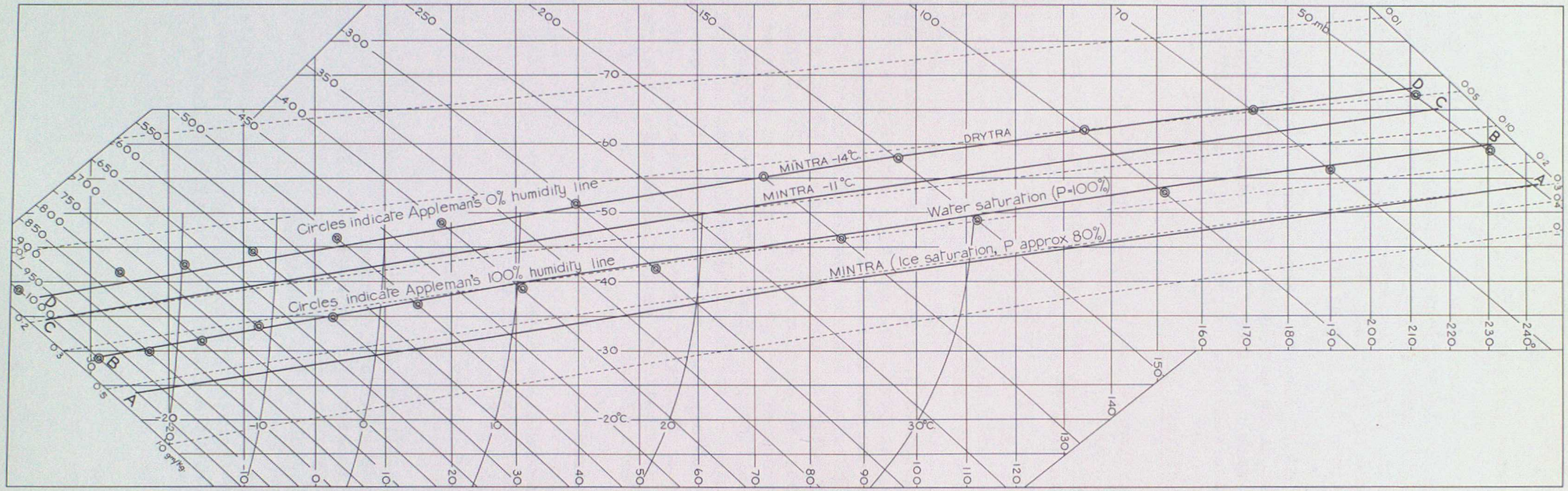

FIGURE 19.5 LINES OF CRITICAL TEMPERATURE
AA - GOLDIE[5]; BB, DD - APPLEMAN[16,17]; CC, DD - HELLIWELL AND MACKENZIE[19] (REVISED)

where it was seen that a few per cent of the observations of trails (including nil reports) occur on the wrong side of the respective critical line. No humidity measurements were available for those tests, but measurements made by the Meteorological Research Flight indicate that the occurrence of trails in this region shows little if any connection with humidity. In the preceding section, the relationship between Appleman's diagram and an empirical method based on Helliwell and MacKenzie's observations has been noted.

A modification of Appleman's diagram for use with piston-engined aircraft is described in an Air Weather Service Manual[18]. The modification arises mainly because of the greater dissipation of heat of combustion outside the wake, in comparison with a jet aircraft.

19.6.3. Estimating the humidity

For use with Appleman's method, and also in border-line cases with the empirical method of Section 19.6.1, it is desirable to have some estimate of the ambient humidity. Since direct observations are unlikely to be available, inferences have to be made on considerations as discussed in Chapter 15, Sections 15.5 and 15.6. It is unnecessary in this connection to consider the lower half of the troposphere, partly because radio-sonde observations are available here, partly because condensation trails in this part of the atmosphere are infrequent and in any case are unlikely to be of operational importance. In the upper troposphere, estimates of humidity should be weighted according to the presence or absence of cirriform cloud. In the stratosphere, relative humidity is usually (but not invariably) very low and decreases further as the height above the tropopause increases. There are several useful diagrams in Chapter 15, showing variation of relative humidity (or frost point) with height in various circumstances; in particular, Figure 15.24 of that chapter gives a good idea of the range of frost-point depression observed by aircraft of the Meteorological Research Flight. Since for forecasting condensation trails a rough estimate of the humidity is all that is required and all that is likely to be obtainable, rule of thumb methods are useful in giving a quick and fairly satisfactory answer. For example, Clodman[22], from a review of available observations, estimates average relative humidities with respect to ice as follows:

	%
In the troposphere:	
no close association with cloud	50
close association with cloud	80
in thin cloud	100
In the stratosphere:	
close association with cloud	60
no close association with cloud:	
just above the tropopause	20
5000 ft above the tropopause	10
8000 ft above the tropopause	7

A rough conversion to relative humidity with respect to water may be made by multiplying these figures by 0.65. If no cloud information is available, then for example Tables 15.5 or 15.6 of Chapter 15 may be used.

19.6.4 Variation with height

Trails may form at any height attained by an aircraft provided the temperature is suitable. At 1000 mb. the critical temperature is about -30°C., so that trails can occur (and have been observed) at ground level in high latitudes. In lower latitudes the usual decrease of temperature with height explains why trails are mostly confined to the upper troposphere and lower stratosphere. At Larkhill the average height at which trails would be expected to form, on the basis of Figure 19.5., ranges in February from about 26,000 ft. to at least 70,000 ft.; in August the range is from 30,000 ft. to only 45,000 ft., since the stratospheric temperature is then too warm for trails to form above that height. The difference in average height range between February and August illustrates the fact that when an aircraft is climbing and making trails, these at times continue to form (even when persistent) well into the stratosphere (up to the ceiling height of the aircraft), while at other times they become short or cease altogether, sooner or later, after entering the stratosphere. The cessation of trails in these cases is perhaps favoured by the very low humidity often occurring in this region; in accordance with the theories described above, cessation on reduction of humidity is most likely to occur when the stratospheric temperature lies between the two critical values, but if the temperature lies outside this range, then the presence or absence of trails is, in theory at least, unrelated to the humidity. In general, therefore, cessation of trails when aircraft enter the stratosphere, or climb farther into the stratosphere, should occur if, and only if, it would be expected on the basis of a forecast diagram such as Figure 19.5.

In the troposphere, an apparent association between persistent trails and the presence of cirriform cloud at the same level has been noted by various writers, and similarly short trails or absence of trails are said to show some relation to absence of cloud - the presence of cloud being taken to indicate a high relative humidity at that level. This association with cloud, in so far as it holds, has an obvious bearing on the variation of trail formation with height. There are of course numerous exceptions, in practice, to the suggested association; as with ascents into the stratosphere, it is considered that humidity is important only when the temperature, in relation to the critical values, leaves some doubt as to the likelihood of trail formation. Nevertheless there is likely to remain a residue of observations both of formation and of non-formation of trails which is hard to account for by existing theories.

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