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# THE VERTICAL GRADIENT OF WIND VELOCITY IN THE LOWEST LAYERS OF THE ATMOSPHERE

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# THE VERTICAL GRADIENT OF WIND VELOCITY IN THE LOWEST LAYERS OF THE ATMOSPHERE

By N. K. Johnson, K.C.B., D.Sc.

**Introduction.**—When the investigation of eddy diffusion in the lower atmosphere was begun at the Experimental Station, Porton, in 1921, it was early realised that the vertical gradient of wind velocity was an important factor. A series of field experiments was therefore carried out to measure this quantity accurately, and in particular to study how it varied with the degree of stability of the atmosphere. It was found that the wind velocity, $u$, could be represented with considerable accuracy by an expression of the form

$$u = A \log (z + c) + B$$

in which $z$ is height, and $A$, $B$ and $c$ are coefficients determined by the nature of the ground and the meteorological conditions.

As the mathematical theory of Roberts[1*] began to take form, it became clear that it would be much more convenient from the mathematical point of view if an expression of the form

$$u \propto z^m$$

could be adopted. Simultaneous records of the vertical gradients of velocity and temperature were therefore analysed to ascertain whether the data could be represented by a law of this latter type with sufficient accuracy for practical purposes.

These two investigations were described in two Porton Reports " On the variation of wind velocity with height ", dated February 14, 1923, and March 19, 1925. The two reports are reproduced below, unchanged except for a few minor omissions, and for the conversion, where necessary, of all linear dimensions into metric units and temperatures into degrees Fahrenheit.

## FIRST REPORT ON THE VARIATION OF WIND VELOCITY WITH HEIGHT

The only observations which appear to have been made on the variation of wind velocity with height close to the ground are those published by Taylor and Cave[2]. Results submitted in the present note differ from those found by Taylor and Cave in two respects. First, the linear increase in the wind velocity with height suggested by them is not confirmed. And secondly, the above authors failed to take into consideration the question of vertical temperature gradient which it will be shown is a factor of primary importance.

**The wind in the lowest 3 metres.**—The following twelve experiments were carried out early in 1921. The greatest precautions were exercised to obtain the maximum accuracy. All the air meters were carefully calibrated against each other over a wide range of wind velocity. Four air meters were mounted on slender but rigid supports at heights of 0·305 m. (1 ft.), 0·91 m. (3 ft.), 1·83 m. (6 ft.) and 3·05 m. (10 ft.)—the same supports being used as were employed when calibrating. To eliminate any error due to obstruction by the observers, each instrument was read from a distance by the aid of binoculars. All the experiments were conducted in a flat open field, the surface consisting of short grass. Inasmuch as the series of experiments was completed in 17 days, changes in the nature of the surface were negligible. Each experiment consisted normally of a run of 20 minutes, with the exception of experiments VIII and XII, when the lengths of run were 16 and 10 minutes respectively. In the absence of properly aspirated thermometers, the temperature gradient was measured by means of simultaneous readings of three thermometers

* The index figures refer to the bibliography on p. 15.

mounted at heights of 0·305 m. (1 ft.), 1·22 m. (4 ft.) and 3·05 m. (10 ft.) above the ground on one of the air-meter standards. The bulb of each thermometer was situated in the centre of a silvered glass tube 10 cm. (4 in.) long and about 2·5 cm. (1 in.) diameter. The connecting support between the tube and the thermometer was made as small as possible, so as not to obstruct the flow of air through the tube. The whole arrangement was mounted horizontally facing into the wind.

Some experiments that were made later showed that moving one of these instruments from bright sunshine into shade caused a change of 0·5°F. in the thermometer reading. This effect is not large ; and since the three instruments were always used differentially it is probable that the recorded temperature gradients are fairly accurate.

TABLE I

| Experiment | Date | Time | Wind direction | State of sky | | Air Temperature at 0·3m. ($\theta_{0\cdot3}$) | Temperature gradient ($\theta_3-\theta_{0\cdot3}$) | Length of run | Velocity as percentage of $u_{3\cdot05}$ | | | Wind velocity at 3·05 m. ($u_{3\cdot05}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Amount | Cloud type | | | | $u_{0\cdot30}$ | $u_{0\cdot91}$ | $u_{1\cdot83}$ | |
| | | G.M.T. | | tenths | | °F. | °F. | min. | % | % | % | m./sec. |
| I .. | 21.2.21 | 1200 | E. | clear | | 56·8 | −3·2 | 20 | 59·2 | 80·4 | 91·5 | 5·63 |
| II .. | 21.2.21 | 1730 | E. | clear | | 41·1 | +3·8 | 20 | 30·0 | 66·0 | 87·4 | 1·92 |
| III | 22.2.21 | 1110 | SE. | clear | | 56·0 | −3·05 | 20 | 58·5 | 79·1 | 90·8 | 4·74 |
| IV .. | 22.2.21 | 1148 | SE. | clear | | 58·4 | −2·55 | 20 | 60·0 | 80·4 | 92·8 | 5·33 |
| V .. | 23.2.21 | 1435 | SE. | clear | | 61·4 | −2·05 | 20 | 59·3 | 81·0 | 92·9 | 5·18 |
| VI .. | 23.2.21 | 1550 | SE. | clear | | 56·2 | −0·6 | 20 | 58·3 | 81·0 | 92·6 | 4·94 |
| VII | 23.2.21 | 1725 | SE. | clear | | 49·0 | +2·7 | 20 | 48·9 | 68·2 | 85·4 | 3·26 |
| VIII | 27.2.21 | 1236 | W. | 2 | As | 57·6 | −2·3* | 16 | 69·8 | 83·8 | 96·9 | 1·82 |
| IX | 27.2.21 | 1145 | W. | 2 | As | 55·0 | −1·2* | 30 | 74·4 | 86·0 | 94·9 | 1·60 |
| X .. | 5.3.21 | 1100 | SSW. | .. | St, Nb | 51·1 | −1·0 | 20 | 53·0 | 76·9 | 91·0 | 5·08 |
| XI .. | 5.3.21 | 1145 | SSW. | slight rain | | 50·1 | −0·5 | 20 | 54·5 | 79·5 | 92·2 | 4·90 |
| XII | 10.3.21 | 1605 | SE. | clear | | 51·8 | −1·45 | 10 | 55·3 | 74·7 | 90·6 | 7·59 |

* The temperature gradient was complex in these two cases.

The results of these twelve experiments are summarised in Table I. The wind velocity in metres per second at a height of 3·05 m. is given in the last column ($u_{3\cdot05}$). The three preceding columns give the velocities at the other heights expressed as percentages of the velocity at 3·05 m. In the column headed "Temperature gradient" is shown the difference between the thermometer readings at 0·305 m. and 3·05 m. A negative sign indicates a lapse and a positive sign an inversion.

With regard to the accuracy of the above observations, the following points may be noted in passing. Experiment IV was a repetition of experiment III, with all four air meters interchanged. It is obvious that the calibration of the instruments is satisfactory. Again, the results obtained in experiments IV and V on two different days under nearly identical conditions are even more striking.

A glance at the velocity percentages shows at once that the two experiments (II and VII) performed with temperature inversions yield widely different results from the others. On the other hand there were seven experiments (I, III, IV, V, VI, X and XI) in which the velocity at 3·05 m. was nearly the same, and in all of which the temperature decreased with height. Although the temperature lapses differ widely in these experiments, yet the wind velocity-gradient figures do not show any systematic variation with them. This leads to the somewhat surprising conclusion that, provided there is a temperature lapse, its magnitude does not affect the wind velocity gradient.

Considering these seven experiments further, therefore, we may take a mean of their results to examine the law of the variation of wind velocity with height. In this way, for a mean velocity of 5·13 m./sec. at 3·05 m. we find the following figures :—

| | | | | |
|---|---|---|---|---|
| Height in metres .. .. | 0·30 | 0·91 | 1·83 | 3·05 |
| Percentage velocity .. | 57·5 | 79·6 | 92·0 | 100·0 |

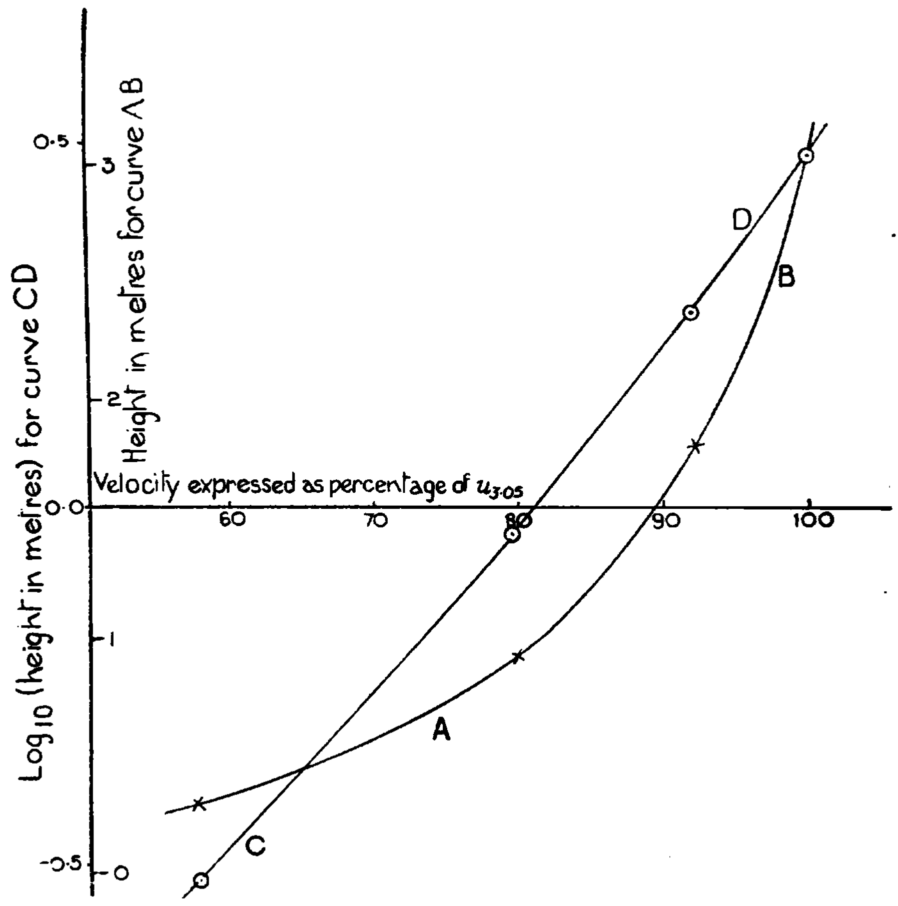


FIG. 1—INCREASE OF WIND VELOCITY BETWEEN 0·305 M. (1 FT.) AND 3·05 M. (10 FT.)
Mean values of experiments I, III, IV, V, VI, X and XI

These mean percentage velocities are plotted against height in Fig. 1. They yield the curve AB, which is seen to differ widely from the straight line found by Taylor and Cave (*loc. cit.*). The shape of the curve suggests the possibility of it conforming to the logarithmic law, found by Chapman[3], for greater heights. The curve CD shows the result obtained when the percentage velocities are plotted against the logarithm of the height. The approximation to a straight line is close, although there is still a slight curvature, indicating that the increase of velocity is rather more rapid close to the ground than the logarithmic law requires.

The question of the effect of the wind velocity itself upon the velocity gradient is not well answered by these experiments. In experiments VIII and IX the wind velocity was lower, but there was a complex temperature gradient. A rather steep lapse in the first 1·22 m. was surmounted by an inversion. Taylor and Cave (*loc. cit.*) found that the percentage velocities were independent of the actual velocity. Experiments X and XII, and more particularly XV and XVI, described below, tend to confirm this.

This being so, it is possible now to examine the cases represented by experiments II and VII, in which there were temperature inversions. The velocity gradient in these cases is seen to be much steeper than when the temperature decreases with height. Moreover, the larger the inversion the steeper the velocity gradient. With regard to the law of variation of velocity with height in these cases, experiment II gives a very close approximation to the logarithmic law mentioned above. The results of experiment VII, on the other hand, lie about midway between the linear and the logarithmic laws. In this connexion it may be observed that the conditions were not steady during experiment VII, the velocity gradient being in a state of flux, owing to the growth of the temperature inversion. In illustration of this point Tables II and III are given. Table II shows the results of experiment VI, and gives an indication of the degree of accuracy achieved in these experiments. The 20-minute run is arranged in the form of 10 runs, each of 10 minutes duration, by taking the air flows from 0–10 min., 1–11 min., 2–12 min., etc.

In this experiment the temperature gradient was nearly steady, and the velocity-gradient figures are seen to repeat themselves consistently.

Table III is constructed from experiment VII in the same manner. As the temperature inversion becomes stronger, the regular decrease in the percentage flows is apparent.

TABLE II

Experiment VI—February 23, 1921, 1550–1610 G.M.T.

| Time interval | Velocity as percentage of $u_{3\cdot05}$ | | | $u_{3\cdot05}$ |
|---|---|---|---|---|
| | $u_{0\cdot30}$ | $u_{0\cdot91}$ | $u_{1\cdot83}$ | |
| min. | | | | m/sec. |
| 0–10 | 58·8 | 81·0 | 92·3 | 4·51 |
| 1–11 | 59·2 | 81·3 | 93·7 | 4·48 |
| 2–12 | 59·0 | 81·0 | 92·2 | 4·43 |
| 3–13 | 58·9 | 81·1 | 92·2 | 4·23 |
| 4–14 | 58·2 | 80·9 | 92·2 | 4·30 |
| 5–15 | 57·8 | 80·3 | 92·1 | 4·31 |
| 6–16 | 57·3 | 81·0 | 91·7 | 4·33 |
| 7–17 | 57·2 | 80·8 | 91·6 | 4·28 |
| 8–18 | 58·7 | 80·9 | 92·2 | 4·22 |
| 9–19 | 57·8 | 80·8 | 92·2 | 4·23 |
| 10–20 | 58·1 | 80·3 | 92·2 | 4·20 |

TABLE III

Experiment VII—February 23, 1921, 1725–1745 G.M.T.

| Time interval | Velocities as percentage of $u_{3\cdot05}$ | | | $u_{3\cdot05}$ |
|---|---|---|---|---|
| | $u_{0\cdot30}$ | $u_{0\cdot91}$ | $u_{1\cdot83}$ | |
| min. | | | | m./sec. |
| 0–10 | 50·4 | 71·7 | 88·3 | 3·01 |
| 1–11 | 49·3 | 70·7 | 87·9 | 2·94 |
| 2–12 | 48·7 | 70·1 | 87·6 | 2·84 |
| 3–13 | 48·0 | 69·6 | 87·3 | 2·80 |
| 4–14 | 47·3 | 69·9 | 88·2 | 2·72 |
| 5–15 | 44·8 | 67·2 | 85·5 | 2·72 |
| 6–16 | 43·1 | 65·0 | 84·8 | 2·63 |
| 7–17 | 41·7 | 63·3 | 83·8 | 2·57 |
| 8–18 | 40·6 | 62·2 | 82·4 | 2·60 |
| 9–19 | 40·2 | 61·8 | 81·4 | 2·67 |
| 10–20 | 39·9 | 61·4 | 81·2 | 2·70 |

**The temperature-gradient effect in the lowest 20·1 m. (66 ft.).**—A number of experiments have been performed with air meters at heights of 0·91 m., 3·05 m. and 20·1 m., but they are too few to establish definitely any exact quantitative relationships. The following five experiments, however, have been selected to show the effect of the temperature gradient upon the velocity gradient. The particulars are given in Table IV. The observations were all made at the same place with an open exposure to windward. In the column headed "Temperature gradient" are given the differences in simultaneous readings of Assmann psychrometers mounted at heights of 1·22 m. and 18·3 m. (60 ft.). As before, a negative sign indicates a lapse, and a positive sign an inversion.

TABLE IV

| Experiment | Date | Time | Wind direction | State of sky | | Air temperature at 0·3 m. ($\theta_{0\cdot3}$) | Temperature gradient ($\theta_s - \theta_{0\cdot3}$) | Length of run | Velocity as percentage of $u_{3\cdot05}$ | | Wind velocity at 3·05 m. ($u_{3\cdot05}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Amount | Cloud type | | | | $u_{0\cdot91}$ | $u_{20\cdot1}$ | |
| | | G.M.T. | | tenths | | °F. | °F. | min. | % | % | m/sec. |
| XXVI .. | 30. 5.22 | 1830 | SW. | 7 | Ci | 66 | +3·8 | 10 | 57·2 | 207 | 2·13 |
| XXVII .. | 30. 5.22 | 1905 | SW. | 7 | Ci | 66 | +3·8 | 4 | 59·4 | 238 | 2·11 |
| XXVIII | 31. 5.22 | 1850 | SSW. | 4 | Ci | 77·0 | +2·7 | 5 | 64·5 | 209 | 1·57 |
| XXX .. | 21. 9.22 | 1330 | SW. | 6 | Ci | 69·3 | −0·8 | 20 | 78·1 | 140 | 5·61 |
| XXXI .. | 6.12.22 | 1140 | W. | 8 | Ac | 45·8 | +0·4 | 10 | 78·8 | 142 | 6·45 |

In the first three of these experiments the surface consisted of hay about 30 cm. high, whilst in the last two the grass was only about 7 to 10 cm. high. On this account the first three readings at 0·91 m. are probably about 10 per cent. lower than they would have been if the surface had been short grass. Even allowing for this, the percentage velocity gradient is seen to be much steeper in the cases in which there were large temperature inversions.

The **velocity gradient from 0·3 m. to 305 m. (1,000 ft.).**—On three occasions, measurements of the velocity gradient in the first 13·9 m. (45 ft.) have been combined with pilot balloon ascents. Details are given in Tables V and VI. In experiment XV, three pilot balloon ascents were made, and in experiments XVI and XVII, two ascents each. The two-theodolite method of observing the balloon was employed. The figures in Table VI are the means of all the ascents on each occasion.

TABLE V

| Experiment | Date | Time | Wind Direction | State of sky: Amount | State of sky: Cloud Type | Air temperature at 0·91 m. ($\theta_{0\cdot91}$) | Temperature gradient ($\theta_{13\cdot1}-\theta_{0\cdot91}$) | Length of run |
|---|---|---|---|---|---|---|---|---|
| | | G.M.T. | | tenths | | °F. | °F. | min. |
| XV .. | 24.10.21 | 1500 | NNW. | clear | | 45·9 | 0·0 | 15 |
| XVI .. | 27.10.21 | 1450 | NW. | 10 | Sc | 55·3 | −2·2 | 10 |
| XVII .. | 1.11.21 | 1525 | NW. | 7 | St.,Cu | 52·3 | +0·3 | 10 |

TABLE VI

| Experiment | Wind velocities at stated heights in metres as percentages of $u_{3\cdot05}$ | | | | | | | | | $u_{3\cdot05}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0·30 | 0·91 | 1·83 | 7·63 | 13·7 | 30·5 | 61·8 | 152·5 | 305·0 | m./sec. |
| XV .. .. | 55·6 | 76·9 | 91·3 | 109 | 120 | — | — | 169 | 185 | 5·67 |
| XVI .. .. | 53·4 | 74·0 | 90·7 | — | 129 | 147 | 153 | 171 | 188 | 2·94 |
| XVII .. .. | 57·4 | 73·9 | — | 115 | 131 | 154 | 171 | 192 | 208 | 2·06 |

The temperature gradients given in Table V are the values found by the use of Assmann psychrometers at heights of 0·91 and 13·1 m. (43 ft.), with the same convention as to sign as before.

The results in Table VI are plotted in Fig. 2. The crosses represent the mean of the values of experiments XV and XVI, percentage velocities being plotted against the logarithm of the height. These two experiments are seen to conform to the logarithmic law between 0·3 m. and 305 m. with considerable accuracy. The circles in Fig. 2 represent the results of experiment XVII. Over the lowest 12 m. (40 ft.), the relationship coincides precisely with the mean of experiments XV and XVI. Above this height the two curves diverge, although experiment XVII is seen to fit a logarithmic law between 12 m. and 305 m. In Fig. 2 the two logarithmic lines of experiment XVII are connected by a smooth curve.

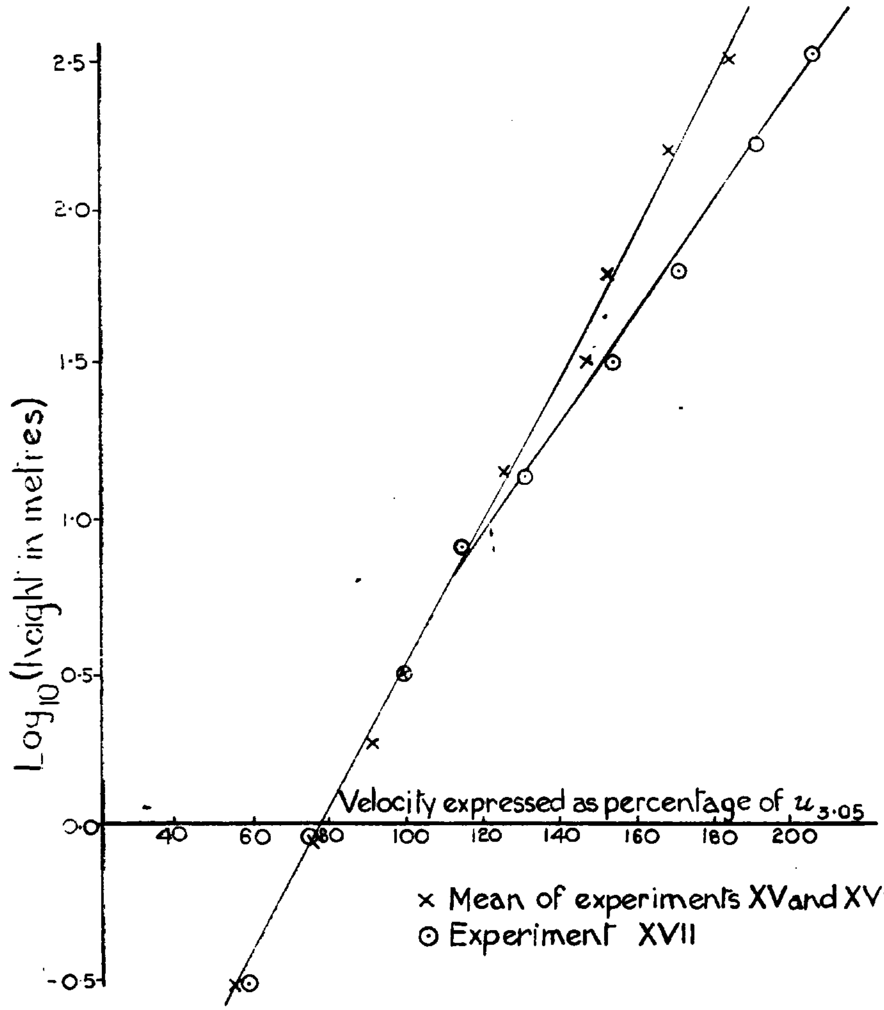


FIG. 2—INCREASE OF WIND VELOCITY BETWEEN 0·305 M. (1 FT.) AND 305 M. (1,000 FT.)

It is, of course, out of the question to make any general deductions from only three sets of observations. Nevertheless it appears that, in some cases at least, a single logarithmic law can be employed to represent the increase in wind velocity from 0·3 m. to 305 m. above the ground. A single uniform law would naturally not be expected with a complex temperature gradient.

Attention has already been drawn to the close agreement of the percentage-velocity figures in experiments XV and XVI, although the actual velocities in the first case are double those in the second.

In conclusion, it may be remarked that the present note does not claim to be anything more than a preliminary introduction to the problem of the variation of wind velocity with height. The various points brought to light in the above pages are considered, however, to be of sufficient importance to justify their discussion at the present stage.

## SECOND REPORT ON THE VARIATION OF WIND VELOCITY WITH HEIGHT

**Object of the investigation.**—In the first report on the variation of wind velocity with height it was shown that the relation between wind velocity and height above the ground could be represented with considerable accuracy by an expression of the form

$$u = A \log_{10}(z + c) + B \qquad \ldots.(1)$$

in which $u$ is wind velocity, $z$ is height, and $A$, $B$ and $c$ are coefficients determined by the geostrophic wind, the nature of the ground, the vertical gradient of temperature in the air, and possibly by other factors.

In the development of the mathematical theory of eddy diffusion, however, it has been found that the above expression is inconvenient from the mathematical point of view, and Mr. Roberts has adopted a more tractable expression of the form

$$u = Bz^m \qquad \ldots.(2)$$

in which $B$ and $m$ are the variable coefficients. The purpose of the present report is to ascertain whether the observational data can be represented by equation (2) with sufficient accuracy for practical purposes, and if so, to determine the manner in which the coefficients $B$ and $m$ depend upon various meteorological factors.

**Accuracy of equation (2).**—It is proposed to consider in the first instance the accuracy with which observational data can be represented by equation (2). In Fig. 3 is shown a series of curves based upon results obtained at Porton. Autographic records were obtained from three electric cup anemometers mounted at heights above the ground of 5·0 m., 13·4 m. and 21·7 m. during the months of January and February, 1924. Hourly values have been extracted from these charts and grouped according to the vertical temperature gradient. Velocities less than 3 m./sec. have been excluded owing to the inaccuracy of the anemometers at low velocities. Fig. 3 shows log velocity plotted against log height. The mean temperature gradient to which each curve refers is shown against them. A positive sign for the gradient indicates that the temperature increases with height (i.e., an inversion) whereas a negative sign indicates a fall of temperature with increasing height.

For equation (2) to be satisfied, it is necessary that the series of points plotted in Fig. 3 should lie on straight lines. It will be observed that for temperature inversions this condition is satisfied approximately, although not quite within the probable error of the observations. As the value of the

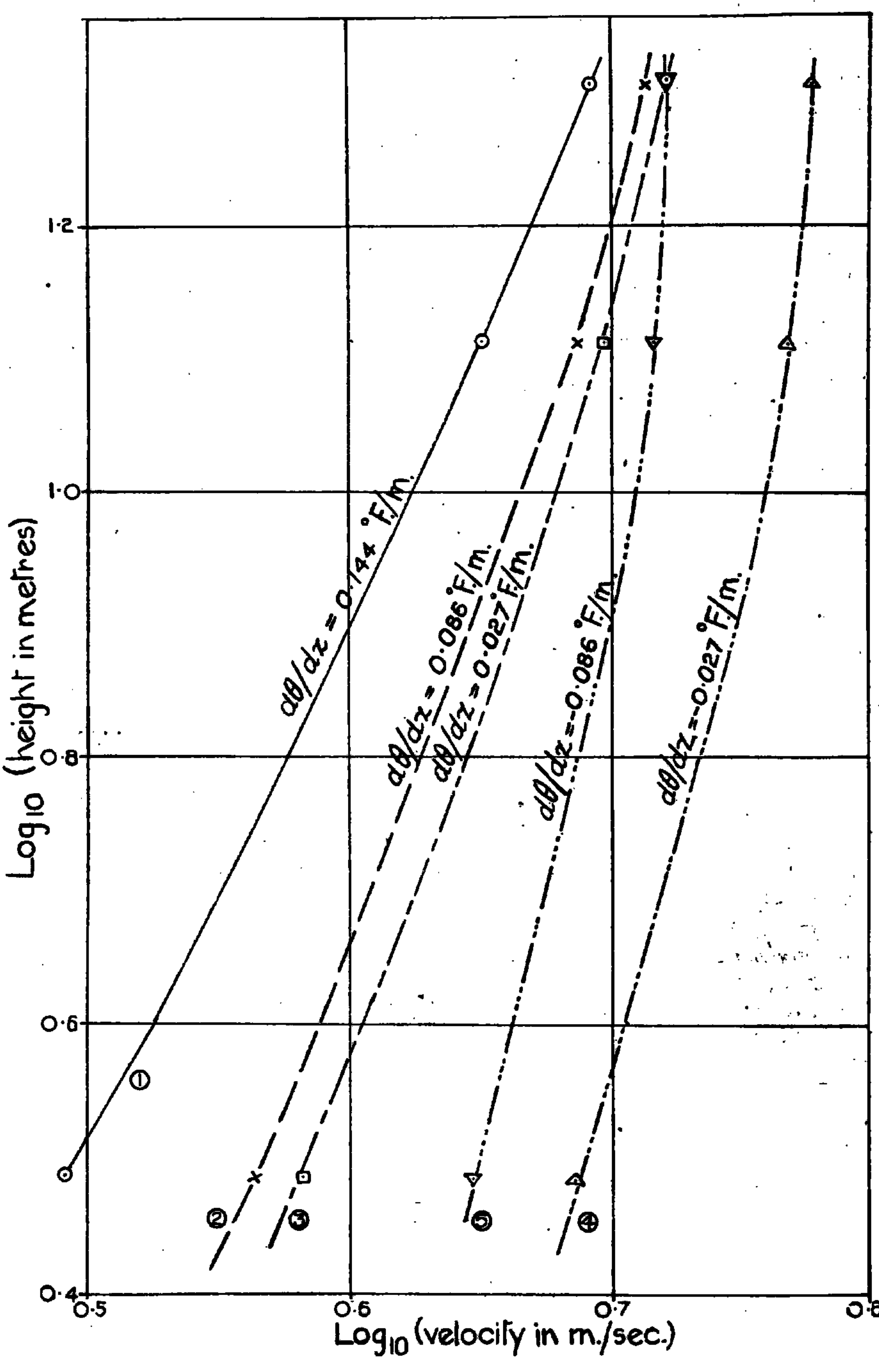


FIG. 3—RELATION OF WIND VELOCITY TO HEIGHT FOR VARIOUS TEMPERATURE GRADIENTS

temperature gradient becomes negative, however, the curvature becomes more pronounced although even then no serious error need be feared for our present purpose if the optimum straight line is used in place of the curves shown.

It will further be observed that if the optimum straight lines are drawn through each series of points, then the slope of these lines increases regularly as we proceed from inversions through zero gradient to lapses. The quantitative aspect of this feature will be considered subsequently.

An expression ot the same form as equation (2) has been suggested by Archibald[4], and also by Hellmann[5].

The former assigns to $m$ the value of 0·25 to represent his observations over the height interval 300 ft. (91·5 m.) to 2,000 ft. (610 m.). Hellmann, on the other hand, adopts the value of 0·20 for the height interval, 2 m. to 32 m., but as he was using mean values without separating out the effects of wind velocity and temperature gradient, his results are of limited value.

**The velocity gradient at a constant temperature gradient.**—In Fig. 4 is shown plotted the ratio of velocity at 13·4 m. ($u_{13\cdot4}$) to the velocity at 3 m. ($u_3$) as a function of the former and for mean temperature gradients over this interval lying between 0·0 and −0·027°F./m. The data are taken from the analysis of the autographic records referred to in the previous section. It will be observed that for values of the wind velocity in excess of 6 m./sec. the ratio has a constant value of 1·185. But below 6 m./sec. the ratio shows a tendency to increase as indicated by the broken line in the figure. The points in this region are admittedly rather scattered, but it is considered that they do indicate a genuine upward tendency of the curve. Independent evidence of the same phenomenon will be given later.

It is seen, therefore, that the velocity gradient defined by $u_{13\cdot4}/u_3$ is not always proportional to the wind velocity even at a constant temperature gradient. The significance of this in terms of viscosity will be discussed later.

**The variation of velocity gradient with temperature gradient.**—The results contained in Fig. 3 have been plotted in a different manner in Fig. 5. Mean temperature gradient in degrees Fahrenheit per metre is plotted as ordinate and velocity expressed as a percentage of the velocity at 13·4 m. is plotted as abscissa. Thus the left-hand curve of the graph represents the velocity at 3m. ($u_3$) expressed as a percentage of $u_{13\cdot4}$ for various temperature gradients, and the right-hand curve shows the velocity at 21·7 m. similarly treated. The actual observations are shown as squares. In addition to these points, which were obtained from the analysis of the autographic records, the crosses represent the results of experiments carried out with delicate air meters in the manner described in the " First report on the variation of wind velocity with height." The coincidence of these points with the curves obtained from the autographic records indicates that the results possess a moderate degreė of accuracy.

The points just referred to relate in every case to occasions on which the velocity was greater than 6 m./sec., so that we are here well away from the region represented by the upturned portion of the curve in Fig. 4. A number of experiments have been performed, however, with the delicate air meters at lower wind velocities, and, when these results are plotted in Fig. 5, it is found that these points lie to the left of the left-hand curve and to the right of the right-hand curve. Moreover, the lower the wind velocity, the greater is the departure of the points from the curves of Fig. 5. Although low, these velocities are amply high enough to ensure that the air meters are giving accurate readings.

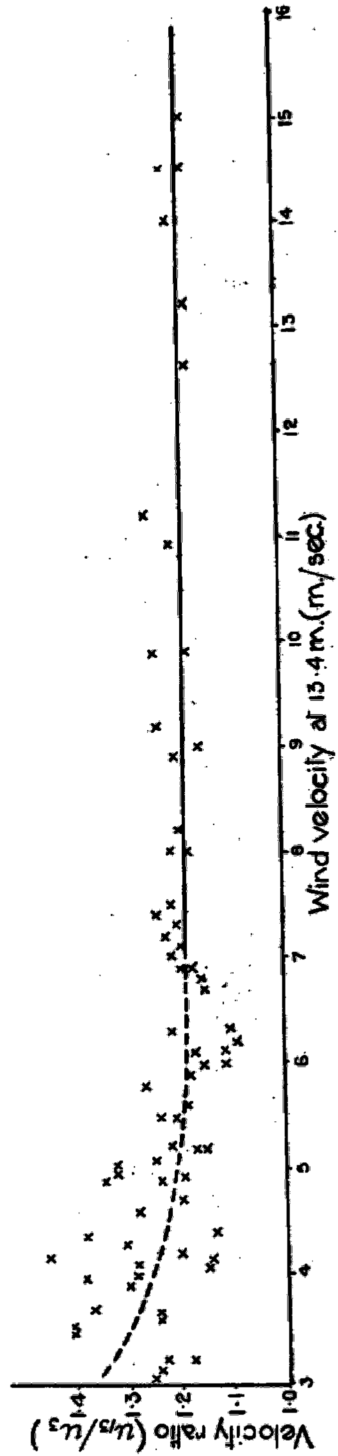


FIG. 4—RELATION BETWEEN VELOCITY RATIO (WIND AT 13·4 M. TO WIND AT 3 M.) AND VELOCITY AT 13·4 M.

For mean temperature gradients between 0·0 and −0·027°F./m.

We have here then definite confirmation of the suggestion made above that the velocity gradient defined by $u_{13\cdot4}/u_3$ increases at low wind velocities. The examination of the quantitative aspect of this phenomenon is forming the subject of a further investigation.

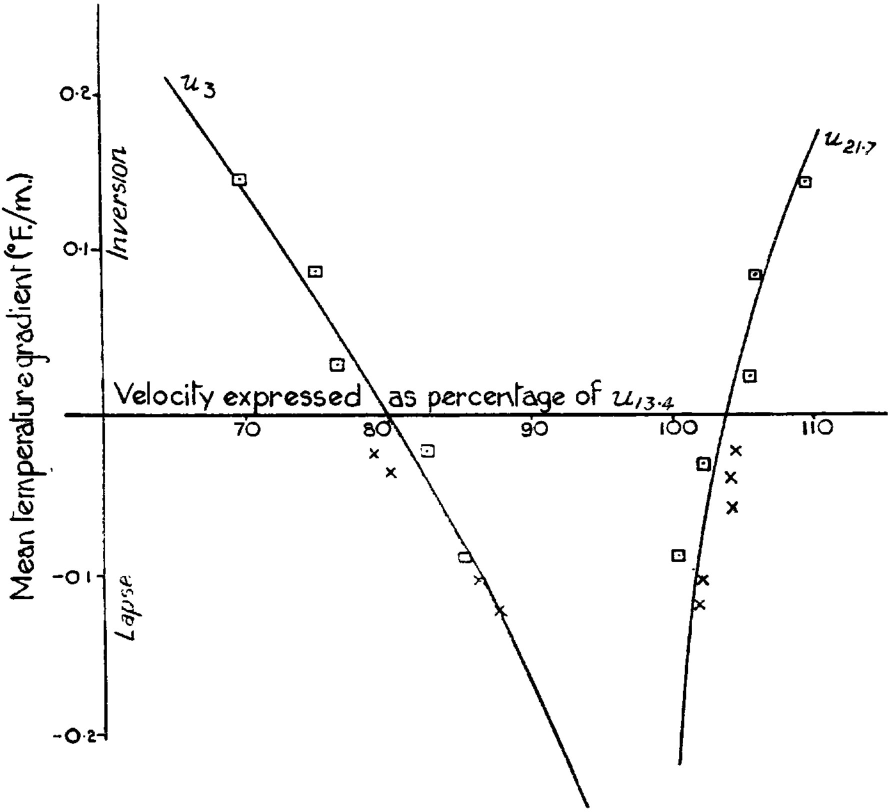


FIG. 5—RELATION OF VELOCITY GRADIENT TO TEMPERATURE GRADIENT

**Values of the coefficient *m*.**—The fact that at above 6 m./sec. the velocity gradient is proportional to the velocity at a fixed temperature gradient shows that, above this limiting velocity, the coefficient $m$ is independent of the velocity. At lower velocities, this is not so. We can examine the variation of $m$ with temperature gradient for the first case. If $u_1$ and $u_2$ are the velocities at heights $z_1$ and $z_2$, then it follows from equation (2) that

$$m = \frac{\log u_1 - \log u_2}{\log z_1 - \log z_2} \quad \dots.(3)$$

Reverting to Fig. 3, it will be seen that the slopes of the optimum lines through each group of points will be almost identical with the slopes of the straight lines down through the top and bottom points. The latter lines have therefore been used and the values of $m$ computed from them for each temperature gradient.

The value of $m$ thus found is therefore the value which gives the closest fit to the observed velocity gradient over the height interval 3 m. to 20 m. The results are plotted against the mean temperature gradient in Fig. 6. The value of $m$ will be seen to vary from 0·08 for a mean lapse of —0·1°F./m. to 0·26 for a mean inversion of + 0·18°F./m.

**Discussion of the results.**—It is probable that Hellmann's result $m \doteq 0{\cdot}20$ refers to an average temperature gradient which is not far removed from zero. The corresponding value found at Porton is $m = 0{\cdot}13$.

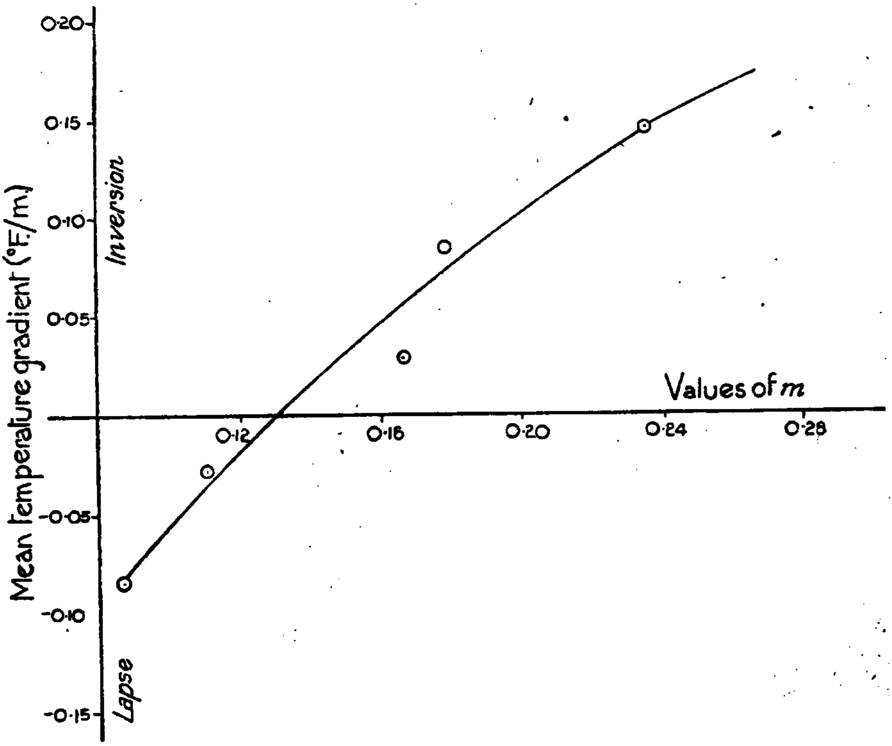


FIG. 6—RELATION OF $m$ TO TEMPERATURE GRADIENT

It seems probable that the difference in these cases is to be attributed to differences in the roughness of the ground surface, for variations in surface roughness will necessarily give rise to differences in the eddy viscosity of the air and hence to different velocity gradients. Neglecting variations in the temperature gradient, which may be regarded as constant for this purpose, it will be seen that the upward trend of the curve in Fig. 4 at low velocities is due to the same cause. A low wind velocity will correspond to a small eddy viscosity and consequently to a steeper relative velocity gradient near the surface.

The results contained in the present report must therefore be regarded as applying only to that particular type of surface over which the experiments were carried out. Further experiments are needed to determine the velocity gradient over surfaces of different degrees of roughness.

**Acknowledgments**

The work leading up to the two reports given above was carried out by a team which included Messrs. O. F. T. Roberts, F. J. Scrase, A. J. Lander, W. A. Toms and R. E. Booth. The writer wishes to acknowledge the various contributions which they made towards the calibrating and mounting of the instruments, the making and reduction of the observations, and the discussion of the results.

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