M.O. No. 210b.

METEOROLOGICAL OFFICE.

THE
FREE ATMOSPHERE IN THE REGION OF THE BRITISH ISLES

GEOPHYSICAL MEMOIRS, No. 2.

FURTHER CONTRIBUTIONS TO THE INVESTIGATION OF THE UPPER AIR,

COMPRISING

THE VERTICAL TEMPERATURE DISTRIBUTION IN THE ATMOSPHERE OVER ENGLAND, WITH SOME REMARKS ON THE GENERAL AND LOCAL CIRCULATION.

Abstract of a paper printed in Volume 211 of the Philosophical Transactions, Series A;

AND

TOTAL AND PARTIAL CORRELATION COEFFICIENTS BETWEEN SUNDRY VARIABLES OF THE UPPER AIR.

BY

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METEOROLOGIST IN CHARGE OF INVESTIGATIONS OF THE UPPER AIR, FOR THE METEOROLOGICAL OFFICE.

WITH A PREFACE BY W. N. SHAW, Sc.D., F.R.S.,

DIRECTOR OF THE METEOROLOGICAL OFFICE.

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Photographs by G. A. CLARKE, Aberdeen Observatory.

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FIG. 2. $2^h\ 7^m$ p.m. Alto-cumulus.

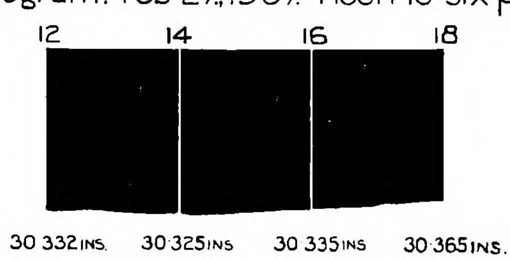
FIG. 3. $2^h\ 10^m$ p.m. Alto-cumulus.

FIG. 4. $2^h\ 12^m$ p.m. Alto-cumulus and Strato-cumulus.

FIG. 5. $2^h\ 20^m$ p.m. Strato-cumulus.

FIG. 6. $3^h\ 20^m$ p.m. Heavy Strato-cumulus.

Barogram. Feb 27, 1907 noon to six p.m.



"Wet and dry" Thermogram. Feb 27, 1907 noon to six p.m.

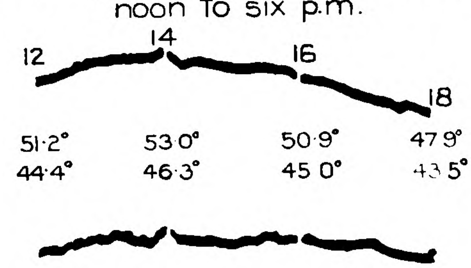


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PLATE II.—ILLUSTRATIONS OF THE CONSECUTIVE STAGES IN THE FORMATION OF CLOUD AT SHERINGHAM, SEPTEMBER 23, 1911.

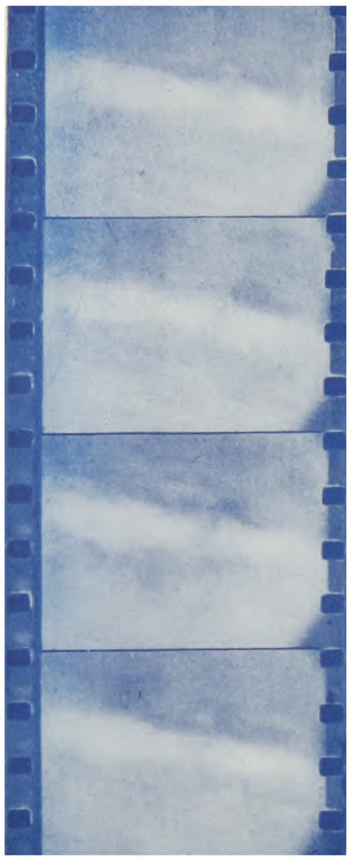

Fig. 1. 4h 30m p.m.
Cirrus.

Fig. 2. About 4h 40m p.m.
Light Cirro-cumulus.

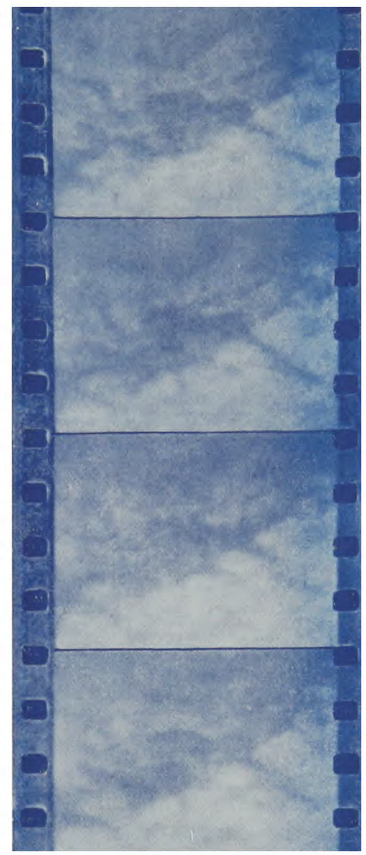

Fig. 3. About 4h 50m p.m.
Heavier Cirro-cumulus
(without shadows).

Fig. 4. About 5h 10m p.m.
Alto-cumulus
(with shadows).

The illustrations are selected from different parts of a kinematograph film, representing three quarters of an hour of the life history of clouds, which developed into rain clouds in the course of the evening of September 23, 1911. The camera was pointed towards the West. The interval between the successive pictures was five or ten seconds.

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THE FREE ATMOSPHERE IN THE REGION OF THE BRITISH ISLES.

(SECOND REPORT.)

PREFACE

BY THE DIRECTOR OF THE METEOROLOGICAL OFFICE.

THE papers which are included in the present Report represent Mr Dines's further contributions to the study of the Upper Air, in continuation of the Report which forms the subject of Publication No. 202 of the Meteorological Office.

I take the opportunity afforded by the presentation of the Report to make some notes on the results which Mr Dines's work discloses, because, as it seems to me, they mark at least the end of a long chapter in meteorological history, if not the commencement of a new era in meteorological work.

The first point to notice is that which is brought out in the paper on vertical distribution of temperature. It is shown there that, on the average of the large number of soundings of which account is taken, the upper regions of cyclonic areas are colder than those of anticyclonic areas, and that the temperature of the column up to a height of 9 kilometres is greater the higher the pressure at the surface. In the second paper it is shown that the temperature changes are practically proportional to those of pressure at the top of the column, and that the five quantities—pressure at ground level, mean temperature of the first 9 kilometres, pressure at the level of 9 kilometres, height of the troposphere, and temperature of the stratosphere—are all closely related.

I select one of these relations in particular, namely, that between pressure at ground level and pressure at 9 kilometres. In Table I. (p. 33) Mr Dines gives the total correlation coefficient for these two quantities for various groups of soundings. For all the groups, except the first, the coefficient has a value ranging from ·67 for the last available hundred soundings on the Continent, to ·88 for soundings in England grouped for the winter season. For the first group, that of the 106 Continental observations of the year 1907, the correlation coefficient is only ·29, and is therefore insignificant. At the same time, the correlation coefficient between the surface pressure and the temperature of the column of 9 kilometres is practically zero, although the value of the coefficient is considerable for all the other groups. Hence the exceptional character of the first group may be regarded as due to some peculiarity of the measures of the surface pressure. It shows itself again in the table of standard deviations. With reference to this exceptional

Wt. 31050/641—6—6—6—6—625—5/12.—N. & Co., Ltd. Gp. XV.

group Mr Dines has recently made the following note: "I have examined the "observations on which these values are based, and find the discrepancy is due to the "following causes. Out of the total of 106 ascents 31 occurred during the last week "of July. During this week the conditions were normal, but it is a time when the "seasonal variations are at the greatest. The temperature is 7° above its annual "mean, and the barometer on the Continent in the summer is below its mean. In "consequence, a large negative element was brought into the product-sum from which "r_{12} was found, and the absence of any large barometric variation in 30 per cent. of "the whole set of observations reduced the standard variation. As I have previously "pointed out, the concentration of the ascents in the neighbourhood of fixed dates "makes averages uncertain, and greatly raises the magnitude of the probable error "deduced from a set of observations." We may therefore for the time being consider the first group as on a different footing from the others.

It follows from the high values of the correlation coefficients between the pressure changes at 9 kilometres and those at the ground that there is approximate proportionality between them. The statement is most fully justified for the case in which the soundings are grouped for locality and season. The proportionality implies that we can obtain the variations of pressure at ground level by multiplying those at 9 kilometres by a constant factor. From the table of the magnitudes of the standard deviations we find, further, that the magnitudes of the changes at the two levels are of the same order. The Continental ascents again differentiate themselves from the English, because the variations of pressure at the surface in the former are less than those at the 9-kilometre level, whereas for the English groups the standard deviations for the high level are smaller than those for the surface by differences ranging from 13 per cent. to 30 per cent.

It has to be remembered that the pressure at the upper level must be regarded as transmitted to the surface, and the variations therefore form part also of the variations at the surface. If this part of the surface variation of pressure be attributed to fluctuations of pressure produced at the 9-kilometre level, very little is left to be attributed to the changes in the density of the column of air, 9 kilometres thick, between the two levels, or indeed to any other cause. Hence we may infer that the dominant cause of the sequence of pressure changes at the surface is the sequence of pressure changes at 9 kilometres. This is in spite of the fact that the temperature of the 9-kilometre column being relatively low over a low pressure and high over a high pressure, tends to counterbalance at the surface the effect of the pressure difference at the 9-kilometre level. Apparently the effect of high pressure upon the density of the column does rather more than counterbalance the effect of high temperature (including humidity); so that on the whole the pressure changes at the surface gain a little in intensity from the effect of the 9-kilometre column.

These statements are statistical, and the processes which they represent are operative in that sense. The conclusions which we may draw are, first, that the pressure changes at the surface are mainly a reproduction of the pressure changes at the 9-kilometre level, and that they must be regarded as produced not by, but in spite of, differences of temperature in the air.

The relation between the pressure at the 9-kilometre level and the temperature of the column beneath is equally remarkable. The correlation coefficients for this relation (r_{23} of Table I., p. 33) range from ·90 to ·96 for the different groups of soundings herein referred to. Such high coefficients mean that the variations of temperature of the columns are directly proportional to the variations of pressure at the top instead of showing an inverse relation with the variations of pressure at the bottom, such as one might expect if changes of pressure at the surface were due to changes of temperature in the air above.

We cannot, therefore, any longer seek to explain the distribution of pressure, as shown on a weather map, as we explain, for example, the changes of pressure between summer and winter over the continents of the Northern hemisphere, on the hypothesis that warm air forms cyclonic depressions, and cold air, anticyclones.

This view is not a new one.* There have been opponents as well as advocates of the view that cyclones originate with an ascending column of warm air, but at least we may say that it presents the evidence against that view in a new and convincing light.

The pressure changes to which Mr Dines's calculations are applied are the differences of individual readings from their mean value; they are not therefore the pressure changes between consecutive readings at one station, nor the pressure differences shown at different places on the map for the same epoch; but it is clear that, if the number of soundings is sufficient to make the inference perfectly general for the limited region of, say, the area of the British Isles, the main features of the consecutive changes of pressure and of the geographical distribution of pressure must be referred to the corresponding changes at the 9-kilometre level. This may be regarded as an alternative to the view that has now been shown to be untenable, namely, that the pressure changes are surface-bred and represented by columns of warm, light air over cyclonic areas, and *vice versâ*. We have next to make it clear that the alternative view, that the sequence of pressure changes at the surface is mainly due to the changes initiated at the 9-kilometre level, is corroborated by other well-established meteorological facts and solves some important meteorological problems.

We may take first the consideration of the pressure gradient for the horizontal motion of air as shown at the surface and at the 9-kilometre level. If for any region the pressure changes at the surface are a reproduction of those at 9 kilometres, the gradients for horizontal winds will be the same. But for steady horizontal motion the gradient is proportional to the product of the velocity of the moving air and its density. Hence the alternative view leads to the conclusion that the product of the wind velocity and density is the same at the surface as at 9 kilometres. That this statement is true on the average has already been established. Messrs. Hildebrandsson and Teisserenc de Bort † quote figures for Bossekop, Upsala, Trappes, Blue Hill, and Washington, and follow them with the statements, "Ainsi dans la zone tempérée *la quantité d'air déplacée dans le vent est constante à toutes les hauteurs*, ou : *La vitesse moyenne du vent varie avec la hauteur en raison converse de la densité de l'air.*" The same facts may be represented in a more expressive manner, from the point of view of the daily weather map, by the simple statement that, *on the average, the barometric gradient is the same at all altitudes.* Whether or not the total quantity of air passing a station is the same at all levels seems to be a matter of interest merely to the higher dynamics, but the alternative

* See v. Hann, *Lehrbuch der Meteorologie*, 2nd Ed., Book V. Chap. II., and the footnote to *Meteorologische Zeitschrift*, vol. 29 p. 63, 1912, in which v. Hann refers to his original statement of this view, *Met. Zeitschr.*, 1890, p. 226.

† *Les Bases de la Météorologie Dynamique*, vol. ii. p. 320 (Chap. VII.).

expression of the same facts, namely, that the gradient at the surface is the same as that at 9 kilometres, brings the matter at once into relation with the problems of daily weather, because it enables us to assert that not only as regards the chronological sequence of pressure values represented in the balloon soundings, but also as regards geographical distribution, the surface distribution of pressure is an echo or impress of that at 9 kilometres.

I have used the level of 9 kilometres because Mr Dines selected it for his computations as a level just under the stratosphere. It represents practically the top layer of the region of convection in the earth's atmosphere, and may therefore be regarded as the top of the atmosphere for all such phenomena as ascending currents. It is probably the region where ascending currents have to spread out and take a horizontal course, since the convection cannot penetrate the stratosphere. It may, however, deform the lower boundary of that region, thereby causing changes of temperature with the retention of a structure of isothermal columns as shown in a note *On the perturbations of the stratosphere* in the previous Report, No. 202.

The conclusion that we have arrived at is that on the average the pressure gradient at the surface is the same as at 9 kilometres, and that the individual departures from the mean value, as represented by the balloon soundings, are also the same. It will be well to point out what stands in the way of a conclusion which seems to be the next step, namely, that in our region, and perhaps generally in the temperate zone, the pressure changes at the surface are simply dictated by the changes which take place at 9 kilometres. In dealing with the subject of wind velocity or pressure gradient at the high level, nothing has been said about the direction of the wind or gradient. Now, on the one hand, the resultant motion of the air at the different levels, as indicated in Hildebrandsson's chapter "Sur la circulation générale de l'atmosphère,"* is different in direction. The windroses for the motion of the upper clouds are relatively deficient in easterly winds;† indeed, the windroses for the air motion at 10,000 feet (3 kilometres) for Blue Hill‡ show all the wind directions practically confined to the quadrant between W.S.W. and N.N.W. Moreover, M. Teisserenc de Bort has computed the average distribution of pressure at the 4-kilometre level and found it to be a general westerly circulation round the pole, and he has indicated that, in certain cases at least, in individual maps the northern or cold sides of cyclonic depressions would open out at higher levels, while the southern or warmer sides retain their intensity, and that at higher levels there would be no longer any northern side to interfere with the general circulation round the pole.

We know also from numerous observations with pilot balloons that an easterly current which forms the northern side of a cyclone is often only a shallow current with a south-westerly or westerly current overhead, while a westerly, south-westerly, or north-westerly current generally increases aloft. On the average of the month the regular circulation round the pole as computed by M. Teisserenc de Bort is interfered with by cold surface air over the continents to such an extent that the closed curves surrounding the pole do not appear on the surface at all, or only about the latitude of

* *Les Bases*, vol. ii. Chap. IV.

† See, for example, "*Observations néerlandaises pour les études internationales des nuages en 1896–1897.*"

‡ A. Lawrence Rotch and A. H. Palmer, *Charts of the Atmosphere for Aeronauts and Aviators.*

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‡ A. Lawrence Rotch and A. H. Palmer, *Charts of the Atmosphere for Aeronauts and Aviators.*

40°. In place of them we have closed curves of anticyclonic areas over the continents, and closed curves of cyclonic areas over the oceans.

What conclusion, then, might we draw from the facts that statistically the pressure changes of the balloon soundings are practically the same at the surface and at the 9-kilometre level, and that the pressure gradient is on the average the same at the high level as at the surface, in face of the clear evidence of differences between a surface map and a high-level map of pressure distribution? Not, I think, that the relations between the high level and the surface are illusory or accidental, but that the distribution of pressure at 9 kilometres does dictate certain main outlines of the pressure distribution at the surface; that these outlines are distorted by local areas of cold air of greater or less thickness, and in some cases so far distorted as to form closed curves either of high pressure or of low pressure which do not exist as closed curves in the upper air.

Suppose, then, that we proceed upon the working hypothesis that the chief outlines of the pressure distribution of our latitudes are governed by pressure changes initiated just under the stratosphere, leaving for the present the difficult question as to how to detect in a weather map for the surface which of its features are to be regarded as chief outlines coming from above, and which are the distortions due to the lower layers; let us consider what light this hypothesis throws on some important meteorological questions. I choose as an example the question of the formation of clouds. The height of 9 kilometres, 30,000 feet, roughly indicates the level of the highest cirrus clouds. The hypothesis enables us to regard the layers of the atmosphere in which clouds and rain are formed as being subject to compression and rarefaction in consequence of pressure changes introduced at the top, without themselves being called upon to do otherwise than respond to the changes.

The effect of compression is warming of the air in accordance with the laws of thermodynamics; the warming $\delta\theta$ for a small increase of pressure δp is in accordance with the formula

$$\delta\theta/\theta = \cdot 29 \delta p/p\ ;$$

θ being expressed in absolute units. Now, within the limits of 9 kilometres θ ranges from about 290°A to 240°A, while p varies from 1 bar. (or 1 c.g.s. atmosphere) to about 0·3 bar. For the same difference of pressure δp, the fraction $\delta p/p$ will be about three times as great at the top as at the bottom, whereas the temperature will be about five-sixths of the surface temperature. It follows that the dynamical changes of temperature at the top will be about two and a half times as great as at the bottom, and for intermediate layers intermediate values will be found. The reverse is the case with the corresponding rarefaction. Hence a *fall* of pressure at the height of 9 kilometres will cause a fall of temperature which is most marked at the higher layers. A continuous fall will reduce the temperature to the point at which condensation will take place. The rarefaction will soon mark the place where saturation first occurs by the formation of a cloud, and in ordinary circumstances, unless the distribution of relative humidity is very unequal, it will be a high cloud of which cirrus is the most conspicuous example. Hence the first noticeable effect of a fall of pressure at the level supposed will be the spontaneous formation of cloud at a high level where saturation is most likely first to be reached. The form of the cloud is most noteworthy. The gradual diminution of pressure, by marking those points where saturation is reached and distinguishing them by cloud formation, will display the intricate structure of the air masses much in the same way as a developing solution brings out the latent image in a photographic plate. We find, in fact, that cirrus cloud is precisely that kind of cloud which displays structure most clearly, and the development of the structure by the gradual reduction of pressure is a natural consequence.

We may carry our inference further and suggest that cirrus cloud is the expression of air structure, and would exhibit similar characteristics at whatever level the condensation was developed. It is thus a type of cloud formation rather than an appanage of particular levels. It is found most frequently at the highest levels, because there the change of temperature for a difference of pressure originating under the stratosphere is greatest.

If the reduction of pressure is continued, clouds may begin to form at other levels, provided that at any level there is a specially humid layer of air. In the first instance it should be similar to the cirrus in its mode of formation and hence in its structure. "False cirrus" may be an example of this formation. But at lower levels the quantity of water available is greater, and the structure of the cloud is perhaps more transient in consequence.

We have thus a useful explanation of the spontaneous formation of clouds at different levels, beginning with cirrus, which is a noticeable feature of the advance of a cyclonic depression.

If we suppose the diminution of pressure carried further, we must remember that the dynamical cooling of air in which condensation has commenced is less than it is in air free from cloud. At the surface the amount of cooling of cloudy air is only about six-tenths of the cooling of dry air for the same pressure difference; but higher up, where the temperature is lower, the difference of cooling is less marked.

Whether it be greater or smaller, however, it is sufficient to cause marked meteorological effects. Under falling pressure the cloud cools less fast than the air in which it floats, and in consequence there is instability set up at the upper surface of the cloud. The cloud will on that account begin to bulge upwards at one or more points, and we get a transition from the thread-like structure of cirrus to the dome shape of clouds of the cumulus type, the little bulging masses of cirro-cumulus, the more marked domes of alto-cumulus, and the great masses of cumulus. There are other causes for instability besides the gradual diminution of pressure of air containing floating clouds; so that, while we may fairly say that the continued diminution of pressure may, by the creation of instability, transform clouds from the cirrus to the cumulus type, we cannot say that all cumulus cloud is to be attributed to that cause. It may be added that if the diminution of pressure is further continued the condensation will produce rainfall.

That on some occasions the transition does actually take place is, I believe, certain. I watched the process at Sheringham with the aid of a kinematograph camera for three-quarters of an hour on September 23, 1911, while a rain depression was coming on from the west. The record is available in illustration of the suggestion. The photography on that occasion was in Mrs Shaw's charge, and she has been good enough to supply me with enlargements (see plate facing p. 13) from four different parts of the film, beginning with a small patch of cirrus by which the camera was directed, and finishing with alto-cumulus which had already begun to show dark shadows.

Later on another layer of clouds began to form at a lower level, and about two hours afterwards rain began to fall.

A set of consecutive photographs by Mr G. A. Clarke of Aberdeen Observatory shows a similar sequence continued for a longer period on February 27, 1907. These are reproduced in a plate facing p. 12. A curious point must be mentioned. Although the clouds may reasonably be referred to a deep cyclonic depression to the eastward, the centre of which was passing southward over Scandinavia, the barometer at Aberdeen, as shown by the trace at the foot of the pictures, was not falling at the time. An explanation of this apparently disconcerting circumstance may perhaps be found in the fact noted at the time by Mr Clarke, that the air was becoming colder. The change in the 24 hours amounted to 10° F. It may be that the aggregate increase of pressure of the colder air in the layers below 9 kilometres was sufficient to compensate for the fall of pressure in the upper air associated with the development of the distant cyclonic depression. It is also interesting to note that on this occasion the development of cloud, although it produced a threatening sky, stopped short of the production of rain. None was recorded at Aberdeen the next morning.

If this explanation be permitted the suggestion of the commencement of pressure changes just under the stratosphere enables us to explain some important questions connected with the formation of cloud, and the tendency to rainfall that is characteristic of low pressure, in a way that is not possible if the changes of pressure are surface-bred. It leads us also to add a word of caution as to the determination of the direction and speed of motion of air currents by observations of cirrus cloud. The passage of a phase of low pressure over a district may develop clouds in successive localities and suggest an apparent motion which is not a real motion of the air.

The phenomena attending an increase of pressure are less easily exhibited, because they would be primarily a raising of the temperature of the whole mass between the region of incidence of the increased pressure and the ground, and the consequent dispersal of clouds. But it should be noticed that the dynamical warming of a cloud-bearing mass of air is retarded by the evaporation of the water, just as the dynamical cooling of the mass is retarded by the condensation. Hence there must be instability produced at the lower surface of a floating cloud when it is in an air current subject to increasing pressure from above. That should probably result in the descent of pockets of cloudy air such as are exhibited in the pocky cloud, the "mammato-cumulus"; but whether the forms of clouds vanishing under increasing pressure generally exhibit such characteristics and otherwise show the reverse of the sequence of formation, I have not yet had sufficient opportunity to decide.

If the facts and reasoning here put forward be sufficient to justify us in adopting as a working hypothesis that the 9-kilometre layer, or "sub-stratosphere," is the layer of origin of the changes of pressure which are the dominant features of our weather maps, it remains for us to consider what is the régime day by day of the 9-kilometre layer. We have, unfortunately, very little effective evidence upon the matter.

We know that in the temperate zone the cirrus clouds, and indeed the upper clouds generally, move as a rule from some westerly point ranging between S.W. and N.W., and the average for the month indicates a circulation round the pole. If the mean velocities of upper clouds given by Hildebrandsson in his cloud report (*Les Bases*, vol. ii. p. 316) can be regarded as representing average westerly motion, the air would complete the circuit round the pole in summer in about eight days at latitude 70°, and about ten days for the zone of latitude 40° to 60°; although the observations at Trappes and Blue Hill give exceptional results, the motion at the former being exceptionally slow and at the latter exceptionally fast. Above Washington, with latitude just under 40°, the period is increased to sixteen days, and for Manila the drift is in the opposite direction, and would take forty-five days to complete the circuit. The winter figures would indicate for the 40° to 60° zone a completed circuit in about eight days.

We are thus able to picture to ourselves as an average result a belt of air making its way round the earth's axis and moving at the 9-kilometre level at about 22 metres per second. This would correspond with a gradient of about 1 millibar per degree of latitude, or a fall of pressure of 20 millibars (about six-tenths of an inch) between the southern and northern limits of a 20° zone. Further north the fall is steeper, further south much less steep.

But observations on individual days do not bear out the suggestion that there is a uniform drift to eastward going on; and, indeed, if there were, our explanation of the origin of the surface changes of pressure, which are very capricious, would disappear. The observations of the direction of motion of upper clouds show extensive changes from day to day. Two consecutive observations at one of the Dutch stations show cirrus from the west on one occasion and from the east on the next. Cirrus moving from the east is not uncommon, though the general direction is from the west. Hence the westerly movement must often be interrupted by cross currents, but observations are not sufficiently numerous for us to say what the cross currents are. Most frequently the oscillation of direction is between S.W. and N.W., but currents from other quarters have to be accounted for.

Another way of approaching the solution of the question of the régime of the 9-kilometre layer is to try to identify those parts of the surface circulation which agree with the high level circulation, but we have no satisfactory means of eliminating the disturbance due to the intermediate layers. If we take, for example, A. W. Greely's *International Charts of Mean Monthly Pressures and Wind Directions for the Years* 1882 *and* 1883, published by the U.S. Signal Service, we find that a characteristic shape for the isobar of 29·9 in a winter month is a figure of 8, of which the bows of low pressure are over the North Atlantic and North Pacific, and the cross might be at the pole; or there might be inflexions near the pole without crossing. We know that this figure of 8 is the result of the surface disturbance due to the winter cold of the continents, and in the upper air the westerly current probably goes over the high pressures. If we take the isobar of 29·9 in the charts for individual days, the general shape is retained but the outline is often indented or extended by loops, and no effective correction for the level of the upper air seems possible.

Mr Dines infers, from the result that there is no statistical correlation between the temperature at the high level and the direction in which the balloon travels, that there is no relation between temperature and wind-direction in the upper levels, and he regards the distribution of temperature in the upper air round a low pressure centre as symmetrical. It is not easy to reconcile a more or less persistent symmetrical suction of air in all directions outwards from above the low pressure centre with an average westerly drift of air at the rate of about 36° of longitude a day. It seems more likely that the lowering of pressure is due to the passage over the spot of a sheet of air with its low pressure on the left-hand side.

Allowing that the rate of motion of the air is inversely proportional to the density, so that the gradient is maintained, we must picture to ourselves a state of air-currents in which the air motion is about six times as fast as that of the surface winds and yet the surface distributions are sufficiently pronounced to be recognisable as existing for days together. The times given for the completion of the westerly circulation at different latitudes show that the average motion of the upper layers is not very different from that of a belt between 40° and 60° moving as a whole, but the displacements of the belt necessary to give the observed changes in direction of motion of the upper clouds would be very large.

Since the preceding paragraphs were written Mr C. J. P. Cave * has supplied me with material for a further important generalisation as to the relations between the surface and the 9-kilometre level. He has prepared a number of models and diagrams from the results of soundings with pilot balloons covering 9 kilometres or more, which show that the sub-stratosphere is sometimes, though not always, the region of maximum wind velocity. In such cases the wind falls off in the stratosphere above and towards the earth below. Occasionally the direction of the wind is reversed between the sub-stratosphere and the ground. The relation between the wind in the higher levels and near the ground, apart from any question of surface friction, is a complicated one. Perhaps without sufficient justification, I have always looked upon a shallow current of Easterly wind in the lowest layers as a specially cold surface layer probably separated from the Westerly currents of the upper air by some discontinuity of temperature if not of motion, and probably formed of air coming from a cold or polar quarter. If, however, the results of Mr Cave's soundings be examined by analysis of the winds at each level into W—E and S—N components, it then appears that generally there is going on between the sub-stratosphere and the ground, a process of modification of the wind at high level which proceeds continuously as one goes downwards, with only such occasional irregularities as may fitly be termed "embroidery." The modification of the W—E component, which in the sub-stratosphere may be either positive or negative, can always be represented by the continuous addition of Easterly component as one goes downward. The current in the sub-stratosphere is sometimes from the East, but less powerfully from the East than it is at the surface. With the S—N component, on the other hand, the variation, though continuous, may be in either direction. These results fall in easily with the explanation that the surface wind is the 9-kilometre wind as modified, first by the effect of the inevitable differences of density between top and bottom in relation to the gradient at the different levels, and secondly by the pressure-distribution set up by the geographical distribution of temperature in the troposphere. As regards West and East, the successive layers of the troposphere introduce a gradient for Easterly winds, which gradually increases down to the surface owing to the distribution of temperature in latitude. An Easterly wind results at the surface, unless the Westerly component of the wind of the sub-stratosphere is strong enough to bear the adverse gradient without reversal; in other words, the height at which reversal takes place is not dependent upon the region from which the lower air has come, but first upon the original wind velocity and therefore upon the pressure gradient in the sub-stratosphere, and secondly upon the temperature-distribution in latitude at the several levels. Thus an Easterly wind at the surface may be transformed into a Westerly one in the sub-stratosphere simply from the fact that the Westerly wind and gradient have become stronger, and

* *The Structure of the Atmosphere in Clear Weather*, Cambridge University Press.

vice versâ, without any modification of the composition of the lower layers. As regards North and South, however, the changes depend on the meteorological conditions and may be in either direction, according as the cold current, generally Northerly, is on the east or west of the warmer current, generally Southerly.

What is most important is that the modification seems to be continuous from the sub-stratosphere to the ground, or just above it. There are differences to be found on different occasions in the rate of diminution (speaking algebraically) of the W—E component, as one descends from the sub-stratosphere, and in the changes in the S—N component, which, though also continuous in magnitude, may be in either direction; but there is no marked discontinuity. All the varieties of general structure of the upper air, including reversals, gradual veering and spiral twists through regions of light wind, can apparently be represented by the combination of continuous variations in the two components. In the sub-stratosphere the W—E component has its algebraic maximum.

These conclusions are based upon the observations made when a balloon can be followed with theodolites to a height of 9 kilometres, and may only be applicable to clear weather. In cloudy weather the variations in velocity with height near the ground are so rapid that it would seem hardly possible for them to be continued in the same sense and the same degree up to the stratosphere. In such cases there may be discontinuity, which is not to be found in clear weather. Such rapid variations, and possible discontinuities, may belong to the class of phenomena associated with rainfall, which must be attributed mainly to instability in the lower layers of the atmosphere. Rain is frequently the accompaniment of the falling barometer, or of the cyclonic depression, but there is no numerical proportionality between the pressure changes and the rainfall. Rain often falls in heavy showers which are associated with sudden slight increases of pressure,* and it seems reasonable to regard rainfall as the result of physical processes going on in lower layers of the atmosphere and related to what I have elsewhere called the "embroidery" of the barogram, rather than with the great surges of pressure which must be associated with changes in the sub-stratosphere. It appears that we must regard the sub-stratosphere and the regions above as the dynamical laboratory of the atmosphere where the main causes of pressure change originate, and the troposphere beneath the sub-stratosphere as the physical laboratory of the atmosphere where cloud, rainfall, and other physical phenomena are produced by local causes induced, in some cases, by the effect of the dynamical changes in the upper regions.

Whatever may be the ultimate explanation of the origin of the pressure changes immediately under the stratosphere, the working hypothesis of locating there the origin of the dominating changes of pressure shown at the surface seems likely to be productive of useful results, and it is important, therefore, that we should make further efforts to picture what the régime of that layer is. The most hopeful method to begin with seems to be closer attention to observations of pilot balloons and of the upper clouds, and to the related pressure distribution over the globe.

The results which are now put forward support the contention that in dealing with the general principles of weather study it is necessary to treat the globe as a whole; and the determination of the régime of currents and their changes at the level of about 9 kilometres, the investigation of the "substratosphere," seems to be, for the time being, the most promising line of meteorological research.

* See Shaw and Dines on *The Study of the Minor Fluctuations of Atmospheric Pressure*, Quarterly Journal, Roy. Met. Soc., vol. 31 p. 39, 1905.

THE VERTICAL TEMPERATURE DISTRIBUTION IN THE ATMOSPHERE OVER ENGLAND, WITH SOME REMARKS ON THE GENERAL AND LOCAL CIRCULATION.

Abstract of a paper contributed to the Royal Society, and published in the Philosophical Transactions Series A, Volume 211, *pp.* 253-278.

BY W. H. DINES, F.R.S.,

METEOROLOGIST IN CHARGE OF INVESTIGATIONS OF THE UPPER AIR, FOR THE METEOROLOGICAL OFFICE.

THE paper contains a summary of the results of about 250 soundings of the upper air made with registering balloons in England according to the method described in the first official Memoir on "The Free Atmosphere in the Region of the British Isles," Meteorological Office, publication No. 202.

The observations dealt with have been already published in the Weekly Weather Report. Of the soundings 105 were obtained by the Meteorological Department of the University of Manchester, chiefly through the liberality of Professor Schuster, 145 balloons being used. Of 125 sent up from Pyrton Hill, 16 miles S.E. of Oxford, or from Criman, Argyllshire, the official stations of the Meteorological Office, 81 were recovered; Mr C. J. P. Cave of Ditcham Park near Petersfield secured 42 out of 70 in spite of the proximity of the station to the South Coast. The remainder were contributed by the Joint Committee of the Royal Meteorological Society and the British Association.

The paper first gives a table of mean temperatures at successive levels up to 14 kilometres for each month as derived from the soundings properly sorted (Table I.*), and another table in which the temperatures are corrected for differences of barometric pressure at the surface (Table II.*). The values as smoothed with the aid of a sine curve are shown in a third table (Table III.), and in a fourth table comparison is made between the results and those obtained by investigators in the Continent of Europe (Table IV.). A fifth table gives the fall of temperature per kilometre for successive steps in the different months of the year (Table V.). The diurnal variation of temperature as recorded in the upper air is next considered, and attention is directed to the differences of the results of morning and evening soundings which may possibly be attributed to solarisation.

The relation of temperature at the various levels to the barometric pressure at the surface is dealt with, and the variations in the height, and in the closely related temperature, of the base of the stratosphere are set out. A suggestion is added as to the existence of wave motion at the base of the stratosphere in association with pressure waves of short period sometimes shown on the microbarograph. The paper concludes with a discussion of the results obtained in relation to the general and local circulation of the atmosphere.

In this abstract atmospheric pressures are given in millibars (1000 dynes per

* Tables I. and II. are not reprinted in this abstract.

sq. cm.) as well as in millimetres; temperatures are given in the absolute or Kelvin scale of centigrade degrees and are marked °A. The figures are obtained by adding 273 to the centigrade temperatures, but in the columns of the tables the first figure, 2, is omitted. Heights are given in kilometres. The term stratosphere has been used for what is also called the isothermal or advective region, which is to be found just above the region of convection. In that region the atmosphere is practically free from water vapour on account of its low temperature, and in marked contrast with the region beneath there is little or no fall of temperature with height.

Further, the variation of temperature *within the day* is spoken of as the *diurnal* variation, and the variation *within the year* as the *seasonal* variation.

The tables referred to are based on the ascents of registering balloons made in England during the period January 1908 to December 1910. The object has been to utilise the observations, which number about 250, so as to obtain the most reliable means for the temperature and hence also for the pressure at different levels in each month of the year, and also to compare their values under cyclonic and anticyclonic conditions.

The difficulty of dealing with these questions arises from three conditions. Firstly, many of the ascents are concentrated into short periods during some of which unusual conditions were prevalent. Secondly, during certain months, notably April, July, and December, the days for which observations are available were days on which the barometric conditions at the surface were unusual, and a correction to meet this is required. Thirdly, in trying to ascertain the effect of the surface pressure on the temperature of the higher strata, the fact had to be faced that very low pressures occurred only in the winter months, that is at a time when the seasonal variation of temperature shows its maximum departure, and the effect of the seasonal variation has to be separated from the variation due to the barometric height.

How the author has endeavoured to avoid these difficulties is fully explained in the paper.

TABLE III.—Mean Monthly Temperatures at each Height. Smoothed.

Height in kilometres.	Ground.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
MONTH.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.	°A.
January .	200+ 76	71	67	63	57	50	43	37	30	24	20	17	17	16	16
February .	„ 76	71	66	62	56	49	43	36	29	23	20	17	18	17	17
March .	„ 77	73	67	63	57	50	44	37	30	24	20	17	19	19	19
April . .	„ 82	76	70	65	59	52	46	39	32	26	22	19	20	21	21
May . .	„ 85	79	73	68	62	56	49	42	36	29	24	20	21	22	22
June . .	„ 88	82	76	71	65	59	52	45	38	31	25	21	22	23	23
July . .	„ 89	83	78	73	67	61	55	47	41	34	26	22	22	23	22
August .	„ 89	83	79	74	68	62	55	48	41	33	26	22	21	21	21
September	„ 86	81	78	73	67	61	54	47	41	33	26	21	21	19	19
October .	„ 83	79	75	70	64	58	51	45	38	31	24	20	19	18	17
November	„ 80	75	72	67	61	55	49	41	35	28	23	19	18	17	16
December	„ 77	72	69	64	58	52	45	38	32	25	21	18	17	16	15

Table III. gives what the author considers to be the most likely values for the British Isles. The seasonal course at each height has been made to fit the first term of a Fourier series, which has been formed to represent the values, due weight being given to the value for each month in accordance with the character of the distribution and with the number of observations for that month.

Table IV. shows the comparison between the British and Continental results, and the close agreement except for Pawlowsk is very striking, especially when it is remembered that for the British ascents a totally different type of instrument is employed. The differences shown at Pawlowsk are explained as due to the difference of latitude.

TABLE IV.

Height in kilometres.	Mean temperature.				Amplitude or half-range.				Date of minimum.	
	B.	G. & H.	W.	R.	B.	G. & H.	W.	R.	B.	G. & H.
	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°C.	°C.	°C.	°C.		
0	82·6	—	80·9	76·1	6·7	—	9·5	12·3	January 20	—
1	77·0	78·1	77·6	70·9	6·3	8·3	8·1	9·5	,, 27	January 25
2	72·6	72·8	73·1	66·2	6·0	6·5	7·0	8·1	February 15	February 5
3	67·7	67·6	68·0	60·9	5·8	6·1	6·2	8·1	,, 8	,, 8
4	61·7	61·8	62·3	55·0	6·1	6·4	7·0	8·0	,, 13	,, 9
5	55·5	55·6	56·1	49·0	6·4	6·7	7·5	7·5	,, 11	,, 8
6	48·9	48·7	49·7	42·6	6·2	7·3	7·7	7·8	,, 15	,, 9
7	41·8	41·5	42·2	35·9	6·0	7·5	8·0	8·0	,, 13	,, 9
8	35·0	34·1	35·0	29·6	6·3	7·6	8·0	7·6	,, 11	,, 8
9	28·9	27·3	28·6	24·9	5·4	7·0	6·9	6·3	,, 15	,, 9
10	23·1	22·3	23·4	22·4	3·3	5·6	5·6	5·0	,, 10	,, 6
11	19·6	19·0	20·2	21·7	2·4	4·7	4·6	4·2	January 29	January 24
12	19·6	18·3	18·8	22·2	2·7	4·6	4·3	4·6	December 28	,, 11
13	19·5	19·1	18·5	—	3·1	4·1	4·0	—	,, 19	December 26
14	19·8	19·1	18·6	—	3·6	4·0	4·2	—	,, 15	,, 29

B.—Results from 200 ascents in the British Isles, 1908, 1909, and 1910.
G. & H.—Messrs GOLD and HARWOOD's results: 400 ascents, mostly on the Continent, to end of 1908.
W.—Dr. WAGNER's results, on the Continent entirely; 1902 to 1907, 380 ascents.
R.—M. RYKATCHEFF's results; 90 ascents at Pawlowsk.

Table V. gives the approximate values of the temperature gradients in terms of the fall of temperature for each kilometre of height. The figures are rounded off to the nearest whole degree to avoid an appearance of accuracy to tenths of a degree which the observations would not warrant.

TABLE V.—Approximate Mean Fall of Temperature with Height at Successive Levels, in Degrees Centigrade per Kilometre.

Level in kilometres.	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	Mean 0–9 km.
	°C.	°C.	°C.	°C.	°C.	°C.	°C	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.
January	5	4	4	6	7	7	6	7	6	4	3	0	1	0	5·8
February	5	5	4	6	7	6	7	7	6	3	3	–1	1	0	5·9
March	4	6	4	6	7	6	7	7	6	4	3	–2	0	0	5·9
April	6	6	5	6	7	6	7	7	6	4	3	–1	–1	0	6·2
May	6	6	5	6	6	7	7	6	7	5	4	–1	–1	0	6·2
June	6	6	5	6	6	7	7	7	7	6	4	–1	–1	0	6·3
July	6	5	5	6	6	6	8	6	7	8	4	0	–1	1	6·1
August	6	4	5	6	6	7	7	7	8	7	4	1	0	0	6·2
September	5	3	5	6	6	7	7	6	8	7	5	1	1	0	5·9
October	4	5	5	6	6	7	6	7	7	7	4	1	1	1	5·9
November	5	3	5	6	6	6	8	6	7	5	4	1	1	1	5·8
December	5	3	5	6	6	7	7	6	7	4	3	1	1	1	5·8
Year	5·3	4·8	4·8	6·0	6·3	6·6	7·0	6·6	6·8	5·3	3·5	–0·1	0·2	0·3	6·1

The author then discusses the question of a diurnal temperature variation, and gives a Table (VI.) which shows the differences between the temperatures observed at sunset and at 8 A.M. of the succeeding or preceding morning. This table shows that in the winter slightly lower temperatures are found in the morning hours, but that in the summer the temperatures at 8 A.M., by which time the sun is high, are decidedly the higher. The opinion is expressed that the difference may be due to solar radiation acting, not directly on the instruments, but on the balloon, and hence warming the air through which the instrument is just going to pass. Whatever the cause may be, the difference accounts for the greater annual range at high levels on the Continent, since nearly all the Continental ascents were made in the daytime but many of the English were made at or after sunset.

The relationship between the atmospheric pressure at the surface and the temperature at each height is then discussed. All the available observations divided into six groups according to the barometric pressure at the surface and the temperatures, after correction for the seasonal variation as exhibited in Table, III. are averaged. The results are given in Tables VII. and VIII.

TABLE VII.—Temperature at each Height for Various Barometric Readings.

Pressure at Surface.		Heights in kilometres.														
		Grd.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Millibars.	mm.*	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+	°A. 200+
984	738 C	79	75	69	63	55	48	40	32	27	26	25	25	25	26	24
992	744	81	76	70	64	57	50	43	35	28	25	24	23	24	24	23
1005	754	83	79	74	67	61	54	47	39	33	27	21	20	20	21	21
1013	760	82	76	72	67	61	55	49	42	35	28	21	18	19	21	22
1021	766	82	79	75	70	64	57	51	44	37	30	25	20	17	15	15
1031	773 A	82	79	77	72	67	61	54	47	40	33	26	21	17	15	15
Difference "A–C"		3	4	8	9	12	13	14	15	13	7	1	– 4	– 8	–11	– 9

TABLE VIII.—Fall of Temperature per kilometre of Height on Occasions of Low and High Barometer.

Mean Pressure.		Heights in kilometres.													
		0·5.	1·5.	2·5.	3·5.	4·5.	5·5.	6·5.	7·5.	8·5.	9·5.	10·5.	11·5.	12·5.	13·5.
Millibars.	mm.*	Fall of Temperature per kilometre (°C.).													
984	738	4	6	6	8	7	8	8	5	1	1	0	0	–1	2
992	744	5	6	6	7	7	7	8	7	3	1	1	–1	0	1
1005	754	4	5	7	6	7	7	8	6	6	6	1	0	–1	0
1013	760	6	4	5	6	6	6	7	7	7	7	3	–1	–2	–1
1021	766	3	4	5	6	7	6	7	7	7	5	5	3	2	0
1031	773	3	2	5	5	6	7	7	7	7	7	5	4	2	0

* The correction for latitude is not applied.

Table VII. shows in a very plain manner the close relation between the barometric pressure at the surface and the temperature of the upper air, for if the temperature be plotted against the mean barometric height in each group, quite smooth curves are formed at all heights beyond the second kilometre. At 7 kilometres a cyclone is shown to be much colder than an anticyclone, the difference being closely proportional to the difference of the intensities; at 10 kilometres the intermediate type of weather is the coldest, and at 13 kilometres the anticyclone has become by far the coldest.

The variation in the height of the base of the stratosphere is then discussed with regard to the annual range and also with regard to the atmospheric pressure at the surface.

The influence of the surface pressure, which is by far the more important, is allowed for, and then, smoothing by means of a sine curve, the following figures are given for the height in kilometres of the base of the stratosphere according to months:—

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
10·7	10·6	10·5	10·6	10·9	11·2	11·4	11·6	11·6	11·5	11·3	11·0

Excepting that the mean value for the height of the base of the stratosphere, H_c, is rather lower on the Continent, these figures are in close agreement with the Continental averages. In the paper the opinion is expressed that the difference is a genuine one, but a subsequent year's observations in England tend to show that the difference is only casual, and will be effaced when sufficient observations are available.

The value of T_c, the temperature of the base of the stratosphere, is then discussed, and finally Table IX. is given.

TABLE IX.—Mean Heights reached and Distances travelled by Balloons in different Months.

	Mean of Extreme Heights of Ascent in km.	Mean Excess of Temperature of the Base of the Stratosphere over the Temperature at the Highest Point.	Length of Trajectory of the Balloon in miles.
		°C.	
January	15·7	0·5	50
February	14·3	1·1	56
March	14·2	5·0	56
April	14·5	2·0	75
May	16·3	6·0	65
June	16·5	5·5	50
July	17·0	1·0	86
August	16·3	7·7	44
September	16·6	2·5	62
October	16·7	2·5	62
November	17·1	0·0	45
December	15·1	1·0	56

TABLE X.—Mean Heights reached and Distances travelled by Balloons for different Surface Pressures.

Surface Pressure.		Mean of Extreme Heights of Ascent in km.	Mean Excess of Temperature of the Base of the Stratosphere over the Temperature at the Highest Point.	Length of Trajectory of the Balloon in miles.
Millibars.	mm.			
			°C.	
984	738	14·6	−1·0	36
992	744	15·6	0·6	60
1005	754	16·0	3·0	66
1013	760	16·1	7·0	59
1021	766	16·2	3·5	58
1031	773	15·6	4·0	54

The rapid variation to which the height of the base of the stratosphere is subject is pointed out, and the opinion expressed that these variations may be connected with certain minor fluctuations of pressure that are sometimes plainly marked on the trace of a microbarograph.

The second part of the paper deals with the theoretical aspect of the question.

Mr Gold's theory of the isothermal conditions in the stratosphere is briefly stated, and the conclusion is drawn from it, and from the fact that changes of temperature produced by radiation must be slow, that places where the upper air is cold in comparison with other places in the same level are places where air has recently ascended, and conversely.

The low temperature at great heights over the tropics is then commented on, and a proof given that the circulation in the tropical and equatorial areas cannot be treated as independent of that of temperate latitudes. This proof rests on the fundamental principle of the conservation of angular momentum. The angular momentum of the atmosphere as a whole remains practically constant. It is constantly increased by the friction of the trades and lessened by the friction of the westerly winds of temperate latitudes. Hence there is an interchange of angular momentum, or, in other words, an interchange of air between the tropical and temperate zones.

The local circulation is then discussed, and figures 1 and 2 are given to show in a graphical form the information about the relation between pressure and temperature previously exhibited in the tables.

Figure 1 is a direct representation of Table VII., and in it the horizontal distances represent changes in the surface pressure. In figure 2, assuming average conditions, the attempt is made to proportion the horizontal scale to geographical distances, the distance from side to side being intended to represent 1000 miles. The full slightly sloping lines show the height at which definite pressures—900 millibars, 800 millibars—etc., are likely to be met with. The lines — · — · — · denote isotherms, the lines — — — — the magnitude of the departure of the temperature from its mean for the particular level.

Starting from the assumption that a negative sign indicates air that has lately risen, and conversely, the conclusion is drawn that the region of recently ascended air in a cyclone is roughly represented by a cone with its vertex downwards and its base at the level of about 9 kilometres; that the current is most intense at a point in the axis about 7 kilometres high, whence it spreads outwards and slightly upwards; that the descending current of the anticyclone also roughly forms a cone with its vertex upwards at about 11 kilometres; that it is less intense than the ascending current of the cyclone; that it spreads outwards and becomes very feeble some time before the ground is reached.

Various facts supporting these conclusions are brought forward.

A possible cause for the distribution of temperature in cyclones and anticyclones is then suggested.

It is shown that differences of pressure between two masses of air cannot exist for more than a minute or two unless there is a sufficiently powerful force acting along every possible line of communication between the two masses.

NORMAL ISOPLETHS OF PRESSURE IN MILLIBARS AND TEMPERATURE IN THE AIR FROM SEA-LEVEL TO 20 KILOMETRES, WITH CURVES OF LOCAL DIFFERENCE FROM THE MEAN TEMPERATURE FOR EACH LEVEL.

Referred to Height and Surface Pressure.

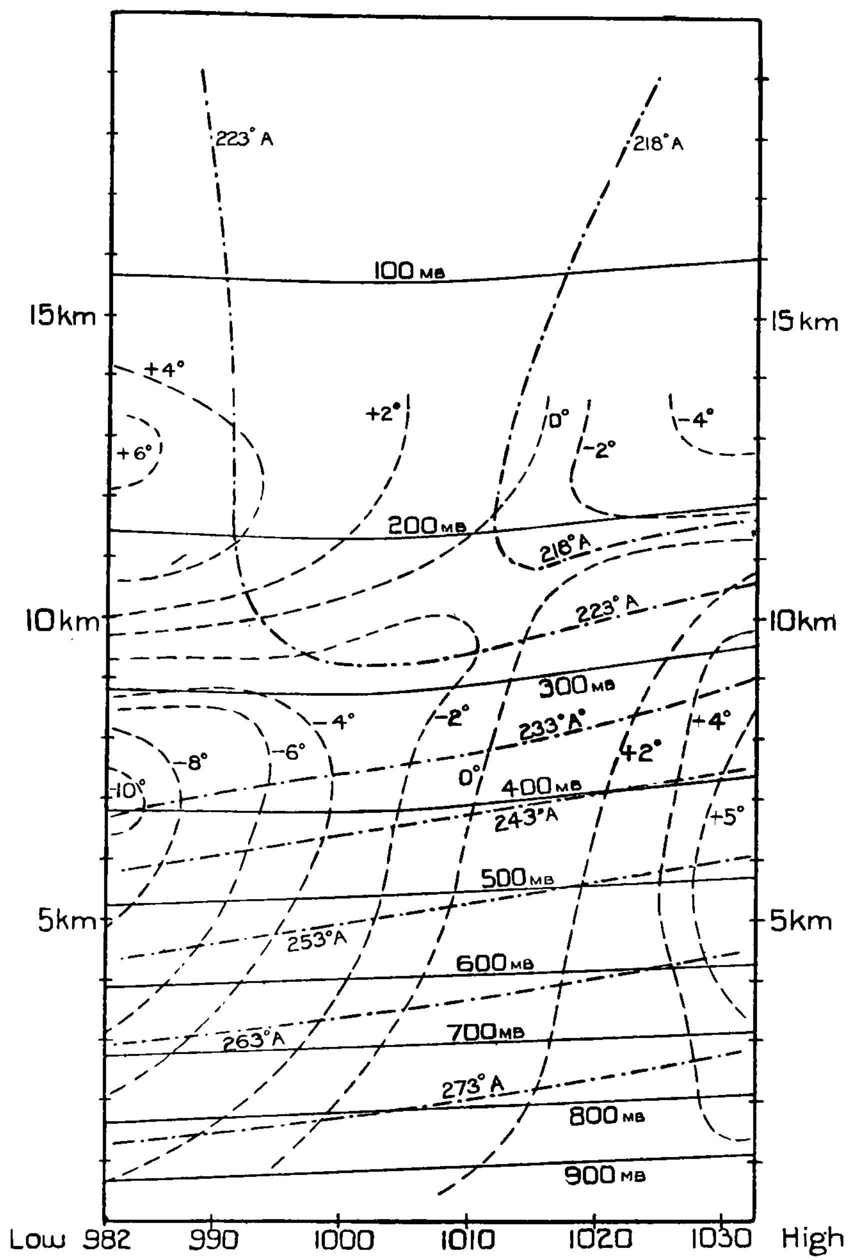


FIG. 1.

Referred to Height and Estimated Horizontal Distance.

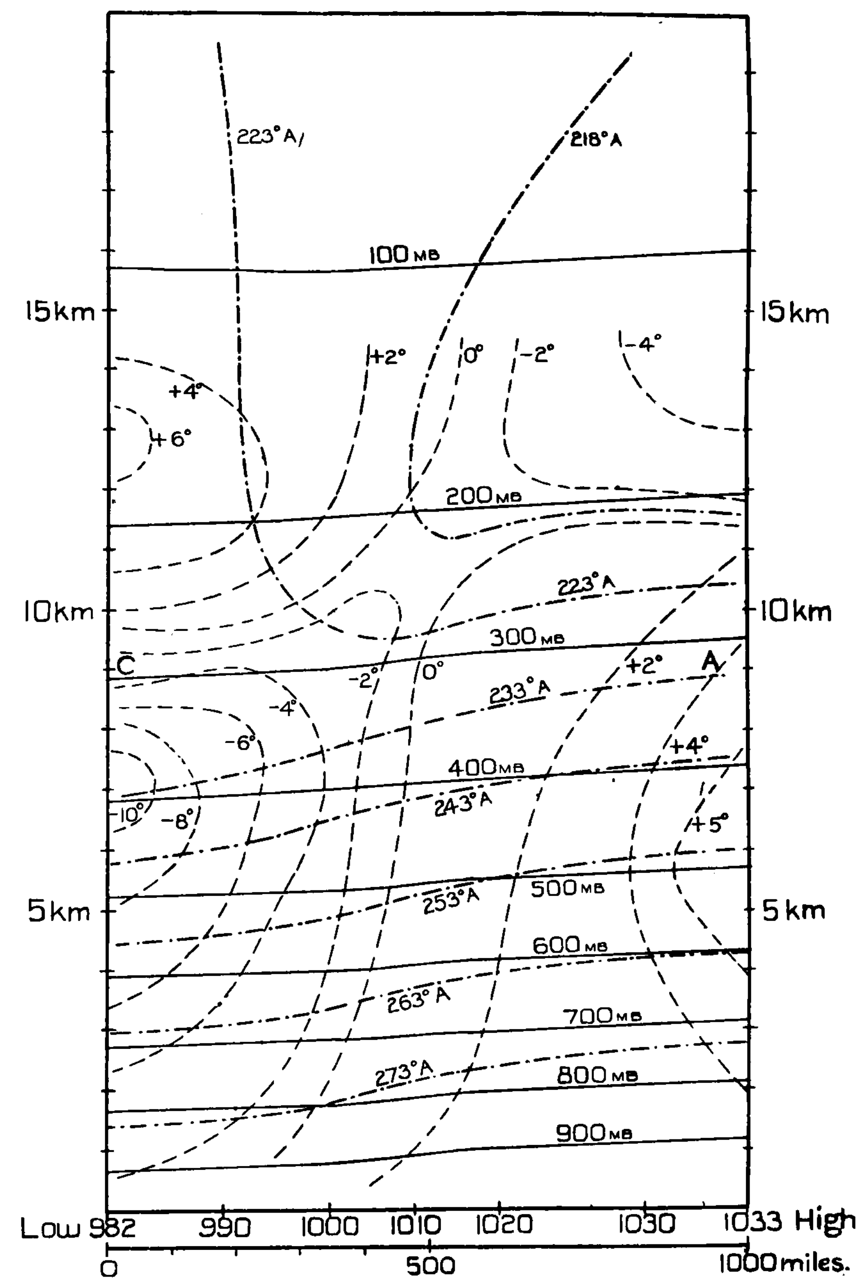


FIG. 2.

It is suggested that for all lines of communication that remain on or near the same equi-potential gravity surface connecting two particles of air on this surface, the necessary force is supplied by the lateral acceleration of strong winds, for this is known to be the case where differences of pressure occur near the ground. However this may be, if a difference of pressure comes into existence between two places A and C on the same level, and it is not possible for this difference to be equalised by the passage of air horizontally from A to C, then a distribution of temperature will be set up that will effectually check the passage of air by any other path.

For, assuming the higher pressure to be at A, air would endeavour to pass from A to C by first falling from A, then travelling horizontally underneath the strata where the direct passage is opposed, and then rising to C. The result of this motion is to warm the air under A and cool that under C, and the effect of gravity upon the two columns of equal height, but of unequal temperature and therefore of unequal density, is to check the motion. Hence a low pressure would produce a high temperature above it and a low temperature beneath it, a high pressure would produce a low temperature above and a high beneath it. Taking the points A and C at about 9 kilometres, this is the actual distribution of pressure and temperature that is found to exist, whether we take A in an anticyclone and C in a neighbouring cyclone, or A over the equator and C in latitude 60° N.

It is then shown that the explanation given to account for the distribution of temperature carries with it also an explanation of the rise in height of the stratosphere over an anticyclone and the fall over the cyclone.

Finally, the strength of the upper winds is discussed and the conclusion reached that strong winds may on occasions extend upwards to at least 16 kilometres and probably higher, although it is certain that on the average the wind velocity decreases rather than increases above 10 or 12 kilometres.

TOTAL AND PARTIAL CORRELATION COEFFICIENTS BETWEEN SUNDRY VARIABLES OF THE UPPER AIR.

By W. H. DINES, F.R.S.

THE method of dealing with statistics that has been developed in recent years appears to be particularly suitable for meteorological problems, although it does not remove the fundamental difficulty produced by the number and complexity of the different agencies that influence all meteorological data. It is here applied to the phenomena of the upper air.

My object in calculating the values given in the following tables and equations has been to ascertain the mutual influence of the various quantities upon each other, and to endeavour to discriminate between cause and effect.

In dealing with statistics it is most important to choose suitable variables, and when the work is done it often appears as though others would have been better.

The following have been chosen :—

P_s, the pressure in mm. at sea-level, denoted throughout by the suffix 1.

T_m, the mean temperature of the air column from 1 to 9 kilometres. It has been formed from the temperatures at 2·5, 5·0, and 7·5 kilometres in degrees C., and is denoted by the suffix 2.

P_9, the pressure at 9 kilometres. Denoted by suffix 3.

H_c and T_c, which have their usual meanings, viz. the height and temperature of the isothermal, and are denoted by the suffixes 4 and 5. The suffixes may be remembered by noting that they run in order from the ground upward.

Of these variables, P_s, T_m, and P_9 are not independent, for P_9 is in each separate case calculated from the values of P_s and T_m. Assuming T_m to be the true mean of the air column, we obtain the following relation by differentiating Laplace's formula for the height, viz. $9 = kT_m (\log_{10} P_s - \log_{10} P_9)$, where k is a constant, and substituting the mean values of P_s, P_9, and T_m. This gives

$$\delta P_9 = \cdot 33\ \delta P_s + 1\cdot 08\ \delta T_m \qquad . \qquad . \qquad \text{I.}$$

This equation is not rigorously exact, for T_m may differ by 1 or 2 degrees from the true mean of the column; but it seemed better to avoid introducing the temperature at the surface, since this is so often influenced by local causes and by the time of the ascent. The error introduced by using this formula is of about the same magnitude as the errors of observation.

The special height 9 kilometres is chosen, so that it may be as high as possible, but yet as a rule clear of the isothermal layer. Since 11 kilometres is the average, 9 kilometres is too low, but on the other hand the commencement of the isothermal is sometimes below 8 kilometres.

The observations used are the following :—

(1) 106 C. One hundred and six observations, the total available for the year 1907, on the Continent. Denoted by 106 C.

(2) 100 C. One hundred observations on the Continent taken backwards from June 1909. This is not a suitable selection, but I wanted 100 observations and took the last hundred that had been published. They extend nearly over a year.

(3) 66 E. The observations made in the British Isles, 66 in number, that have been worked up at Pyrton Hill, and that occurred between July 1909 and June 1911.

(4) E A. The same set of 66, but taken as departures from the mean for the month in which each occurred, instead of as departures from the mean for the whole set.

(5) 29 E. Twenty-seven ascents in England in the winter months, December 1 to March 7, with two from November. This last set embrace large barometric but small temperature variations and are chosen for that reason, but they include every available ascent in the period covered.

The observations made at Manchester unfortunately do not always give the value of H_c to tenths of a kilometre, the unit adopted for height, and hence are not suitable for correlation.

The observations have been taken separately to show what errors may be produced by using too small a number. Where the three sets show the same result, one may certainly accept that result with confidence; where they differ, the difference must be due to different conditions or to errors from too small a sample. In many cases the differences that occur are readily explained.

The knowledge of statistical science required to understand these tables can be acquired from an elementary text-book on the subject.

The following notation is adopted:—

σ_1 denotes the standard deviation of P_s viz. $\sqrt{\Sigma(\delta P_s)^2/n}$,
σ_2 ,, ,, ,, T_m,

and so on.

r_{12} denotes the correlation coefficient between P_s and T_m, viz. $\sqrt{\Sigma(\delta P_s \delta T_m)}/(n\sigma_1\sigma_2)$,
r_{35} ,, ,, ,, P_9 and T_c,

and so on.

b_{12} denotes the coefficient of T_m in the regression equation $\delta P_s = a\delta T_m$, or, as we may write it,

$$P_s = 760 + a\delta T_m.$$

In this equation δT_m denotes the departure of T_m from its mean value, and a, or b_{12}, is chosen, so that if we know the value of δT_m, the equation gives us the most likely value of P_s. The mean value of T_m may be taken as $-18°$ C., and thus if we knew that on a certain occasion T_m was equal to -21, the most likely value of P_s on that occasion, in so far as we can judge from the experience of the special set of observations concerned, was $760 - 3a$ millimetres.

Similarly, b_{21} denotes the coefficient β in the equation $\delta T_m = \beta\delta P_s$, where β is of such a value that it fits in best with the result of the whole set of observations concerned. The values are given in Tables I. and II. (Note that $b_{12} \times b_{21}$ does not as a rule equal unity.)

Various conclusions may be drawn from Tables I. and II., but first it should be stated that the result of the errors of observation is to lower the value of all the correlation coefficients and to raise those of the standard deviations. The effect will be small unless the probable error of observation is appreciable in comparison with the standard deviation, and in these tables I do not think it can in any case reach 5 per cent. This point is discussed later on (p. 36).

There is some discrepancy between the two sets of the Continental observations.* That there should be a difference between the English and Continental sets is reasonable, for the Continental sets embrace results from such different climates as

TABLE I.—Total Correlation Coefficients.

	106 C.	100 C.	66 E.	29 E.	E. A.	Mean.
r_{12}	·04	·52	·47	·74	·55	·46
r_{13}	·29	·67	·68	·88	·80	·66
r_{14}	·34	·64	·73	·92	·81	·69
r_{15}	− ·55	− ·38	− ·71	− ·65	− ·65	− ·59
r_{23}	·88	·96	·90	·90	·94	·92
r_{24}	·78	·80	·74	·87	·68	·78
r_{25}	− ·02	− ·31	− ·28	− ·65	− ·60	− ·39
r_{34}	·82	·79	·86	·92	·77	·83
r_{35}	− ·22	− ·27	− ·48	− ·76	− ·71	− ·49
r_{45}	− ·46	− ·60	− ·60	− ·86	− ·73	− ·65

TABLE II.—Standard Deviations.

		106 C.	100 C.	66 E.	29 E.	E. A.	Mean.
P_s	σ_1	5·6	7·4	10·5	13·0	10·4	9·4
T_m	σ_2	7·3	8·0	7·6	6·5	5·7	7·0
P_9	σ_3	8·0	9·7	9·2	10·0	8·9	9·2
H_c	σ_4	12·4	14·8	14·7	18·2	14·1	14·8
T_c	σ_5	7·2	6·5	6·1	7·4	5·7	6·6

The units are degrees C for the temperature, mm. of mercury for the pressures, and 100 metres for the height H_c.

Pavia and St Petersburg. Also, the difference between the 29 winter ascents and the 66 annual is what might be expected.

With few conceptions, all the correlation coefficients are high, many very high, and all those with the suffix 5, that is, all those involving T_c, are negative, all others positive. With a small exception, presently to be noted, all the corresponding standard deviations are almost identical in value. The conclusion to be drawn is, that all these variables are mutually dependent in some way one upon another, or are all very closely dependent upon some other quantity which is not considered; the problem is to discover the form of such mutual dependence.

From the nature of the case, P_9 is very closely dependent on P_m, and it is not surprising that the correlation coefficient should exceed ·9; but this close connection renders it difficult to separate the influence of T_m and P_9 upon the other quantities, a point which was my primary object when undertaking this investigation.

Suppose that we had two water butts of the same size, fed by the same roof area, and utilised by two cottagers living side by side, and both watering the same-sized garden from their butts. If we found the correlation coefficients between the

* See top of p. 14.

rainfall, and the depth of water in each butt, and also between the depths in the two butts, we should certainly find three very high values, particularly between the amounts of water in the two butts, and, not knowing the facts, we might be led to the utterly false conclusion that the amount of water in one butt was dependent upon that in the other. We must be careful not to draw equally false conclusions from the figures of Table I.

In the tables we have the mean values of $r_{23} = \cdot 92$, $r_{34} = \cdot 83$, and $r_{24} = \cdot 78$. The cause of the first high value is known, but what is the cause of the other two? (It should be noted that in meteorology, and indeed in statistics in general, ratios exceeding ·75 are very seldom found.) The other high values are $r_{14} = \cdot 69$, the connection between the surface pressure and the height of the isothermal; $r_{13} = \cdot 66$, which is partly accounted for by the connection between P_s and P_9 shown in equation I.; and $r_{45} = -\cdot 65$, the connection between the height of the isothermal and its temperature.

There are two influences at work in the production of the figures in the tables; the more important one is the variation of barometric pressure, the result of which is shown in the set 29 E. The other is the effect of the seasonal variation, which in all the annual sets is superadded to the barometric variations. In summer, T_m and H_c are considerably, and T_c slightly, increased, and P_9 in virtue of the change in T_m is also increased. P_s is unaltered in England, but is decreased on the Continent. The result of the seasonal variation should therefore be to increase in the algebraical sense the value of the correlation coefficients r_{23}, r_{24}, r_{34}, r_{25}, r_{35}, and r_{45}; to decrease on the Continent all the r's with a suffix 1, but to leave them unaltered in England. However, the effect is trifling save for the coefficients r_{25}, r_{35}, and r_{45}, where it may amount to as much as ·30 or ·40, and perhaps also for r_{12} and r_{13} on the Continent. The fact that all the r's with suffix 5 are negative, whereas the annual range should make most of them positive, shows that barometric variations exercise a more important influence on the temperature of the upper strata than direct and indirect solar radiation, a fact emphasised by the low temperature of the isothermal over the equator.

On the supposition that the seasonal variations of the various quantities form sine curves with the same phase angle for each one and with known amplitudes, it is possible to eliminate the effect of the seasonal variations from the standard deviations and the correlation coefficients. This has been done for the coefficients r_{23}, r_{24}, r_{25}, r_{34}, r_{35}, and r_{45} in the sets 106 C, 100 C, and 66 E.

The result is shown below, and the values for E A are repeated for comparison with the row above it.

	r_{23}.	r_{24}.	r_{25}.	r_{34}.	r_{35}.	r_{45}.
106 C	·83	·76	− ·24	·82	− ·50	− ·64
100 C	·94	·80	− ·58	·79	− ·54	− ·77
66 E	·86	·72	− ·59	·87	− ·85	− ·78
E A	·94	·68	− ·60	·77	− ·71	− ·73

It would be tedious to discuss the value of each coefficient in detail, but I hope that anyone who has a theory to test may find them useful; the two following points may be worth noting.

The figures were submitted to Dr Shaw, who made the following comment:—

The standard deviations of P_s and P_9 are practically equal and the correlation coefficient between them is high and positive. It follows that barometric variations, our cyclones and anticyclones, extend unchanged up to at least 9 kilometres, and are therefore manufactured in the upper strata and imposed upon the lower. A most important generalisation, if it can be substantiated.

A point that occurred to me is the following:—

The standard deviations of all the variables are virtually the same on the Continent as in England, excepting that the variation of P_s is less on the Continent, a fact well known otherwise. Since the standard deviation of the pressure is a fair measure of the number and intensity of the cyclones that occur, it follows that at 9 kilometres cyclones are as prevalent over the Continent as over England, but that on the Continent they do not extend downwards to the surface so readily. This conclusion is supported by the fact that registering balloons on the Continent do not travel a noticeably less distance than in England, although the surface winds are lighter; and it may perhaps be due to the fact that the greater humidity of an oceanic climate renders the lower strata less stable, for the adiabatic gradient for moist air is much nearer to the value of the actual gradient than in the case of dry air.

The values of the regression coefficients have not been formed, but this can be easily done by the formula

$$b_{pq} = r_{pq}\frac{\sigma_p}{\sigma_q}.$$

Suppose, for example, we wish to find the effect of P_9 upon H_c: we require the value of a in the equation $\delta H_c = a\delta P_9$, and with the notation adopted a is b_{43}.

$$b_{43} = r_{43}\frac{\sigma_4}{\sigma_3} = \cdot 83\ \frac{14\cdot 8}{9\cdot 2} = 1\cdot 33.$$

That is to say, that where P_9 is 1 mm. above its average value, H_c will most likely be 133 metres above its average; and since the correlation coefficient is high and the number of observations large, viz. 272, the estimate is not likely to be far out. Similarly we have $\delta P_s = b_{13}\delta P_9 = \cdot 88\ \frac{13}{10}\ \delta P_9 = 1\cdot 14\ \delta P_9$, if we take the figures of the set 29 E, or $\delta P_s = \cdot 67\ \delta P_9$ if we use the mean values.

Magnitude of the Errors due to Paucity of Observations.—It seems superfluous to print the probable error for each separate case, but the following formulæ taken from G. U. Yule's book * are given for reference, and from them the probable error for any coefficient can be readily obtained. The formulæ refer to a normal distribution, which is practically the case here.

Take n as the number of observations—

Probable error of a standard deviation $(\sigma) = 2\sigma/(3\sqrt{2n})$

Probable error of a correlation coefficient $(r) = 2(1 - r^2)/3\sqrt{n}$

Probable error of b_{12}, a regression coefficient $= 2\sigma_1\sqrt{1 - r_{12}}/(3\sigma_2\sqrt{n})$

The meaning of the term "probable error" is that about half the results will differ from the value that would be obtained from a very great number of observations by less than the probable error, and that it is unlikely that any one result will differ by as much as four times the probable error. Take, for example, a standard deviation $\sigma = 15$ based on 50 observations. The probable error is $\frac{2 \times 15}{3 \times \sqrt{100}} = 1$. This means that if we calculate σ from some other 50 observations, the observations taken in another

* G. Udny Yule, *An Introduction to the Theory of Statistics*, Griffin & Co., Ltd., London. 10*s*. 6*d*. net.

year, for instance, we should be quite likely to find $\sigma = 14$ or $\sigma = 16$, but that we should not be at all likely to find σ below 11 or above 19.

Magnitude of the Observational Errors.—A point in connection with these figures may well be mentioned here. Doubt about the accuracy of the upper air observations has been very freely expressed in the past, and is perhaps not extinct, but the high values of the coefficients in Table I. afford an absolute proof of the smallness of the errors of observation.

A correlation coefficient cannot exceed 1, and a value of 1·0 between two quantities implies not only perfect correlation, but also perfect accuracy in all the measurements of the two quantities, if we exclude the impossible supposition of perfect correlation between all the casual errors of observation. Let x and y be the observed departures from the mean of two variables, a and b the errors of observation, σ_x and σ_y the observed standard deviations, and r the calculated correlation coefficient; σ_a and σ_b are of necessity less than σ_x and σ_y. It is easily shown that $r\sigma_x\sigma_y/\sqrt{(\sigma_x^2 - \sigma_a^2)(\sigma_y^2 - \sigma_b^2)}$ is the true coefficient, which must be less than or equal to 1. Obviously, if $r = 1$, $\sigma_a = \sigma_b = 0$, and if r is nearly 1, σ_a and σ_b must be small compared with σ_x and σ_y. Suppose the observational errors equally divided between the two variables, that is $\sigma_x/\sigma_a = \sigma_y/\sigma_b$— this is practically the case,—then the following table is easily obtained.

Probable error	=	$\frac{1}{2}$	standard deviation,	r must be		$<$·45
,,	,,	$\frac{1}{3}$	,,	,,	,,	$<$·75
,,	,,	$\frac{1}{4}$	,,	,,	,,	$<$·86
,,	,,	$\frac{1}{5}$	,,	,,	,,	$<$·91
,,	,,	$\frac{1}{6}$	,,	,,	,,	$<$·94.

In the English ascents, excluding the values of r_{23}, there are four coefficients of the approximate value of ·91, and it is therefore impossible that the probable error of observation for a variable should exceed one-fifth of the standard deviation. This gives probable errors of 1·5° C. for the temperature, 2 mm. for the pressure at 9 km., and 300 m. for the value of H_c. The figures for the Continent admit of rather larger values. However, we have assumed perfect correlation, and in view of the complexity of the conditions this can hardly be. Also in some ascents H_c is decidedly indefinite, so that it is very surprising that the coefficients involving the suffix 4 can be so high. The rule has been to measure H_c to the point where the decrease of temperature becomes 1° C. or less per kilometre. Had 2° C. been substituted for 1° C. many values of H_c would have been different, some by at least a kilometre. Perfect correlation is therefore impossible, and the probable errors of observation must be less than those given above. The conclusion is that both in England and on the Continent the probable error is insignificant when compared with the standard deviation.

Table III. gives the partial correlation coefficients of the first order, and, in conjunction with Table IV., affords a considerable amount of information with regard to the mutual influence upon each other of the various quantities.

Thus, for example, the total correlation r_{12}, which equals ·46, shows the actual connection between 1 and 2, that is between P_s and T_m, no matter whether the connection be direct or indirect; but the partial coefficient $r_{12\cdot3}$, the mean value of which from the table is $-$·54, shows the connection when the influence of P_9 (3) is excluded. We know otherwise that this ratio should be negative, because, P_9 being constant, P_s will rise as T_m falls, and it is satisfactory to find the fact plainly shown in the tables. Indeed, it is useless working out the coefficients that involve only the three suffixes 1, 2, and 3, because we have already the known relation between them in equation I. It was done only as a test, and it will be seen that in every case the sign + or − is correct. The inference is that where in the tables such another connection is shown, that connection is a genuine one, although as yet we cannot assign it with certainty to its true cause.

TABLE III.—Correlation Coefficients of the First Order.

Suffix.	106 C.	100 C.	66 E.	29 E.	E. A.	Mean.	Mean.
12·3	− ·47	− ·59	− ·45	− ·25	− ·96	− ·54	
12·4	− ·38	− ·39	− ·16	− ·31	− ·02	− ·25	− ·08
12·5	·03	·64	·40	·55	·21	+ ·37	
13·2	·49	·71	·67	·74	·98	+ ·72	
13·4	·02	·35	·15	·22	·47	+ ·24	+ ·51
13·5	·21	·64	·55	·78	·63	+ ·56	
14·2	·49	·49	·64	·84	·72	+ ·64	
14·3	·19	·25	·39	·59	·51	+ ·39	+ ·53
14·5	·12	·56	·54	·94	·65	+ ·56	
15·2	− ·55	− ·25	− ·68	− ·33	− ·48	− ·46	
15·3	− ·52	− ·28	− ·60	·06	− ·20	− ·37	− ·31
15·4	− ·47	·01	− ·50	·71	− ·15	− ·09	
23·1	·90	·96	·90	·78	·99	+ ·91	
23·4	·67	·89	·76	·52	·88	+ ·74	
23·5	·90	·96	·91	·82	·91	+ ·90	
24·1	·82	·82	·66	·72	·46	+ ·70	
24·3	·22	·26	− ·18	·24	·24	+ ·16	+ ·59
24·5	·87	·83	·75	·80	·43	+ ·74	
25·1	·00	− ·08	·09	− ·33	− ·38	− ·14	
25·3	·34	− ·18	·40	·12	·27	+ ·19	+ ·07
25·4	·61	·44	·30	·39	− ·22	+ ·30	
34·1	·82	·64	·73	·59	·62	+ ·64	
34·2	·45	·16	·67	·64	·52	+ ·49	+ ·63
34·5	·83	·81	·81	·80	·55	+ ·76	
35·1	− ·08	− ·02	·01	·52	− ·33	− ·19	
35·2	− ·39	·11	− ·54	− ·53	·53	− ·36	− ·14
35·4	·31	·42	·08	·16	− ·34	+ ·12	
45·1	− ·35	− ·50	− ·17	− ·88	− ·46	− ·47	
45·2	− ·71	− ·67	− ·61	− ·79	− ·55	− ·67	− ·52
45·3	− ·50	− ·67	− ·41	− ·64	− ·41	− ·53	

Table IV. gives such of the regression coefficients as have been worked out, and from them the regression equations can be readily formed. The notation, which is taken from G. Udny Yule's book, *An Introduction to the Theory of Statistics*, is very simple and convenient; $b_{12\cdot3}$ denotes the coefficient of T_m (suffix 2) in the equation $\delta P_s = b_{12\cdot3}\delta T_m + b_{13\cdot2}\delta P_9 + \ldots$, $b_{21\cdot3}$ the coefficient of P_s in the equation $\delta T_m = b_{21\cdot3}\delta P_g + b_{23\cdot1}\delta P_9 + \ldots$

It seems superfluous to set out all the equations, but the following connection between the temperature of the air column from 1 to 9 kilometres (T_m), the pressure at 9 kilometres (P_9), and the temperature of the isothermal (T_c) deduced from the 66 ascents in England, is given as an example :—

$$T_m = -\ 18^\circ + \cdot73\,\delta P_9 + \cdot25\,\delta T_c,$$
$$P_9 = 231 - \cdot42\,\delta T_c + 1\cdot13\,\delta T_m,$$
$$T_c = -\ 57^\circ + \cdot64\,\delta T_m - \cdot71\,\delta P_9.$$

The first of these equations means that if we exclude any influence that T_c may have upon T_m, then for every millimetre of pressure that P_9 is above its average value, T_m will in England most likely be raised by ·73° C. of temperature.

TABLE IV.—Regression Coefficients. First Order.

	106 C.	100 C.	66 E.	29 E.	E. A.	Mean.	B.
$b_{21\cdot3}$	−·31	−·21	−·53	−·11	−·31	−·30	−·40
$b_{31\cdot2}$	·34	·46	·66	·38	·33	·43	·44
$b_{12\cdot3}$	−·72	−1·44	−·38	−·54	−(3·00)	−·79	−·59
$b_{32\cdot1}$	·95	1·54	·42	·85	1·03	·96	·73
$b_{32\cdot4}$	·67	1·10	·76	·64	1·26	·89	·68
$b_{32\cdot5}$	·97	1·15	1·13	1·07	1·21	1·11	·84
$b_{42\cdot3}$	·45	1·05	−·38	·62	−1·09	·13	·10
$b_{42\cdot5}$	1·31	1·27	1·29	1·51	·87	1·25	·59
$b_{52\cdot3}$	·67	−·51	·64	·26	·53	·32	·34
$b_{52\cdot4}$	·43	·41	·29	·34	−·20	·25	·27
$b_{13\cdot2}$	·71	1·09	1·22	1·48	(3·25)	1·12	1·09
$b_{23\cdot1}$	·85	·60	1·08	·72	·95	·84	1·10
$b_{23\cdot4}$	·66	·72	·76	·42	·61	·63	·83
$b_{23\cdot5}$	·83	·80	·73	·62	·66	·73	·96
$b_{43\cdot2}$	·92	·51	1·69	1·32	1·64	1·22	·76
$b_{43\cdot5}$	1·15	1·03	1·24	1·18	·80	1·08	·67
$b_{53\cdot2}$	−·71	·23	−·71	−·68	−·78	−·53	−·56
$b_{53\cdot4}$	·45	·36	·08	·15	−·25	·14	·15
$b_{24\cdot3}$	·11	·06	·06	·10	−·05	·03	·06
$b_{24\cdot5}$	·59	·54	·43	·43	·20	·43	·91
$b_{34\cdot2}$	·22	·05	·27	·31	·16	·20	·32
$b_{34\cdot5}$	·60	·65	·54	·55	·34	·54	·87
$b_{54\cdot2}$	−·66	−·47	−·34	−·50	−·24	−·44	−·98
$b_{54\cdot3}$	−·54	−·46	−·27	−·43	−·18	−·38	−·81
$b_{25\cdot3}$	·17	−·07	·25	·07	·13	·11	·10
$b_{25\cdot4}$	·83	·41	·29	·34	−·20	·35	·32
$b_{35\cdot2}$	−·21	·05	−·42	−·41	−·37	−·27	−·25
$b_{35\cdot4}$	·21	·48	·08	·14	−·50	·08	·06
$b_{45\cdot2}$	−·75	−·95	−1·10	−1·26	−1·24	−1·06	−·47
$b_{45\cdot3}$	−·47	−·97	−·61	−·95	−·92	−·78	−·35

B. Mean value with standard deviation of each variable taken as unit of measurement for that variable. The figures enclosed in brackets are not used in forming the means.

It is obvious in these equations that the magnitude of the coefficients, unlike that of a correlation coefficient, depends upon the unit of measurement; if, for example, we measured the pressure in inches instead of in millimetres, we should have to multiply the coefficients of P_s and P_9 by 25, because then a change of a single unit in P_9 would have 25 times as much effect as before. It follows that we cannot compare the effect, say, of P_9 and T_c upon T_m by looking at the magnitudes of the coefficients. To some extent this difficulty is insuperable, but it may be partly met by the device of using the standard deviation of each quantity as the unit of measurement for that quantity.

The means shown in the column headed B are thus expressed, and are therefore comparable *inter se*. Thus transformed, the above equations become

$$T_m = -\ 18^\circ + \cdot 91\,\delta P_9 + \cdot 20\,\delta T_c,$$
$$P_9 = 231 - \cdot 27\,\delta T_c + \cdot 90\,\delta T_m,$$
$$T_c = -\ 57^\circ + \cdot 80\,\delta T_m - 1\cdot 12\,\delta P_9.$$

From this it appears that the average variation of P_9 produces very nearly the full average variation of T_m, whereas the full average variation of T_c is associated with only one-fifth the average variation of T_m, and hence we may say from this particular equation that P_9 has on T_m $4\frac{1}{2}$ times the effect of T_c.

It would appear at first sight as though no coefficient could be greater than one, and when we are dealing with two variables only this is the case, since the regression coefficient becomes equal to the correlation coefficient, which must be less than one. It is not so with more than two variables. For example, in the equation

$$\delta T_c = 80 \delta T_m - 1 \cdot 12\ \delta P_9,$$

suppose δP_9 is a positive variation of unit magnitude. Taken alone, the average variations of T_c are above the average—an absurd statement, but the explanation is that P_9 is positively associated with T_m, that when δP_9 is positive δT_m is also positive, and therefore the ·80 of the T_m suffices to cancel out, or more, the excess of $-\cdot 12$ of the P_9 and thus reduce the variation of T_c.

For the sake of those not acquainted with the modern theory of statistics, it may be well to add a few further words of explanation, and in the following remarks I follow Dr Walker of Simla.

We have, say, four quantities P_s, T_m, H_c, and T_c, and by a certain mathematical process given in G. U. Yules' book we get the following equation :—

$$\delta P_s = b_{12 \cdot 45} \delta T_m + b_{14 \cdot 25} \delta H_c + b_{15 \cdot 24} \delta T_c.$$

This equation denotes that P_s is a quantity, the variation of which (*i.e.* δP_s, the departure from its average value) is to some extent independent, and to some extent dependent upon each of the other variables T_m, H_c, and T_c. The extent to which δP_s depends upon T_m is shown by the magnitude of $b_{12 \cdot 45}$, its dependence upon H_c by $b_{14 \cdot 25}$, and so on. If one of the coefficients be zero, say $b_{15 \cdot 24}$, this proves that P_s is independent of T_c. The values of $b_{12 \cdot 45}$, etc., are such that the sum of the squares of the expression $(\delta P_s - b_{12 \cdot 45} \delta T_m - b_{14 \cdot 25} \delta H_c - b_{15 \cdot 24} T_c)$ shall be the smallest possible when the individual values of $\delta P_s -$ etc., for each occasion are substituted in the expression. Or, put in another form, we may say that $b_{12 \cdot 45}$, etc., are chosen so that the correlation between δP_s and $b_{12 \cdot 45} \delta T_m + b_{14 \cdot 25} \delta H_c + b_{15 \cdot 24} \delta T_c$ shall be the greatest possible.

The coefficients are not reliable unless they are based upon a large number of observations, but except in the set 29 E the number of observations is fairly large, and throughout the whole set of correlation coefficients given all probable errors are less than ·1.

The use of the partial coefficients is to avoid the risk of a false assumption, such as has been referred to in the case of the two water butts.

It would be tedious to go through the whole series of figures and comment upon each ; they are only published in the hope that they may be of use to anyone who has a theory on the atmospheric circulation to test, but there are a few points that are worthy of notice, some of which admit of explanation and some of which do not.

Firstly, it is necessary to see what effect the ordinary laws of thermodynamics have upon the various correlations and coefficients.

Other things being the same, the effect of an increase of pressure of the air is to raise the temperature—an important fact, but very commonly overlooked excepting when the change of pressure is produced by change of height. At the surface the amount is about 1° C. to 10 mm., at 9 kilometres it is 3° C. to 10 mm., and at 11 kilometres, the height of the isothermal, it is 4° C. to an increase of 10 mm. pressure.* An increase of P_9 is, of necessity, associated with an increase of pressure in the column below 9 kilometres and also with an increase in the pressure at 11 kilometres; we have, therefore, apart from any other action, a positive correlation due to this alone between P_9 and T_m and P_9 and T_c, and we may perhaps take a value of + ·20 in the regression coefficients of the form $b_{23\ldots.}$ and of + ·35 in those of the form $b_{53\ldots.}$ as due to this cause.

Secondly, we have to consider the amount by which a mere expansion of the air column by heat raises the value of H_c without altering the pressure at which H_c occurs. This is important in connection with Mr Gold's theory of the dependence of H_c on radiation, for taking, as no doubt he is justified in doing, the radiating layers of air as plane and not spherical, the distance between them is immaterial, since the effect of radiation from an infinite plane is the same at all distances. His effect depends upon the mass of the radiating layer, that is upon the density, and therefore chiefly upon the pressure, and thus in seeking to test his theory it would have been better to have taken the pressure instead of the height of the commencement of the isothermal. The objection was that in considering the circulation of the air we require the slope of the surfaces and therefore the height of the isothermal.

The effect is easily allowed for.

The mean value of T_m is −18° C., or 255° A. Taking the column to which T_m refers as 8 kilometres, an increase of 1° C. in T_m expands the column and raises H_c by $\frac{8}{255}$ kilometres, and from this cause alone $\delta H_c = \cdot 31\ \delta T_m$; hence, in seeking for the effect of T_m on H_c that is produced by radiation, we must first deduct ·31 from all coefficients of the form $b_{42\ldots}$ It must not, however, be forgotten that radiation and absorption depend very greatly upon humidity, which is not here considered. We have, unfortunately, no reliable means of measuring humidity in the higher strata.

Tables V. and VI. give the correlation coefficients and the regression coefficients of the second order. They do not very greatly alter or enlarge the information supplied by Tables III. and IV. save in one or two points to be presently noted.

TABLE V.—Correlation Coefficients. Second Order.

	106 C.	66 E.	29 E.	E. A.	Means.
$r_{12\cdot45}$	− ·14	0	− ·95	− ·07	− ·29
$r_{13\cdot45}$	·20	·32	·12	·44	·27
$r_{14\cdot25}$	·17	·39	·99	·63	·53
$r_{14\cdot35}$	− ·10	·20	·82	·48	·35
$r_{15\cdot24}$	− ·33	− ·48	·95	− ·15	...
$r_{15\cdot34}$	− ·50	− ·52	·70	·01	...
$r_{23\cdot45}$	·63	·78	·50	·89	·70
$r_{24\cdot15}$	·88	·67	·97	·36	·70
$r_{24\cdot35}$	·47	·02	·43	− ·13	·20
$r_{25\cdot14}$	·53	·27	·93	− ·22	·37
$r_{25\cdot34}$	·54	·37	·37	·18	·38
$r_{35\cdot14}$	·83	·74	·33	·22	·53
$r_{34\cdot25}$	·25	·51	·42	·37	·39
$r_{35\cdot14}$	·36	·19	·01	·31	·19
$r_{35\cdot24}$	− ·12	− ·23	− ·02	− ·33	− ·18
$r_{45\cdot12}$	− ·61	− ·31	− ·97	− ·34	− ·56
$r_{45\cdot13}$	·36	·19	·01	− ·31	·06
$r_{45\cdot23}$	− ·64	− ·38	− ·69	− ·38	− ·52

* For fuller details, see *The Statical Changes of Pressure and Temperature in a Column of Air that Accompany Changes of Pressure at the Bottom*, Q. J., Roy. Met. Soc., vol. 38, pp. 41–50, 1912; or the abstract of the paper printed on p. 48.

TABLE VI.—Regression Coefficients. Second Order.

	106 C.	66 E.	29 E.	E. A.	Mean.	B.
$b_{21·45}$	− ·04	·0	− ·59	− ·05	− ·17	− ·25
$b_{31·45}$	·18	·14	·14	·39	·21	·23
$b_{41·35}$	− ·19	·21	·73	·65	·35	·23
$b_{41·25}$	·14	·45	·53	·70	·45	·30
$b_{51·24}$	− ·17	− ·31	·85	− ·16	·05	·08
$b_{51·34}$	− ·56	− ·35	·54	·01	− ·08	− ·12
$b_{12·45}$	− ·40	·0	− 1·57	− ·10	·52	− ·35
$b_{32·45}$	·75	·77	·65	1·15	·84	·62
$b_{42·15}$	2·05	·99	·85	·64	1·18	·53
$b_{42·35}$	·91	·04	·85	− ·32	·38	·17
$b_{52·14}$	·34	·22	1·36	− ·36	·39	·39
$b_{52·34}$	·97	·52	·50	·34	·59	·59
$b_{13·45}$	·21	·35	·12	·50	·29	·27
$b_{23·45}$	·53	·77	·37	·68	·59	·80
$b_{43·15}$	1·72	1·30	·27	·34	·91	·56
$b_{43·25}$	·42	1·20	·62	1·15	·85	·52
$b_{53·14}$	·43	·20	·01	− ·24	·10	·14
$b_{53·24}$	− ·19	− ·33	− ·07	− ·46	− ·26	− ·36
$b_{14·35}$	− ·05	·18	·92	·35	·35	·52
$b_{14·52}$	·22	·32	1·85	·55	·74	1·12
$b_{24·15}$	·38	·44	1·13	·24	·54	1·20
$b_{24·35}$	·25	·01	·22	− ·05	·11	·24
$b_{34·15}$	·40	·42	·41	·14	·34	·55
$b_{34·25}$	·16	·21	·29	·11	·19	·31
$b_{54·12}$	− ·40	− ·17	− 1·63	− ·33	− ·63	− 1·41
$b_{54·13}$	− ·27	− ·15	− ·71	− ·18	− ·43	− ·96
$b_{54·23}$	− ·60	− ·24	− ·48	− ·84	− ·54	− 1·20
$b_{15·24}$	− ·62	− ·74	1·05	− ·13	− ·11	− ·07
$b_{15·34}$	− ·44	− ·76	·92	·01	− ·07	− ·05
$b_{25·14}$	·81	·32	·64	− ·13	·41	·41
$b_{25·34}$	·30	·25	·27	·10	·23	·23
$b_{35·14}$	·30	·18	·01	− ·40	·02	·01
$b_{35·24}$	− ·08	− ·13	− ·07	− ·22	− ·13	− ·09
$b_{45·12}$	− ·91	− ·56	− ·57	− ·35	− ·60	− ·27
$b_{45·13}$	− ·82	− ·40	− ·97	− ·71	− ·72	− ·32
$b_{45·23}$	− ·67	− ·59	− ·97	− ·17	− ·60	− ·27

The following matters, amongst many others, seem to me to be worth noting.

THE MUTUAL ACTION OF T_m AND P_9.

It has already been pointed out that the value of P_9, a value obtained by calculation and not like the other quantities by observation, is chiefly dependent on T_m, but this does not exclude the supposition that T_m may be influenced in a direct way by P_9. There is strong evidence that this is the case, for the whole set of coefficients $b_{23·1}$, $b_{23·4}$, $b_{23·5}$, are high and very consistent. The mean value is ·73, but of this ·20 is due to the adiabatic rise of temperature due to increased pressure. A value of ·53 is left for the direct action of P_9 upon T_m apart from the mere adiabatic increase of temperature, which, expressed in terms of the standard deviations as units, becomes ·70. For the nature of this action, see p. 30.

MUTUAL ACTION OF T_m AND H_c.

The determination of this point was the object with which I undertook the investigation, and it seems fairly clear that the direct action is very small, so small that one cannot with certainty determine the sign. The total correlation coefficients are all large; the mean is ·78, but the first order ratios show that the action occurs through P_9, for we have $r_{24\cdot3} = \cdot16$ only, while $r_{24\cdot1} = \cdot70$, and $r_{24\cdot5} = \cdot74$. The regression coefficients are $b_{42\cdot3} = \cdot13$, $b_{42\cdot35} = \cdot38$, but $b_{42\cdot5} = 1\cdot25$ and $b_{42\cdot15} = 1\cdot18$. It is perfectly obvious that the effect occurs through P_9 (3). We have further to deduct ·31 if we wish to consider the radiation effect. This gives a mean of $\frac{\cdot13+\cdot38}{2} - \cdot31 = -\cdot06$ for the regression equation $\delta H_c = -\cdot06\ \delta T_m + \ldots$

The reverse action of H_c on T_m appears to be large but also indirect and to act by means of P_9.

The Mutual Action of T_m and T_c.

All the total correlation coefficients (r_{25}) between T_m and T_c are negative, and this is the more remarkable when we consider that the summer raises and the winter lowers both quantities together. When, however, we look at the regression coefficients $b_{52\ldots\ldots}$ we see the reason, for we notice that $b_{52\cdot34}$ has a high positive value, that $b_{52\cdot3}$ is positive and fairly high, and $b_{52\cdot14}$ and $b_{52\cdot4}$ are also positive. The negative influence is therefore introduced by the action of 3 or 4. P_9 and H_c are closely correlated, and it is not easy to distinguish between them; but in this case the evidence is clear that it is chiefly P_9, *i.e.* (3), that is important, since it is the exclusion of the 3 that gives the highest positive value to the coefficients.

The direct action of T_m on T_c is therefore positive, either by means of the direct radiation of heat from the lower to the higher layers of air, or because solar radiation raises or its absence lowers both quantities at the same time.

Mutual Action of P_9 and H_c.

The connection between these two quantities is very close and direct, for the total correlation coefficients are high, and the partial ratios are also high whatever other variable may be excluded. It is, however, reduced by excluding T_m except in the E. A. set. The influence of P_9 on H_c seems to be greater than that of H_c upon P_9, but the difference in the magnitude of the regression coefficients is not very great.

Mutual Action of P_9 and T_c.

All the total regression coefficients, and the partial ones which exclude the suffix 2, are negative. The partial coefficients which do not exclude 2 are mostly positive. Excluding 2 means, to some extent, excluding the effect of the annual variation, for the standard variation of T_m (2) is about equal to half the annual range. This is further shown by the fact that $r_{35\cdot4}$ and $r_{35\cdot14}$ in the E. A. set are negative though positive in the other sets. In the E. A. set the annual variation is excluded.

We have already seen that the adiabatic warming produced by increasing P_9 adds an amount estimated at ·35 to all the coefficients of the form $b_{53\ldots\ldots}$ It follows that since the action is negative notwithstanding these inducements for a positive action, there must be a large direct negative action of P_9 upon T_c.

The values of $b_{35\cdot12}$ and $b_{35\cdot24}$, which are consistently low throughout, show that the reverse action of T_c on P_9 is trifling.

Mutual Action of H_c and T_c.

The mutual action of these two quantities is more pronounced than in any other case: it is direct and negative, and it must be remembered that the annual range of the two quantities tends to give a positive connection. Inasmuch as the regression coefficients in column B for $b_{54\ldots}$ are greater than 1, there must be also an indirect action of a positive kind. This may occur through T_m. A plus value of δH_c makes a plus value of δT_m, and this in turn raises T_c. At least this conclusion may be drawn from the regression coefficients $b_{54\cdot 2}$ and $b_{45\cdot 3}$, but it is not supported by the values of $b_{54\cdot 12}$, $b_{54\cdot 13}$, and $b_{54\cdot 23}$, which seem to show that the action takes place through P_s.

The reverse action of T_c on H_c is small, the mean value of the regression coefficients being only $-\cdot 29$ instead of $-1\cdot 18$.

Action of P_s.

The surface pressure P_s does not seem to be an important variable. This is unfortunate, for it is the only one of the series that we are able to measure when and where we will. Judging from the partial regression coefficients of the second order, we see that it has a direct negative effect upon T_m and a positive effect upon P_9 and H_c. It seems to be positively influenced by H_c and P_9.

To sum up the results so far obtained—

T_m, the temperature of the air column from 1 to 9 kilometres, has a negative effect upon P_s, the surface air pressure, a large positive effect upon P_9, the pressure at 9 kilometres, no direct effect upon H_c, and a moderate positive effect upon T_c.

P_9, the pressure at 9 kilometres, has a positive effect upon P_0 and T_m, is very closely and positively correlated with H_c, but whether as cause or effect is somewhat uncertain, and has a negative effect upon T_c.

H_c, the height of the isothermal, has a positive effect upon P_s, no direct effect upon T_m, is closely correlated with P_9, and has a very distinct negative effect upon T_c.

T_c, the temperature at the commencement of the isothermal, has but little effect upon any of the other variables.

There are two points in the tables that require explanation. In the set E. A. the correlation ratios involving 1, 2, and 3 are all very high. This is perhaps because in calculating the ratios, δP_9, instead of being entered from the figures given for each ascent, was written down at once from the formula

$$\delta P_9 = \cdot 33\ \delta P_9 + 1\cdot 08\ \delta T_m.$$

That is to say, that for this particular group T_m represented the true mean of the air column from 0 to 9 kilometres, and P_9 was calculated without appreciable error. In the other sets the errors in calculating T_m might easily be 1 or 2 mm., and T_m is not the true mean of the column, since the temperature of the first kilometre is not considered.

The second point is the high correlation coefficients of the second order involving 1, 2, 4, and 5 in the 29 E set. I cannot discover any mistake or offer any explanation, unless it be that 29 is not a sufficient number of observations on which to base coefficients of the second order.

It should be added that all the partial correlation coefficients are obtained by two independent methods, so that there is a perfect check upon their accuracy. The regression coefficients and the correlation coefficients are connected by the equation $r^2_{pq \ldots} = b_{pq \ldots} \times b_{qp \ldots}$ with the same secondary suffixes in each. It cannot be contended that there are no mistakes, but all the figures, and, more especially, the noticeably discordant ones, have been carefully examined.

All calculations have been made with a slide rule, which gives accuracy to the second figure, but I do not consider that any importance should be attached to the figure in the second decimal place of any coefficient. I think the first figure is worthy of credence.

A diagram (Plate 12) is also given on semilogarithmic paper showing the mean temperatures and pressures over England in the summer (August) and the winter (February). The temperatures (degrees Absolute) are written in at the sides at each kilometre of height; the pressures are given in bars (megadynes per sq. cm.) by the vertical lines which denote the logarithms. Conditions at sea-level are represented by the point A; at 10 kilometres in August and February by the points C and D respectively. The curves pass from A to C and D through B, reappearing at B′, and they are continued from C and D to G and H (20 km.) by the broken portions C′E, E′G and D′F, F′H for summer and winter respectively. The pressure at B and B′ is 1·000 bar; at C and C′ it is 0·274 bar; at E and E′ it is 0·100 bar; and at G it is 0·060 bar. The lines I. S. and I. W. show the usual position of the isothermal in summer and winter.

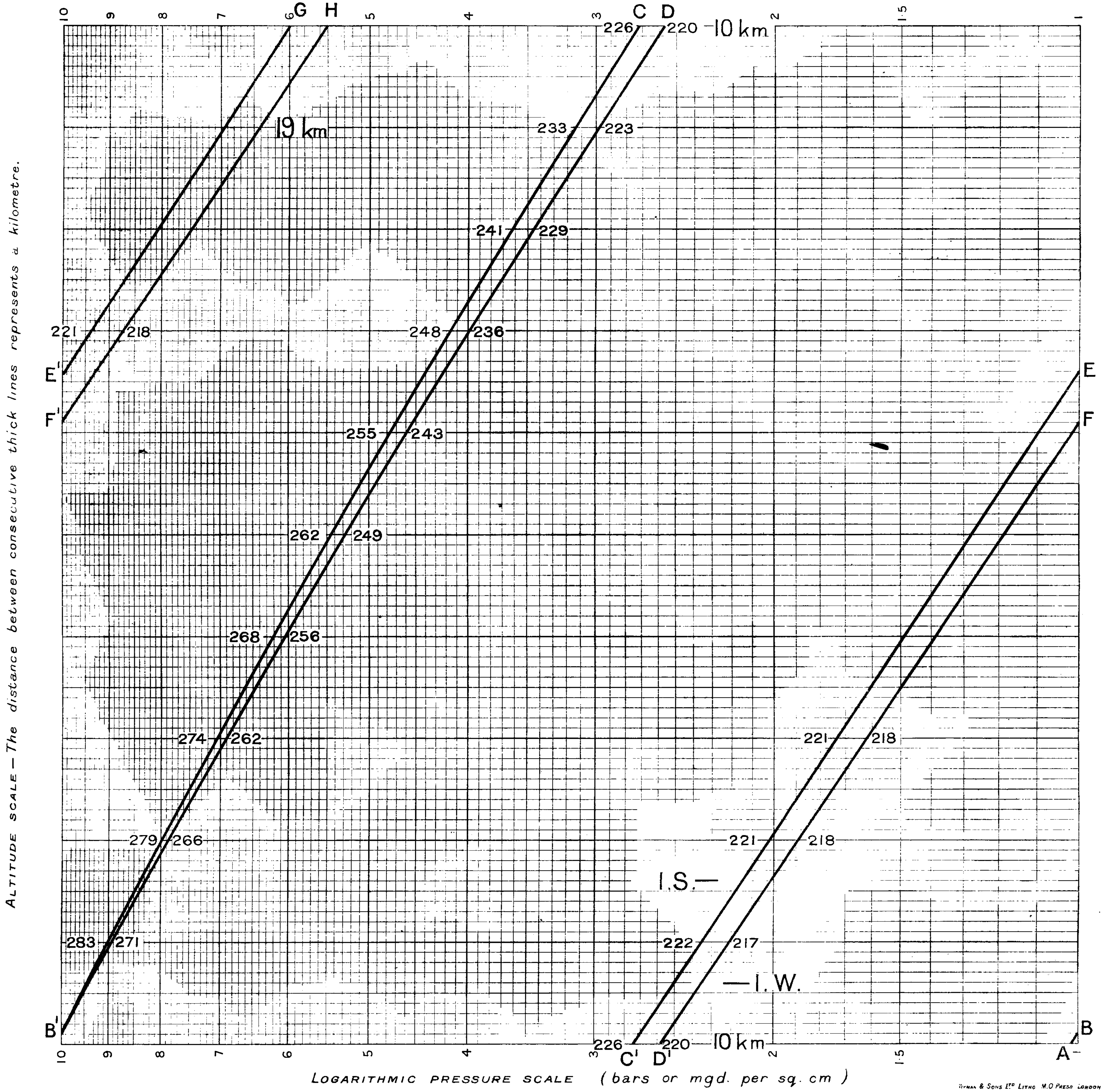
The above series of total and partial correlation coefficients were obtained last summer and submitted to Dr Shaw and others, to whom I am indebted for useful suggestions. Since then it has seemed desirable to ascertain the extent to which the various quantities are dependent upon the direction and magnitude of the upper currents, and for this purpose the values given in the following tables have been calculated.

The force and direction of the wind at some definite height are only known in some few exceptional instances, but the general drift of the air up to 12 to 20 kilometres on the date of each successful registering balloon ascent is shown by the bearing and distance of the falling place of the balloon.

The N—S and E—W components of this drift have been correlated with the temperatures, at the surface, at 4 kilometres, and at 8 kilometres, and also with the height of the isothermal. In general, the balloons remain in the air from 2 to $2\frac{1}{2}$ hours in England and less on the Continent, but in some cases they do not burst and the time is much longer. The distance they run depends upon the time, and it is not desirable to introduce into the correlation the exceptionally long runs which may occur when the balloon fails to burst, neither is it always possible to know if it did burst. To avoid this, all cases where the run exceeds 200 kilometres have been treated as equal to 200 kilometres. Falls to the North and East are treated as positive, to the South and West as negative. Thus a balloon falling 100 kilometres N. 30° E. would be entered in the correlation tables as + 9, + 5, 10 kilometres being taken as the class interval.

Winter has been taken as November to April, summer as May to October.

DIAGRAM SHOWING THE NORMAL RELATIONS BETWEEN PRESSURE, TEMPERATURE AND HEIGHT OVER ENGLAND IN SUMMER AND WINTER. (for explanation see p. 44.)



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	English, 1908–11.			Continental.			Means.	Gradient Wind.		Surface Wind.
	Winter.	Summer.	Combined.	1909.	Winter, 1907–08.	Summer, 1907–08.		Winter.	Summer.	Winter.
Correlation Coefficients.										
N and T_0	− ·16	·01	·18	·26	− ·01	·09	·06	− ·22	− ·16	− ·53
N and T_4	− ·13	− ·16	·10	·18	− ·08	·00	− ·02	− ·01	·12	...
N and T_8	− ·17	0	·06	·21	·18	− ·08	·03	− ·12	·14	...
N and H_c	− ·26	− ·11	− ·06	·04	− ·05	− ·18	− ·10	·03	− ·01	...
E and T_0	·18	·07	·13	− ·12	·36	− ·24	·06	− ·57	− ·17	− ·42
E and T_4	− ·17	·12	·07	− ·28	·09	− ·31	− ·08	− ·31	·05	...
E and T_8	− ·08	·17	·16	− ·26	− ·06	− ·13	− ·03	− ·02	− ·18	...
E and H_c	− ·31	·08	− ·09	− ·08	·06	− ·23	− ·09	− ·11	− ·11	...
H_c and T_0	·22	·42	·43	·49	·09	·15	·30	...	...	...
H_c and T_4	·60	·57	·67	·75	·68	·55	·64	...	...	...
H_c and T_8	·75	·72	·71	·76	·82	·66	·74	...	...	...
No. of Obs.	(46)	(93)	(139)	(80)	(83)	(105)	(407)	(38)	(55)	(64)
Standard Deviations.										
T_0	4°·8	3°·5	...	6°·5	6°·2	4°·6	5°·1	4°·1	3°·6	3°·9
T_4	6°·0	4°·5	...	7°·2	5°·4	4°·0	5·4	5·9	4·5	...
T_8	6°·8	4°·7	...	6°·7	6°·8	5°·7	6·1	6·3	5·3	...
H_c	1·82	1·04	...	1·32	1·18	1·19	1·31	1·79	1·25	...
N	53	71	...	65	60	72	64	7·5	7·0	2·1
E	73	63	...	78	60	67	68	11·0	9·0	2·7
Means.										
T_0	3°·6	14°·1	...	8°·5	0°·1	13°·6	...	3·6	13·5	...
T_4	− 16°·4	− 6·4	...	− 10·5	− 13·5	− 4·6	...	− 16·4	− 6·3	...
T_8	− 43°·2	− 33·8	...	− 38·5	− 43·4	− 31·2	...	− 44·1	− 34·3	...
H_c	10·6	11·5	...	10·9	10·6	11·7	...	10·3	11·6	...
N	− 8	32	...	− 30	− 29	− 29	...	− 1·4	− 1·8	− ·4
E	20	29	...	52	34	41	...	+ 1·0	2·5	− ·9

N.—Northerly component in kilometres, metres per second or Beaufort numbers.
E.—Easterly component.
T_0.—Temperature (° C.) at the surface.
T_4.—Temperature at 4 kilometres.
T_8.—Temperature at 8 kilometres.
H_c.—Height of isothermal in kilometers.

It will be seen from the table that the drift of the balloon is practically without effect upon the temperature of the air and has but a trifling effect upon the height of the isothermal. When dealing with 400 observations the probable error of a small correlation coefficient is ·033, and hence the correlation between H_c and the drift of the air is about three times the probable error. We may infer a connection, the isothermal being lower when the drift is towards the North East, that is to say, when the general direction of the wind taking all strata up to about 16 kilometres into account is South West. But the connection is most probably a slight one, and the correlation coefficients are not large enough to make sure that it is not fortuitous.

The correlation between H_c and the mean temperature of the air column below the isothermal layer has been already discussed. The values in the table show that there is an intimate connection between H_c and the temperature even down to the surface. The correlation coefficient for the surface is not large, and it is partly due to the fact that T_0 and H_c increase together in summer and decrease in winter; but cutting out the annual sets, the mean ·22, with a probable error of ·036, is still large enough to prove a connection, since it is over six times the probable error. Strictly, the mean ·22 of the four sets is not the correlation coefficient of the combined observations; but the error is not serious.

The small value ·06 between the surface temperature and the general drift is very noticeable, for a South wind at all seasons is most obviously warmer than a North. The inference is that the general drift does not represent the surface wind. To check this the temperatures have been correlated with the gradient wind obtained from the weather chart for 7·0 A.M. on the same day. The ascents at Ditcham Park and Pyrton Hill during the four years 1908–1911 have been used for the purpose. The components of the gradient wind have been expressed in metres per second, and it must not be forgotten that the sign of the coefficients should be just opposite to those obtained for the drift of a balloon, because the falling place of a balloon lies in the opposite direction to the designation of the wind which carried it there.

A much closer connection is shown in this case. Owing to the incurvature of the surface wind we may take the actual wind direction as two points at least on the left of the gradient, and thus the E—W component really means ENE—WSW. This explains why the winter E. wind (– ·57) appears so much colder than the N. wind (– ·22).

The balloon ascents did not all occur at 7·0 A.M., the time (8·0 A.M. for the earlier years) at which the gradients were observed, and hence as a further check the 9·0 A.M. temperature of every third day for two winter quarters was correlated with the force and direction of the wind at the same time; the force of the wind being expressed in the Beaufort Scale. The result is what one would have expected, about half the variation of temperature at the ground level being credited to the direction of the wind.

The following points also appear from the investigation. On the average, the balloons fall 18° to the right of the gradient wind. Sixty per cent. fall within 4 points (45°) of the gradient direction, and 20 per cent. have their motion reversed and fall more than 8 points away. The connection between the steepness of the gradient and the distance the balloon goes is very slight.

The object of this investigation is to ascertain if the distribution of the upper temperature is symmetrical in the various segments of a cyclone.

If we look upon a cyclone or anticyclone as comprising purely surface phenomena, the segment in which any balloon ascent occurs is fixed by the direction of the gradient wind on that occasion; if, on the other hand, we regard a cyclone as a disturbance reaching up to 10 or 20 kilometres, though not necessarily with a vertical axis, then the segment is fixed by the mean direction of the drift of the balloon, and in both cases the magnitude of the gradient or drift is a good measure of the intensity of the cyclone. In both cases the figures show that there is either no connection, or only a very slight connection, between the temperatures away from the surface, or the isothermal, and the special segment. In other words, the upper temperatures and the value of H_c in a cyclone are almost, or quite, symmetrical with regard to the points of the compass.

This is a most important point in regard to the local circulation, and it seems quite well established. Were there a well-defined connection, it could not fail to appear from the figures. Take, for example, the connection between H_c and T_4 with a mean correlation of ·64. This is of quite a different character. The connection is shown in every single group; the whole set lie between ·55 and ·75, notwithstanding the fact, already stated, that there is a good reason for a positive correlation in the annual sets, which is absent, or nearly so, from the six month sets.

The pressures and temperatures of the atmosphere and also the height of the isothermal are most intimately connected over the British Isles and Mid Europe, but, at least for a first approximation, the strength and direction of the air currents do not share in the connection.

A few words may be added about the standard deviations and means. The variability of the temperature increases with increasing height, notwithstanding the fact that the amplitude of the daily variation decreases. The deviations for the N—S and E—W components of the wind are much the same; dividing by 2 hours, we get about 30 kilometres per hour or 8m/s. as the standard deviation of the velocity of the drift, which is about the same as the deviation of the gradient velocity.

The centre of the falling points of the balloons on the Continent is 30 kilometres S. and 42 kilometres E., denoting a NW. by W. wind. Taking the standard deviation as 60, the standard error for a mean of 80 observations is under 7, and for the mean of the whole 268 it is under 4 kilometres. We may accept it, therefore, as practically certain that the point given by the values 30 and 42 is within 15 kilometres of the point that would be given by a very large number of observations. Hence there exists over Mid Europe in summer and winter a general drift of the atmosphere up to some 15 kilometres from between WNW. and NW. with a mean velocity of about 7 m/s. It would be of extreme interest to know if the Northerly component of this drift exists over the whole parallel of 50° North Latitude, or if it be a local phenomenon. We cannot infer much from the English figures, because the Channel lies close to Ditcham Park and Pyrton Hill on the South, and, besides leading to the loss of many balloons, its proximity introduces a systematic error into the mean falling point of the English balloons, which is thereby displaced to the north. To this cause too may be assigned the mean easterly component of the gradient winds on the dates of the successful ascents at Ditcham Park and Pyrton Hill. If, as seems likely, this Northerly drift is not local, it follows that there must be a compensating current from the South at altitudes beyond the reach of our balloons, and the existence of such a circulation carries with it a ready and simple explanation of the low upper temperature over the equatorial regions and the high temperatures found in the isothermal in high latitudes.

THE STATICAL CHANGES OF PRESSURE AND TEMPERATURE IN A COLUMN OF AIR THAT ACCOMPANY CHANGES OF PRESSURE AT THE BOTTOM.*

THE effect of change of pressure at the surface upon changes of temperature and pressure in a column of dry air, which is initially under the same conditions in regard to temperature and pressure as those which exist in the average atmosphere over England, depends upon the way in which the change is brought about.

Case I.—If the column of air is supposed to be enclosed in a vertical cylinder, the pressure at the surface may be increased by diminishing the radius of the cylinder by 1 per cent. The following changes in pressure (δP) and temperature (δT) then occur at the heights specified.

km.	δP	δT
	mm.	°C.
0	15·2	1·6
5	9·7	1·7
10	5·7	1·6
15	3·1	1·3

Case II.—The pressure at the surface can be increased by the same amount as before (15·2 mm.) by inserting a column of air 170 m. thick at the bottom, of such a temperature that the usual temperature gradient is maintained. The corresponding changes at different levels are

km.	δP	δT
	mm.	°C.
0	15·2	1·0
5	9·2	1·1
10	5·1	0·8
15	2·4	0·0

Case III.—The pressure at the surface may be *reduced* by 15·2 mm. by the withdrawal of air in the upper part of the column. The air may be withdrawn at any level, and the changes below that level will remain the same as if the air were removed at the top. The changes induced in the column are the following:—

km.	δP	δT
	mm.	°C.
0	−15·2	− 1·6
5	− 9·9	− 2·3
10	− 6·2	− 4·2
15	− 5·0	−12·0

In each of the above cases it is assumed that the changes of temperature are adiabatic. Reasons are given for supposing that in practice this is the case.

* Abstract of a paper by W. H. Dines, F.R.S., in the *Quarterly Journal, Roy. Met. Soc.*, **38**, pp. 41-50, 1912.

Observations in the winter months in England, selected on account of cyclonic and anticyclonic conditions, lead to the following conclusion.

If the surface pressure drops by 15 mm. the chances are many thousands to one that the following variations will occur.

The pressure at 9 km. will fall by about 10 mm., and the mean temperature of the air column from 1 to 9 km. will fall by about 5° to 6° C. The figures given are the averages for the various quantities, and the departure from such average in any individual case is not likely to be great, *i.e.* not likely to be more than 20 per cent. of the given value.

The question is, will either Case I., II., or III., or any combination or repetition of them, fit the known changes of pressure and temperature?

A cyclone is known to be accompanied by a very slowly rising air column in the lower strata; how far up this rising current extends has not been known with any certainty. Provided the rate of ascent is quite small so that no appreciable change of pressure can be ascribed to the vertical motion, and this certainly is the case, this rising current is strictly equivalent to a combination of Cases II. and III., repeated if necessary.

Let us take Case III. repeated three times and Case II. repeated twice and see the combined effect upon the temperature and pressure.

Cases III. and II. together have the effect shown in the following table:

km.	δP	δT
0	−15·2 +15·2	−1°·6 +1·0
5	− 9·9 + 9·2	−2·3 +1·1
10	− 6·2 + 5·1	−4·2 + ·8

Repeated twice this gives—

km.	δP	δT
0	0	−1°·2
5	− 1·4	−2·4
10	− 2·2	−6·8

Adding Case III. again, finally we get—

km.	δP	δT
0	−15·2	− 2°·8
5	−11·3	− 4·7
10	− 8·4	−11·0

These values give a very good fit for the calculated and observed changes of pressure up to 10 km., and also, excepting that the temperatures are rather too low, for the temperatures up to 8 km. The temperatures ought to be too low because we have taken the air as dry, whereas a certain amount of cloud and rain, and therefore a setting free of latent heat, always occurs.

It may appear arbitrary to combine Cases II. and III. a definite number of times, but it is not so in reality. On the ordinary principle of the superposition of small quantities, it is immaterial to the result whether we suppose II. and III. to occur at different times or simultaneously, and taking the five operations as occurring simultaneously we get an ordinary ascending current and decrease of pressure.

The whole process involves the ascent of an air particle from the ground to a height of 3×170 metres, say 1750 feet, and if 18 hours be taken as the time, this gives about 100 feet per hour for the rate of the ascending air near the surface, about the right value, and the 18 hours is itself a reasonable time for the development of a deep cyclone.

Of course, by supposing Cases II. and III. to occur the right number of times, it would be possible to make the temperature or the pressure fit any values at any definite height, but we could not so make both fit over a considerable range of height.

The inference drawn is that a cyclone is produced by the withdrawal laterally of the air at a height of from 8 to 10 kilometres; for if we choose this height the observed and the theoretical variations of pressure and temperature agree, whereas they would not do so if any other height were chosen for the outflow of the air which undoubtedly flows in along the earth's surface.

The values of δT that have been found depend upon the initial distribution of temperature. Had the initial distribution been 1° C. per 100 metres, the adiabatic gradient, all three methods would have given the same result, viz. $\delta T = 1{\cdot}7$ at each height. No repetition of II. and III. alternately would alter the temperature, but starting with any possible distribution the repetition of II. and III. tends to make an adiabatic gradient.

Strictly, the second application of II. and III. in the supposed series of operations does not give quite the same result as the first; but for two only, the error is not serious.

The term "ascending current of a cyclone" has been used, but it appears to be incorrect. The actual phenomenon seems rather to be a bulging upward of the strata between 1 or 2 km. and the isothermal, a bulging downward and a lateral contraction of the strata above the isothermal, accompanied with a lateral contraction over a large area of the lower strata and a lateral expansion of the strata below the isothermal.

The author doubts whether the actual vertical motion of any air particle involved in a cyclonic circulation often exceeds 2 km. (6600 ft.).

The results due to the condensation of vapour are not considered. The result of such condensation is to reduce the fall of temperature by some 40 per cent. in the parts of the lower strata where it is occurring; *i.e.* in the actual region of condensation the value ·29 in the equation $\frac{\delta T}{T} = {\cdot}29 \frac{\delta P}{P}$ should be replaced by a value of about ·20.

Some idea of the magnitude of the error caused by ignoring the humidity of the air may be gained by noting that a fall of rain of about ·16 in. sets free sufficient latent heat to raise the temperature of the whole mass of air over the rainfall area 1° C. The effect of the humidity is therefore to lessen the negative changes of T generally, and increase the negative changes of P at 5 and 10 km.