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STABLE NIGHT SPECTRAL ANALYSIS

by

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1. INTRODUCTION:

Results published by Kaimal et al (1972) showed that, in the surface layer, most spectra and cospectra obey similarity theory in the range $.005 < f < 20$, [where f is the reduced frequency $f = nz/\bar{u}$]. All the stable spectra measured during the 1968 Kansas experiment consistently obeyed similarity laws down to about $f = .005$, in contrast to results obtained by other workers, e.g. McBean (1971), who observed a lot of scatter in the spectra at $f < .1$. Busch (1973) found that unavoidable effects from nonstationarity and large scale inhomogeneities set a lower frequency limit, below which Monin-Obukhov similarity did not seem to hold. The present paper describes some spectra and cospectra which were obtained at Cardington under stable conditions and compares them with results obtained by these other investigators.

The data were collected on the night of 8th - 9th November 1972 over the period 2300 GMT (8/11/72) to 0415 GMT (9/11/72). A "pre-warm front" situation existed with a south-westerly flow over the British Isles. At Cardington the Balthum ascents (Painter 1972) showed that there was a 5°C surface inversion which extended to a height of about 200 M. Although the relative humidity increased from midnight to dawn at no time did it reach 100% so no fog formed.

Brunt-Vaisala frequencies were calculated at 8M (using 20 minute profile data to obtain gradient values), 46M and 91M (from the Balthum ascents). They showed very little sign of variation with time between 0000 GMT and 0600 GMT. The midnight results are given in Table 1. Although these will of course differ from the observed frequencies of any waves, [due to the wind introducing a Doppler shift] they give some idea of the order of magnitude expected.

Three-component Cardington Turbulence Probes [Readings and Butler 1972] were used to measure the instantaneous values of temperature, total wind speed and the inclination of the wind flow to the horizontal. The measurements were made at 91M (on a balloon cable), 46M (on the top of a tower) and 8M (on the top of a mast) and the data were recorded on paper tape. Each variable was sampled once per second, setting an upper frequency limit of 5×10^{-1} Hz . The total period of 5¼ hours was divided into nine 35 minute segments, and the spectral analysis was carried out using a Fast Fourier Transform technique [Rayment 1972].

2. SPECTRA FOR THE TOTAL PERIOD AT ALL HEIGHTS:

Spectra for the total period (5¼ hours) were obtained by averaging the data in sets of nine and then applying the Fast Fourier Transform to the whole run. Figs. 1(a), (b) and (c) show the spectra obtained at each height for the three components u_H (horizontal wind speed), W (vertical wind speed) and T (temperature). The Brunt-Vaisala frequencies are marked and, in all cases, lie in the low frequency energy band, reflecting possible wave motions. A series of harmonics is clearly seen on the u_H spectra and can also be detected on the W and T spectra reinforcing the idea of low frequency wave motions. The cospectral peaks also occur in the vicinity of the Brunt-Vaisala frequency, but a coherence and phase spectral analysis did not indicate the presence of any dominant wave frequency.

3. RESULTS AT 8M

3.1 Spectra for 35 Minute Segments

When the high frequency records were considered it was found convenient to divide the total record into three distinct periods, Table 2. This classification is clearest for the temperature traces but is reflected in all the other traces. A graph of Ri v Time at 8M - Fig 2 - reinforcesthe classification. Ri is about .3 for weak turbulence, varies from about .1 to .7 for sporadic turbulence and falls to below .1 in the period of semi-continuous turbulence.

The spectra for the individual segments in each period were plotted all together, with the abscissae normalised to $f = nz/\bar{u}$ and the ordinates left in arbitrary units on a log scale so that shapes could be compared, (except for the T spectrum in period 1, when the high frequency signal was at noise level). Figs 3 and 4 show the T spectra for periods 2 and 3. In Fig. 3 (sporadic turbulence) the fall-off in the high frequency range is -0.77 for $f > .5$. Similar slopes (i.e. > -0.66) have been observed by Readings et al (1973) in their study of entrainment at an inversion interface and may be caused by a burst of turbulent activity which increases the energy at a particular frequency to a level above that of the background trace. This has been discussed by Roth (1971) who considered the superposition of an energy source on an equilibrium spectrum. In Period 3 (semi-continuous turbulence) the cascade appears to have become established and a slope of -0.67 for $f > .5$ is observed, as would be expected if an inertial subrange existed. Figs. 3 and 4 also show the positions of the peaks obtained by Kaimal et al (1972) for values of $z/L = .1$ and $.3$. The position of the observed high frequency peak appears to agree reasonably well with the spectra corresponding to $z/L = .2$ from Kaimal's curves. For period 3 z/L was calculated from the high frequency end of the spectrum (i.e. $f > .05$) and was found to be about .19, again in reasonable agreement with Kaimal et al's results.

Following Kaimal et al, the results from period 3 have also been normalised with respect to the variance (estimated from the high frequency spectral output) and this is compared with the curve obtained by Kaimal et al (1972) in Fig. 5. The abscissa is f/f_0, where f_0 is the value of the reduced frequency where the extrapolated $\frac{2}{3}$ region intercepts the line $nS_T(n)/\sigma_T^2 = 1$. The shapes of the two curves are very similar, the difference in magnitude implying that the estimated σ_T^2 is too large, probably due to a contribution from wave motions. Using the relationship $(f_o)_T = .83\, Ri$ /_Kaimal (1973)_7 Ri is found to be .008, which is not unreasonable (see Fig. 2). However it is relevant to note that in all cases Kaimal et al obtained only a high frequency peak, probably due to the flatness of their site, (see introduction).

The same procedure was carried out for U_H and W. No high frequency fall-off was observed for W in any period due to the upper frequency limit of .5 H . However the position of the high frequency peak agreed with that obtained by Kaimal et al (1972) for $z/L = .2$.

For U_H there was a fall-off of about -0.66 in all periods (c.f. T). This may be because the U_H fluctuations were actively producing the T fluctuations /_Busch (1973)_7 and so the U_H spectra had reached equilibrium before T. The high frequency peak again agrees with that of Kaimal et al (1972) for $z/L = .2$ and the relation $(f_o)_u = .5\, Ri$ gives Ri = .012, which is also reasonable (c.f. Fig. 2).

3.2 <u>Cospectra at 8M</u>

$U_H T$ (horizontal heat flux), WT (vertical heat flux) and $U_H W$ (momentum flux) cospectra were plotted for each of the nine segments, which were then grouped together as before. Fig. 6 shows the $U_H T$ cospectra for the three different periods, plotted on a log-linear scale with the abscissae again normalised to $f = nz/\bar{u}$, and with the cospectral amplitude, $n\, C_{U_H T}(n)$, as ordinate. In period 1 the high frequency amplitude is negligible, but it builds up in period 2 and becomes well established in period 3. The high frequency peak lies between the positions of Kaimal's peaks for $z/L = .1$ and $z/L = .3$, agreeing well with the calculated value of .19. Integration over the intervals $f < .05$ and $f > .05$ shows that in period 3 less than 10% of the energy occurs in the low frequency region (i.e. $f < .05$); the main contribution to the spectrum coming from the high frequency region. Very similar features were observed in the $U_H W$ and WT cospectra.

A coherence and phase angle analysis has also been performed on the various components. This showed that the coherences were high at the low frequency end of the spectrum during the first period, which may indicate that some type of wave motion was present. However as the high frequency amplitudes increased during the other 2 periods the coherences became random at all frequencies, possibly reflecting the presence of a mixture of waves and turbulence. The phase angles were random throughout the run, implying that no dominant waves were present, but it is not clear whether there was a mixture of waves and turbulence present or just a mixture of waves.

4. RESULTS AT 46M:

The high frequency peak would be expected to remain stationary, on a reduced frequency scale, up to a height of about 100M [Panosfsky 1969], while the low frequency peak would be expected to shift to higher values of f with increasing height (Table 1). Hence the separation of the two peaks should decrease with increasing height. This was observed in all the spectra at 46M. The high frequency peak had not moved relative to its position at 8M, while the low frequency peak was located at higher values of f, as expected. Although two peaks could still be discerned it was more difficult to distinguish between them than at 8M, hence the separation of waves and turbulence proved to be impractical.

The T spectra at 46M had the same trends at high frequency as at 8M, with slopes of about -0.74 in period 2 and about -0.68 in period 3. (Period 1 could not be compared). The cospectra showed larger fluctuations than at 8M, but two peaks could still be distinguished, with greater amplitude than at 8M.

5. RESULTS AT 91M:

Following the discussion in the previous section the low frequency peak at 91 M would be expected to shift to even higher values of f, with the high frequency peak unchanged in position Hence the peaks will be even closer than at 46 M and may perhaps be actually superimposed. This was the case for and W, where the low frequency peak had moved so far to the right that two peaks could no longer be identified. On the T spectra again only one peak was seen, but the high frequency fall-off followed the same pattern as at the other two heights. The cospectra showed great fluctuations and no large peaks could be identified.

6. CONCLUDING COMMENTS:

In general the spectra show two peaks one at low frequency and one at high frequency, the low frequency peak occurring at frequencies comparable to the Brunt Vaisala frequency. The higher coherences at low frequency and the existence of harmonics seem to indicate that some kind of wave motion was present, although the analysis indicates that no dominant frequency was present. This low frequency peak was not observed by Kaimal et al (1972) but McBean (1971) found that for $f < .1$ the observed spectra were so scattered that no meaningful average curve could be drawn. Busch (1973) also observed a deviation from Kaimal's curves at low frequency, due to effects from nonstationarity and large-scale inhomogeneities. In this study the spectra deviate from those obtained by Kaimal et al (1972) at values of $f < .05$, with greater scatter than for $f > .05$ and an identifiable peak. The shapes of the spectra for $f > .05$ are very similar to the curves obtained by Kaimal et al (1972) and the positions of the peaks agree well with the estimated value of $z/L = .19$.

The high frequency peak remains stationary on the reduced frequency scale as the height increases and appears to be due to turbulent activity. However the simultaneous slow decrease in the Brunt-Vaisala frequency with height shifts these low frequency peaks to higher values of f, so that the two peaks become increasingly difficult to separate as height increases.

The attempt to estimate z/L, σ_w/u_*, σ_T/T_* etc for period 3 was not very satisfactory because of the superposition of two processes (i.e. waves and turbulence) which prevented the unambiguous separation of the spectra into two distinct components. At 8M the results were of the right order of magnitude, but at the other heights it was not possible to attempt an estimation as there was no clear separation of the peaks.

The conclusions reached in this paper are only tentative and more stable night spectra need to be studied before they can be substantiated. The structure of the bursts must also be considered in greater detail to try and establish a relationship between the variation in Ri and the occurrence of the bursts, and to find the energy transfer in one of these bursts.

It may also be possible to find the sequence of Ri values for a typical stable night and relate these to the degree of turbulent activity present.

TABLE 1

Height	Time	Brunt-Vaisala Frequency (N)	$f = nz/\bar{u}$
8M	0000 G.M.T.	.0107 HZ	0.026
46M	0000 G.M.T.	.008 HZ	0.048
91M	0000 G.M.T.	.007 HZ	0.080

Brunt-Vaisala Frequency $N = \frac{1}{2\pi}\sqrt{\frac{g}{\theta_v}\frac{\partial \theta_v}{\partial z}}$ HZ

TABLE 2

Segment No	Time Interval	Period No	Description of Turbulence
1	2300 - 2335 G.M.T.	One	Low frequency oscillation, no high frequency fluctuations
2	2335 - 0010 G.M.T.		
3	0010 - 0045 G.M.T.		
4	0045 - 0120 G.M.T.	Two	Randomly spaced bursts of high frequency fluctuation superimposed on the low frequency oscillation. Bursts last from about half a minute to three minutes and have significantly greater amplitude than the background level.
5	0120 - 0155 G.M.T.		
6	0155 - 0230 G.M.T.		
7	0230 - 0305 G.M.T.		
8	0305 - 0340 G.M.T.	Three	Fluctuations are fairly continuous at high frequency, though some low frequency fluctuations are still visible.
9	0340 - 0415 G.M.T.		

LIST OF FIGURE CAPTIONS:

Figure 1(a):- u_H spectra at 8M, 46M and 91M. The letters B-V indicate the position of the Brunt-Vaisala frequency

1(b):- W spectra at 8M, 46M and 91M.

1(c):- T spectra at 8M, 46M and 91M.

Figure 2:- Plot of Richardson number against Time at 8M [using 20 minute profile data].

Figure 3:- Temperature (T) spectra at 8M (9/11/72) 0045-0305 GMT - Sporadic Turbulence. The dotted line represents a $\frac{2}{3}$ fall-off.

Figure 4:- T spectra at 8M (9/11/72) 0305-0415. Semi-continuous Turbulence.

Figure 5:- Comparison of T spectrum at 8M for category 3 with curves obtained by Kaimal et al (1972) under stable conditions.

Figure 6:- $u_H T$ cospectra at 8M (9/11/72) for each of the 3 categories.

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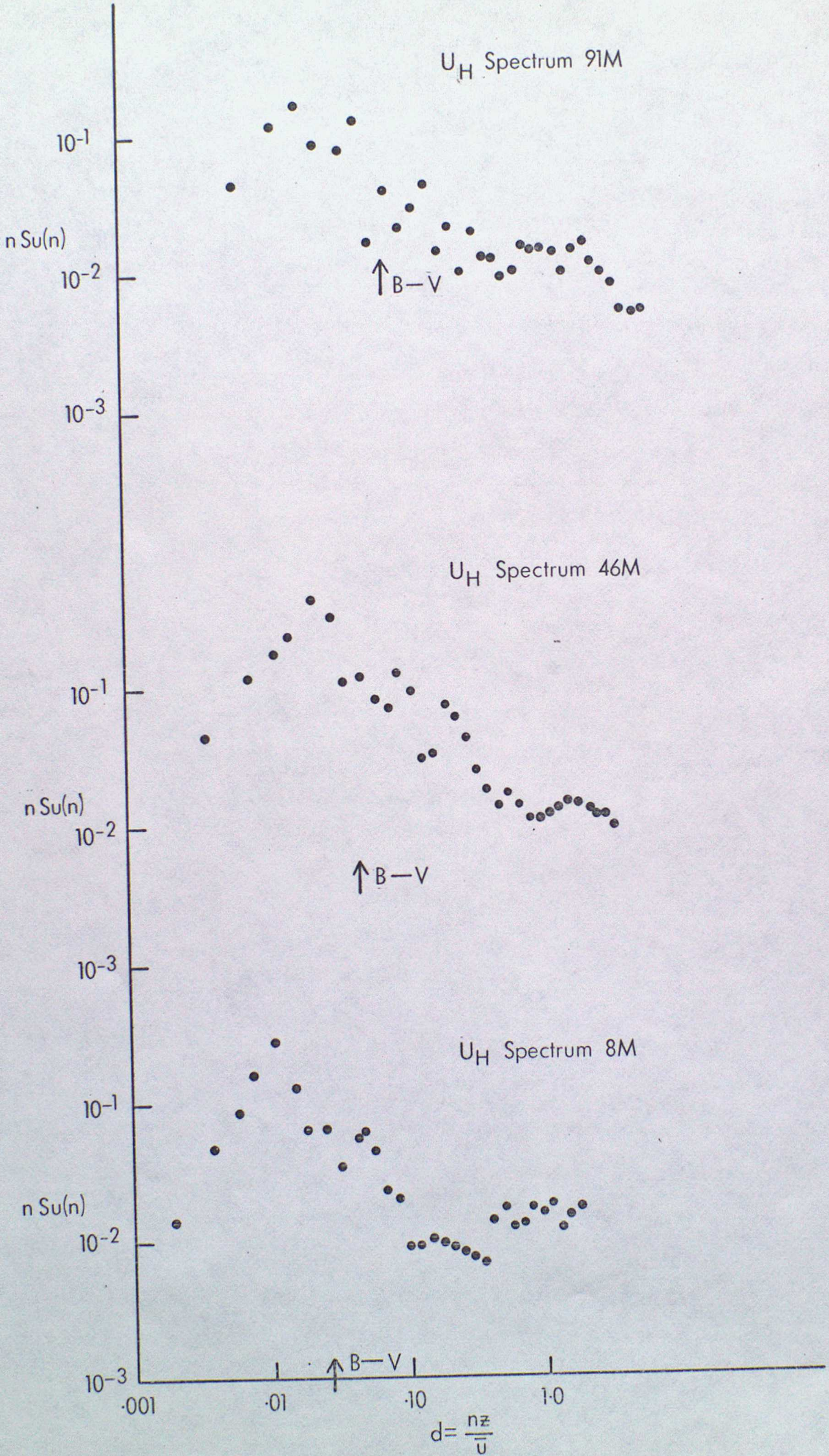
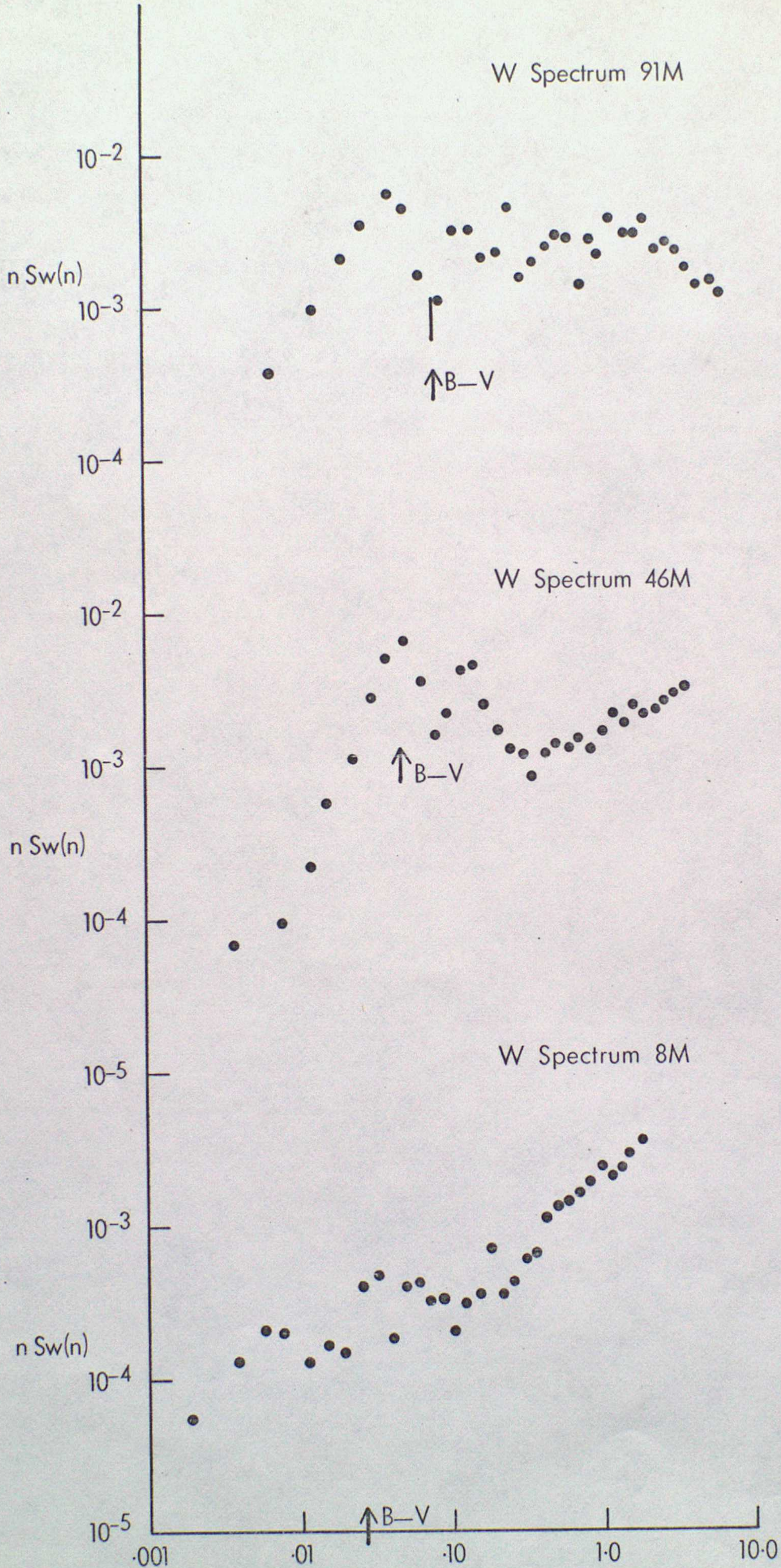


U_H Spectrum 91M
10-1
n Su(n)
10-2
↑B—V
10-3
U_H Spectrum 46M
10-1
n Su(n)
10-2
↑B—V
10-3
U_H Spectrum 8M
10-1
n Su(n)
10-2
10-3
↑B—V
·001
·01
·10
1·0
d= nz/U

Figure 1(a)



W Spectrum 91M
W Spectrum 46M
W Spectrum 8M
n Sw(n)
n Sw(n)
n Sw(n)
B—V
B—V
B—V
10⁻²
10⁻³
10⁻⁴
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻³
10⁻⁴
10⁻⁵
·001
·01
·10
1·0
10·0

Figure 1(b)

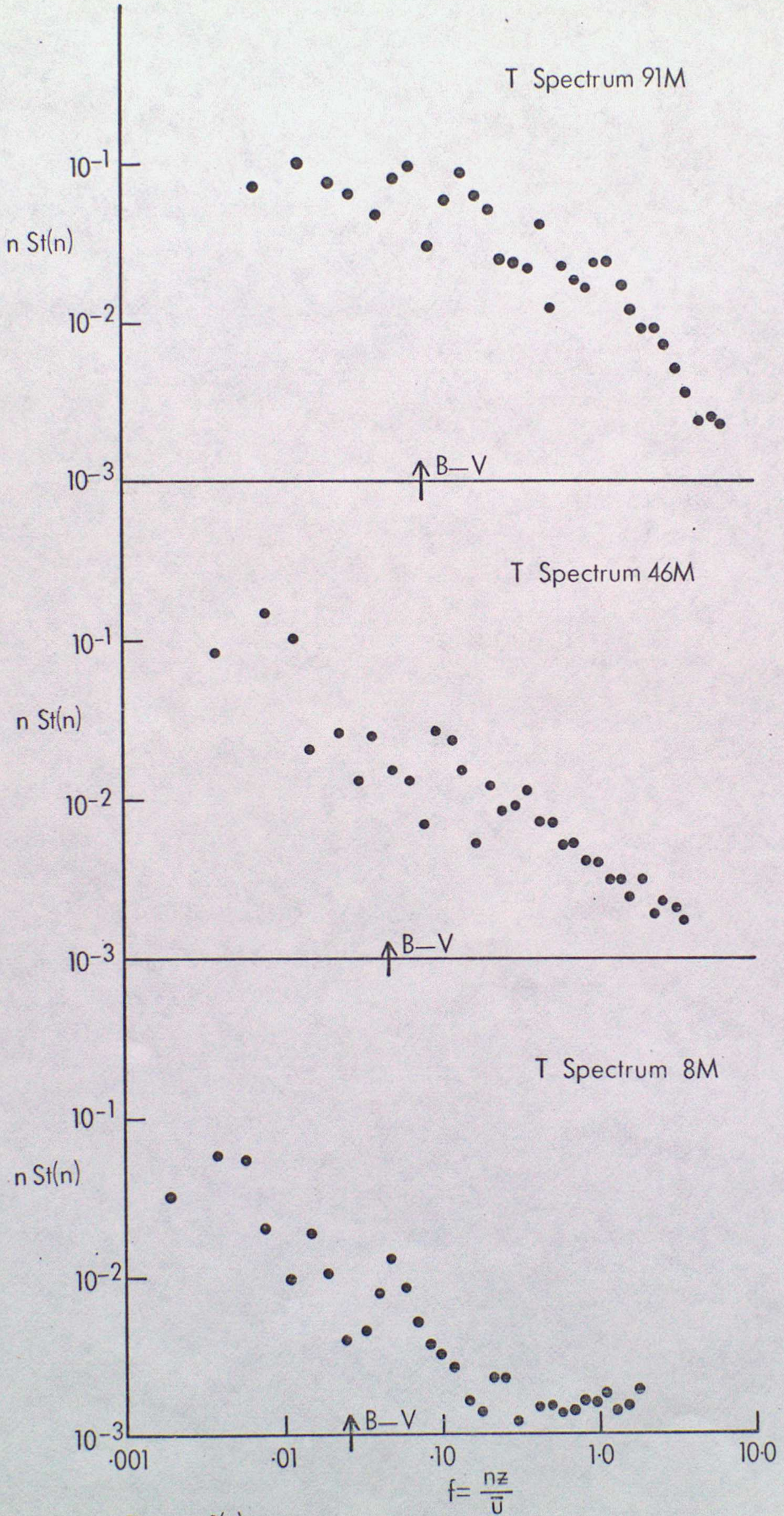

T Spectrum 91M
T Spectrum 46M
T Spectrum 8M
n St(n)
n St(n)
n St(n)
10−1
10−2
10−3
B—V
·001
·01
·10
1·0
10·0
f= nz/ū

Figure 1(c)

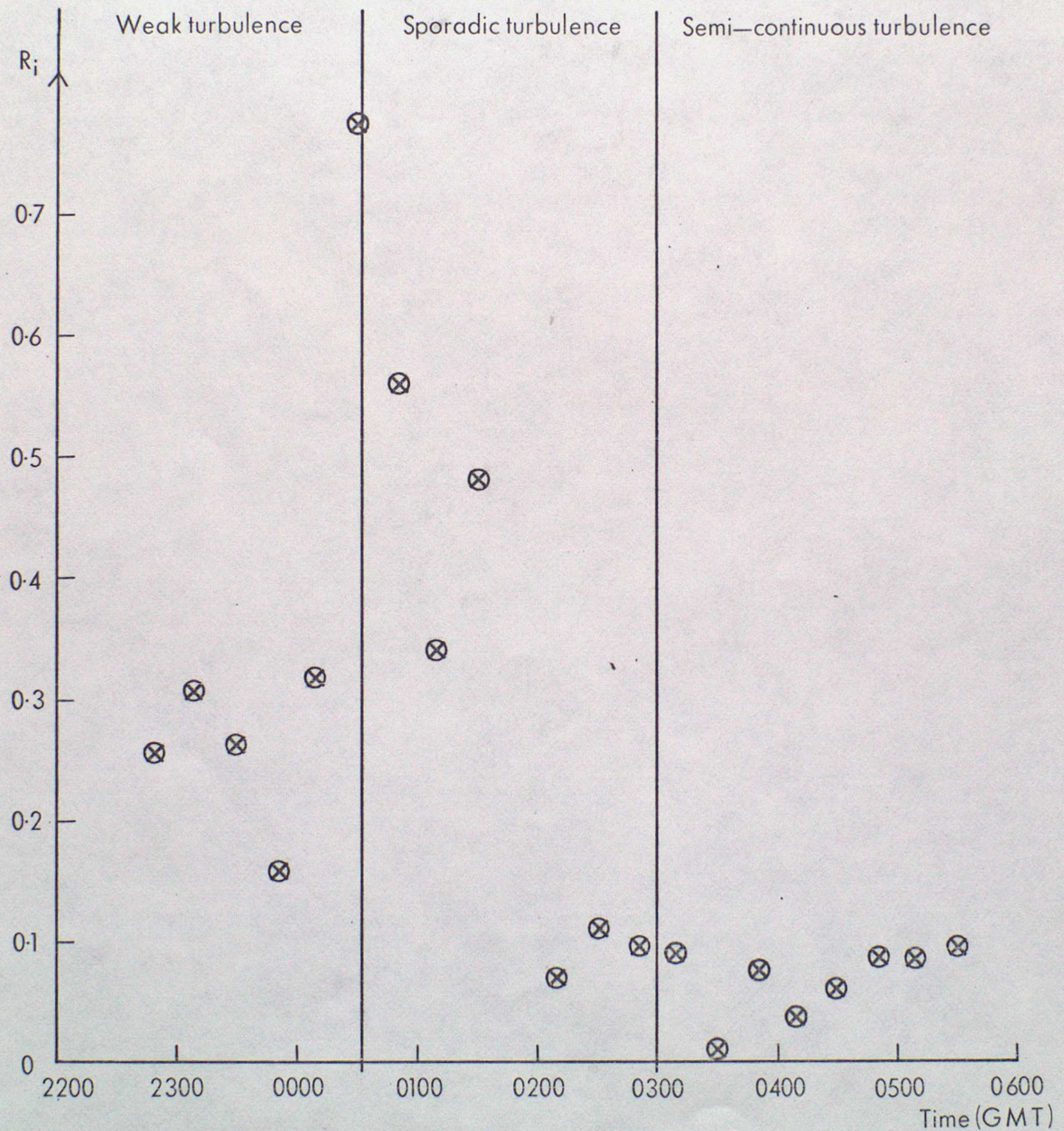
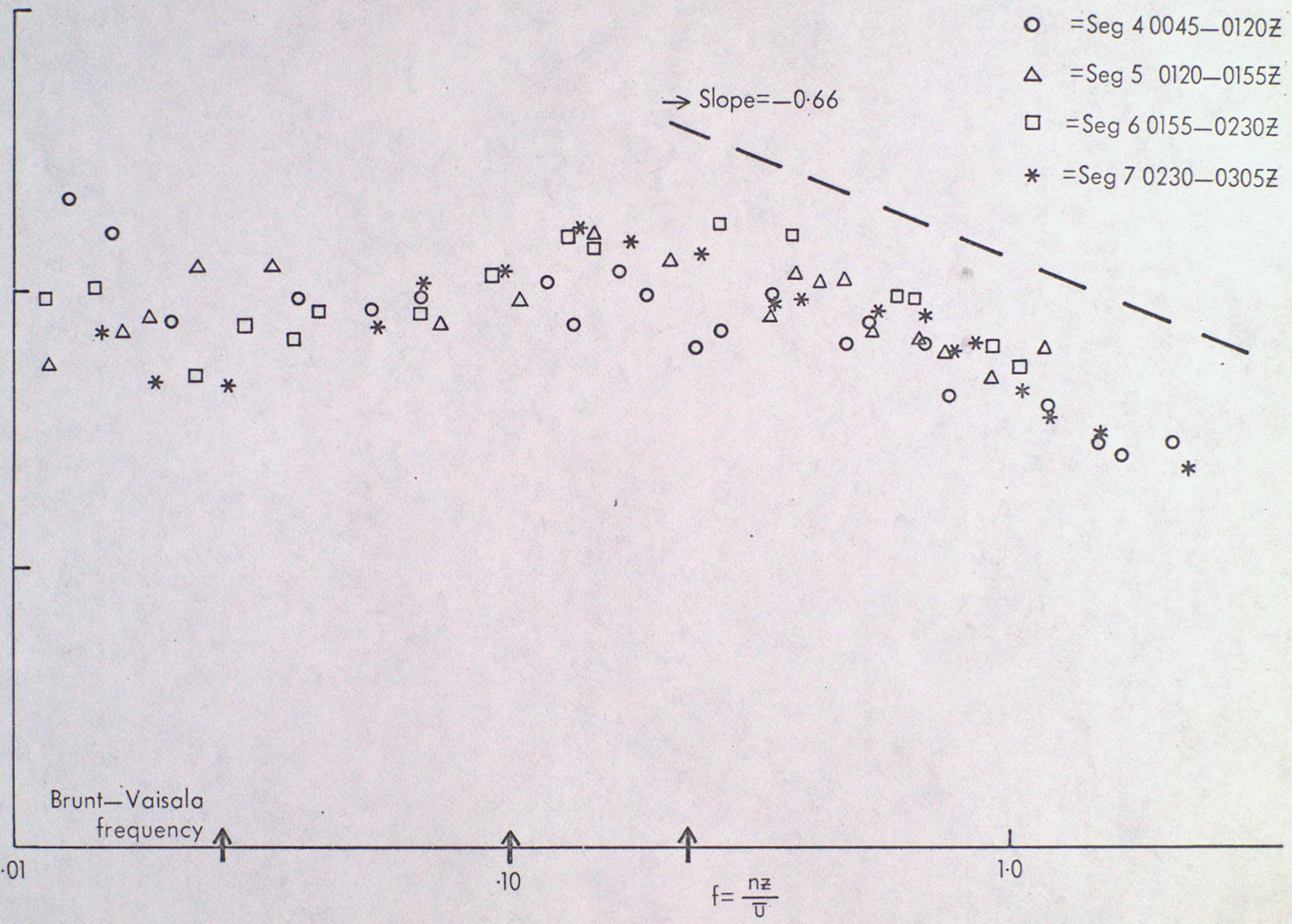
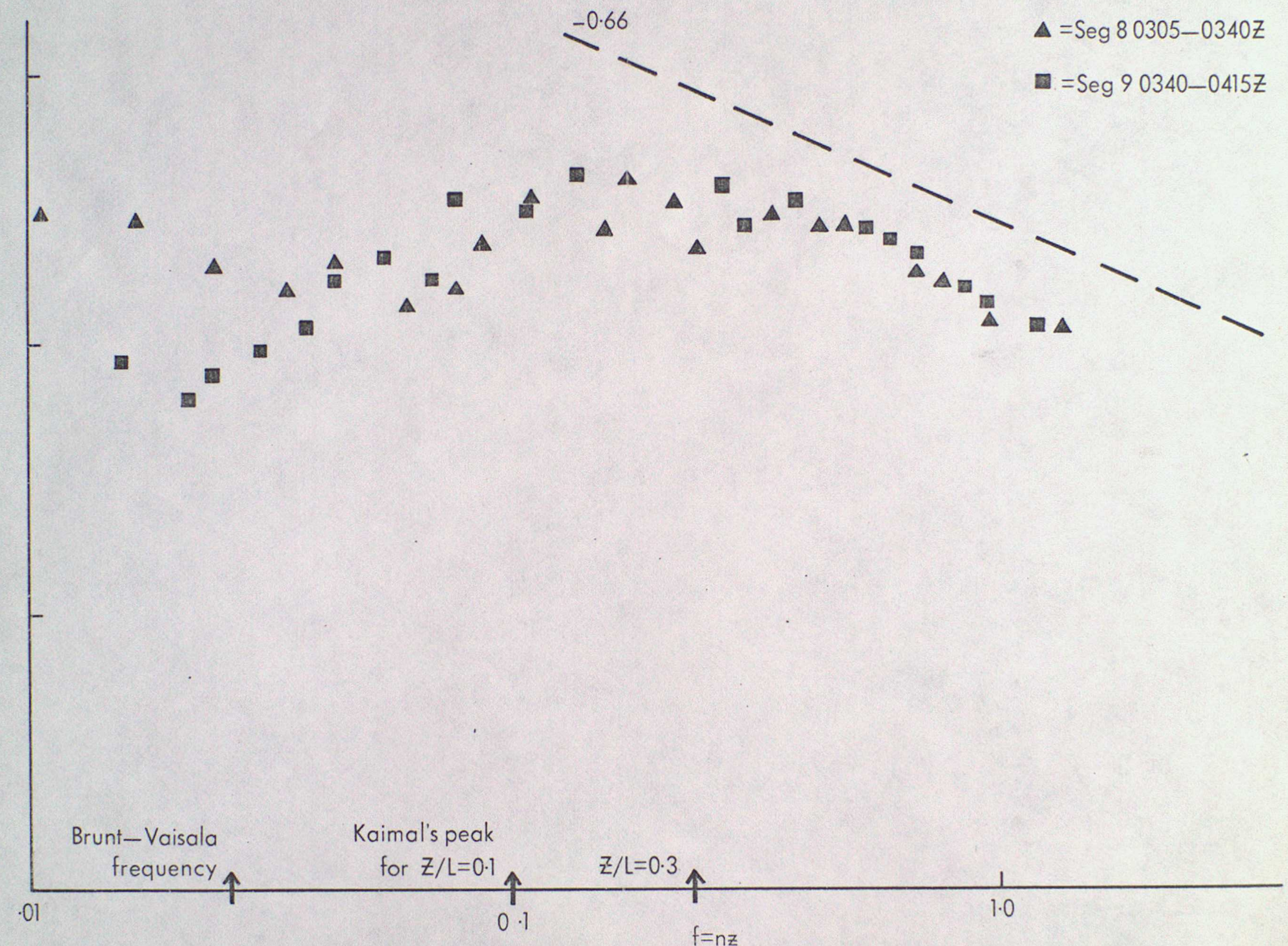

Weak turbulence
Sporadic turbulence
Semi—continuous turbulence
R_i
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
2200
2300
0000
0100
0200
0300
0400
0500
0600
Time (GMT)

Figure 2



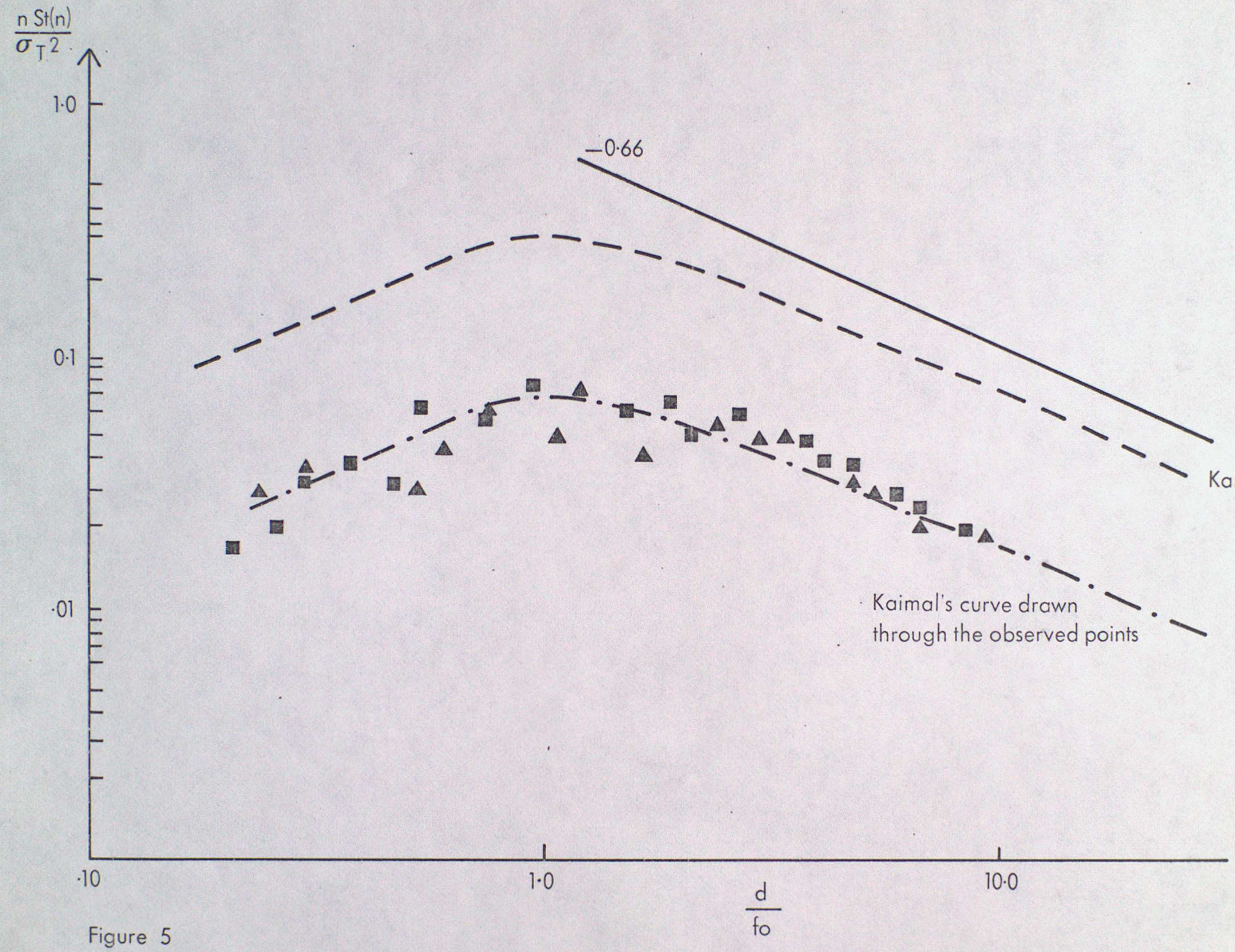
○ =Seg 4 0045—0120Z
△ =Seg 5 0120—0155Z
□ =Seg 6 0155—0230Z
∗ =Seg 7 0230—0305Z
→ Slope=—0·66
Brunt—Vaisala
frequency
·01
·10
1·0
f= nz/U

Figure 3



−0·66
▲ =Seg 8 0305—0340Z
■ =Seg 9 0340—0415Z
Brunt—Vaisala frequency
Kaimal's peak for Z/L=0·1
Z/L=0·3
·01
0 ·1
1·0
f=nz/U

Figure 4



n St(n) / σT²
1·0
0·1
·01
−0·66
Kaimal Et Al
Kaimal's curve drawn
through the observed points
·10
1·0
10·0
d / fo

Figure 5

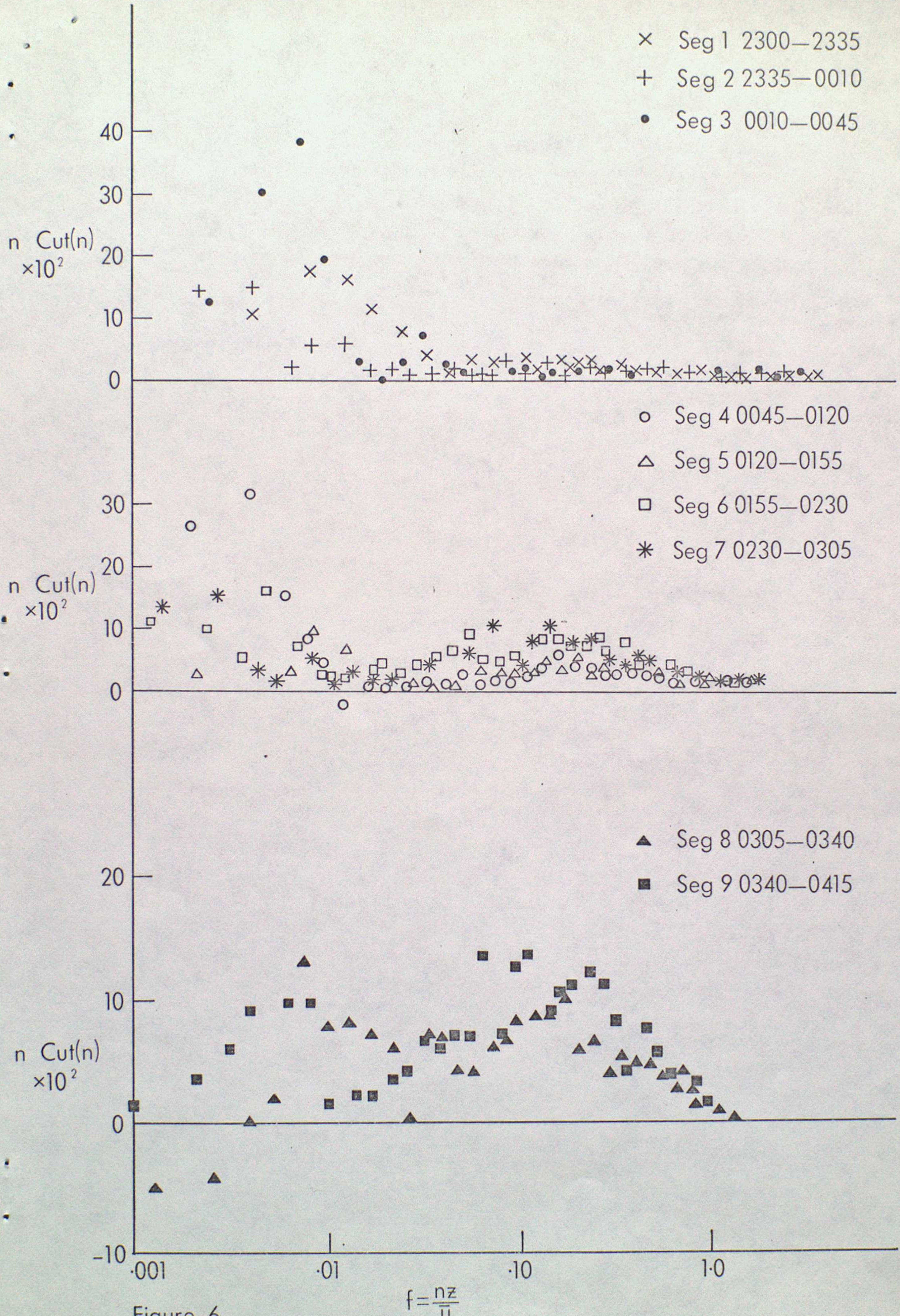

Seg 1 2300—2335
Seg 2 2335—0010
Seg 3 0010—0045
Seg 4 0045—0120
Seg 5 0120—0155
Seg 6 0155—0230
Seg 7 0230—0305
Seg 8 0305—0340
Seg 9 0340—0415
n Cut(n) ×10²
40
30
20
10
0
-10
·001
·01
·10
1·0
$f = \frac{nz}{\bar{u}}$

Figure 6