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PROBABILITY OF AIRCRAFT ENCOUNTERS WITH HAIL

by J Briggs

1. Introduction

Assessments of the chances that aircraft will meet hail are necessary for design purposes. It seems reasonable to base such assessments on meteorological parameters for which long term records are available and which can take account of the regional differences which undoubtedly exist. This note presents estimates of hail probabilities which are based on the incidence of thunderstorms at points on the ground though it is necessary in obtaining these estimates to make several assumptions, notably concerning the typical hail cell dimensions as well as variations in the probability of hail occurrence which are due to differences in height. Some comparisons with actual aircraft experience suggest that the estimates are reasonably realistic and can give a fair indication of likely experience of hail encounters supposing that avoiding action, based (say) on airborne or surface radar, is not taken.

2. Method

If the probability of hail at a point on the ground, or on a particular height level, is P_p and the corresponding probability of hail at any point inside unit area is P_a then, taking the average hail cell radius as R ($R \ll 1$)

$$P_a \simeq P_p / R^2$$

If now no avoiding action is taken the chance of an aircraft encounter with a hail cell of radius R somewhere during the crossing of an area of unit radius is given by $\frac{4P_a}{\pi} R$ i.e. $\frac{4P_p}{\pi R}$

But if the aircraft speed is V
then the time taken to cross the area is 2/V
and so the chance of a hail encounter in unit time is

$$\frac{2VP_p}{\pi R} \qquad (1)$$

Now suppose the number of thunderstorms per year at a point on the ground is N and that the average duration of hail at the point is t whilst the probability of occurrence of hail of diameter x inches or more during the storm is P_x

then for stones of diameter x or more $P_p = \frac{NP_x t}{8760}$ (2)

(t in hours). Crossley (1) suggests t as about 1/10.

So taking V as 500 knots (1) and (2) indicate that the number of encounters with hail of diameter x or more per flight hour, assuming no avoiding action, is given by

$$3.6 \times 10^{-3} \frac{NP_x}{R} \qquad (3)$$

In (3) N will vary with the locality whilst P_x will vary with the locality and height.

3. Variations with locality

Long period records provide incidence of thunderstorm days on a world wide basis so relative values of N are given almost directly by the number of thunderstorm days. On the other hand there are few data on which estimates of P_x can be based.

Topographical and other factors play important parts in the production of hail and there are some regions where hail occurs fairly frequently whereas others in the same general climatic regime only rarely have hail. Reports of hail are much less reliable than those for thunder though isopleths of hail incidence can be produced for most areas. Relative values of P_x for different places can be obtained by consideration of the relative values of the hail day/thunderstorm day ratio though this approach has to be used with care for there is no unique hail/thunder relation even for a given place (eg at Heathrow the peak incidence of hail is in April whereas that for thunder is in June). However, if any small hail is excluded and if comparisons are restricted to places with similar temperature regimes then the hail/thunder ratio can give a good estimate of the P_x variation.

Beckwith (2) used a network of 79 stations in the vicinity of Denver, Colorado over a period of 10 years. He had 829 reports of hail of which about 80 referred to hail of diameters one inch or larger. These figures suggest that $NP_1 = 0.1$. Another estimate applicable to central areas of the United States is due to Souter and Emerson (3). During flights through thunderstorms they found that 800 traverses were required for each occurrence of one-inch diameter hail or larger. So at the flight levels, around 12000 ft, $P_1 = 0.00125$. These two estimates agree if $N = 80$ and since this is in close accord with the reported storm incidence for the Denver area we may assume that in this area $N = 80$ and $P_1 = 0.00125$ for levels from the ground to about the middle troposphere.

In the tropics values of N are often large but hail incidence at the ground is usually low. Frisby and Sansom (4) indicate values of the hail/thunderstorm ratio for Central Africa and Malaysia which are usually less than 1/5 of the Denver values. This suggests that even for the worst places in these areas P_1 is about 0.00025 in the lower troposphere.

For NE India values of thunderstorm and hail incidence are both near the values for the central plains of the US. Indeed these two areas appear to have about the highest incidence of hail in the world. The Denver value of P_1, 0.00125, can be taken as typical of these two areas.

In the United Kingdom hail/thunder ratios are higher than for Denver but, as indicated above, small hail is a complicating factor. It seems unlikely that P_1 values for the United Kingdom will be higher than those for Denver, at least at usual flight levels and indeed it is considered that the Denver value of P_1 can be used for all temperate inland areas though the values of N will generally be considerably reduced.

For high latitudes and for temperate latitudes over the sea all the evidence indicates that though small hail is relatively frequent large hail is almost unknown. No reliable values for P_1 are available but it is suggested that United Kingdom values will be reduced by an order of magnitude or more.

4. Variations of P_x with hailstone size

There is little reliable information on the variation of P_x with x. Estimates have been made of the probable size distribution of hail using reasonable assumptions as to equivalent rainfall rates and of the distribution of the ice-mass amongst stones of different sizes. These estimates (RF Jones - unpublished) indicate broadly that the chance of a stone of given size decreases by an order of magnitude for each doubling of the stone diameter. Summers and Paul (5) reporting on Alberta hail studies in the priod 1957-66 show that for stones of diameter exceeding about $\frac{1}{2}$ inch the doubling of size is equivalent to a frequency decrease of nearly an order of magnitude. A formula which fits these data well is $P_x = P_o 10^{-x}$ (x in inches). This formula will be assumed here for each area considered. Thus in the lower troposphere and for the Denver/NE India areas it will be assumed that

$$P_1 = 0.00125$$

$$P_2 = 0.000125$$

$$P_3 = 0.0000125$$

5. Variations of P_x with height

The process of hail formation is not yet understood fully but is seems certain that large updraughts are needed and the stronger the updraughts the more probable are large stones. In a storm cloud the strong currents are likely to penetrate the bulk of the cloud, dying away only towards the cloud top. Thus, in a particular cloud which generates large hail, the chance of a stone of given size occurring at a particular level may be expected to remain fairly constant until towards the cloud top and then to diminish rapidly.

Radar studies of the precipitation content of storm clouds give support to this idea of the distribution of hail. For example, Donaldson (6) found that the median profiles of radar reflectivity for hailstorms over New England showed a concentration of precipitation at 20000/25000 ft with a rapid fall-off in precipitation above this region. Similarly, Marshall et al (7) found, averaging over summer storms around Montreal, that the hours in excess of a given intensity of precipitation decrease by about a factor of ten for each rise of 10000 ft through the upper levels of a storm.

Radar reflectivity is a measure of the numbers and sizes of both large rain-drops and hailstones so that reflectivity profiles do not readily indicate the way in which hailstone occurrence varies with height. However, as a first estimate it is reasonable to consider that the chance of a stone of given size is closely indicated by the average reflectivity profile.

Another factor which must affect the overall probability of hail at a given height is the chance of the cloud reaching that height. All clouds which generate large hail are likely to reach near to or beyond the tropopause but even so the frequency with which these cloud tops occur is likely to decrease sharply as the height concerned approaches and exceeds the tropopause. For example, studies of the height of the highest radar echo in the vicinity of Singapore showed the following percentage frequencies:-

Height (km)	% Frequency of highest echo	Height (km)	% Frequency of highest echo
≥ 9	85.1	≥ 16	16.6
≥ 10	83.0	≥ 17	9.7
≥ 11	77.7	≥ 18	4.0
≥ 12	68.2	≥ 19	1.4
≥ 13	53.2	≥ 20	0.5
≥ 14	40.2	≥ 21	0.2
≥ 15	27.6		

These figures for cloud-top frequency suggest that the probability of occurrence of a hailstone of given size is likely to decrease even more rapidly with height than is indicated by the radar reflectivity profiles. However, for planning purposes the radar profiles may be taken as safely indicating the variation of the hail probability (P_X) with height. Thus, P_X may be taken as constant up to a height in the region 20000-30000 ft but then it decreases by a factor of ten for each rise of 10000 ft. The depth of the near constant layer must be related to the average storm depth and so to the tropopause, the lower value (20000 ft) corresponding to temperate latitudes and the higher value (30000 ft) corresponding to tropical latitudes. On this basis the variation of the probability of one-inch hail, P_1, with height for the various areas discussed will be assumed as follows:-

Denver		United Kingdom		Singapore/Central Africa	
0-25000 ft	0.00125	0-20000 ft	0.00125	0-30000 ft	0.00025
35000 ft	0.000125	30000 ft	0.000125	40000 ft	0.000025
45000 ft	0.0000125	40000 ft	0.0000125	50000 ft	0.0000025

6. Comparison with aircraft experience

British Aircraft Company height and speed profiles for several aircraft can be combined with our assumed values for N, P_X and R to determine the probable hail experience of the aircraft and to compare with the actual hail experience. In these comparisons the radius of the typical hail cell (R) will be taken as 1 n mile since the average hail-cell diameter is 1-3 n miles, Crossley (1).

a. Britannia aircraft

BAC height and speed profiles indicate the following breakdown of each 30000 flight hours:-

2000 hours	at	0-10000 ft	speeds about 300 kt
14000 "	"	10-20000 ft	" " " "
14000 "	"	20-30000 ft	" " 350 kt

with Denver/NE India values of N,P_X equation (3) then gives the following probabilities for one-inch hail encounters in 30000 hours

0-10000 ft	0.4
10-20000 "	3.0
20-30000 "	2.9
Total	6.3

Thus every 30000 hours flight over Denver/NE India areas should give 6 encounters with one-inch hail. Similarly there should be 0.6 encounters with two-inch hail or about 18 encounters with two inch hail in 800000 hrs.

Similarly United Kingdom values of N and P indicate 1.9 encounters with two inch hail in 800000 hrs whilst Central Africa/Singapore values indicate about 5 encounters in the same time.

Britannia experience over all routes is 2 encounters with two inch hail in 800000 hours.

b. Viscount aircraft

BAC profiles indicate all flight between 0 and 20000 ft at speeds averaging about 233 kt.

Denver/NE India values then indicate			9	encounters with three inch hail in 5.5×10^6 hours					
United Kingdom	"	"	$1\frac{3}{4}$	"	"	"	"	"	"
Singapore/Central Africa		"	2	"	"	"	"	"	"
whereas actual Viscount experience is			1	"	"	"	"	"	"

c. Caravelle aircraft

This distribution of each 30000 flight hours has been indicated as follows.

2000 hrs	0-10000 ft	speeds about 300 kt
2000 hrs	10-20000 ft	speeds about 350 kt
2000 hrs	20-20000 ft	speeds about 425 kt
24000 hrs	30-40000 ft	speeds about 450 kt

Denver/NE India values then give			66	encounters with one inch hail in 900000 hrs							
United Kingdom	"	"	7	"	"	"	"	"	"	"	"
Singapore/Central Africa	"	"	30	"	"	"	"	"	"	"	"
whereas actual Caravelle experience is			2"		"	"	"	"	"	"	"

The hail risk is extremely variable from region to region so that the above comparisons are of very limited value without more knowledge of the route histories of the aircraft concerned - for example the hail risk must be extremely low over most sea areas and over areas such as N Africa and the Middle East. Nevertheless the comparisons indicate that probabilities based on United Kingdom values are reasonably close to overall experience whilst probabilities for the worst hail areas are likely to be one or two orders of magnitude avove overall experience.

7. Estimates for the SST

Concorde height and speed profiles have been used together with the appropriate values of N and P_x to estimate the hailstone diameter which corresponds to one encounter in 10^4, $^x10^5$ and 10^6 hours of flight at a given level. Figure 1 presents these estimates and shows how the size of stone varies with height for the areas of the United Kingdom, Singapore/Central Africa and Denver at a, b and c respectively.

8. Summary

The estimates presented above depend on several assumptions and clearly cannot do more than indicate probabilities of hail within about an order of magnitude. Nevertheless comparisons made with actual aircraft experience are encouraging. The comparisons are least satisfactory for the aircraft, the Caravelle, flying at the highest levels and so the extrapolation to the levels of most concern to the SST may be less realistic though at these levels the estimates seem more likely to be pessimistic than optimistic as regards the probabilities of large hail being met.

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References

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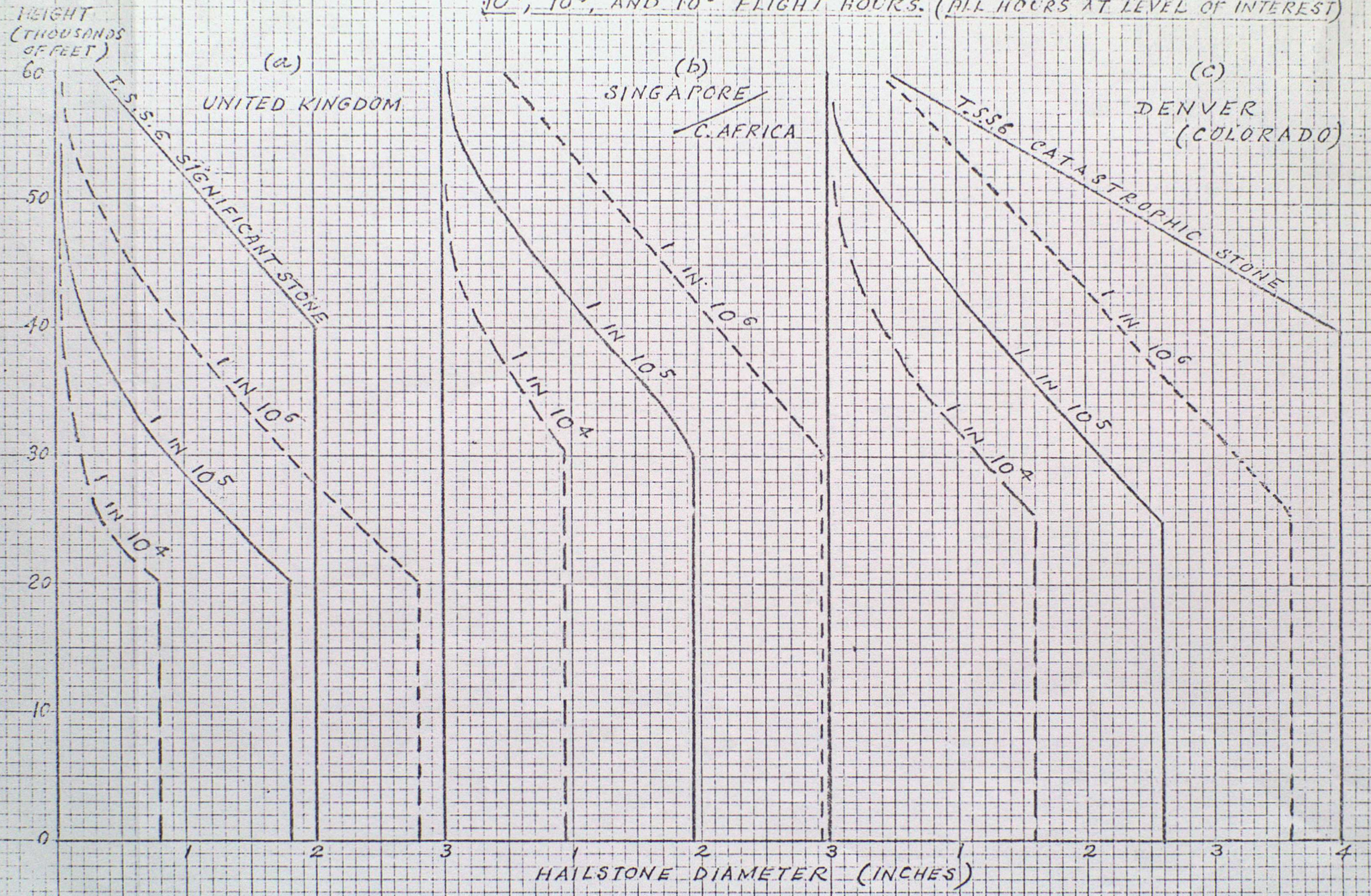
FIGURE 1 HAILSTONE DIAMETER FOR ONE ENCOUNTER IN 10^4, 10^5, AND 10^6 FLIGHT HOURS. (ALL HOURS AT LEVEL OF INTEREST)



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