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THE TRAVEL OF CIRCULAR DEPRESSIONS AND TORNADOES

AND THE

RELATION OF PRESSURE TO WIND FOR CIRCULAR ISOBARS.

BY

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PREFACE.

THIS memoir is the result of an endeavour to ascertain the part which "revolving fluid" plays in the phenomena which are represented or implied in weather-maps. By revolving fluid in this connection is meant a column or disc of air which spins about a vertical axis, and at the same time travels with a velocity of translation which is common to every part of the column or disc, and which, therefore, does not alter the relative motion of the air about its axis. The idea that we have in mind may be illustrated by supposing a disc rotating about its axis in a travelling railway carriage, or a rotating platform carried along a street on a cart; but when it is air that is under consideration we have to remember, first, that rotation, "like a solid" with the same angular velocity for successive rings, is only a special case: there may be cases in which the angular velocity of rotation of successive rings is not equal; and, secondly, that in consequence of the effect of the rotation of the earth the motion of the column or disc as a whole introduces an alteration in the distribution of pressure with regard to the axis which is comparable with the distribution of pressure associated with the rotation.

In considering the dynamics of moving air we may proceed in one or other of two ways: we may either take for granted that the facts and particulars of the motion are defined and endeavour to assign the forces which are a necessary part of the defined state of motion, or take for granted that the initial state of motion and the distribution of forces are defined, and endeavour to calculate the subsequent states of motion.

The second is the commoner form of dynamical exercise, but it is the former alternative which is adopted in this memoir. By the study of weather-maps we are led to idealise a cyclonic depression of special type as indicated by a series of circular isobars which travels for a day or several days without change along a path which approximates to a great circle, and we ask ourselves the question, What are the conditions necessary for that sequence of events? We know that the ideal is not frequently, perhaps never, perfectly realised. In actual cases the isobars are not a set of perfect concentric circles, and they change in various ways as they move; but if we can form a definite idea of the conditions necessary for the ideal case, we shall have taken an important step towards the comprehension of the real case.

Or again, we take the ideal case of a disc of fluid rotating about its axis, like a solid, or according to some other defined law, and suppose that all its parts are affected by a motion of translation, the same for all, and that in consequence the revolving mass moves bodily, while preserving its own relative motion of rotation: we ask ourselves the question, What are the conditions necessary for the persistence of this state of motion? Again, we know that the ideal state of things is not frequently, perhaps never, perfectly realised, but if we can define the conditions for the ideal case we shall

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put ourselves in a favourable position for examining the actual state of things which a weather-map represents.

It may be remarked that this method of approaching difficult questions of dynamics has for a precedent the procedure of Newton's solution of the problem of the motion of the heavenly bodies. The discovery of the laws of gravitation was preceded by a definition of an ideal of planetary motion according to Kepler's laws, determined by laborious observation, but in the end only an approximation, though a very close one, to the real motion : the approximation in our case is much more crude. We cannot hope that the winds of the atmosphere can be effectively represented by so simple a system as the motions of the planets, yet the method is not inapplicable, and it helps us to form useful ideas of the actual sequence of events in the atmosphere.

We start from the idea of a disc of air at a sufficient distance above the ground for the effect of surface friction to be reduced to negligible dimensions, with circulating winds, which have a velocity at each point proportional to the distance of the point from the axis. *So long as the axis is stationary* the circulation can be and must be balanced by a distribution of isobars which will form a series of curves differing only from circles concentric with those of the winds on account of the variation of a factor depending on the latitude which comes into computation. In this memoir we neglect the variation and therefore deal with phenomena belonging to a flat earth revolving about a vertical axis instead of a spherical or a spheroidal earth revolving about its polar axis.

So long as the axis round which the winds circulate remains stationary, the pressure system which maintains the circulation is easily calculated by the ordinary gradient equation ; but it is the purpose of this memoir to consider two cases in which the axis moves forward with a velocity of travel easily measurable by noting the position of the axis on successive weather-maps.

We deal with two cases, the travelling cyclone and the travelling "revolving fluid," which appear at first to be distinct, but are proved subsequently to be simply two separate aspects of the general case of the travel of revolving fluid. In the consideration we use certain propositions regarding the combination of rotation and linear translation and the combination of a system of circular isobars with a system of straight isobars. These propositions are quite elementary and are probably well known as class-exercises for mathematicians, but so far as I know their application to the problem of the travel of cyclones has not yet been brought to the notice of meteorologists.

The propositions are as follows :—

1. The motion of air in instantaneous rotation "like a solid" about a travelling centre, which is regarded as a normal type of travelling cyclone, is equivalent to the permanent relative motion of the same intensity round another centre, combined with a translation of the air as a whole with the velocity of motion of the travelling centre.

2. The same motion is produced by subjecting the disc of air which is in instantaneous rotation to a uniform acceleration transverse to the disc, the law being that a similar disc of rotating air will be formed at successive points along the line at right angles to the direction of the acceleration from right to left of a person looking along the line along which the acceleration would move the air, if it were at rest.

3. A system of circular isobars centred at any point can be resolved into another system of exactly similar circular isobars centred at any other point, and a system of linear isobars at right angles to the line joining the two points.

It is upon these three propositions that the argument of the memoir depends, and it leads up to the conclusion that the isobars appropriate to a travelling cyclone can be resolved into a similar system of isobars round the centre of the cyclone with a transverse field of linear isobars that is not represented by any equivalent of velocity, or alternatively into a similar system of isobars round the centre of relative motion or centre of " revolving fluid," which exists for every travelling circular cyclone of the prescribed type, together with a transverse linear field of pressure which is the geostrophic equivalent of the velocity of translation of the cyclone.

But the subject was not approached in this systematic manner. It began with the consideration of the conditions of translation of a circular cyclonic depression without change, and it was only subsequently discovered that the consideration of that problem came in as part of the general problem of the travel of " revolving fluid." The subject is set out in the memoir in the order in which it was developed, because it is thought that meteorologists will find that method of dealing with it more acceptable than a more formal theoretical discussion, as bringing more directly under review the various aspects of the subject that are presented to the student of weather-maps. On that ground the author may be excused if the scaffolding is sometimes more in evidence than the finished structure.

The conclusions have an important bearing upon the calculation of the velocity of the wind in the upper air from the gradient of pressure at the surface when the isobars are curved. For many years now we have regarded the gradient wind as probably, at least, in accord with the actual wind when the isobars are approximately coincident with great circles, but when the isobars are curved the computation has been indefinite, and we now see how the computation should be modified in the special case of travelling circular cyclones.

NAPIER SHAW.

METEOROLOGICAL OFFICE, LONDON,
December 6, 1917.

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§ 1. THE PROBLEM OF REVOLVING FLUID IN THE ATMOSPHERE.

SINCE the cyclone and anticyclone were identified as the most characteristic features of weather-maps, many papers in meteorological journals have been devoted to them. Some have dealt with the statistical associations of such elements as the surface-temperatures, or the surface-winds, or the rainfall, with concentric circles described round the centre of the isobars which represent a cyclone or anticyclone at the surface; and others have dealt in a somewhat similar manner with the association of the temperature in the upper air, or of the motion of clouds, with the isobaric centre on the earth, while some have given dynamical theories of the formation of cyclones. So far at any rate as the experience of the Meteorological Office is concerned, the theoretical and statistical results have found little practical application in dealing with the realities of weather, and it has often been a matter of surprise that so much labour and ingenuity of speculation should have so little practical result. It has been customary to excuse this condition of things by the reflection that meteorology was a science of slow growth and that if the patient accumulation of observations went on and developed, there would in time be observations enough to put the facts about cyclones and anticyclones on a footing that would enable the student of weather-maps to get a true insight into the dynamics and physics of their behaviour.

But besides lack of pertinent observations there are certainly two other obstacles which have stood in the way of practical knowledge of the subject: one is the tendency to seek an explanation of the cause and origin of the cyclone instead of its persistence and travel, the features which we really notice on maps; and the other is the want of appreciation of the difference of scale between the cyclone of the atmosphere and the corresponding phenomena of fluid moving in circles in a laboratory.

From the first has come the obsession that not only in its origin, but also at every subsequent stage of its career, a cyclone must consist of ascending air with some sort of focus at the centre that might cause it, while an anticyclone must still be the focus of descending air, however many days may have elapsed since the necessary dynamical operations first set up the circulation and high pressure. And from the other has come the unfortunate habit of regarding the travel of a cyclone as something which could be treated independently of the cyclone itself, because in laboratory experiments on revolving fluid the spin is many thousand times greater than what occurs in the atmosphere. If a section of the core of a most destructive tornado were brought into the laboratory it might present the appearance of a column that took six minutes to make a revolution, whereas what the experimenter looks at gets round its axis in a second or thereabout. The motion round the centre of an ordinary cyclone would take eight hours to accomplish, and what is unrecognisable in a second may become of importance in six minutes and still more so in eight hours. M. C. L. Weyher overlooks this when he writes in the preface to his book *Sur les Tourbillons* : " Ainsi un cyclone est de l'air tournant en rond sur un très grand rayon autour d'un axe plus ou moins vertical et se déplaçant dans le sens horizontal.

" Mais que le diamètre du tourbillon soit grand ou petit, que la giration se fasse dans un sens dans un hémisphère, et en sens inverse dans l'autre ; que la trombe se déplace horizontalement ou non : au point de vue de l'étude des effets, ces circonstances n'ont aucune importance, du moment que le mouvement de rotation autour d'un axe subsiste malgré le déplacement horizontal du tourbillon.

" Étudions donc un tourbillon pendant qu'il est stationnaire ; les effets qui en résultent seront certainement les mêmes pendant le déplacement ; à ce détail près, qu'ils auront lieu à chaque instant sur un point nouveau du sol ou de la mer."

The same kind of omission is made by a professor of physics writing recently in a " Flying " journal, and drawing a barbed circle crossed by barbed straight lines pointing eastward as " an eddy in the stream of air flowing from the Atlantic over Western Europe," and adding that " it is these eddies which form in the stream of air flowing over the country which are responsible for the variable winds experienced in these latitudes." Later on he says : " It must be realised that meteorology never can be regarded as an exact science, and that on any particular occasion there may be conditions of temperature or some local influence which will interfere with the general nature of the disturbance." And certainly meteorology never can be an exact science so long as its exponents are content to make a vague representation of the general nature of a cyclone and then blame nature for not working to the pattern.

Equally vague was a distinguished professor of physics who, with the gravest assurance, once asked me, when I professed surprise that some people regarded a travelling cyclone as a horizontal cart-wheel that rolled along, " Well, isn't it ? I always teach my class that it is and base upon that hypothesis the two lectures a year in which I dispose of meteorology."

Quite recently Lord Rayleigh, moved thereto by Dr John Aitken's papers on the dynamics of cyclones and anticyclones, has contributed a paper to the Royal Society setting out the conclusions that can be drawn from the theory of revolving fluid because " so much of meteorology depends ultimately upon it." It behoves us therefore to consider what part " revolving fluid " does play in meteorology. Most scientific people are content with supposing that cyclones and anticyclones are somehow or other examples of revolving fluid and that the conclusions from the theory must apply to them, without troubling to consider what really is meant by revolving fluid and how precisely the particular portions of the atmosphere revolve.

Lord Rayleigh gives the necessary precision to the conditions in which the results which he deduces can be applied. The motion must be symmetrical about the axis ; a ring of particles with its centre at the axis keeps its shape : it may shrink in size if fluid is withdrawn along the core, but if there is no change of that kind the ring rotates like a wheel. With a fluid like air we need not assume that the consecutive rings which form a disc all revolve together like a solid. That would be a special case. In others, rings near the core may revolve faster or slower than those further out, sliding one within the others, but the particles which form one of the spinning rings remain in a ring. If the set of rings travels all must travel together and keep their relative positions. Every particle has a velocity of translation, the same for all, superposed upon its motion of rotation, so that the actual motion of each particle is a composite result of combining the velocity of translation and the relative velocity. The relative velocity is the velocity of rotation about the centre, and the whole is carried along as if by a stream, which carries the disc along with it. Many people think that these words would really describe a travelling cyclone, forgetting that any motion of the air is wind, and if a revolving disc of air moves along bodily all its winds are affected.

The first question to be decided is where, on a weather-map, we should look for a specimen of "revolving fluid." The well-developed circular travelling cyclone will not satisfy the condition, because, as M. Angot points out in his elementary treatise on meteorology, when allowance is made for the motion of translation common to all parts of the cyclone, the "relative motion" is not represented by circles round the centre of isobars. In a paper before the Royal Society on June 21, 1917, on "Revolving Fluid in the Atmosphere," I have explained that a disc of travelling revolving fluid would be represented on a map, not by a series of circular isobars like a travelling cyclone, but by a distortion of the isobars of a large cyclone such as we are accustomed to call a small secondary. We have therefore been in the habit of looking for centres of revolving fluid in the wrong part of the map. We ought not to expect to find them at the centres of cyclones, but in those regions of the cyclone where small secondaries appear. It is pointed out that in order that a column of revolving fluid may be permanent, the winds which carry it along must be in proper adjustment, having no variation with height, otherwise the head of the revolving column would be torn away from the base in a very short time. The condition for uniformity of velocity with height, according to Mr W. H. Dines's formula (*Nature*, vol. 99, p. 24, 1917), is that the isobaric surfaces should be isothermal too, and whatever may be the general physical interpretation of this condition, it is most likely to be satisfied in that part of a cyclone where the churning of the air by convection has been most active, where the lapse of temperature with height approaches most nearly to the adiabatic limit. These conditions are most likely to be satisfied in the southern section of a cyclone advancing from the West, where small secondaries are generally found. Further, it seems probable that the column must extend from the ground to the boundary of the stratosphere, where alone, according to one of my own formulæ in the *Illusions of the Upper Air*,[1] a cap for the rotating column can be found. Thus the full development of revolving fluid requires the satisfaction of a number of conditions, and is not likely to obtrude itself frequently in the weather-map.

Small secondaries sometimes develop winds of extreme violence with which the name of tornado has been associated, and which are doubtless cases of revolving fluid for which the necessary conditions have been satisfied. Two examples of revolving fluid of this character were suggested and represented by the weather-maps for March 24, 1895, when a very destructive small secondary, a very large column of revolving fluid, swept across the British Isles from the South-west of Ireland to Norfolk, and for October 27, 1913, when a most violent tornado of very narrow

[1] *Nature*, vol. 97, pp. 191 and 210, 1916.

dimensions did great damage in South Wales. A paper by Professor R. de Courcy Ward in the *Journal of the Royal Meteorological Society* at the same time pointed out that the tornadoes of the United States are formed in corresponding positions in travelling cyclones, and we may accordingly call the centre of "revolving fluid" a tornado-centre to distinguish it from the centre of a travelling cyclone. The difference between the motion in the two cases is of fundamental importance in meteorology. In the case of the tornado we have a disc of fluid revolving in rings round its axis, while by some means the whole is simultaneously transported bodily along, with the velocity of translation of the tornado; and in the case of the travelling cyclone we have a distribution of winds, which is at the moment arranged symmetrically in rings, but which, for a reason which will be explained later, is not free to go on moving in the circles which it marks out but develops a like distribution about a new centre in advance of the old. Of course, if the centre of the cyclone is stationary, the motion persists in the circles, and we have a case of fluid revolving about a fixed axis.

In the paper before the Royal Society, following the precedent set in the *Life-History of Surface Air-Currents*,[1] the distribution of velocity in the cyclone was taken to be uniform. In what follows the case of a system in which the velocity is proportional to the distance from the centre is considered. In that case a stationary cyclone or "revolving fluid" rotates like a solid, or, to use a more homely analogy, like a cart-wheel. Later on we shall call a cyclone which satisfies the condition of proportionality of velocity to radius a normal cyclone.

§ 2. Difference between "Revolving Fluid" and Cyclonic Motion.

In the discussion of the paper on "Revolving Fluid in the Atmosphere" before the Royal Society on June 21, 1917, the President raised the question of "circulation,"[2] in the mathematical sense, in relation to revolving fluid and cyclones. As explained in the introductory sentences of the paper, revolving fluid was understood to be restricted to the cases in which the motion is symmetrical with regard to an axis. The motion of air in a travelling cyclone, which for brevity we may call cyclonic motion, has not the necessary quality of symmetry, because it is the motion *relative to its axis* which must be symmetrical if the inferences to be drawn from the theory of revolving fluid are to be applied to atmospheric processes, not merely the instantaneous motion with reference to a moving centre. The mathematical definition is therefore not sufficiently explicit for immediate use in the study of weather-maps. For example, in the case of a disc of air in instantaneous rotation, with uniform vorticity equal to v/r, about an axis which is travelling, the relative motion will certainly be "symmetrical" about a point and the fluid will therefore technically be "revolving fluid," but the point of symmetry is not the apparent centre but a centre at a distance therefrom proportional to the rate of travel; it may even be beyond the boundary of the moving disc. The disc in instantaneous rotation may, in fact, be regarded as a circular fragment of a larger travelling disc of "revolving fluid," and in order to make the results of the theory applicable in practice the effect of leaving out the remainder of the revolving disc has to be considered.

[1] M.O. Publication, No. 174.

[2] Circulation round a closed curve means the integral of the tangential component of the velocity taken round the closed curve. In the case of a normal cyclone the circulation for a ring radius r and velocity v round the centre is $2\pi rv$.

Thus in fig. 1 an instantaneous velocity at P proportional to PO and at right angles thereto, or rotation about O, is equivalent to a velocity proportional to PO′ and at right angles thereto—that is, rotation about O′, together with a velocity proportional to OO′ and at right angles thereto. And this will be persistently true if O and O′ move parallel with the velocity of translation. P continuing to rotate round O′ and translated with the same velocity as O, will always be instantaneously moving at right angles to the instantaneous radius OP.

The motion relative to the centre in the case of a travelling cyclone depends upon the distribution of velocity in the cyclone. I have been unable to find, in the authorities on vortex motion within my reach, any specification of the distribution of velocity in a vortex ring or other actual example of vortex motion on a small scale that could be adopted as a guide. The observations of surface-winds which are given in weather-maps are too much affected by local causes to make their numerical difference a satisfactory basis of calculation. The region near the centre of a storm was generally regarded by seamen as the place where the winds are strongest, and Dr Aitken had called attention to the influence of the conservation of moment of momentum in magnifying the velocity near the centre in the case of convergence of the air due to convection in the central region. The isobars for a travelling circular storm, perhaps for want of knowledge of detail, are drawn at approximately equal distances apart. Accordingly, in the *Life-History of Surface Air-Currents* we took a uniform distribution of velocity as representing a mean result, and the assumption was reasonably well borne out by the slightness of the variations of velocity shown in the actual trajectories of air in travelling storms. With a uniform distribution of velocity the motion of the air relative to the axis of a cyclone is planetary motion about the kinematic centre as focus, the excentricity being the ratio of the velocity of translation to the velocity in rotation.[1]

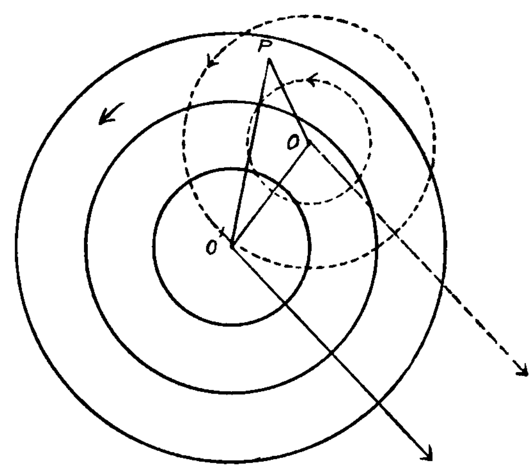


FIG. 1.—Relation of cyclone centre O to tornado centre O′.

§ 3. A CYCLONE OF UNIFORM VORTICITY.

In his paper on the dynamics of revolving fluid Lord Rayleigh[2] deals with an example in which the ratio of the velocity to the vorticity is of the form $r + A/r$, and the motion represents the combination of motion as a solid body with that of a simple vortex increasing in intensity towards the centre. I have not yet compared this distribution with the winds of a travelling cyclone, but further inquiry into the subject has led me to examine the case of a cyclone of uniform vorticity in which the velocity is proportional to the radius from the centre, partly because Dr Chree suggested that distribution as the most suitable for a general description of the phenomena of vortex-motion, and partly because the assumption simplified the calculations which

[1] G. T. Bennett, *Life-History of Surface Air-Currents*, part iv. p. 97, 1906.
[2] *Proc. Roy. Soc.*, A, vol. 93, p. 153, 1917.

are given later on in this paper, with results that seem to be of practical value in meteorology.

The motion of air with velocity proportional to distance from the axis implies that the air is rotating like a rigid solid, the necessary stresses being provided by the distribution of pressure; but in the case of the moving cyclone the analogy of the motion about the centre of the cyclone with that of a rigid solid is only instantaneous. In that case, however, since the vorticity is uniform, the relative motion is also circular motion, though it is about a centre which is not the kinematic centre of the actual movement, but is distant from that point by an amount depending upon the velocity of translation. Thus, if the law of distribution of instantaneous velocity is $v = \zeta r$ and the cyclone is travelling with velocity V, the relative motion obtained by the vectorial subtraction of V from the motion of all the elements of the cyclone is in concentric circles with the same law $v = \zeta r$, but with their centre at a point on the axis of y at a distance $-V/\zeta$ from the centre of co-ordinates taken at the instantaneous centre, the axis of x being taken along the direction of motion of the centre.

The case is an interesting one, because later on it will be proved that when the earth's rotation is taken into account, the distribution of pressure in a travelling cyclone which has the given distribution of velocity may be regarded as a series of concentric circles with the common centre also displaced from the centre of instantaneous rotation, so that a travelling cyclone of the normal type may be regarded as having "symmetry" with regard to three different centres: first, the centre of instantaneous motion, which I have already referred to as the *kinematic centre*, with regard to which the *instantaneous motion* is symmetrical; secondly, the centre of relative motion or *tornado-centre*, at a distance V/ζ from the first: with regard to this centre the *relative motion* is symmetrical; and thirdly, the centre of isobars or *dynamic centre*, with regard to which the *distribution of pressure* is symmetrical: the position of this centre, depending upon the earth's rotation, is at a distance $V/(2\omega \sin \phi + \zeta)$ from the kinematic centre. In a particular case described later the dynamic centre works out to be nearly halfway between the kinematic centre and the revolving-fluid-centre.

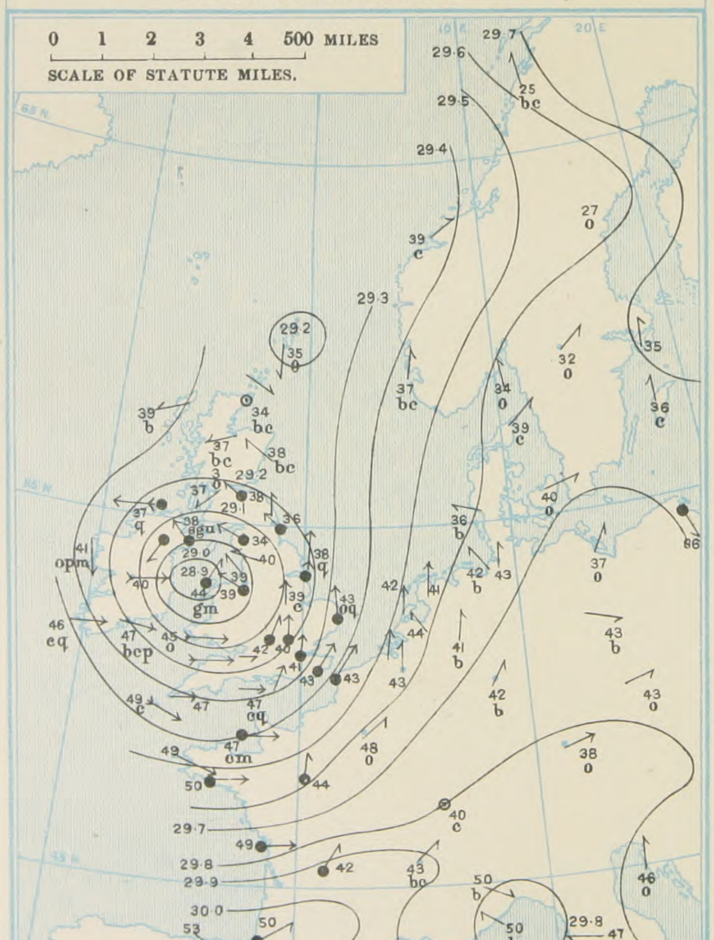
But for the fact that the rotation of the earth itself brings in a pressure-gradient as compensation for velocity of translation, the dynamical centre would always coincide with the tornado-centre or centre of relative motion, and the (ordinary) properties of revolving fluid would hold good with regard to that point; but the dynamical complication necessary to make a disc of atmosphere revolve about a travelling axis instead of a fixed axis makes the dynamical inferences from the theory of revolving fluid not immediately applicable to the central region of a travelling cyclone.

In order to apply these considerations to actual weather-maps it is necessary to specify the boundary of the cyclone and the ratio of velocity to radius within it. We may have a cyclone of small dimensions with a high ratio of velocity to radius, in which case the map will show gales and strong winds close to the kinematic centre; or a cyclone with smaller velocity ratio and covering greater area, so that gales are found in the marginal regions, while the central regions are occupied by relatively light winds. Colourable examples of both these cases might be adduced from actual maps, and the recognition of the dissociation of the kinematic centre from the centre of isobars makes the regularity more apparent, but the phenomena of the more central region which may be affected by convection are still a subject rather of conjecture than of observation.

Plate II. Fig. 1.

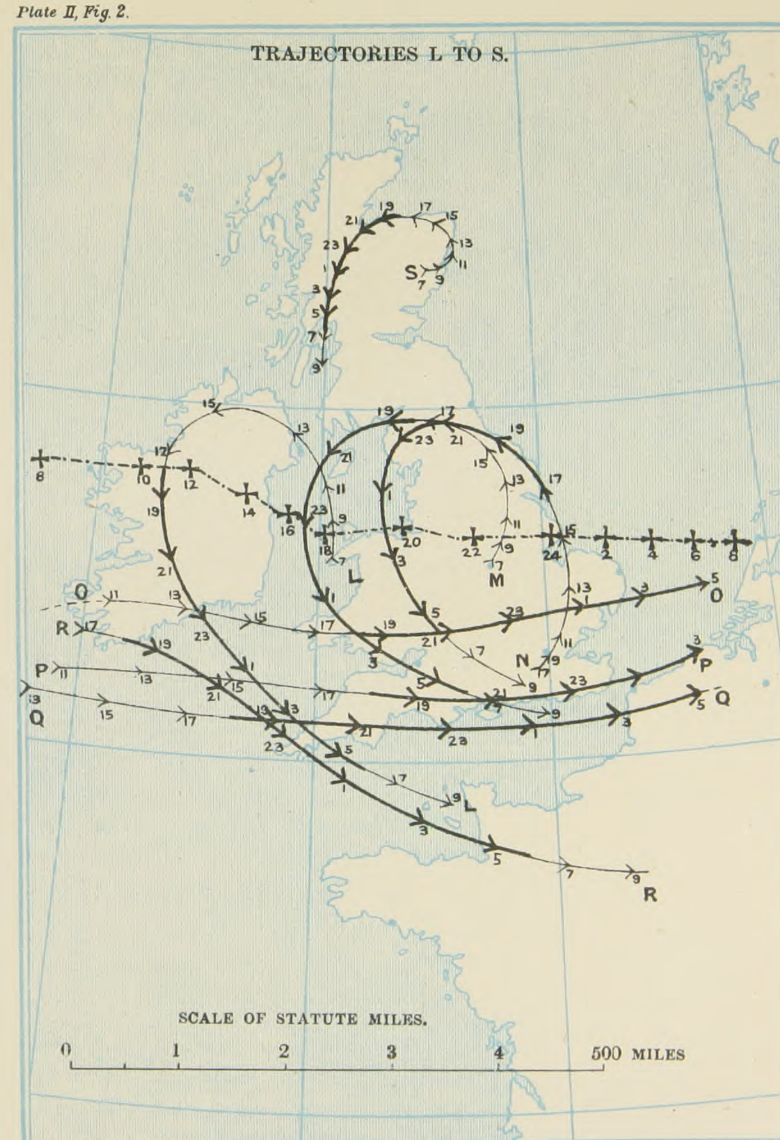
CIRCULAR STORM OF MARCH 24-25, 1902.

6 P.M. [18 H.] MARCH 24.



ISOBARS, WINDS, TEMPERATURE AND WEATHER.

Plate II, Fig. 2.



TRAJECTORIES OF AIR.

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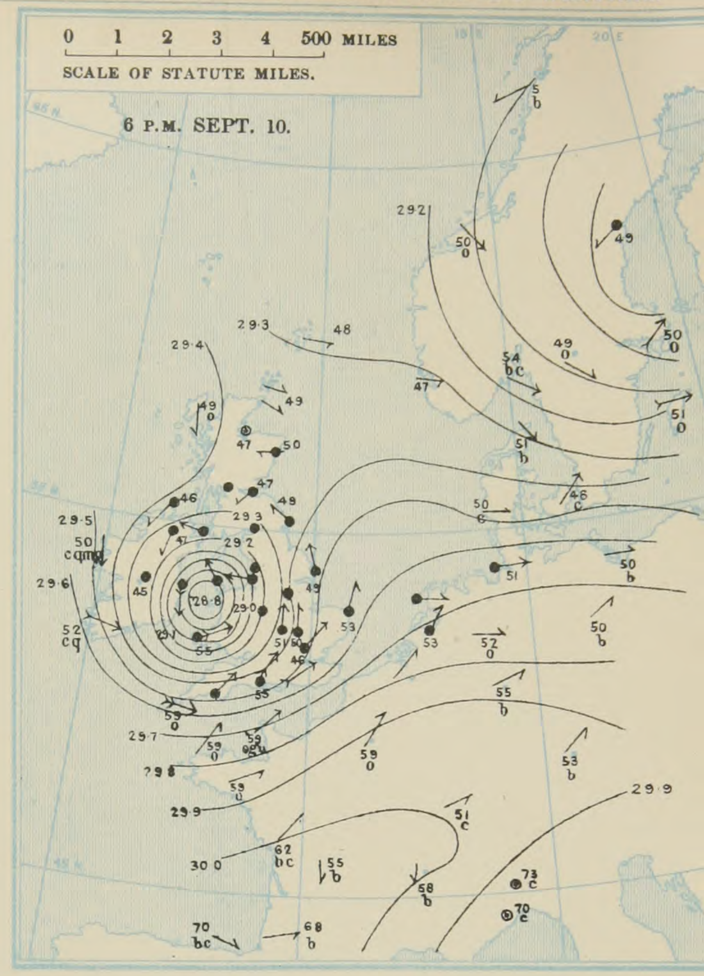
Ps. 347. 4591. S.O.P. 470. 1000. 2/18.

CIRCULAR STORM OF SEPTEMBER 10-11, 1903.



§ 4. The Analogy of Wave-Motion and Cyclonic Motion.

It may here be remarked that the trajectories or actual paths of air in the case of a travelling cyclone are different according to the law of distribution of velocity within the boundary of the cyclone. When the distribution of velocity in the cyclone is uniform the trajectories are what Professor Hudson called anemoids, and are referred to in the *Life-History of Surface Air-Currents.* But when the cyclone is made up of fluid with uniform vorticity (equal to the ratio of the velocity to the radius), the trajectories are the trochoids formed by the rolling of the disc of relative motion along the line of motion of the instantaneous centre, the path of the cyclone. The conditions are set out in Plate I. (frontispiece). Thus the relation between the instantaneous motion and the pressure, for the normal cyclone is the reciprocal of the relation between the motion and the pressure in the trochoidal theory of waves of water. In the case of waves the paths are circles and the pressure-lines are trochoidal curves, whereas in the case of a travelling cyclone of the type mentioned, the pressure lines are circular and the actual paths are trochoids.

Examples of trajectories drawn for actual specimens of well-developed travelling cyclones are given in the *Life-History of Surface Air-Currents*, which are reproduced in Plates II., III., here, and in Plate IV., facing p. 41. They are suggestive enough of the shapes of trochoids to make the association of the two plausible; but it must be remembered that the travel of a cyclone with any distribution of velocity must give trajectories similar in general shape to trochoids, and the actual curves drawn are obtained mainly from surface measurements of wind which would not give exact representation of the curves required. The trajectories drawn in the plates give little help in deciding what was the law of distribution of velocity near the centres of the cyclones investigated.

§ 5. The Dynamical Conditions for Steady Rotation about a Fixed Centre and a Travelling Centre.

Whatever be the law of distribution of velocity, a series of concentric rings of fluid in cyclonic motion certainly simulates instantaneously the properties of "circulation" of fluid revolving about a *fixed* axis, because the line integrals, which take account only of instantaneous velocity, are indistinguishable from those of a fluid in persistent revolution about the position of the instantaneous axis of the cyclone. The difference between the two types of motion, namely, rotation about a fixed axis and about a travelling axis respectively, becomes apparent when the conditions of the subsequent motion are considered.

The rotation of the rings of fluid will persist and the fluid will be "revolving fluid" with a fixed axis if the pressure-gradient for each ring is appropriate to the latitude and to the radius of the ring, but not otherwise. With the usual notation for the computation of gradient winds, the relation for persistent motion in this case is

$$\gamma_r = 2\omega v\rho \sin \phi + \rho \frac{v^2}{E} \cot r \quad . \quad . \quad . \quad . \qquad (1)$$

Considering an area so small that the surface may be regarded as plane, we may write the equation

$$\gamma_r = 2\omega v\rho \sin \phi + \rho v^2/r \quad . \qquad (2)$$

where r is the radius vector from the axis to the point at which the velocity is v, and in this case is also the radius of the circular isobar.

If instead of persistence as rotating fluid we have the case of a cyclone of the same instantaneous winds travelling with velocity V, we have for steady conditions

$$\gamma_c = 2\omega v\rho \sin\phi + \rho\frac{v^2}{E}\cot X \quad . \quad . \quad . \quad . \quad (3)$$

where X is the angular radius of the small circle osculating the path of the air. Assuming in like manner the surface to be plane, we may write

$$\gamma_c = 2\omega v\rho \sin\phi + \rho v^2/X \quad . \quad . \quad . \quad (4)$$

where X is the radius of curvature of the path. We have only to suppose the path to be that which an element of a cyclone should pursue and the cyclonic motion is thereby intrinsically provided for, if the velocity is adjusted to the required value.

Hence the difference between the condition for steady motion as a stationary cyclone and for a travelling cyclone is that the cyclostrophic component of the gradient should be adjusted in the one case to the curvature of the circle which is identical with the isobar and in the other to the curvature of the path. The difference between the two depends upon the velocity with which the cyclone travels and *vice versa*.

§ 6. The Expression of the Difference as a Superposed Field of Barometric Pressure which transforms "Revolving Fluid" into a Travelling Cyclone.

In his report on *Barometric Gradient and Wind-Force*,[1] Gold has shown that in a travelling cyclone which consists of rings of fluid in instantaneous rotation about a moving axis, the relation between X, the radius of curvature of the path, and r, the radius of the instantaneous circle, is given by the equation

$$X = r/(1 + V\sin\alpha\, v)$$

where α is the angle which the radius makes with the line of path of the kinematic centre.

Hence equation (4) becomes

$$\gamma_c = 2\omega v\rho \sin\phi + \rho\frac{v^2}{r}(1 + V\sin\alpha\, v)$$

$$= \gamma_r + \rho V \sin\alpha\, v/r.$$

Hence

$$\gamma_c - \gamma_r = \rho V \sin\alpha\, v/r \quad . \quad (5)$$

The factor $\sin\alpha$ merely gives a component, along the radius, of the fixed quantity $\rho Vv/r$ as the radius sweeps round through the circle. The direction of the component is inwards from $\alpha = 0$ to $\alpha = 180°$ when $\sin\alpha$ is positive, and outwards for the rest of the circle when $\sin\alpha$ is negative.

If, therefore, we regard $\rho Vv/r$ as a field of force due to a pressure-gradient β over the whole circle radius r, the component of the field along the radius will give the acceleration which is necessary to transform the motion of fluid revolving about a fixed axis into motion about an axis travelling with the velocity V. The tangential component will provide for the adjustment of the velocity in proportion to the radius. Or, in other words, if upon a disc of revolving fluid with motion tangential to the isobars there be superposed a transverse field of pressure-gradient represented by $\rho V\zeta$ (where $\zeta = v/r$), the superposed field will give the acceleration which is necessary to transform the rotation of the disc about its axis into the motion of the several parts of a travelling cyclone. It will be noticed that this superposed field at any point is not balanced by the existing velocity of the air at that point, but in any actual case must

[1] M.O. Publication, No. 190.

be regarded as continuously superposed upon the air instantaneously in the condition of revolving fluid about its own kinematic centre as axis. The necessary continuity of superposition will be satisfied if the cyclone moves without change along the isobars of a fixed superposed field. Such a state of things might easily arise from changes of pressure in the upper regions of the atmosphere, or from the differences of the velocity of layers of air at different levels, which would certainly alter the distribution of pressure upon a layer near the surface. In fact, the details of the circulation of the atmosphere can hardly be otherwise regarded than as streams of air with their associated isobars moving under the influence of superposed fields of pressure arising from the relative motion of different layers. For the purpose of fixing ideas we suppose the air instantaneously in the condition of revolving fluid to be a layer extending from the level of about half a kilometre to one kilometre above the surface, and the problem before us is how to keep this layer half a kilometre in thickness in the dynamical condition necessary for the kinematical sequence of a travelling cyclone. The half-kilometre between it and the ground will be affected by the friction of the ground. According to G. I. Taylor[1] (under certain conditions not of course universally applicable) the relation between the surface wind and the gradient wind is $\cos\theta - \sin\theta$, where θ is the deviation of the direction of the surface wind from the gradient wind. In practice it is difficult to make a satisfactory allowance for the friction on account of the difference of conditions between day and night, which shows itself as a large diurnal variation in the velocity of wind at all low-level stations (see, for example, the diagrams in *The Weather-map*, pp. 94–97), due entirely to the reduction of the eddy motion by the lack of convection in the night hours. The atmosphere above the layer from one kilometre upwards is liable to changes in the distribution of pressure from various causes, which cannot be precisely defined except from the kinematical results. If, in consequence of the operation of these causes, an unbalanced field of pressure-gradient is superposed upon the layer which is in the condition of instantaneous rotation, the acceleration thereby caused transforms the revolving fluid into a travelling cyclone. The fact that the motion of a travelling revolving disc can be represented by the superposition upon simple rotation of a uniform acceleration at right angles to the line of travel can be demonstrated in various ways, of which the following is perhaps the simplest.

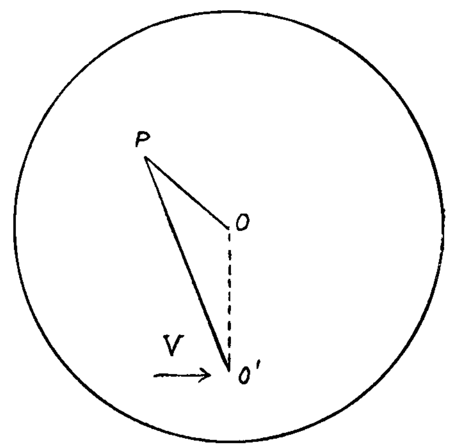


FIG. 2.—Relative acceleration with regard to the cyclone centre O and the tornado centre O′.

Let O (fig. 2) be the kinematic centre, O′ the tornado-centre, P a point of the disc and ζ its angular velocity, V the velocity of translation, so that $OO' = V/\zeta$. Then the instantaneous motion of P is rotation round O with radial acceleration $\zeta^2 OP$. The effect of the superposed acceleration is to make P travel with velocity V and at the same time rotate about O′, which will require a radial acceleration $\zeta^2 O'P$. But geometrically PO′ is the sum of PO and OO′, so that the result of the change is the uniform velocity V, which has no effect on acceleration and the geometrical addition of the acceleration $\zeta^2 OO'$. So the effect is the same as an acceleration $\zeta^2 OO'$ for each point of the disc, and since OO′ is V/ζ, the impressed acceleration is $V\zeta$.

[1] *Phil. Trans.*, A, vol. 215, p. 16, 1915.

§ 7. General Description of the Motion under the Prescribed Conditions.

When this acceleration is operative each portion of the air of the cyclone will describe the path necessary for its part in the evolutions of the cyclone instead of continuing in its own circle. The paths of the particles of air are trajectories formed by successive steps of rotation round a moving centre. At each successive instant the particles group themselves into a fresh set of circles. They go through a kind of dance or march; they take one step in the instantaneous circle of the moment, the next step in a new circle, infinitesimally smaller or greater in radius according as the centre is approaching or receding. As in the reciprocal case of a travelling wave, the march is so orderly that the particles never get in each other's way, yet all are moving.

The path of each particle is formed of a succession of steps in successive circles, but it is not itself a circle: it can be constructed, in the case of the cyclone of uniform winds, by superposing the uniform translation of the cyclone upon the planetary motion with reference to the centre of the cyclone as focus, which has already been referred to. It must be remembered in that case that of all the conic sections with the centre of the circle as focus, only a limited number of the ellipses will be entirely contained within the circle that represents the boundary of the cyclone. All the parabolas and hyperbolas and a certain number of the ellipses will cut the boundary. And similarly, in the case of the cyclone of uniform vorticity v/r, some at least of the arcs of revolution round the displaced centre will cut the circle which apparently marks the boundary of the cyclone,[1] which has to be adjusted with due regard to continuity of pressure to an environment having its own conditions. At the instant all the particles lying on the circle which forms the boundary will be moving tangentially, but the next step takes those in the rear outside the boundary circle and those in the front within it; and this fact illustrates a very important characteristic difference between a travelling cyclone and revolving fluid. In a travelling cyclone the air is constantly being renewed from the outside; it is drawn into the cyclone from the front and thrown out in the rear, whereas revolving fluid, like a tornado, keeps itself to itself. The rings may shrink in consequence of the disappearance of air from the centre, but a ring always remains a ring and is always composed of the same particles. This is not true of the limited portion of a disc of revolving fluid which defines a cyclone.

Thus the transformation which is effected by superposing the acceleration of a transverse field is a many-sided one; there may be changes in the relation of velocity to radius, or in the strength of the superposed field or disturbance of local character, and it is no wonder that uniformity is not generally maintained and that changes and modifications accompany the phenomena which are included in the description of the progress of a travelling cyclone in actual experience.

§ 8. The Cyclone Centre and the Tornado-Centre.

If we pursue further the kinematic development of a cyclonic area by representing the progress of the several particles in trochoids formed by the rolling of a circle along the line of motion of the centre of the cyclone, it is evident that the centre of the rolling circle is the point of the cyclone which makes uniform progression and is

[1] It will be shown subsequently, § 14, that the cyclone, in spite of the apparent circular boundary, is probably to be regarded as part of a larger disc of revolving fluid moving as a whole with the velocity of translation of the cyclone.

therefore the centre of relative motion round which the air may be regarded as revolving like a solid carried along with the wind. It is therefore a centre of revolving fluid and is at a distance V/ζ from the centre of the cyclone. This centre of relative motion will be within the area of the cyclone if V/ζ is less than the extreme radius: that is to say, if the cyclone is travelling at a speed below that of its marginal winds. That is usually the case, and we may therefore say that a fully developed cyclonic system of normal type has within itself, besides the cyclone centre, a centre of relative motion, or tornado centre, about which the air is actually revolving as a solid, and similarly, if a tornado or travelling disc of revolving fluid is sufficiently extended in diameter for the velocity in the margin to exceed the velocity of travel, the tornado will contain a cyclonic centre at a distance V/ζ from the tornado-centre, while the centre of isobars is between the two. The relation of the tornado-centre to the cyclonic centre is illustrated by the diagrams of paths and of the distribution of instantaneous motion, relative motion, and pressure in the plate which forms the frontispiece.

Thus it appears that meteorologically a cyclone and a tornado are not alternative terms: a cyclonic system which is developed to a sufficient area according to the law $v=\zeta r$ always contains both. Whether the tornado, which exists in any case, develops so much energy as to become the most prominent feature of the whole depends upon the manipulation of the fluid in the neighbourhood of the tornado-centre, where convection is quite likely to occur, with the consequences to be inferred from the laws of revolving fluid. Meanwhile, the cyclone travels on so long as it keeps its centre on an isobar of an uncompensated field. It will be noticed that this suggestion of the dual revolutionary character of a cyclonic area is well supported by the fact that the point which has been defined here as the tornado centre is at the region of a cyclone where tornadoes are actually found in real cyclones.

§ 9. Numerical Values.

Let us therefore look further into the suggestion that a travelling cyclone is in reality a layer of air with the instantaneous properties of revolving fluid under the influence of a superposed and uncompensated transverse field of pressure-gradient represented by $\rho Vv/r$. First let it be noticed that if the superposed gradient is from North to South the motion of the centre is from West to East, in the direction opposite to that of the equivalent geostrophic flow of air of the superposed field, the effect reminding one of gyroscopic movements which are perhaps also dynamically similar, though in place of the rigidity of the gyroscope we have the flow of air through the cyclone.

Next, let us examine the numerical values for the revolving fluid and the superposed field.

From equation (2)

$$\gamma_r=\rho v(2\omega \sin \phi + v/r) \qquad . \qquad . \qquad (6)$$

and the gradient of the superposed field γ_s is equal to $\rho Vv/r$. It will evidently simplify the consideration if v/r is a constant, ζ, because then, for a given value of the velocity of translation, V, the superposed field required will be uniform over the whole cyclone. It is a most natural supposition in view of the circumstances in which the superposed field is likely to occur in practice. In what follows, therefore, the normal cyclone of uniform vorticity with a superposed uniform field is considered, and the cyclone of uniform winds is excluded.

If we neglect the small variations in ρ, the density of the air, the supposition that, in equation (6), v/r is constant would give a gradient proportional to v, and therefore to r, because the quantity within the bracket of equation (6) is constant.

Substituting numerical values $\rho = \cdot 0012$, $\omega = \cdot 000073$, $\phi = 55°$, $\sin \phi = \cdot 82$, we get

$$\gamma_r = \cdot 0012(1 \cdot 2 \times 10^{-4} + \zeta)\zeta r$$

whence

$$p - p_0 = \cdot 0006(1 \cdot 2 \times 10^{-4} + \zeta)\zeta r^2 \qquad (6a)$$

The distribution of velocity in a cyclone proportional to the distance from the centre is not the one which, at first, suggested itself from the experience of weather-maps, but in reality the maps give us very little detail of the central region of a cyclone where the variation of v with r might be conspicuous; further, from the centre the variation with distance becomes hardly noticeable; and it will be seen later on that the hypothesis does, in fact, explain some of the apparent eccentricities of the central regions of real cyclones which hitherto have been disregarded.

If we take, for example, the wind (undisturbed by surface friction) to be a gale (20 m/s) at 200 k from the centre of instantaneous motion (which is only 1 cm. on the larger chart of the Daily Weather Report), then in c.g.s. units

$$\zeta = \frac{v}{r} = \frac{20 \times 10^2}{200 \times 10^5} = 10^{-4}$$

$$p - p_0 = \cdot 0006(1 \cdot 2 \times 10^{-4} + 1 \times 10^{-4})10^{-4} r^2 = 1 \cdot 32 \times 10^{-4} r^2,$$

or in millibars per hundred kilometres,

$$\mathrm{P} - \mathrm{P}_0 = 1 \cdot 32 \mathrm{R}^2 \qquad (7)$$

where R is the radius in hundred kilometres; the superposed field

$$\gamma_s = 1 \cdot 2 \times 10^{-7} \mathrm{V}.$$

For a velocity of translation, equal to 10 metres per second,

$$\gamma_s = 1 \cdot 2 \times 10^{-7} \times 10^3 \text{ dynes/cm.}^2 = 1 \cdot 2 \text{ mb./100 k.}$$

Hence, for a cyclone with a velocity of 10 m/s at 100 k from the centre, and a pressure-distribution of $1 \cdot 32 \times \mathrm{R}^2$ mb at R(100 kilometres) from the centre, a superposed field of $1 \cdot 2$ mb per hundred kilometres will cause the fluid to rotate about an axis travelling at 10 m/s instead of a fixed axis. Or, conversely, a cyclone of the type specified and travelling at the rate mentioned is exposed to conditions which are equivalent to a superposed field of pressure of $1 \cdot 2$ mb per 100 k.

§ 10. The Resultant of the Superposition of a Linear Pressure-field upon a Field of Circular Isobars.

It is clear that when this uncompensated pressure-gradient is superposed, the resultant pressure-distribution must not be expected to be in agreement, point for point, either in direction or in velocity, with the winds as shown on the map in the manner in which it is customary to think of them. The winds will represent the instantaneous condition of revolving fluid with a calm centre, and the pressure will be the field of circular isobars *corresponding therewith*, together with the superposed transverse field. Let us, therefore, consider the effect of superposing a uniform transverse field upon a field of circular isobars. Generalising the case of equation (7), we have, taking the centre of the circular isobars as origin,

$$p - p_0 = kr^2 = k(x^2 + y^2), \text{ where } k = \tfrac{1}{2}\rho\zeta(2\omega \sin \phi + \zeta) \qquad (7a)$$

Adding a superposed field

$$\delta + \beta y, \text{ where } \beta = \zeta\rho V \tag{7b}$$

we get the resultant pressure

$$\left.\begin{aligned} p' &= p_0 + \delta + k(x^2+y^2) + \beta y \\ x^2 + y^2 + y\beta/k &= (p' - p_0 - \delta)/k \end{aligned}\right\} \tag{8}$$

Of the superposed pressure-field we only want the gradient: the value of δ, the increase of field at the centre, only means a general increase of pressure over the whole field, and need not be considered. For the rest, it is clear from equation (8) that when successive values of pressure are assigned to the resultant field, p', the isobars are still concentric circles with a centre at a point on the map $\frac{1}{2}\beta/k$ "below" or to the right of the path of the centre of the circles of instantaneous revolving fluid. Hence the effect of superposing the field which transforms the instantaneous revolving fluid into a normal cyclone is to produce a set of circular isobars not coincident with the wind system but having its centre "below" the centre of the wind system. The general formula for the displacement D is

$$D = V/(2\omega \sin\phi + \zeta) \tag{9}$$

For the numerical values already given $\beta = 1{\cdot}2$ mb/100 k and $k = 1{\cdot}32$ (where the units are mb and 100 k); the displacement of the centre will be $\frac{1}{2}\beta/k = 1{\cdot}2/2{\cdot}64(100\text{ k})$, *i.e.* 45 kilometres for a rate of travel of 10 metres per second and proportionately greater for greater rates of travel or for a smaller value of ζ.

Transferring the origin to this centre, the equation of isobars becomes

$$k(x^2+y^2) = P' - P_0 - \delta + \tfrac{1}{4}\beta^2/k^2, \tag{10}$$

which differs from the original equation ($6a$) of isobars of the fluid in instantaneous rotation only in the value of the pressure at the centre. So far as differences of pressure from that at the centre are concerned, the two systems are identical.

Hence the representation, on a map, of a travelling cyclone differs from that of a stationary cyclone with a similar system of isobars only in the fact that with the stationary cyclone or revolving fluid, the common centre of the isobars is coincident with the kinematical centre of the winds, whereas in the travelling cyclone the system of isobars has the same gradients, but its centre is displaced relatively to the kinematical centre of winds; the direction of displacement is at right angles to the direction of consequent motion of the cyclone in the sense that if the centre of isobars is to the South of the centre of winds the cyclone is moving to the East, and the velocity of travel is directly proportional to the displacement, so that with a system of winds of 10 m/s at 100 k radius a dislocation of the centre through 45 k implies a travel at 10 m/s.

§ 11. Application to Weather-Maps.

Apart from the disturbance due to turbulent motion, the relation of the position of the centre of isobars to the kinematical centre might in favourable circumstances be definitely located on a map, because the identification of the centre of a circle is the most rudimentary of geometrical processes. Surface winds are, of course, very ill adapted for the purpose because of the capricious mechanical effects of surface friction, which complicate the effect of any meteorological turbulence due to thermal convection. If the atmosphere is sufficiently free from such thermal effects as to give a barogram without "embroidery," the velocity of pilot balloons at 500 metres, or of the lower clouds, should give all the information necessary for constructing the kinematical centre, and the pressure values, with the isobars drawn in the usual way, give directly the dynamical centre. Hence the identification of travelling cyclones of normal type ought not to present any serious difficulty. The reasoning seems to present a perfectly possible case of cyclonic motion directly related to a special case of revolving fluid. There may be some more general explanation of cyclonic motion requiring no assumption as to the law of distribution of velocity in the cyclone, but in accounting for the persistence of the cyclone it would have to make provision for the superposition of a properly adjusted field. There are also the cases in which the isobars are not, properly speaking, circular, but take some oval shape. These may also come under the same general principle when it becomes possible to examine the details. Meanwhile it seems desirable to examine the application of the general principle, stated in the form that for any group of curved isobars, whether forming closed curves or not, the travel of the group can be inferred from a measure of the displacement of the kinematical centre of the group from the centre of the isobars.

§ 12. The Computation of Gradient Winds.

The effect of this hypothesis upon the computation of the gradient wind from the distribution of pressure when the isobars are curved requires some explanation. The cyclonic depression must be regarded as having three centres: one the kinematic or cyclone-centre to the radii from which the winds are at right angles; the second the centre of relative motion or tornado-centre round which the air of the cyclone is revolving like a solid; and the third the dynamic centre of the isobars which give the gradient. The system of winds computed in the usual manner from the distance apart and the curvature of the isobars should agree with the actual system of winds, but the computed winds are not to be looked for at the points at which the gradients were taken, but at points distant from them by a step at right angles to the path of the kinematic centre equal to $V/(2\omega \sin \phi + \zeta)$, which can be computed at once for any set of isobars if the kinematic centre and its path can be identified and the proper ratio of the velocity to the distance from the kinematic centre determined. A rough approximation will have to serve provisionally if surface winds are to be used to identify the cyclone. In this connection we may use the formula which Major G. I. Taylor has given connecting the deviation of the surface wind from the "undisturbed" upper wind under the influence of the pressure-gradient with the reduction of velocity at the surface due to eddy motion. From this formula we get

$$\text{Surface wind} = (\cos \theta - \sin \theta) \times \text{gradient wind}.$$

For the well-exposed stations from which the observations for the Daily Weather Report are obtained the value 20° for the deviation and the associated ratio 0·6 of surface to gradient wind may be provisionally accepted. In other words, in order to identify the kinematic centre the velocity of the surface wind should be increased by one half and its direction veered through 20° or approximately through two points.

With this arrangement the wind computed from a point on the map within the cyclonic area should give the wind, not at the point itself, but at 45 kilometres (or other appropriate distance) along the line at right angles to the path equal and parallel to the displacement of the kinematic centre from the centre of isobars.

It must be noted that this process is not quite the same as taking a point on the path of a particle of air and computing the gradient from the velocity and the curvature of the path. That would give the true gradient at the point, which differs from the resultant gradient of the cyclone with dual centres by the omission of the tangential component. But the calculation of the gradient wind from the displaced circular isobars is an accurate representation of the wind if the point is properly selected. Experience with maps will tell us which is the more accurate representation of the actual conditions, and thus enable us to decide whether the superposition of an extraneous field of pressure upon a disc of revolving fluid is a fair representation of the dynamical condition of a travelling cyclonic depression.

In any case we have to note that a distinction should be drawn between the kinematic centre and the dynamic centre of a travelling cyclone, because the angle α in Gold's formula refers to the radius from the kinematic centre, not the centre of isobars, unless the two are coincident, which is not the case with a travelling cyclone. The distinction is of great importance in the region near the centre, and gradually loses its importance as the distances from the centre increase.

An endeavour to obtain the position of the kinematic centre of the cyclone of March 24–25, 1902, described and mapped in the *Life-History*, reproduced here in Plate II., gave a displacement of the kinematic centre through 35 kilometres to the North of the centre of isobars, the figures being obtained from the following data, taken from the particulars given in the text or maps :—

Average velocity of travel (V) . . .	26 miles per hour.
Extreme surface velocity taken as being in the outermost ring	45 ,, ,,
Equivalent undisturbed velocity	67·5 ,, ,,
Diameter of the disc, taken as the diameter of the largest circular isobar on the map for 6 p.m. .	380 miles.

The computed dislocation agrees well enough with the general appearance of the chart for the neighbourhood of the centres at 6 p.m. ; the one discordant note is a moderate South-easterly wind at Donaghadee, which would point to a kinematic centre some 40 kilometres to the West of the calculated point, and it is curious that the path of the centre of isobars which is charted shows a sudden alteration of direction at that time. The kinematic centre was above the point of change two hours previously, and if the observer at Donaghadee had been an hour in advance with his observation the wind would fit. It is considerations of this kind which make the study of this subject tedious.

§ 13. Application to Smaller Whirls.

The dislocation of a series of circular isobars through a horizontal distance by the superposition of a uniform pressure-field in accordance with the formula $D = V/(2\omega \sin\phi + \zeta)$ may have other applications than that to which reference has already been made. We have supposed the circulation to be in being and an outside field superposed upon it which had no other compensation in corresponding motion. We should arrive at the same result as regards the distribution of pressure by supposing a system of concentric isobars to begin to operate upon a layer of air beneath it which is already under the influence of a uniform field of pressure, and is therefore simply a steady current of wind. The superposition of the circular isobars will result in the transmission of a pressure-field with similar circular isobars to the ground, *not vertically underneath* the superposed whirl but displaced from the vertical towards the lower pressure through a distance $V/(2\omega \sin \phi + \zeta)$, where ζ is the spin of the whirl and V may be taken to be $\beta/(\zeta\rho)$ where β is the gradient of the layer between the superposed field and the surface.

With β equal to about 1 mb per 100 kilometres and ζ, 10 metres per second per 100 kilometres, we have seen that the displacement is 45 kilometres. With a small whirl and the same value of β, ζ may be 1000 times as great, giving a spin of 10 m/s at 100 metres, and the dislocation will then be 45 metres. With a larger value of β, we may get a dislocation of some 50 metres with a whirl which has gale force quite close to the axis, as a whirlwind has. Hence if a whirl arises above a layer of moving air, the effect of the whirl in producing a local set of concentric isobars will be transmitted to the surface at a dislocated point, and its effect, which will be in the first instance convergence and convection at the core, will be felt at the displaced point and not immediately underneath the whirl. Hence the similitude of water-spouts to elephants' trunks may be simply another example of the superposition of a circular and a linear field of pressure.

§ 14. The Distribution of Pressure referred to the Tornado Centre.

Hitherto we have concentrated attention on the cyclonic centre as being the point round which the instantaneous winds are symmetrically arranged, and therefore the most easily sought for on the weather-map. We have seen that the dynamic centre or centre of isobars of a normal cyclone travelling with velocity V is at a distance $V/(2\omega \sin \phi + \zeta)$ on the right of the path of the cyclone centre, and the displacement of the centre of isobars is equivalent to the superposition of a field of pressure transverse from left to right of the path of which the gradient is $\rho V \zeta$. So we have regarded the distribution of pressure as made up of a circular field represented by

$$p - p_0 = k(x^2 + y^2), \text{ where } k = \tfrac{1}{2}\rho\zeta(2\omega \sin \phi + \zeta),$$

and a linear field represented by $\delta + \rho V \zeta y$, where the origin is the kinematic centre and the axes are along the path of the centre and at right angles thereto. By the same reasoning the resultant field of pressure can be resolved into a field of circular isobars *with any other point as centre* and a linear field properly adjusted to give the necessary displacement of the centre. An examination of the diagrams of instantaneous motion, paths of air, and relative motion, included in Plate I., leads us now to transfer our attention to the tornado centre, and regard a circular field round that centre as one of the components of the resultant field, and to inquire what the other component will be in that case.

We shall show that the second component will be a linear field of which the gradient is $2\rho\omega V \sin \phi$—that is to say, the geostrophic gradient for the velocity of translation of the cyclone. So that the cyclone (together with a certain addition on the right-hand side of its path) may be regarded as a large area of revolving fluid, under the influence of a field of pressure which consists of the isobars appropriate to the rotation round the tornado-centre, and, in addition, the linear field which is necessary to maintain a velocity (along a great circle) equal to the travel of the cyclone. It follows that the system which we know as a cyclone can hardly be regarded as having a separate existence like revolving fluid, but merely represents the phenomena belonging to that part of a disc of travelling revolving fluid round the point of the disc which is instantaneously at rest.

The steps leading to this important conclusion are as follows:—On referring to the diagrams of Plate I., it will be noticed that the paths of the air in the various parts of the cyclone are the trochoids traced by points attached to a circular disc, of which the radius is O′O (where O is the cyclone centre and O′ the tornado-centre), and which rolls along the path of the cyclone-centre. If we regard the air under consideration as defined by the region of circular isobars shown on the map, those points which are within the circle whose radius is O′O will form circles of revolving fluid with O′ as centre, rotating like a solid and travelling bodily with the velocity of translation of the cyclone. The points belonging to the cyclonic system which are outside the circle of which the radius is O′O will form looped trajectories, and will represent air which enters the cyclonic system from the outside in the front and leaves the system in the rear of the cyclone. It has already been remarked that the disc of the cyclone must be fitted into a suitable environment with due regard to the continuity of pressure. The diagrams of trajectories for the normal cyclone, compared with actual trajectories taken from the *Life-History*, show that the continued existence of the travelling cyclone requires air to come in from and to pass out to *the right* of the path of the centre. In actual examples of travelling cyclones the necessary provision is made by the apparent extension of the cyclone on the right of the path by means of isobars, which curve round the dynamic centre of the cyclone, but do not form closed curves within the region of the map. It appears that this extension of the cyclone on the right of the path may really represent the completion of the disc of revolving fluid, of which the cyclonic region of circular isobars forms a part; and what we have to deal with is, in fact, a large area of revolving fluid defined by a circle, of which the tornado-centre is the centre and the radius is the line drawn from the tornado-centre to the margin of the circular isobars on the left of the path. It therefore exceeds the apparent radius of the circular margin of the cyclone by the distance between the cyclone-centre and the tornado-centre.

Let us, therefore, look upon the cyclone with the added isobars on its right as forming a disc of revolving fluid rotating like a solid about the tornado-centre and travelling with the velocity of travel of the cyclone.

We have already obtained (equation 7*a*) the distribution of pressure which is necessary for the travelling cyclone as a system of circular isobars according to the law

$$p - p_0 = \tfrac{1}{2}\rho\zeta(2\omega\sin\phi + \zeta)(x^2 + y^2)$$

centred at the dynamic centre, which is at a distance $V/(2\omega\sin\phi + \zeta)$ from the kinematic centre towards the tornado-centre. A distribution according to the same law about the tornado-centre will give the distribution necessary to maintain the circulation of the revolving fluid during its travel as a solid. If now we use x and Y as co-ordinates with the tornado-centre as origin (instead of the dynamic centre), the distribution will be

$$p - p_0 = k(x^2 + Y^2) \text{ where } k = \tfrac{1}{2}\rho\zeta(2\omega\sin\phi + \zeta) \text{ and } Y = y + O'O''.$$

If we add the geostrophic field corresponding with the velocity of translation of the revolving fluid, we must add the gradient $-2\rho\omega V\sin\phi$. The negative sign is prefixed because the gradient is from right to left across the path. Making the addition by adding the pressure-field

$$p' - p'_0 = -2\rho\omega V\sin\phi Y,$$

we get the resultant pressure field

$$p + p' - p_0 - p_0' = k(x^2 + Y^2) - 2\rho\omega V \sin\phi Y,$$

or

$$P - P_0 = kx^2 + k(Y - \rho\omega V \sin\phi/k)^2 = kx^2 + k\left(Y - \frac{2\omega V \sin\phi}{\zeta(2\omega\sin\phi + \zeta)}\right)^2 \quad . \quad (11)$$

This means a pressure-field of circular isobars with the same law of distance from the centre as before, but about the centre which is defined by the co-ordinates $x = 0$, $Y = \dfrac{2\omega V \sin\phi}{\zeta(2\omega\sin\phi + \zeta)}$. The point thus formed is easily seen to be identical with the dynamic centre of the cyclone, because the distance from the tornado-centre to the cyclone-centre is V/ζ, and the distance from the cyclone-centre to the dynamic centre is $V/(2\omega\sin\phi + \zeta)$. Hence the distance from the dynamic centre to the tornado-centre (the difference of these two) is $V/\zeta - V/(2\omega\sin\phi + \zeta)$, or $\dfrac{2\omega V \sin\phi}{\zeta(2\omega\sin\phi + \zeta)}$, the expression which we have already obtained for the distance from the tornado centre to the centre of isobars resulting from the addition of the field of translation to the field of rotation of the revolving disc. Thus, the distribution of pressure, which we have shown to be necessary for a travelling cyclone, is, in fact, the same as the distribution obtained by compounding a similar field of rotation about the tornado-centre with the geostrophic field corresponding with the velocity of translation.

In the paper on revolving fluid in the atmosphere I noted that I was unable to assign the pressure-distribution which was appropriate to the figures therein obtained for the combination of a velocity of translation with the velocity of rotation. For the construction of those figures the linear velocity was taken as uniform over the area. What is given here is a solution of the problem when the angular velocity of rotation is uniform, and it is curious that the result obtained shows a distribution of pressure which may be regarded as either

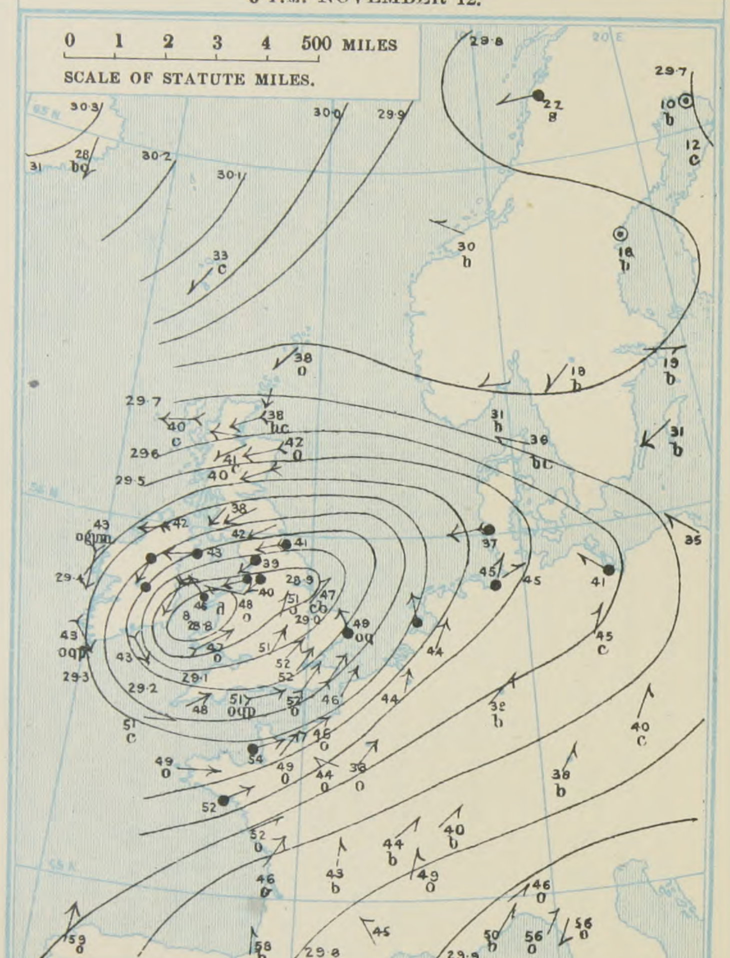
(1) A system of circular isobars round the cyclone-centre with an uncompensated field of pressure $\rho V\zeta$ *from left to right*, or (2) a similar system of circular isobars round the tornado-centre compounded with a field of pressure *from right to left*, represented by $2\rho\omega V\sin\phi$, the geostrophic equivalent of the translation V.

The second mode of regarding the distribution throws new light upon the phenomena of cyclones, in particular of those which have a rate of travel comparable with their own extreme winds. It would appear that the phenomena which we have been accustomed to associate with cyclones do not really form an independent system. They are included in the phenomena of revolving fluid formed within a broad stream of air and carried along with it. If the area and velocity of the revolving fluid are sufficiently large there will be a portion in which the instantaneous velocity is opposite to the direction of travel and one point which is instantaneously at rest: the phenomena of the cyclone are simply the phenomena of the portion of the revolving fluid in the neighbourhood of the centre of instantaneous rest. That point has attracted attention, because it is the instantaneous motion (modified to some extent by friction) which is represented on a map of winds.

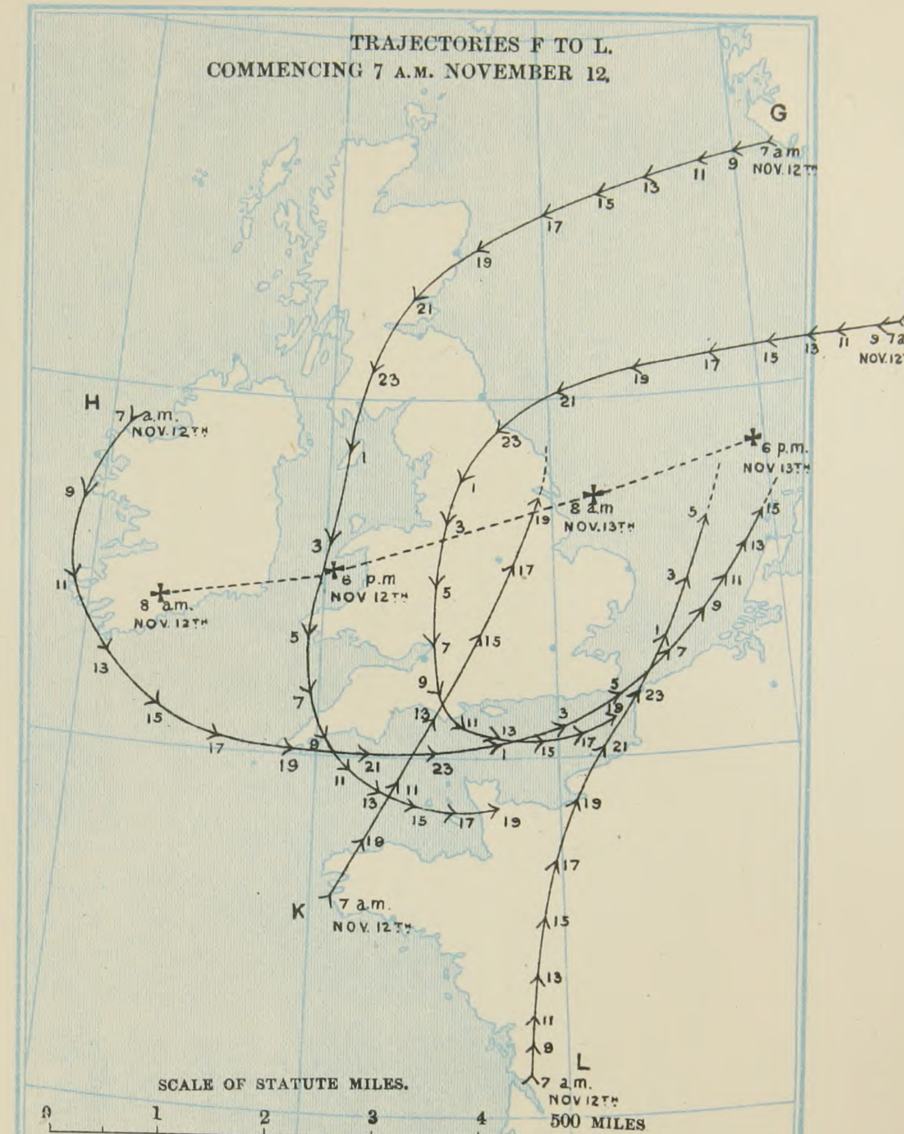
If we suppose a disc of revolving fluid to be formed in a current of air and gradually to increase in diameter (or in its velocity of rotation), a cyclonic centre will be formed automatically on the left-hand side of the path and will be indicated on the map as the point of instantaneous rest round which the instantaneous winds appear to circulate

SLOW TRAVELLING DEPRESSION, NOVEMBER 11-13, 1901.

6 P.M. NOVEMBER 12.



ISOBARS, WINDS, TEMPERATURE AND WEATHER.



TRAJECTORIES OF AIR.

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with such incurvature as may be due to surface friction. This kinematic centre can be identified. The dynamic centre can also be identified as the centre of the circular isobars; but the tornado centre escapes attention altogether: its position can, however, be calculated from the position of the dynamic centre and the velocity of translation if an estimate can be formed of the value of ζ. For, as we have seen, the distance from the tornado-centre to the dynamic centre is $\dfrac{2\omega V \sin\phi}{\zeta(2\omega\sin\phi+\zeta)}$.

If these considerations are valid for actual cyclonic depressions, we ought to find the representation of a travelling cyclone to consist of a circle representing the area of revolving fluid, of which the cyclonic depression forms a part, set in a region of straight isobars which have the velocity of travel of the depression as their geostrophic equivalent. If we examine the cases figured in the *Life-History of Surface Air-Currents* we find this state of things well exemplified by the second example of a fast travelling storm, that of September 10–11, 1903, of which the rate of travel is given as 16 m/s, and which gives a circular area joined up on the East to straight isobars which correspond with a travel of 17 m/s. With the first example, that of March 24–25, 1902, the isobars just outside the circular area on the East run nearly North and South across the path of the depression, but on the West, in the rear, there are straight isobars with a geostrophic velocity of 13 m/s, corresponding very well with the actual rate of travel, which is 12 m/s. So either of these two cyclones might be regarded as belonging to a column of revolving fluid formed in a stream represented by straight isobars. In the example of the slow travelling depression of November 11–13, 1901, the setting of the depression in its environment is of an entirely different character; it lies between two areas of high pressure.

We cannot proceed satisfactorily with the interesting questions which these three examples present without taking into consideration the general question of the environment of cyclonic depressions. I know of no general answer to the question. Hildebrandsson suggested the idea of a large cyclone round the polar axis carrying travelling depressions along with it as representing the general circulation of a hemisphere; and in a paper contributed to the Royal Society in 1903 I gave reasons for thinking that a general circulation of that character belonged only to the level above four kilometers in height, while at the surface the general circulation might be regarded as an anticyclonic circulation round the pole surrounded by a cyclonic circulation further South. Both these suggestions are worth considering in relation to the general question of the environment of cyclones in our own latitudes.

§ 15. Limitation to Incompressible Fluid moving in two Dimensions.

With reference to what has been written here it is necessary to make three remarks. First, that in considering the horizontal motion of the air the variations of density in its own level have been ignored; to that extent the air has been regarded as an incompressible fluid. At a fixed level the variations of density in the course of the evolutions of a cyclone are very small, and they are therefore of secondary importance in the general features of cyclonic motion. Secondly, no mention has been made of the convection of air and its influence in originating or maintaining a cyclonic circulation: the question has been treated as one of motion in two dimensions. The ultimate justification for that must be looked for in the test of the conclusions arrived at. The evidence that we have of the irregularity of the incidence and the distribution of rainfall in the region of cyclonic depressions seems to show that rainfall is not essential to the structure of a travelling cyclone. It may be the original cause of the circular arrangement of the winds, but in a cyclonic depression once established, rainfall, and consequently convection, seem to be more or less an accidental accompaniment of a depression, depending on the physical or thermal rather than upon the primary dynamical conditions. Thirdly, with a cyclone of large diameter the assumption that the same value for the latitude can be taken for the whole of the area of the system is obviously inexact. If we suppose the kinematic conditions to be maintained, that is to say, if we suppose r to be the angular radius instead of the linear radius, we have to deal with a rotating shell travelling over the earth's surface instead of a rotating disc travelling over a plane. The range of latitude will affect the computations in two ways. First, the adjustment of pressure to the velocity will be different, the isobars for the more northern parts of the system being closer than in the more southern parts; and secondly, the pressure-field necessary to transport the system unchanged will not be uniform but will increase with the latitude. For the present we assume both these effects to be relatively small disturbances of the general system.

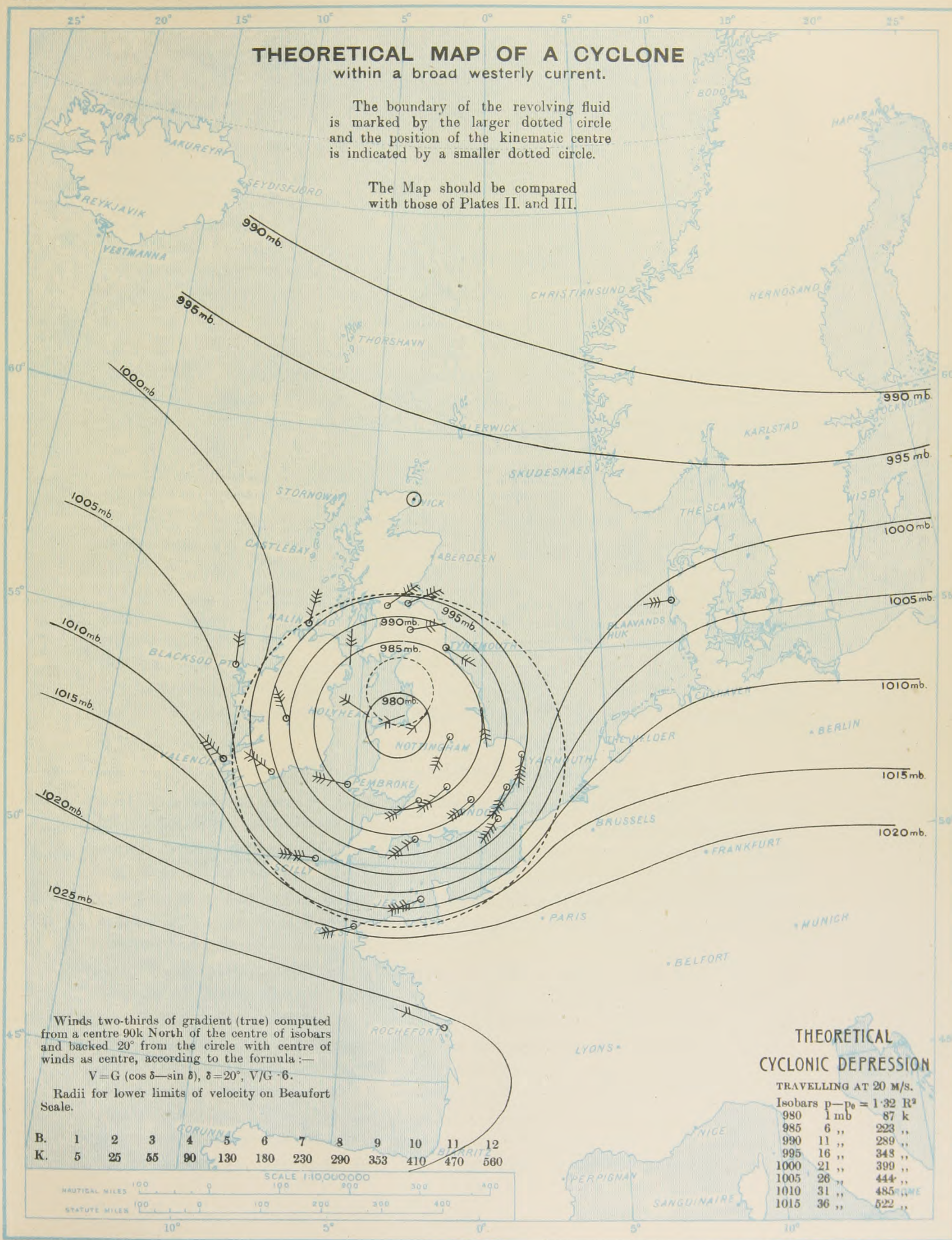
There remains the question of the mode of covering in the travelling cyclone at the top. The distribution of pressure at the top may be varied at higher elevations provided that at each level the winds are suitably adjusted. All that is necessary to maintain a travelling cyclonic disc in being is that the pressure in the superposed atmosphere should give continuity with the pressure-gradient in the cyclone, with a uniform or a properly adjusted gradient in addition. Any mischance which a cyclone may suffer in the way of want of accommodation of its maintenance will be expressed in the first instance by a change in the speed or direction of its travel, except in the most improbable event of its coming under a set of isobars exactly concentric with its own.

§ 16. A Theoretical Map of a Cyclone travelling at 20 Metres per Second.

In illustration of the application of these considerations a map, reproduced as Plate V., has been constructed showing the isobars and winds for a cyclonic depression, consisting of a disc of 300 kilometres radius, in which the winds are proportional to the distance from the kinematical centre, and which travels at the rate of 20 m/s., a very rapid rate of travel. The isobars are a system of concentric circles identical in radius with those which would balance the winds if they were concentric therewith, but the centre of the isobars is 90 kilometres to the south of the centre of the winds; outside the area of the system of winds isobars have been added which represent the velocity of travel of the cyclone according to the considerations set out in § 14. The winds as drawn on the map are surface winds: they have been reduced from the winds corresponding with the gradient of a similar system of isobars properly centred by reducing the velocity to two-thirds of the calculated value and allowing a backing of two points. Within the area of the system of winds the map presents many noteworthy points of agreement with actual maps of fast-travelling cyclones, such as those figured in the *Life-History of Surface Air-Currents.*

§ 17. Secondaries.

It has been shown in § 7 that a normal travelling cyclone has a tornado-centre about which the air of the cyclone revolves like a solid, which in addition to its



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rotation has a superposed motion of translation. It must be understood that the presence of this tornado-centre is not disclosed by any irregularity in the isobars. Everything may be perfectly smooth and circular, and yet one part of the cyclonic disc has the properties of revolving fluid or of a tornado. It should also be noted that the tornado-centre is in the trough-line, is in that part of the cyclonic disc where, judging by our experience of cyclones, convection invariably occurs, and where, therefore, the atmosphere approaches the labile condition. Should the interior portion of the column be removed by convection, the conditions indicated in Lord Rayleigh's paper would be realised: the outer rings of the revolving column would be brought nearer the centre, and the law of distribution of relative velocity, instead of being simply proportional to the distance, would become

$$v = \zeta r + A/r,$$

and "in addition to the motion of a solid body, the fluid acquires the motion of a simple vortex of intensity increasing towards the centre." This is presumably the process of creation of the destructive tornado. In its first stages it means the development of a small secondary, because the pressure will be reduced where the angular velocity is increased, and in its further stages it would lead to the development of a new cyclone-centre, when the area of the revolving fluid became too great for all the air to be within the rolling circle at the speed at which that circle moves.

On a map the most definite evidence of local convection is local rainfall, not exactly over the spot where the convection has occurred, but not far away. Consequently we ought to expect the development of a small secondary, possibly a tornado, and ultimately a new cyclonic centre, to be marked on the map by a line of relatively heavy or maximum rainfall. This expectation is borne out by the association of heavy rainfall with the line of travel of the tornado-centre in both the cases of revolving fluid on March 24, 1895, and October 27, 1913, already cited. It is, in fact, with the tornado-centre that the properties of revolving fluid deduced by Lord Rayleigh and the experimental reasoning of Dr Aitken may be associated, not with the cyclone-centre.

§ 18. The Normal Travelling Cyclone.

If we call the cyclonic depression which has winds at right angles to the radii from a kinematic centre, with velocity proportional to the distance from the centre, a normal cyclone, we may claim that, subject to the correction for variation of latitude and other considerations, a suitable explanation of the travel of a normal cyclone has now been arrived at, and it is not an exaggeration to say that the quest of an explanation of those phenomena has engaged the attention of students of weather-maps for the past half-century.

The explanation, now that it has come, is almost absurdly simple, considering the long period during which it has escaped detection. Properly speaking, a cyclone is simply the name given to the phenomena of travelling revolving fluid as grouped round the point instantaneously at rest, and the proper centre of reference for the phenomena is not the cyclone-centre but another point, the tornado-centre; but accepting the traditional standpoint, we may describe the conditions as follows:—Disregarding internal friction, the persistence of a stationary cyclone, when once formed, only requires a suitable lid on the top, and the maintenance of a suitable field of concentric isobars. Its travel with a uniform speed is simply the expression of a transverse field of pressure superposed upon the conditions of the stationary cyclone: the rate of travel can be calculated from the constant of the cyclone and the strength of an equivalent superposed field, and, *vice versa*, the superposed field can be calculated from the rate of travel and the constant of the cyclone. It must be understood that from whatever cause the superposition of the field may arise, it must be in operation. It transforms the instantaneous rotation round the kinematic or cyclonic centre into relative motion round a second centre which we call the tornado-centre, and at the same time provides that air shall be supplied to the front of the cyclone along trochoidal curves and thrown out along similar curves at the rear, instead of the cyclone remaining always composed of the same fluid, and it is simply the dynamical expression for these kinematic changes, which are themselves simply a technical description of a travelling normal cyclone. The tornado-centre is generally within the boundary of the cyclone itself, and therefore a portion of the cyclone, forming a circle with a point on the trough as centre, consists of revolving fluid that rotates and travels with the cyclone like a solid.

The superposed field also causes a displacement of the circular isobars appropriate to a stationary cyclone, so that the centre of isobars which is so easily identified on a map coincides neither with the tornado-centre nor the cyclone-centre, but in an ordinary case lies about halfway between the two. It is the spin of the earth's rotation which causes the separation between the dynamic centre and the tornado-centre. Neither convection nor any other transformation of energy is necessary for the maintenance of a travelling cyclone, except in so far as energy is required to make good the loss from friction at the surface and interior. These amounts are thought to be very small in comparison with the whole energy of the rotating mass, and consequently no provision of convection can be regarded as necessary.

But any part of the cyclonic disc may become, from local causes, the seat of local convection, and if for any reason convection occurs in the revolving fluid of the trough, and it generally does occur there, it will accelerate the revolution of the fluid round the tornado-centre, so that a secondary to the original circular isobars will be formed there; and if the convection goes on the secondary may develop into a destructive tornado, or before that occurs create a new cyclonic system on its margin with a new constant of rotation.

It must be allowed in conclusion that the large majority of the cyclones shown on our own maps cannot be described as normal because their isobars are not circular; the winds have no apparent common centre, and the fluid of the cyclone has no defined tornado-centre about which it revolves as a solid. But since the cyclones of our experience find it practicable to travel (with more or less of change of shape on the way), and the travel of curved circular isobars is the direct expression of the superposition of a pressure-field, it is clear that the travel of the cyclones of our experience can be referred to the superposition of some pressure-field, but not necessarily the simple one which represents the motion of the normal cyclone. And we are also entitled to the generalisation that we must no longer regard the apparent centre of closed isobars, whatever their shape may be, as a single centre round which all the properties of the cyclone can be symmetrically grouped. What we ought to regard, in the case of the irregular-shaped isobars of ordinary travelling depressions, as the appropriate alternatives for the kinematic centre and the tornado-centre of the normal cyclone must be left for further investigation.

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