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A STATISTICAL STUDY OF THE
VARIATION OF WIND WITH
HEIGHT

By C. S. DURST, B.A.

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A STATISTICAL STUDY OF THE VARIATION OF WIND WITH HEIGHT

By C. S. DURST, B.A.

Summary.—The variation of wind with height is examined by means of vector correlation coefficients in a similar manner to the variation of wind with distance and time published in *Geophysical Memoirs* No. 93.

The correlation coefficients are given for the British Isles up to 100,000 ft., for Habbaniya up to 40,000 ft. as indicating the conditions in middle latitudes, for Nairobi indicating conditions in the tropics and for Barrow, Alaska, indicating conditions in the far north.

In *Geophysical Memoirs* No. 93¹* the discussion centres round the variation of wind with time and distance. The variation with height was originally included but, for various reasons, was excluded from that publication. Since then, Graystone² has given some correlation coefficients between winds at two heights, and others have been given by Ellison and Walshaw³.

In dealing with winds it is possible to resolve the observed winds at any level into components $U + u$ and $V + v$ (where U and V are the mean of a large number of occasions), and then correlate the components at two heights thus forming two coefficients. Alternatively it is possible to use the vector correlation technique which gives a coefficient in some ways easier to use.

It is desired to find that wind at height b which is most likely to be associated with a given wind at height a . It is supposed that we know the mean vector winds at both heights. The desired wind at height b is $\bar{\mathbf{v}}_b + \mathbf{v}_b$ and the observed wind at height a is $\bar{\mathbf{v}}_a + \mathbf{v}_a$. It is supposed that $|\mathbf{v}_b|$ is equal to $R|\mathbf{v}_a|$ and that \mathbf{v}_b is turned through an angle ϕ from \mathbf{v}_a . Then to find the most probable values of R and ϕ we have to minimize the sum

$$\Sigma\{|\mathbf{v}_b|^2 + R^2|\mathbf{v}_a|^2 - 2R|\mathbf{v}_a||\mathbf{v}_b|\cos(\theta_{ab} - \phi)\}$$

taken over the available observations of \mathbf{v}_a and \mathbf{v}_b . θ_{ab} is the angle between \mathbf{v}_a and \mathbf{v}_b . The values of R and ϕ which make the sum a minimum are found in the usual way by equating to zero the partial differential coefficients of the sum with respect to R and ϕ . This gives at once

$$\tan \phi = \frac{\Sigma|\mathbf{v}_a||\mathbf{v}_b| \sin \theta_{ab}}{\Sigma|\mathbf{v}_a||\mathbf{v}_b| \cos \theta_{ab}},$$

$$R = \sqrt{\{(\Sigma|\mathbf{v}_a||\mathbf{v}_b| \sin \theta_{ab})^2 + (\Sigma|\mathbf{v}_a||\mathbf{v}_b| \cos \theta_{ab})^2\} / \Sigma|\mathbf{v}_a|^2}.$$

In the same way as in scalar correlation the total vector correlation coefficient r_{ab} is taken to be $R\sigma_a/\sigma_b$, where σ_a is written for $\sqrt{\{(\Sigma|\mathbf{v}_a|^2)/(n-1)\}}$ and σ_b for $\sqrt{\{(\Sigma|\mathbf{v}_b|^2)/(n-1)\}}$, and so

$$r_{ab} = \frac{\sqrt{\{(\Sigma|\mathbf{v}_a||\mathbf{v}_b| \cos \theta_{ab})^2 + (\Sigma|\mathbf{v}_a||\mathbf{v}_b| \sin \theta_{ab})^2\}}}{(n-1)\sigma_a\sigma_b}.$$

The associated quantities

$$r_{ab}' = \frac{\Sigma|\mathbf{v}_a||\mathbf{v}_b| \cos \theta_{ab}}{(n-1)\sigma_a\sigma_b},$$

$$r_{ab}'' = \frac{\Sigma|\mathbf{v}_a||\mathbf{v}_b| \sin \theta_{ab}}{(n-1)\sigma_a\sigma_b}.$$

* The index numbers refer to the bibliography on p. 11.



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are termed respectively the coefficients of simple stretch and simple turn. It will be noted that

$$r_{ab}^2 = (r_{ab}')^2 + (r_{ab}'')^2.$$

The regression equation connecting \mathbf{v}_b and \mathbf{v}_a can be written

$$\mathbf{v}_b = \frac{\sigma_b}{\sigma_a} r_{ab} \eta_\phi \cdot \mathbf{v}_a,$$

where η_ϕ is the linear vector operator, expressible in the matrix form

$$\begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix},$$

which denotes that the vector \mathbf{v}_b is rotated positively through the angle ϕ from the vector \mathbf{v}_a . The graphical solution of this regression equation when $r_{ab}'' = 0$ is illustrated in Fig. 1 where OA represents the vector \mathbf{V}_a on a particular occasion, OA' represents the average $\bar{\mathbf{V}}_a$ and OB' represents the average $\bar{\mathbf{V}}_b$.

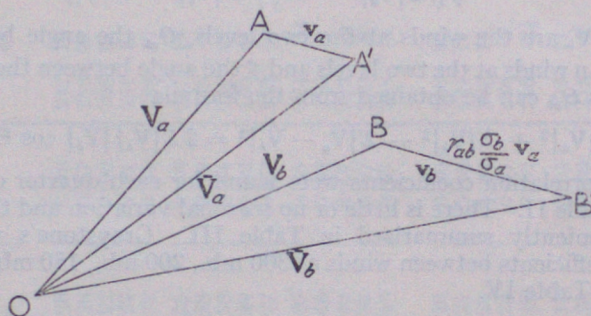


FIG. 1

To obtain \mathbf{V}_b corresponding to this particular value of \mathbf{V}_a join AA', which gives \mathbf{v}_a and draw from B' a line BB' (\mathbf{v}_b) parallel to AA' and of length $r_{ab}'(\sigma_b/\sigma_a)AA'$. Then OB represents in direction and length the probable value of the wind at height b on this occasion.

If r_{ab}'' is not zero the length BB' is $r_{ab}(\sigma_b/\sigma_a)AA'$ and the vector BB' is rotated through an angle α_{ab} which is given by

$$\alpha_{ab} = \tan^{-1} \left\{ \frac{\sum |\mathbf{v}_a| |\mathbf{v}_b| \sin \theta_{ab}}{\sum |\mathbf{v}_a| |\mathbf{v}_b| \cos \theta_{ab}} \right\}.$$

The data used in the initial investigation were winds measured over Liverpool during the period June 1945 to May 1946, and correlation coefficients of simple stretch were calculated between winds at heights of 900, 700, 450, 300, 200 and 100 mb., corresponding approximately to 3,000, 10,000, 20,000, 30,000, 40,000 and 53,000 ft. The calculations were lengthy and before they were undertaken a pilot examination was made to find out whether the correlation coefficient of simple turn was significant. The result is shown in Table I, where correlation coefficients of stretch and turn are shown between 900 mb. and all the other heights.

TABLE I—CORRELATION COEFFICIENTS OF SIMPLE STRETCH AND SIMPLE TURN AT LIVERPOOL, JUNE 1945–MAY 1946

| Lower level | Upper level | Stretch | Summer Turn | Total | Stretch | Winter Turn | Total |
|-------------|-------------|---------|-------------|-------|---------|-------------|-------|
| mb. | mb. | | | | | | |
| 900 | 100 | 0.50 | —0.05 | 0.50 | .. | .. | .. |
| 900 | 200 | 0.54 | —0.14 | 0.56 | .. | .. | .. |
| 900 | 300 | 0.60 | —0.25 | 0.64 | 0.61 | 0.03 | 0.61 |
| 900 | 450 | 0.62 | —0.21 | 0.65 | .. | .. | .. |
| 900 | 700 | 0.86 | —0.15 | 0.87 | 0.87 | 0.14 | 0.88 |

The correlation coefficient of a simple turn is much less than that of simple stretch, and though it does exercise some influence in summer the error in neglecting it is not large.

If, then, the correlation coefficient of a simple turn is neglected it is possible to obtain the coefficient of simple stretch quite easily from the formula

$$r_{ab}' = \frac{\Sigma |\mathbf{V}_a| |\mathbf{V}_b| \cos \Theta_{ab} - n |\bar{\mathbf{V}}_a| |\bar{\mathbf{V}}_b| \cos \beta}{\sqrt{(\Sigma |\mathbf{V}_a|^2 - n \bar{\mathbf{V}}_a^2)(\Sigma |\mathbf{V}_b|^2 - n \bar{\mathbf{V}}_b^2)}}$$

where \mathbf{V}_a and \mathbf{V}_b are the winds at the two levels, Θ_{ab} the angle between them, $\bar{\mathbf{V}}_a, \bar{\mathbf{V}}_b$ the mean winds at the two levels and β the angle between them. Moreover $\Sigma |\mathbf{V}_a| |\mathbf{V}_b| \cos \Theta_{ab}$ can be obtained from the formula

$$\Sigma |\mathbf{V}_a|^2 + \Sigma |\mathbf{V}_b|^2 - \Sigma |\mathbf{V}_a - \mathbf{V}_b|^2 = 2 \Sigma |\mathbf{V}_a| |\mathbf{V}_b| \cos \Theta_{ab}. \quad (1)$$

In this way correlation coefficients were found for each quarter of the year as is shown in Table II. There is little or no seasonal variation and the coefficients can be conveniently summarized in Table III. Graystone's values of the correlation coefficients between winds at 300 mb., 200 mb., 150 mb. and 100 mb. are set out in Table IV.

TABLE III—STRETCH VECTOR CORRELATION COEFFICIENTS BETWEEN WINDS AT VARIOUS HEIGHTS

| | | 900 mb. 3,000 ft. | 700 mb. 10,000 ft. | 450 mb. 20,000 ft. | 300 mb. 30,000 ft. | 200 mb. 40,000 ft. |
|-----|--------|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| mb. | ft. | | | | | |
| 100 | 53,000 | 0.41 | 0.54 | 0.59 | 0.60 | 0.73 |
| 200 | 40,000 | 0.54 | 0.73 | 0.83 | 0.89 | |
| 300 | 30,000 | 0.59 | 0.79 | 0.90 | | |
| 450 | 20,000 | 0.65 | 0.85 | | | |
| 700 | 10,000 | 0.85 | | | | |

By the use of big balloons Scrase⁴ has obtained a number of ascents reaching to 80,000 or 100,000 ft. Correlation coefficients were calculated from these for combinations of the levels 50,000, 80,000 and 100,000 ft. as is shown in Table V and for practical purposes we may consider the pressure levels to be 130 mb., 20 mb. and 10 mb. These are based on a small sample of observations and the correlation coefficients in S.-N. and E.-W. components are clearly not representative; however, the vector correlation coefficients appear reasonably consistent.

Further, a comparison was made between winds at various heights over Habbaniya and Barrow, Alaska, during 1949 and also over Nairobi. The correlation coefficients are given in Table VI.

TABLE II—STRETCH VECTOR CORRELATION COEFFICIENTS OF WINDS AT TWO HEIGHTS

Period: June 1945–May 1946

Liverpool

| Lower level | Upper level | Mean wind | | Standard deviation | | Stretch correlation coefficient | Mean wind | | Standard deviation | | Stretch correlation coefficient | | | | |
|-----------------------------|-------------|-------------|-------------|--------------------|-------------|---------------------------------|-------------|-------------|--------------------|-----|---------------------------------|----|-----|----|------|
| | | Lower level | Upper level | Lower level | Upper level | | Lower level | Upper level | | | | | | | |
| | | | | | | | | | ° | kt. | | ° | kt. | | |
| WINTER (December–February) | | | | | | | | | | | | | | | |
| mb. | mb. | ° | kt. | ° | kt. | | ° | kt. | ° | kt. | | | | | |
| 200 | 100 | 315 | 27 | 301 | 25 | 38 | 23 | 247 | 19 | 231 | 9 | 33 | 20 | 24 | 0.77 |
| 300 | 100 | 289 | 19 | 301 | 25 | 34 | 23 | 253 | 21 | 231 | 9 | 48 | 20 | 40 | 0.66 |
| 300 | 200 | 289 | 24 | 300 | 31 | 45 | 45 | 265 | 23 | 254 | 21 | 49 | 36 | 25 | 0.88 |
| 450 | 100 | 299 | 12 | 301 | 25 | 30 | 23 | 251 | 19 | 231 | 9 | 29 | 20 | 25 | 0.62 |
| 450 | 200 | 286 | 15 | 300 | 31 | 38 | 45 | 256 | 18 | 254 | 21 | 32 | 36 | 19 | 0.85 |
| 450 | 300 | 281 | 24 | 282 | 31 | 45 | 55 | 251 | 18 | 261 | 23 | 34 | 51 | 25 | 0.91 |
| 700 | 100 | 323 | 6½ | 301 | 25 | 24 | 23 | 239 | 8 | 231 | 9 | 22 | 20 | 17 | 0.66 |
| 700 | 200 | 314 | 16 | 300 | 31 | 24 | 45 | 252 | 10 | 254 | 21 | 23 | 36 | 26 | 0.77 |
| 700 | 300 | 279 | 17 | 282 | 31 | 29 | 55 | 253 | 10 | 261 | 23 | 24 | 51 | 37 | 0.82 |
| 700 | 450 | 279 | 17 | 281 | 24 | 29 | 45 | 253 | 10 | 251 | 18 | 24 | 34 | 21 | 0.85 |
| 900 | 100 | 238 | 3½ | 301 | 25 | 22 | 23 | 234 | 5½ | 231 | 9 | 17 | 20 | 19 | 0.50 |
| 900 | 200 | 252 | 7 | 300 | 31 | 22 | 45 | 250 | 7 | 254 | 21 | 18 | 36 | 34 | 0.54 |
| 900 | 300 | 259 | 12 | 282 | 31 | 25 | 55 | 253 | 8 | 261 | 23 | 17 | 51 | 45 | 0.60 |
| 900 | 450 | 259 | 12 | 281 | 24 | 25 | 45 | 253 | 8 | 251 | 18 | 17 | 34 | 29 | 0.62 |
| 900 | 700 | 259 | 12 | 279 | 17 | 25 | 29 | 253 | 8 | 253 | 10 | 17 | 24 | 12 | 0.86 |
| SPRING (March–May) | | | | | | | | | | | | | | | |
| 200 | 100 | 232 | 3 | 339 | 2½ | 26 | 14 | 296 | 12 | 305 | 7 | 38 | 18 | 19 | 0.77 |
| 300 | 100 | 199 | 5 | 339 | 2½ | 33 | 14 | 291 | 10 | 305 | 7 | 39 | 18 | 19 | 0.74 |
| 300 | 200 | 209 | 8 | 259 | 5½ | 35 | 28 | 300 | 14 | 310 | 15 | 45 | 41 | 44 | 0.92 |
| 450 | 100 | 213 | 2 | 339 | 2½ | 24 | 14 | 286 | 5 | 305 | 7 | 31 | 18 | 22 | 0.73 |
| 450 | 200 | 220 | 5 | 259 | 5½ | 26 | 28 | 295 | 8½ | 310 | 15 | 34 | 41 | 20 | 0.89 |
| 450 | 300 | 231 | 7 | 226 | 11 | 32 | 40 | 279 | 11 | 298 | 18 | 35 | 46 | 20 | 0.94 |
| 700 | 100 | 88 | 2½ | 339 | 2½ | 19 | 14 | 215 | 5 | 305 | 7 | 22 | 18 | 20 | 0.62 |
| 700 | 200 | 148 | 2 | 259 | 5½ | 20 | 28 | 257 | 5½ | 310 | 15 | 23 | 41 | 30 | 0.78 |
| 700 | 300 | 204 | 2 | 226 | 11 | 23 | 40 | 252 | 8 | 298 | 18 | 24 | 46 | 32 | 0.84 |
| 700 | 450 | 204 | 2 | 231 | 7 | 23 | 32 | 252 | 8 | 279 | 11 | 24 | 35 | 20 | 0.86 |
| 900 | 100 | 98 | 5 | 339 | 2½ | 17 | 14 | 174 | 6 | 305 | 7 | 17 | 18 | 21 | 0.53 |
| 900 | 200 | 127 | 4½ | 259 | 5½ | 18 | 28 | 201 | 5 | 310 | 15 | 18 | 41 | 39 | 0.57 |
| 900 | 300 | 136 | 3 | 226 | 11 | 19 | 40 | 220 | 6 | 298 | 18 | 19 | 46 | 41 | 0.63 |
| 900 | 450 | 136 | 3 | 231 | 7 | 19 | 32 | 220 | 6 | 279 | 11 | 19 | 35 | 29 | 0.61 |
| 900 | 700 | 136 | 3 | 204 | 2 | 19 | 23 | 220 | 6 | 252 | 8 | 19 | 24 | 15 | 0.80 |
| AUTUMN (September–November) | | | | | | | | | | | | | | | |
| 200 | 100 | 232 | 3 | 339 | 2½ | 26 | 14 | 296 | 12 | 305 | 7 | 38 | 18 | 19 | 0.77 |
| 300 | 100 | 199 | 5 | 339 | 2½ | 33 | 14 | 291 | 10 | 305 | 7 | 39 | 18 | 19 | 0.74 |
| 300 | 200 | 209 | 8 | 259 | 5½ | 35 | 28 | 300 | 14 | 310 | 15 | 45 | 41 | 44 | 0.92 |
| 450 | 100 | 213 | 2 | 339 | 2½ | 24 | 14 | 286 | 5 | 305 | 7 | 31 | 18 | 22 | 0.73 |
| 450 | 200 | 220 | 5 | 259 | 5½ | 26 | 28 | 295 | 8½ | 310 | 15 | 34 | 41 | 20 | 0.89 |
| 450 | 300 | 231 | 7 | 226 | 11 | 32 | 40 | 279 | 11 | 298 | 18 | 35 | 46 | 20 | 0.94 |
| 700 | 100 | 88 | 2½ | 339 | 2½ | 19 | 14 | 215 | 5 | 305 | 7 | 22 | 18 | 20 | 0.62 |
| 700 | 200 | 148 | 2 | 259 | 5½ | 20 | 28 | 257 | 5½ | 310 | 15 | 23 | 41 | 30 | 0.78 |
| 700 | 300 | 204 | 2 | 226 | 11 | 23 | 40 | 252 | 8 | 298 | 18 | 24 | 46 | 32 | 0.84 |
| 700 | 450 | 204 | 2 | 231 | 7 | 23 | 32 | 252 | 8 | 279 | 11 | 24 | 35 | 20 | 0.86 |
| 900 | 100 | 98 | 5 | 339 | 2½ | 17 | 14 | 174 | 6 | 305 | 7 | 17 | 18 | 21 | 0.53 |
| 900 | 200 | 127 | 4½ | 259 | 5½ | 18 | 28 | 201 | 5 | 310 | 15 | 18 | 41 | 39 | 0.57 |
| 900 | 300 | 136 | 3 | 226 | 11 | 19 | 40 | 220 | 6 | 298 | 18 | 19 | 46 | 41 | 0.63 |
| 900 | 450 | 136 | 3 | 231 | 7 | 19 | 32 | 220 | 6 | 279 | 11 | 19 | 35 | 29 | 0.61 |
| 900 | 700 | 136 | 3 | 204 | 2 | 19 | 23 | 220 | 6 | 252 | 8 | 19 | 24 | 15 | 0.80 |

TABLE IV—STRETCH VECTOR CORRELATION OF WINDS AT TWO HEIGHTS

| | Period | No. of obs. | Levels | | Wind at upper level | | | Correlation coefficient |
|---------------------------|------------------------|-------------|--------|-----|---------------------|-----|---------|-------------------------|
| | | | | | Mean | | S.V.D.* | |
| | | | km. | km. | ° | kt. | kt. | |
| Tateno | Dec. 1950, 51, 52 | 86 | 6 | 9 | | 109 | 42 | 0.68 |
| Tateno | Dec.-Feb. 1950, 51, 52 | 242 | 6 | 9 | | 110 | 47 | 0.66 |
| | | | mb. | mb. | | | | |
| Malta | July-Aug. 1950 | 112 | 300 | 200 | | 36 | 24 | 0.70 |
| Liverpool .. | Jan.-Feb. 1951 | 183 | 200 | 150 | 280 | 21 | 25 | 0.91 |
| Langenhagen .. | July-Aug. 1951 | 116 | 200 | 150 | 252 | 22 | 24 | 0.88 |
| Liverpool .. | Apr. 1951, 52 | 211 | 300 | 150 | 274 | 17 | 20 | 0.82 |
| Habbaniya .. | Apr. 1951, 52 | 110 | 300 | 150 | 262 | 50 | 31 | 0.70 |
| Downham Market and Hemsby | Apr. 1951, 52 | 223 | 300 | 150 | 269 | 17 | 19 | 0.72 |
| Liverpool .. | Jan.-Feb. 1951 | 183 | 150 | 100 | 279 | 15 | 22 | 0.88 |
| Downham Market and Hemsby | Apr. 1951, 52 | 223 | 150 | 100 | 270 | 11 | 15 | 0.90 |

* S.V.D. = Standard vector deviation.

TABLE V—CORRELATION COEFFICIENTS BETWEEN WINDS AT VERY HIGH LEVELS OVER SOUTHERN ENGLAND

(Observations obtained by big balloons)

| Lower level | Upper level | Winter (Nov.-Apr.) (25 observations) | | | Summer (May-Oct.) (26 observations) | | | Year Stretch Vector |
|-------------|-------------|---|----------------------|----------------|--|----------------------|----------------|---------------------|
| | | S.-N. compo- nent | W.-E. compo- nent | Stretch Vector | S.-N. compo- nent | W.-E. compo- nent | Stretch Vector | |
| | <i>feet</i> | | | | | | | |
| 80,000 | 100,000 | 0.70 | 0.53 | 0.56 | 0.22 | 0.40 | 0.31 | 0.56 |
| 50,000 | 100,000 | 0.33 | 0.41 | 0.35 | 0.62 | 0.23 | 0.36 | 0.30 |
| 50,000 | 80,000 | 0.65† | 0.09† | 0.34† | 0.23 | 0.55 | 0.42 | 0.31 |

No. of observations † 36.

TABLE VI—STRETCH VECTOR CORRELATION COEFFICIENTS BETWEEN WINDS AT DIFFERENT HEIGHTS

Period: 1949

| Lower level | Upper level | Habbaniya | | | | Nairobi Year | Barrow, Alaska | | | |
|-------------|------------------|-----------|--------|--------|--------|--------------|----------------|--------|--------|--------|
| | | Winter | Spring | Summer | Autumn | | Winter | Spring | Summer | Autumn |
| | <i>millibars</i> | | | | | | | | | |
| 300 | 200 | 0.92 | 0.85 | 0.92 | 0.82 | 0.43 | 0.62 | 0.77 | 0.75 | 0.60 |
| 500 | 200 | 0.62 | 0.67 | 0.43 | 0.57 | 0.09 | 0.34 | 0.71 | 0.60 | 0.51 |
| 500 | 300 | 0.77 | 0.79 | 0.62 | 0.70 | 0.22 | 0.51 | 0.79 | 0.74 | 0.83 |
| 700 | 200 | 0.47 | 0.49 | 0.30 | 0.40 | .. | 0.21 | 0.52 | 0.51 | 0.38 |
| 700 | 300 | 0.64 | 0.69 | 0.21 | 0.43 | 0.04 | 0.45 | 0.53 | 0.58 | 0.49 |
| 700 | 500 | 0.77 | 0.63 | 0.53 | 0.63 | 0.22 | 0.60 | 0.69 | 0.76 | 0.71 |

Using the data given in these tables, diagrams can be constructed showing the variation in correlation coefficients between winds at various heights over the British Isles and over Habbaniya. This has been done, and is shown in Figs. 2 and 3. The lines form a reasonably consistent pattern and show that over the British Isles there is a high correlation between winds at 600 mb. and other levels even with those some distance above the tropopause, no doubt because

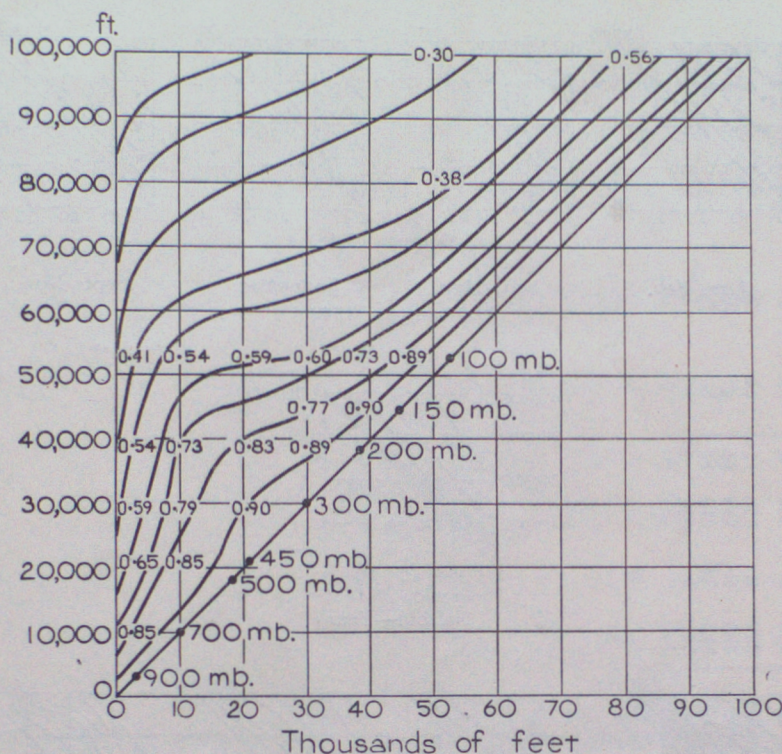


FIG. 2—STRETCH VECTOR CORRELATION COEFFICIENTS BETWEEN WINDS AT TWO HEIGHTS OVER SOUTHERN ENGLAND

the level of 600 mb. is about the centre of mass of the troposphere. It is at present not known how the lines should be drawn in the top left-hand corner of the diagrams, but even there they must follow somewhat on the pattern indicated.

It would seem that the correlation coefficients up to 200 mb. are lower over Habbaniya and very much lower over Nairobi than over the British Isles. This is not unexpected since the effects of geostrophic control are so much smaller towards the equator. One would expect the wind variations due to pressure gradients to affect a considerable depth of the atmosphere simultaneously, but the ageostrophic components of wind are likely to be propagated upwards or downwards by turbulence, a much slower process. In the tropics the ageostrophic components become increasingly predominant. The comparatively small correlation coefficients in the Arctic winter, as illustrated at Barrow, Alaska, are noteworthy.

Provided that the average winds and standard vector deviations are known at different heights it is possible to construct the standard vector deviation of the mean winds up to any height, and this quantity is of interest in a number of problems.

Let \mathbf{V}_z be the average wind at any height z and \mathbf{V}_{nh}/n or $(1/n) \Sigma \mathbf{V}_z$ be the wind averaged from 0 to nh in steps of thickness h . Let \mathbf{v}_z be the departure from the average wind at height z on a particular occasion, and further let it be assumed that this wind is averaged over a step h deep. Then the departure from the average of the mean wind over a layer from 0 to nh is given by $(1/n) \Sigma \mathbf{v}_z$ ($= \mathbf{v}_{nh}/n$, say) and the mean wind up to nh on this occasion is $(1/n) \Sigma (\mathbf{V}_z + \mathbf{v}_z)$.

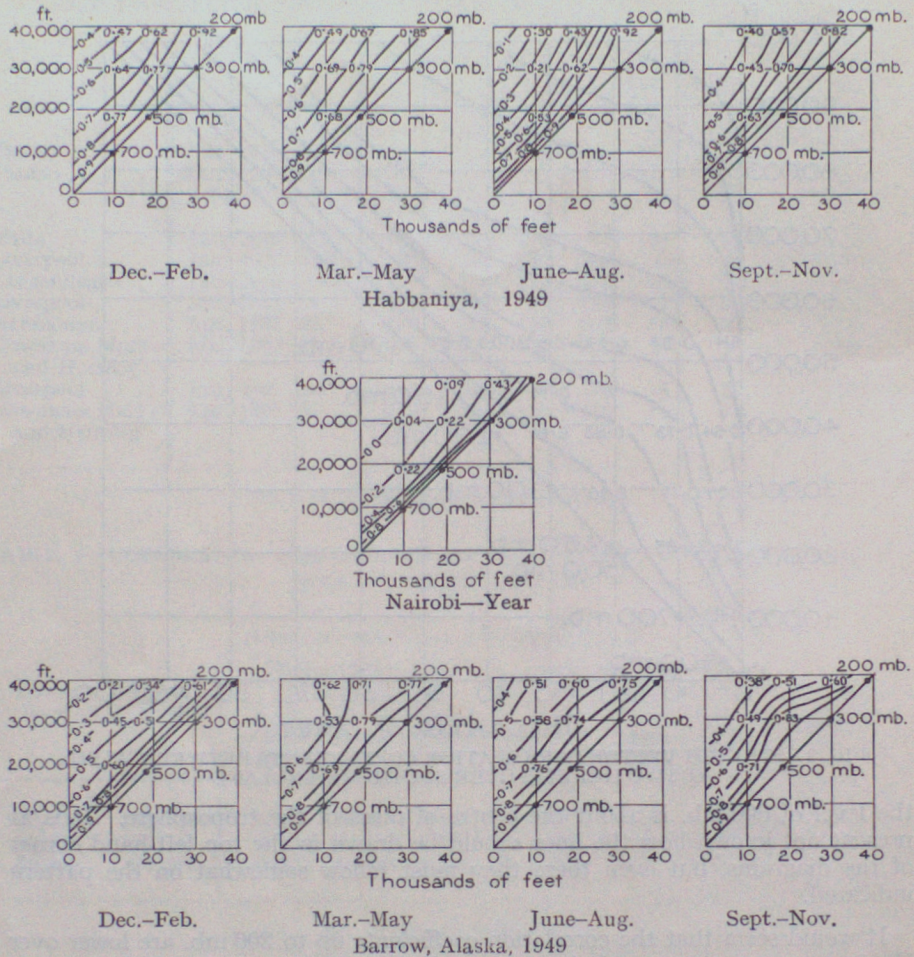


FIG. 3—SEASONAL VARIATION OF STRETCH VECTOR CORRELATION COEFFICIENTS BETWEEN DIFFERENT HEIGHTS

Moreover,

$$\left(\frac{1}{n} \mathbf{v}_{nh}\right)^2 = \frac{1}{n^2} \left(\sum_1^n \mathbf{v}_z\right)^2 = \frac{1}{n^2} (\mathbf{v}_a^2 + \mathbf{v}_b^2 + \mathbf{v}_c^2 + \dots + 2\mathbf{v}_a \cdot \mathbf{v}_b + 2\mathbf{v}_a \cdot \mathbf{v}_c + \dots). \quad (2)$$

If we denote the average of a large number of occasions by a vinculum then the variance of the mean wind up to height nh is given by

$$\begin{aligned} \Xi^2 &= \overline{\left(\frac{1}{n} \mathbf{v}_{nh}\right)^2} \\ &= \frac{1}{n^2} \left\{ \overline{\mathbf{v}_a^2} + \overline{\mathbf{v}_b^2} + \overline{\mathbf{v}_c^2} + \dots + 2\sqrt{(\overline{\mathbf{v}_a^2})} \sqrt{(\overline{\mathbf{v}_b^2})} r_{ab} + 2\sqrt{(\overline{\mathbf{v}_a^2})} \sqrt{(\overline{\mathbf{v}_c^2})} r_{ac} + \dots \right\} \\ &= \frac{1}{n^2} \sum_1^n \sum_1^n r_{pq} \sqrt{(\overline{\mathbf{v}_p^2})} \sqrt{(\overline{\mathbf{v}_q^2})}; \quad \dots \dots \dots (3) \end{aligned}$$

or, proceeding to the limit,

$$\Xi^2 = \frac{2}{n^2} \int_0^x \int_0^y \sigma_x \sigma_y r_{xy} dx dy, \quad \dots \dots \dots (4)$$

where σ_x and σ_y are standard vector deviations of winds at heights x and y and r_{xy} is the stretch correlation coefficient between winds at those two heights, z being written for the height nh . It follows directly that

$$\frac{1}{z^2} \left\{ \int_0^z (\mathbf{V}_z + \mathbf{v}_z) dz \right\}^2 = \frac{1}{z^2} \left\{ \int_0^z \mathbf{V}_z dz \right\}^2 + \frac{2}{z^2} \int_0^z dy \int_0^y \sigma_x \sigma_y r_{xy} dz. \quad \dots (5)$$

The average winds and the standard vector deviation of winds are given month by month for Larkhill and Habbaniya in *Upper air data*^{5,6}. Further, some estimates have been made of the mean wind components at 50,000, 80,000 and 100,000 ft. over southern England, and also at 60 mb. (63,000 ft.) over Habbaniya.

TABLE VII—VALUES OF WIND COMPONENTS AT VARIOUS HEIGHTS OVER SOUTHERN ENGLAND

| | | Dec.-Feb. | | Mar.-May | | June-Aug. | | Sept.-Nov. | | May-Oct. | | Nov.-Apr. | |
|---------|-----|-----------|-------|----------|-------|-----------|-------|------------|-------|----------|-------|-----------|-------|
| | | S.-N. | W.-E. | S.-N. | W.-E. | S.-N. | W.-E. | S.-N. | W.-E. | S.-N. | W.-E. | S.-N. | W.-E. |
| ft. | mb. | knots | | | | | | | | | | | |
| 100,000 | | -11 | +70 | -1 | -6 | -1 | -23 | -6 | +19 | -1 | -15 | -8 | +45 |
| 80,000 | | -14 | +23 | -3 | -6 | -2 | -13 | -4 | +9 | -3 | -8 | -9 | +15 |
| 53,000 | 100 | -5 | +20 | -2 | +10 | +2 | +10 | -3 | +18 | +1 | +11 | -5 | +17 |
| 50,000 | | -3 | +23 | -4 | +11 | -4 | +12 | -2 | +18 | -3 | +13 | -3 | +19 |
| 45,000 | 150 | -9 | +23 | -3 | +13 | +1 | +21 | -3 | +28 | +1 | +21 | -7 | +22 |
| 39,000 | 200 | -11 | +28 | -6 | +17 | 0 | +30 | -5 | +35 | 0 | +29 | -11 | +27 |
| 30,000 | 300 | -10 | +32 | -7 | +21 | +1 | +32 | -1 | +35 | +2 | +30 | -11 | +30 |
| 18,000 | 500 | -3 | +24 | -4 | +15 | +1 | +22 | +2 | +24 | +2 | +20 | -4 | +23 |
| 10,000 | 700 | -1 | +18 | -2 | +10 | +2 | +14 | +4 | +17 | +3 | +13 | -1 | +16 |
| 5,000 | 850 | +2 | +14 | -1 | +6 | +2 | +10 | +3 | +13 | +2 | +9 | +1 | +12 |

TABLE VIII—QUARTERLY VALUES OF WIND COMPONENTS AT VARIOUS HEIGHTS OVER HABBANIYA

| | | Dec.-Feb. | | Mar.-May | | June-Aug. | | Sept.-Nov. | |
|--------|-----|-----------|-------|----------|-------|-----------|-------|------------|-------|
| | | S.-N. | W.-E. | S.-N. | W.-E. | S.-N. | W.-E. | S.-N. | W.-E. |
| ft. | mb. | knots | | | | | | | |
| 63,000 | 60 | +3 | +33 | +2 | +13 | +4 | -15 | -2 | +19 |
| 53,000 | 100 | 0 | +43 | +6 | +35 | +10 | +9 | +6 | +31 |
| 45,000 | 150 | -2 | +63 | +5 | +49 | +11 | +23 | +11 | +48 |
| 39,000 | 200 | -2 | +73 | +12 | +64 | +10 | +28 | +11 | +52 |
| 30,000 | 300 | 0 | +63 | +12 | +53 | +7 | +26 | +9 | +49 |
| 18,000 | 500 | +2 | +33 | +7 | +29 | +3 | +13 | +4 | +21 |
| 10,000 | 700 | +2 | +15 | +3 | +16 | -2 | +10 | +1 | +14 |
| 5,000 | 850 | 0 | +6 | 0 | +5 | -9 | +10 | -2 | +5 |

Tables VII-IX together with the values shown in Figs. 2 and 3 give the data necessary to evaluate equation (4). Table X gives the mean wind to various heights

$$\left(\frac{1}{z} \int_0^z \mathbf{V} dz \right)$$

and the standard vector deviation (Σ) for southern England and Habbaniya in summer and winter, from which expression (4) can at once be derived.

Thus even though the winter winds are on average stronger in the higher levels over Habbaniya than over England the deviations in the mean winds are smaller. Moreover at Nairobi, in the tropics, the correlation coefficients between winds at different levels is much lower than over the British Isles and also the

normal winds at levels to at least 40,000 ft. are quite light, from which it may be judged that the standard vector deviation of the mean winds up to different heights is much smaller in the tropics than over the British Isles.

TABLE IX—STANDARD VECTOR DEVIATIONS OF WIND OVER SOUTHERN ENGLAND AND HABBANIYA

| | | Southern England | | Habbaniya | | | |
|---------|-----|---------------------------|--------------------------|---------------|--------------|---------------|----------------|
| | | Winter (Nov.– Apr.) | Summer (May– Oct.) | Dec.– Feb. | Mar.– May | June– Aug. | Sept.– Nov. |
| ft. | mb. | <i>knots</i> | | | | | |
| 100,000 | .. | 39 | 9 | .. | .. | .. | .. |
| 80,000 | .. | 21 | 8 | .. | .. | .. | .. |
| 50,000 | 100 | 23 | 16 | 23 | 23 | 19 | 22 |
| 45,000 | 150 | 29 | 27 | 32 | 33 | 26 | 28 |
| 39,000 | 200 | 41 | 39 | 41 | 38 | 29 | 32 |
| 30,000 | 300 | 53 | 45 | 45 | 38 | 27 | 30 |
| 18,000 | 500 | 39 | 31 | 30 | 23 | 17 | 19 |
| 10,000 | 700 | 29 | 21 | 19 | 18 | 15 | 15 |
| 5,000 | 850 | 26 | 19 | 16 | 16 | 12 | 13 |

TABLE X—VECTOR MEAN WINDS AND STANDARD VECTOR DEVIATION OF WINDS AVERAGED OVER THE LAYER FROM THE SURFACE TO VARIOUS HEIGHTS OVER SOUTHERN ENGLAND AND HABBANIYA

| | Southern England | | | | | | Habbaniya | | | | | |
|---------|-----------------------|-----|-----|----------------------|-----|-----|-----------------------|-----|-----|-----------------------|-----|-----|
| | Winter (Nov.–Apr.) | | | Summer (May–Oct.) | | | Winter (Dec.–Feb.) | | | Summer (June–Aug.) | | |
| | Mean wind S.V.D.* | | | Mean wind S.V.D. | | | Mean wind S.V.D. | | | Mean wind S.V.D. | | |
| ft. | ° | kt. | kt. | ° | kt. | kt. | ° | kt. | kt. | ° | kt. | kt. |
| 100,000 | 287 | 22 | 22 | 276 | 7 | 15 | .. | .. | .. | .. | .. | .. |
| 80,000 | 287 | 19 | 24 | 271 | 11 | 18 | .. | .. | .. | .. | .. | .. |
| 60,000 | 285 | 19 | 28 | 268 | 16 | 22 | 270 | 44 | .. | 254 | 17 | .. |
| 50,000 | 284 | 21 | 31 | 268 | 17 | 25 | 270 | 45 | .. | 259 | 19 | .. |
| 40,000 | 284 | 23 | 34 | 265 | 19 | 27 | 269 | 41 | 27 | 264 | 18 | 16 |
| 30,000 | 281 | 21 | 34 | 262 | 17 | 26 | 268 | 31 | 24 | 272 | 14 | 14 |
| 20,000 | 275 | 16 | 29 | 258 | 13 | 21 | 266 | 19 | 19 | 288 | 12 | 12 |
| 10,000 | 265 | 12 | 25 | 255 | 9 | 18 | 266 | 7 | 15 | 310 | 13 | 11 |
| 5,000 | 260 | 10 | 24 | 255 | 7 | 18 | 270 | 4 | 15 | 320 | 16 | 11 |

* S.V.D. = Standard vector deviation.

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Erratum

Page 2, line 36; for " $\sqrt{\{(\Sigma|\mathbf{v}_b|^2)(n-1)\}}$ " read " $\sqrt{\{(\Sigma|\mathbf{v}_b|^2)/(n-1)\}}$ "

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