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**Met O 11 Scientific Note No. 13**

**A simple two phase  
precipitation scheme for use in  
numerical weather prediction models**

**by**

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# A Simple Two Phase Precipitation Scheme For Use In Numerical Weather Prediction Models

By B. W. Golding

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## SUMMARY

A simple parametrization of precipitation processes is proposed for use in numerical weather prediction models. It has one supplementary variable, which is liquid water or ice water depending on the situation of the grid box. Glaciation is assumed to occur when the cloud top is colder than  $-15^{\circ}\text{C}$ . Liquid water cloud at warmer temperatures then becomes ice, and the process of ice growth occurs as the critical humidity for condensation becomes saturation over ice. Simplified formulae for evaporation and melting are also presented and their behaviour discussed.

## 1. INTRODUCTION

In the development of numerical weather prediction (NWP) models, the ability to produce forecasts of precipitation has been a major goal (Bushby & Timpson 1967). In mid-latitude depressions, the residence time of air parcels in areas of ascent is long compared with the timescales on which microphysical processes act and so the overall precipitation closely matches the moisture convergence into the system. Most NWP models parametrize this sort of precipitation by setting a humidity threshold (Bell & Dickinson



1987) - usually 100% - above which moisture condenses, releasing latent heat, and then falls to the ground. Evaporation is modelled during fall but no cloud is left behind. Deep convective clouds, predominant in the tropics, are modelled differently, as sub-grid scale phenomena. A variety of parametrisation schemes is used, most relying on some form of parcel calculation (Kuo 1965, 1974, Arakawa & Schubert 1974, Fritsch & Chappell 1980).

As NWP models attempt to represent the atmosphere in more detail, so the need for a more sophisticated treatment of precipitation processes increases. On the large scale, the importance of the large sub-tropical cloud sheets in the earth's radiation budget argues for the explicit prediction of cloud water. At the other end of the size spectrum, the regional forecasting of stratus and fog for transport makes the same demand. An attempt was made by Sundqvist (1978) to resolve this problem. His scheme can be considered a minimal one in that only one extra variable is required, the cloud water mixing ratio, and the equations describing precipitation of this cloud water are simple. However, his scheme fails to represent the important differences between the precipitation from deep, glaciated cloud and from warm stratus and stratocumulus clouds. This difference has been understood for over half a century in terms of the presence or not of ice crystals in the cloud (see e.g. Mason 1971). Indeed Bergeron (1935) asserted that precipitation size particles could only be formed as ice crystals. It is now recognised that other processes can produce rain from clouds with high liquid water content such as tropical convective clouds. However at the scales represented by numerical weather



prediction models, especially in mid-latitudes, there would seem to be a good case for distinguishing between glaciated and liquid clouds in any parametrization scheme.

Cloud modellers have developed many such schemes (see e.g. Pielke 1984) but in general, they involve the inclusion of several additional variables. The present scheme is put forward as a simplified parametrization which, like Sundqvist (1978) requires only one additional variable, but which represents the important differences between glaciated and liquid clouds. The scheme has been developed for use in the UK Met Office Mesoscale Model (Golding 1986) but has recently also been adopted with some modifications for use in a Climate Model (Smith, personal communication).

## 2. OUTLINE OF SCHEME

In order to parametrize the cloud microphysics with only one variable, it is necessary to make simplifying assumptions.

Pielke(1984) describes a scheme with four variables: water cloud, ice cloud, rain, and snow. The assumptions used to reduce this set to one variable are: -

- (i) rain falls to the ground sufficiently quickly to be diagnosed rather than predicted
- (ii) water cloud and ice cloud are mutually exclusive and an algorithm can be devised to determine which is present.
- (iii) ice cloud and snow are indistinguishable except in so far as their ice water mixing ratios differ.



(iv) snow melts sufficiently quickly below the freezing level to be diagnosed rather than predicted.

Since raindrops fall at about  $5\text{ms}^{-1}$ , there is no difficulty in making the first approximation in synoptic scale models. The UKMO mesoscale model has an effective timestep of 2 minutes in the precipitation scheme, so it will take less than two timesteps for rain to fall from a 1km freezing level. The steady state approximation implicit in the assumption is only likely to be violated with such a timestep in the tropics.

In order to justify the second assumption, it is necessary to show that once ice crystals are present in a cloud, it will glaciate rapidly. For deep precipitating clouds, Mason(1971) cites calculations by Wexler & Atlas(1958) and observations by Peppler(1940) showing that such clouds are almost entirely composed of ice down to the  $0^{\circ}\text{C}$  level. Even in convective clouds, where the process of glaciation can be observed, the transition phase between mainly liquid and mainly ice is of relatively short duration.

Peppler (1940) observed that the presence of ice crystals increases rapidly in the temperature range  $-12^{\circ}\text{C}$  to  $-16^{\circ}\text{C}$ . Consistent with this,  $-15^{\circ}\text{C}$  is used as the cloud top temperature threshold for glaciation to occur in the parametrization. The effect of this is shown schematically in figure 1. Where the cloud top is colder than  $-15^{\circ}\text{C}$  all cloud colder than  $0^{\circ}\text{C}$  becomes ice. Elsewhere, the cloud is assumed to be supercooled liquid.



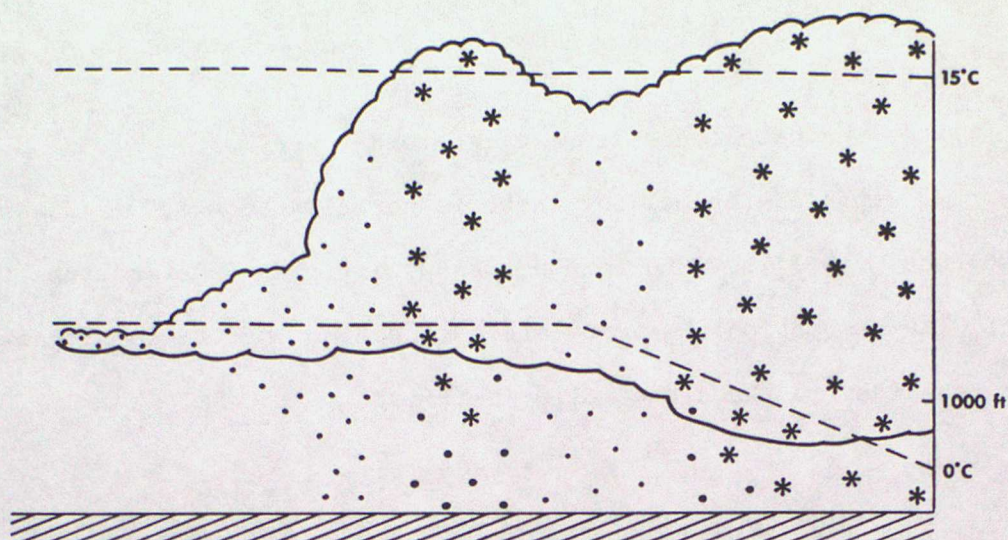


Fig. 1 Conceptual diagram of the behaviour of the proposed scheme

The third assumption is dependent on particle size. Whereas liquid water condenses on cloud condensation nuclei in large numbers and at small supersaturation to produce many small drops, ice is deposited on very few condensation nuclei and requires high supersaturation to do so. Once a crystal has formed, it will grow rapidly so long as the air is supersaturated with respect to ice (Bergeron 1935). Thus for a wide range of ice water mixing ratios, the crystals are able to grow sufficiently to fall at nearly  $1\text{ms}^{-1}$ . Use of a constant fall speed is very attractive. However, if applied to all ice cloud, it results in excessive depletion of cirrus. The observed dependence of fall speed on mixing ratio documented by Heymsfield(1977) is therefore adopted. The processes of ice growth are ignored here since the assumption of instantaneous glaciation requires all moisture in excess of ice saturation to be instantly condensed. Thus, differences between the behaviour of crystals and



snowflakes in these processes are irrelevant.

The depth of the melting layer is normally about 300m (Klaassen 1988), so with a  $1\text{ms}^{-1}$  fall speed, snow will take 2-3 timesteps to melt. The assumption of steady state conditions needed to diagnose its behaviour is therefore fully justified

### 3. IMPLEMENTATION AND BEHAVIOUR

#### (a) *Diagnosis of cloud.*

In order to implement the scheme, a way of diagnosing the cloud water and its phase is needed. In the UKMO mesoscale model a partial cloud formulation using a TKE closure turbulence scheme is employed (Smith 1984, Golding 1987). However, the precipitation scheme will work quite adequately with a simple 100% RH threshold. The critical humidity mixing ratio for condensation depends on whether ice is already present at sub-zero temperatures. If it is not, the critical curve is taken to be saturation over water at temperatures higher than  $-15^{\circ}\text{C}$ , over ice at temperatures below  $-40^{\circ}\text{C}$ , and a linear interpolation between the two for intermediate temperatures. If ice is already present (from the previous timestep or falling from the next higher level), saturation over ice is used at all temperatures.

If the temperature is colder than  $-15^{\circ}\text{C}$  all condensate is ice. Otherwise, the diagnosed phase of the condensate depends on whether ice is already present (as above). Condensate is water if ice was



not present and ice if it was.

The details of the treatment of this transition may need to be varied according to the application to which the model is being put.

(b) *Precipitation of ice cloud.*

The precipitation of ice is determined by its fall speed which is taken from the observations of Heymsfield (1977).

$$V = 3.23 (\rho m)^{0.17} \text{ ms}^{-1} \quad (1)$$

where  $\rho$  is the density of air, and  $m$  is the cloud ice mixing ratio in  $\text{kg. kg}^{-1}$ .

With the fall velocity defined, the precipitation rate may then be written

$$\frac{\delta P_s}{\delta z} = \frac{\delta (\rho V m)}{\delta z} \text{ kg m}^{-3} \text{ s}^{-1}. \quad (2)$$

where  $P_s$  is the snowfall rate and  $z$  is height.

Except when overridden by the evaporation formula below, the snowfall at any level depends only on the ice water mixing ratio in the layer above. If the fall speed is approximated by  $V = 1 \text{ ms}^{-1}$ , the snow rate can be written  $P_s = \rho m \text{ kg m}^{-2} \text{ s}^{-1}$  or for  $\rho = 1$ ,  $m = 1 \text{ g. kg}^{-1}$ , and converting to  $\text{mmhr}^{-1}$ ,  $P_s = 3.6 \text{ mmhr}^{-1}$ . Table 1 shows the dependence of  $P_s$  on  $m$  using the fall speed given in (1) and illustrates the reduction due to lower fall speeds at concentrations such as might be found in cirrus cloud.



TABLE 1 ICE PRECIPITATION RATE FOR VARIOUS VALUES OF CLOUD ICE CONTENT

Cloud ice mixing ratio in g.kg <sup>-1</sup>	.01	.05	.1	.5	1.0
Precipitation rate in mm.hr <sup>-1</sup>	.02	.11	.24	1.6	3.6

(c) *Evaporation of snow*

Equations (1), (2) imply that snow falling into a cloud free layer will evaporate entirely. This is unrealistic except for very deep layers, so in such cases the precipitation equation is overridden by the following simplified evaporation equation based on Rutledge and Hobbs (1983) .

$$\frac{\delta P_s}{\delta z} = \chi P_s (S_1 - 1) \text{ kg.m}^{-3}.\text{s}^{-1} \quad (3)$$

where  $\chi = 3 \times 10^{-3} + 1 \times 10^{-4}T$ , T the temperature in °C

$S_1 = q/q_{s1}$  ,  $q_{s1}$  the saturation humidity ratio over ice, and q the humidity mixing ratio of the environment. A fall velocity of  $1\text{ms}^{-1}$  is assumed here.

This formula controls the depth that snow must fall to fully evaporate. It is independent of precipitation rate and depends only on the the sub-saturation and the temperature. At  $-10^\circ\text{C}$  the depth is  $500/(1-S_1)$  metres.



(d) *Melting of snow*

This is again represented by a simplified form of that in Rutledge & Hobbs (1983). Care must be taken in defining the relevant precipitation rate, however, since the equation appears to transfer precipitation from snow to rain. Matsuo and Sasyo (1981a) showed that in reality, snow flakes absorb the melt water from their outer edges like a sponge so it is appropriate to add together the frozen and melted parts in the melting formula.

$$\frac{\delta P_s}{\delta z} = -\frac{\delta P_r}{\delta z} = -0.0028 T_w (P_r + P_s) \text{ kg. m}^{-2} \text{ s}^{-1} \quad (4)$$

where  $T_w$  is the wet bulb temperature in °C and  $P_r$  is the rain rate. The fall velocity has been approximated as  $1 \text{ ms}^{-1}$  in this equation. It will be noted that this formula uses wet bulb rather than dry bulb temperature to control melting. Following Matsuo and Sasyo (1981b) it is argued that the evaporation of snow in subsaturated air will cool the snow to its wet bulb temperature thus delaying melting until the wet bulb freezing level is reached. This is also consistent with some observational studies (eg Lumb(1963)). The difference is significant where dry low level air undercuts precipitating cloud, but in heavy precipitation the humidity of the sub-cloud layer is soon raised by evaporation.

In neutral conditions the snow melt equation (4) gives total melting about 300m below wet bulb freezing level, consistent with observations both of the depth of the bright band (Klaassen 1988),



and of the surface temperature required for snow (Boyden 1964, Murray 1959). With constant wet bulb lapse rate ( $\Gamma$ ), the equation is quadratic in distance below the freezing level so the melting layer has a depth:

$$\Delta z = \sqrt{1/0.0014\Gamma}$$

Table 2 shows the dependence on lapse rate of this depth.

TABLE 2 DEPTH OF MELTING LAYER FOR VARIOUS VALUES OF WET BULB LAPSE RATE

Wet bulb lapse rate in °C km <sup>-1</sup>	1	2	4	6	10
Melting depth in m	845	598	423	345	267

(e) *Precipitation of liquid cloud*

Since rain is not carried as a variable, its formation and modification as it falls to the ground rely on the steady state assumption. Expressions for these quantities therefore contain height rather than time as the independent variable.

The local production of rain by autoconversion and coalescence within a cloud is represented in the form used by Sundqvist (1978)

$$\frac{\delta Pr_{local}}{\delta z} = c_L \rho_m \{ 1 - \exp(- (m_c/c_m)^2) \} \quad \text{kg m}^{-3} \text{ s}^{-1} \quad (5)$$

where  $c_L = 10^{-4} \text{ s}^{-1}$  is a rate constant

$c_m = 8 \times 10^{-4} \text{ kg kg}^{-1}$  controls the humidity mixing ratio of liquid water required to give significant precipitation



$m_c = m/CF$  is the local humidity mixing ratio within cloud in a partially cloudy layer with cloud fraction CF.

With these settings for the constants there is little production of rain below  $0.4 \text{ g kg}^{-1}$  consistent with observations.

The local production is enhanced by accretion when rain (or melting snow) is falling through a cloud layer. This term has the simple form used by Golding & Machin (1984) based on Rutledge & Hobbs (1983).

$$\frac{\delta Pr_{acc}}{\delta z} = \rho m (Pr + Ps) / c_a \quad \text{kg m}^{-3} \text{ s}^{-1} \quad (6)$$

$$c_a = 1 \quad \text{kg m}^{-3}$$

This term is much more efficient at producing rain than the local production term Eq. 5.

For purely liquid cloud, the rain falling out of cloud base will depend on the depth of cloud and on the whole cloud water profile through the cloud. Stratiform liquid water clouds often have a cloud water profile close to that expected from adiabatic ascent. Table 3 shows the rainfall rate at cloud base for varying depths of cloud assuming such a profile. For the deeper clouds, a maximum of  $1.2 \text{ g kg}^{-1}$  has been imposed. The cloud base is at 610m in an ICAO standard atmosphere and integration is over the standard model resolution at 1010m, 1510, 2110, 2810m. As required, shallow cloud layers produce very little rain which will be evaporated before it reaches the ground. Deeper clouds are able to produce significant, though not intense, rain.



TABLE 3 PRECIPITATION RATE AT CLOUD BASE (610m) FROM LIQUID WATER  
CLOUDS OF VARYING DEPTH

Height of cloud top in m	1010	1510	2110	2810
Cloud depth in m	450	1000	1650	2400
Cloud water at top in g.kg <sup>-1</sup>	0.4	0.9	1.2	1.2
Precip. rate at base in mm.hr <sup>-1</sup>	0.02	0.18	0.8	2.8

(f) *Evaporation of rain*

Various formulae have been proposed for evaporation of rain and the choice is not relevant to the substance of the scheme. However, for completeness, the equation used in the UKMO mesoscale model is given, based on Orville and Kopp (1977) .

$$\frac{\delta Pr_{\text{evap}}}{\delta z} = - C_E (q_s - q) \frac{(40.09 Pr^{0.4} + 443.4 Pr^{0.6})}{5.5 q_s + 0.044/\rho} \quad (7)$$

where  $C_E = 1.5 \times 10^{-5}$

and  $q_s$  is the saturation humidity mixing ratio over water.

The behaviour of this equation is illustrated in Table 4. A dry adiabatic sub-cloud layer has been assumed below cloud with a base at 1010m and a cloud base temperature of 10°C.



TABLE 4 RAIN RATE AS A FUNCTION OF FALL DISTANCE BELOW CLOUD BASE  
FOR VARIOUS INITIAL RAIN RATES AT CLOUD BASE

Height	precipitation rate					
cloud base (1010m)	0.1	0.2	0.5	1.0	2.0	5.0
610m	.055	.135	.395	.848	1.78	4.64
310m	.01	.064	.270	.661	1.5	4.16
110m	-	.027	.192	.536	1.31	3.83
ground	-	.017	.164	.490	1.23	3.69

(g) *Droplet settling*

Although a very small term, this can form an important part of the budget of fog and possibly of stratus. The form used is a linear approximation to that used by Sundqvist (1978). It is currently only applied to fog in the lowest layers.

$$\frac{\delta m}{\delta t} = V_c \frac{\delta m}{\delta z} \quad V_c = 0.01 + 90 m_c \text{ ms}^{-1} \quad (8)$$

This term is not included in the rate of rainfall.

#### 4. CONCLUSIONS

A simple precipitation parametrization scheme requiring only one supplementary variable has been described. It successfully reproduces the main features distinguishing the precipitation characteristics of shallow stratiform and deep frontal clouds. It also gives a realistic representation of snow melt. It has been used successfully in trials of the UKMO Mesoscale model since 1986



and has given good results in that context (Golding 1988). The limitations of the scheme must not be ignored, however. The glaciation process is represented here as a step change occurring when the cloud top becomes colder than  $-15^{\circ}\text{C}$ . This ignores the presence of ice particles in increasing numbers as the temperature falls towards  $-15^{\circ}\text{C}$ . It also ignores the observed presence of supercooled water in stratiform cloud colder than  $-15^{\circ}\text{C}$ . The solution adopted here is a compromise between the detailed representation of physics appropriate for a cloud simulation model and the constraints of efficiency in a weather prediction or climate model. It is put forward as a simple solution to the limitations of previously proposed single phase schemes.

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