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MATHEMATICAL PSYCHOLOGY OF WAR.

By

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Dedicated to my Comrades of the motor ambulance convoy  
known as S. S. Anglaise 13, in whose company this essay  
was mainly written.

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"The cooling advice which we get from others when the fever-fit is on us is the most jarring and exasperating thing in life. Reply we cannot, so we get angry; for by a sort of self-preserving instinct which our passion has, it feels that these chill objects, if they once but gain a lodgment, will work until they have frozen the very vital spark from out of all our mood and brought our airy castles in ruin to the ground. Such is the inevitable effect of reasonable ideas over others - if they can once get a quiet hearing; and passion's cue accordingly is always and everywhere to prevent their still small voice from being heard at all. 'Let me not think of that! Don't speak to me of that!' This is the sudden cry of all those who in a passion perceive some sobering considerations about to check them in mid-career. 'Haec tibi erit janua leti', we feel. There is something so icy in this cold-water bath, something which seems so hostile to the movement of our life, so purely negative, in Reason when she lays her corpse-like finger on our heart and says, 'Halt! give up! leave off! go back! sit down!' that it is no wonder that to most men the steadying influence seems, for the time being, a very minister of death.

The strong-willed man, however, is the man who hears the still small voice unflinchingly, and who, when the death-bringing consideration comes, looks at its face, consents to its presence, clings to it, affirms it, and holds it fast, in spite of the host of exciting mental images which rise in revolt against it and would expel it from the mind. Sustained in this way by a resolute effort of attention, the difficult object ere long begins to call up its own congeners and associates and ends by changing the disposition of the man's consciousness altogether. And with his consciousness, his action changes, for the new object, once stably in possession of the field of his thoughts, infallibly produces its own motor effects".

(William James, in the chapter on The Will in his "Psychology").

The numbers in the margins, pp. 1 to 6,  
should be ignored.

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## I. INTRODUCTION.

The death-grip of opposing forces which the war developed out of the comparatively mild rivalries of peace, demands our best efforts to understand it.

The view here put forward is that the warlike striving of either side is largely, though not entirely, an instinctive reaction to the stimulus of the warlike striving of the opposing side. By an instinct is here meant an inborn tendency to perceive a certain state of affairs, and thereupon to feel in a particular way, and to act towards a corresponding end. A tendency, that is to say, which one might easily follow without considering it; and to resist which, if one judged it desirable to resist, would require an effort of will.

1.

We may make this view of the matter more definite by putting it into symbols. Let  $A_E$  be the warlike activity of the Entente,  $A_G$  the warlike activity of the Germanic alliance. Then the instinctive tendency which has been referred to above, may be expressed by two such equations as the following, in which  $t$  is the time,  $\kappa_E$  and  $\kappa_G$  are positive constants, and  $d$  is the operator of the differential calculus so that  $dA_E/dt$  means the rate of increase of  $A_E$  with time.

$$\frac{dA_E}{dt} = \kappa_E A_G \quad (1)$$

$$\frac{dA_G}{dt} = \kappa_G A_E \quad (2)$$

In other words, a war, once set afoot, is continued and augmented by the tendency to mutual reprisals.

To see further what these equations imply, let us deduce some of their consequences. If, at any instant, both  $A_G$  and  $A_E$  were zero, then, by the equations (1) and (2), their rates of increase would be zero, so that they would both remain eternally nil. This is rather like the view of those who say that armaments tend to produce war, and that if armaments could be abolished, wars would be less likely to arise. Equations (1) and (2) are in fact an exaggeration or caricature of the view that armaments tend to produce war. The equations must be modified to avoid this exaggeration, by taking account of the desires, ambitions, and rivalries which exist and which make for war independently of the reciprocal

2.

2.

instinctive stimulation of the kind represented by (1) and (2). The rivalries are perhaps mainly those of a powerful few, who are guided by intelligent self-interest, after the manner in which the Utilitarians supposed all men to be guided, and who influence the suggestible instinctive Many.

A further logical deduction from equations (1) and (2) is that if  $A_E$  and  $A_G$  were at any instant greater than zero, then they would both increase, becoming large without limit as time went on. It is true that armies and military expenditure increased as the war proceeded, but there are limits to all things human, and so we must modify the equations to take account of fatigue.

Other improvements in the equations will also be attempted. The final results are set out in section VIII2. They are led up to by the intervening argument.

## II. An apology for the use of mathematics

To have to translate one's verbal statements into mathematical formulae compels one carefully to scrutinize the ideas therein expressed. Next the possession of formulae makes it much easier to deduce the consequences. In this way absurd implications, which might have passed unnoticed in a verbal statement, are brought clearly into view and stimulate one to amend the formula. An additional advantage of a mathematical mode of expression is its brevity, which greatly diminishes the labour of memorizing the idea expressed. If the statements of an individual become the subject of a controversy, this definiteness and brevity lead to a speeding up of discussions over disputable points, so that obscurities can be cleared away, errors refuted and truth found and expressed more quickly than they could have been, had a more cumbersome method of discussion been pursued. Mathematical expressions have, however, their special tendencies to pervert thought: the definiteness may be spurious, existing in the equations but not in the phenomena to be described; and the brevity may be due to the omission of the more important things, simply because they cannot be mathematized. Against these faults we must constantly be on our guard. It will probably be impossible to avoid them entirely, and so they ought to be realized and admitted.



Mathematical expressions are in general use in various parts of Sociology, for example in Economics\*, in Anthropometry and in all cases where statistics of large masses of mankind have to be described. It therefore seems reasonable to enquire whether mathematical language can also express the behaviour of people in another case in which they also act in large groups, namely in war.

It is sometimes supposed that mathematical expressions can be used to describe only the action of objects which follow laws of a rigid, mechanical, deterministic type in all particulars. That this is not necessarily so, is shown by the fact that Prof. William James used mathematical expressions to illustrate his defence of the view that our wills are partially free. His example has been followed in this essay in this respect.

A fundamental rule of scientific method is Ockham's "Razor" to the effect that: "Entities are not to be postulated without necessity". For shaving off the superabundant growth of mathematical uncertainties and difficulties I have frequently used an analogous rule: "Formulae are not to be complicated without necessity". For example, if observation shows nothing except that two quantities increase and decrease together, and vanish together, then one quantity has been taken as a simple multiple of the other. Quadratic, cubic, or other more complicated terms have not been introduced without clear evidence to show that they were necessary. The formulae set down are therefore at best only rough approximations. Indeed on account of the difficulty of defining the fundamental quantities, there remains a general vagueness, which may scandalize some of those who have been trained in the exact sciences, but which, in the author's opinion, does not deprive the formulae of meaning, interest and suggestiveness.

In a treatise on mathematical physics it is customary to state hypotheses and then to deduce the consequences. In this essay a very different use is made of mathematical symbols. The successive formulae are not usually deduced from those which precede. Rather each formula has been mentally compared with the miscellaneous facts known to the author, and the succeeding formula is often an improvement, a higher synthesis in the Hegelian sense, and not a deduction.

\* (Vide Marshall's "Principles of Economics" V ed. p. 101 for an appreciation of the service rendered to Economics by mathematics).

### III. Presuppositions.

Professor McDougall, in his very interesting book on "Social Psychology", "propounds a theory of action which is applicable to every form of animal and human effort, from the animalcule's pursuit of food or prey to the highest forms of moral volition".<sup>1</sup> While not accepting this theory as adequate, it will be convenient to attempt to state it as a starting point. If I understand it aright, its central idea is that mankind, all animals, (and plants also) are purposive. They strive towards ends, overcoming obstacles by persistence with varied effort. The animal may or may not be conscious of the aim of its striving. Consciousness of the aim ranges from a dim awareness that something or other is due to happen, to an objective clearly thought out in detail. Some of these aims are inherited and are then called instinctive. Thus observation of human beings and of the higher mammals indicates that mankind has some eight primary instincts. Each of these instinctive dispositions has a threefold structure; it is a tendency to perceive certain objects and thereupon to feel and to act in certain ways. As when, on hearing a very loud unexpected noise, we feel a momentary shudder of fear, and we start as if to take shelter. McDougall's list of primary instincts runs as follows. The name of the instinct is put first and after it that of the corresponding emotion. Flight and concealment with fear. Repulsion with disgust. Curiosity with wonder. Pugnacity with anger. Self-abasement with subjection. Self-assertion with elation. The parental instinct with tender emotion. And lastly the sex instinct.

In addition to the eight foregoing instincts, Prof. McDougall also treats, as fundamental, some other instincts of less well-defined emotional tendency, namely those of gregariousness, acquisition and construction. He also takes, as axiomatic, various other general innate tendencies. Among these there is a group of three divided according as they depend on thought, feeling, or action, namely: suggestibility, in the sense of accepting ideas on authority independently of evidence; sympathy, in the sense of experiencing an emotion because it has been expressed by another person; and thirdly of imitation of the actions of those with whom we have to do. These three tendencies are concerned in the occasional unreasoning spread of panic among a crowd, and with the tendency to uniformity in public opinion generally. Other fundamental innate properties discussed by McDougall are the impulse to rivalry and play, the formation of habits by practice and the tendency to feel pleasure and pain.

1. Social Psychology. Methuen & Co., Ltd. XII edition, page 352.



- From these innate beginnings Prof. McDougall traces the gradual development of the mind of the growing child under the influence of his social environment. The instincts appear in their simplest form at the time of life, at which they mature; but even then they are seldom as simple as the corresponding instincts of animals. Afterwards they become interwoven, and attached to new objects of thought, so as to form complicated emotional dispositions. For instance he considers the parental instinct to be the root also of brotherly love. An example of the formation of a disposition, which has a special bearing on war, may be quoted at some length. It was published<sup>1</sup> before the outbreak of the present war. It deals with "the case of anger roused by an insulting blow and restricted in its expression by fear. Up to the time of the incident, I had been, we may suppose, as nearly as possible indifferent to my assailant; that is to say his presence had evoked in me no well-defined feeling or attitude. But after the painful incident, I cannot think of him without fear, or anger, or both, and without desiring both to avoid him and to get the better of him in some way. Suppose, now, that circumstances repeatedly bring us together, and that his behaviour on such occasions is that of a bully covertly reminding me of the past insult that I dare not avenge. My attitude of blended anger and fear is renewed on each such occasion, and being thus confirmed and rendered permanent, it becomes a full-blown sentiment of hatred" . . . . . This is a linking of the idea of a particular person with the instinct to destroy and to feel angry, and with the instinct to flee away and to feel fear. "The effect of such linkage is not only that whenever the object of the sentiment is forced upon my attention, my thinking of him is coloured or suffused with these emotions, but also that I am rendered peculiarly apt to think of him. If I pass by a crowd of which he is a member, my eye singles him out and watches him furtively; if we both have occasion to attend the same board meeting, I am acutely aware of him and of all he says and does, though I may avoid glancing at him; if I overhear his name mentioned by others in conversation, I am all agog to hear what is said. And this may continue in spite of my best efforts to cast out this demon of hatred and to resume my former attitude of indifference. Again, all my thinking of my adversary is biased by my attitude; whatever I hear to his discredit I accept and retain, and I attribute his actions to the meanest motives; until, by repetition of this process of selective thinking under the guidance of the specialized conative tendency, I come to think of him as a monster of iniquity."

1. "Psychology". Home University Library, page 115.

In many particulars the above description would apply to the way in which the newspapers of each side have written of the other side during this war.

- To return to general social psychology, McDougall attempts to show how the moral and religious sentiments are formed by association of the innate mental tendencies, which have been enumerated above, with various objects of thought, and especially with the individual's idea of himself or herself, under the influence of the society in which he or she grows up. McDougall writes (page 382) "Our 'sense of duty' is, in short, at the lower moral level our sense of what is demanded of us by our fellows, and, at the higher moral level, it is our sense of what we demand of ourselves in virtue of the ideal of character that we have formed". With a scientist's desire to economize hypotheses, McDougall is anxious to avoid introducing, as independent causes, either an inner free self-acting on these instincts, or a moral sense, or a conscience, or anything in the nature of God. McDougall repeatedly uses such words as higher, better, wrong, lower, or unjustifiable, and they give a moral tone to his whole discourse; but one does not see how these terms can logically derive any meaning from his restricted hypotheses, - they appear to have been imported from an outside source. His theory is therefore no argument for making these restrictions. It leaves us free to accept instead the traditional belief in an inner free Self acting on the brain, [as McDougall has elsewhere suggested,<sup>1</sup> by altering the resistance of the synapses] and to believe that this inner self may put itself into communication with the Divine through prayer,<sup>2</sup> and may possibly also communicate with other persons by telepathy.<sup>3</sup>

- Other general properties of the mind, to which we shall need to refer later, are distraction, warming-up, learning-by-practice and forgetfulness. They might have been mentioned earlier, together with habit. With regard to distraction, there is the familiar difficulty about "doing two things at once". Either of them tends to distract attention from the other. A comical example is the difficulty many people experience in simultaneously patting their head and rubbing their chest with a circular motion. On the other hand there are plenty of diverse actions which can be done easily and sweetly together, as for instance to sing while the hands

1. "Physiological Psychology".

2. "The Meaning of Prayer" by H. E. Fosdick, Student Christian Movement.

3. "Human Personality" by Frederick Myers.



ply some familiar routine, as women often do in factories even when working hard on a "piece-rate". Again other actions are not merely indifferent, but might well be called "serial" because the first paves the way for the second; thus to know much about any machine, for example about a gun, makes one eager to see it work. Now whether two particular courses of thought, feeling, and action are "rival" or "indifferent" or "serial" can only be found out by experience and observation. It is an observed fact that anger and tender emotion when directed towards the same object, are rival in this sense. To love a person is almost automatically to cease to hate him and vice-versa. Subsequently, whichever of these two feelings has obtained a footing by displacing its rival, it is increased by "warming-up" and by practice, and tends to become habitual; in contrast with the rejected alternative, which becomes progressively more completely obliterated by forgetfulness. These properties of the mind tend to intensify loves and hatreds. In wartime many other agencies work with them.

Several mental dispositions or tendencies may be illustrated by comparing thought to the flow of water down a bank of soft mud. Wherever water has flowed, a channel is formed, which is analogous to a habit tending to direct any future flow. Wherever the flow ceases in any channel, the mud, owing to its softness, begins to close in, obliterating the groove and thus illustrating forgetfulness. The phenomena of distraction require us to suppose that the supply of water is limited, so that the flow down any one channel can only be made at the expense of the flows down all the other channels together. The Will may be compared to a person who can block up certain channels or scoop out others; but, to make the analogy fit, we must suppose that some of the channels are out of his reach, and that even in those which he can reach, the amount of closing or enlarging, which he can do, is very limited. On this analogy the instincts are to be represented by capacious channels in a harder mud, or in a rock even, so that they do not tend to close up when disused. Across the upper ends of many of these instinctive channels the Will has constructed a barrage of mud. But should a sudden rush of water wash away this barrage, then the instinctive channel, by its depth and breadth, draws all the water to itself, draining the others. Thus it is when a man loses his temper. His struggles to regain equanimity may be compared to pouring mud into the torrent flowing down the instinctive channel. At first the mud may perhaps be washed away as fast as he can pour it in; but as the supply of water becomes exhausted, the barrage is restored

and the remaining flow diverted to other channels.

McDougall's theory of the development of purposes, by the attachment of instincts to new objects, is contrasted by him with the fundamental hypothesis of the older economists, according to which man was a rational creature guided mainly by intelligent self-interest. While admitting that the older economists greatly exaggerated this side of human nature, we must also admit that it is a side which exists, and exists perhaps especially among those, such as philosophers or the heads of businesses, who are accustomed to take long views. This type of man has his instincts well under control and is quick to realize logical necessities.

#### Section IV. Certain General Relations.

##### IV.1. The Vigour-to-War.

The present theory attempts to describe the changes of the Purposes or Intentions of the two opposing sides under the various influences to which they are subjected. This purpose or intention is considered as varying from Eagerness-for-War at one end of the scale, to Peace-At-Any-Price at the other end. It is thus a "quantity", not measurable exactly, but yet having enough of the characteristics of largeness, smallness, positiveness or negativeness about it, to justify one in denoting it by an algebraic symbol. The symbols chosen are  $V_E$  for the Entente,  $V_G$  for the Germanic alliance, or, where the side does not require to be specified, simply  $V$ . It is difficult to find a completely satisfactory name for  $V$ . The neat alliterative phrase the "Will-to-War" comes to mind; but unfortunately the appearance of the word Will in this phrase gives a suggestion that Volition has something to do with it; and that is unnecessary and confusing. Because when, for example, somebody hits me violently on the nose, my tendency to personal combat with him is not a matter of volition at all, but simply of automatic instinct, which the Will has to struggle to resist. Better names for  $V$  would be "Warlike Striving" or "Vigour to War", and by the latter name  $V$  is denoted in what follows. But it will be convenient to restrict  $V$  to mean the instinctive part of the Vigour-to-War, denoting by  $H$  and  $F$  the remaining parts due to higher influences and freewill respectively.



#### IV2. Conditions at the moment of Victory or Defeat.

From the military point of view that side is considered beaten whose vigour-to-war first sinks to zero. When the peace is "by understanding" it might be argued that the vigours-to-war of the two sides reach zero simultaneously at the moment of signing the peace treaty. In contrast to this, one side might continue to invade territory, while the other was willing and eager to make peace. From a non-military point of view that side may well at long last appear as the victor, which has given to humanity most and best.

It is remarkable that rifles, guns, grenades, poison gas, tanks, battleships, submarines and aeroplanes have not made war more deadly than it was when men fought with bows and arrows, swords and spears, horse-chariots and rowing-boats. The limit at which a side admits defeat, as measured by percentage casualties, has remained about the same, thus showing that it depends not on mechanical contrivances, but on human capacity for endurance.

#### IV3. Notation for the two opposing sides.

It will be convenient to distinguish them by subscript letters

E for the Entente

G for the Germanic Alliance

and to use for a quality such, for example, as the Vigour-to-War the same symbol V, distinguishing it as  $V_E$  or  $V_G$ . In fact if any psychological feature is noticeable on one side, it seems best to look for a corresponding feature, possibly much greater or much less, existing on the other side; and to have a symbol ready for it. Thus the differences between the two sides in character and temperament will be left to be expressed by the numerical values of certain algebraic symbols. Lest any extreme nationalists should be scandalized at seeing the same symbol used to represent the ideas of the enemy, even with a different subscript, it may be well to point out that the appropriate method of expressing their dislike of the enemy, in this notation, is to assign low values to the symbols for desirable qualities and high values to the symbols for undesirable qualities, when they bear the enemy's subscript. This aims to be first a method of discussion, in order that,

in the course of years, it may lead ultimately to a statement of agreed fact.

It is sometimes convenient, in general discussions, to distinguish the two sides by the suffixes x and y. It is then to be understood that x may be either of E or G provided that y is the other one. By this device the number of equations, that have to be written down, is halved.

#### IV4. The Individual and the Government.

Before proceeding to consider the behaviour of nations, it will be well to take note of the diversity of individuals in any one nation:— rich and poor, brave and cowardly, annexionists and pacifists, dutiful and selfish, pertinacious and easily fatigued, internationalists and nationalists, mystical and matter-of-fact; we can hardly lump them all together without a blurring and confusion of vital issues. On the other hand, to take separate note of all the diversities of mankind, would lead us into bewildering complexities. So, avoiding infinite-simals, let us seek a division into a few significant finite differences. We can perhaps hardly do better than to follow the customary division into (i) the rich, (ii) the comfortable, (iii) the poor. Any symbol will, when necessary, be distinguished by a subscript 1 for the rich, a subscript 2 for the comfortable, or 3 for the poor. The subscript-number thus increases as the type of individual becomes more numerous, and as his possessions become less. A symbol with no subscript-number is a joint one for all social classes combined. By the Vigour to War of a side we must here understand the effective vigour as organized in the Government, or Group of Governments of that side. It is not to be confused with an average vigour which might be estimated by the counting of heads. The Vigour to War is some sort of balance or resultant — perhaps obtained by biased scales — of the various vigours in the side. The vigours of individuals are the motive power, and their effectiveness depends partly on their numbers, partly on their influence with their government. So let

$n$  be the number of individuals in any class

$i$  be the influence of the average individual with his government

$v$  be the vigour to war of the average individual.

Accordingly we may put

$$V_E = n_{1E} i_{1E} v_{1E} + n_{2E} i_{2E} v_{2E} + n_{3E} i_{3E} v_{3E} \quad (3)$$

$$V_G = n_{1G} i_{1G} v_{1G} + n_{2G} i_{2G} v_{2G} + n_{3G} i_{3G} v_{3G} \quad (4)$$



In a "perfectly democratic" country, governed on the principle of "one man one unit of influence",  $i_1$  would be equal to  $i_2$  and to  $i_3$ . In this connection "influence" and "votes" are not by any means equivalent. Such a "perfectly democratic country", like the "perfect fluid" discussed by mathematicians, does not actually exist. In all actual countries the influence  $i_1$  of an average rich man, is greater than that  $i_3$  of an average poor man. But  $i_3/i_1$  will probably be larger in countries commonly called democratic than in those known as autocratic. Even in the latter  $i_3$  is not negligible; for however autocratic a government may be in form, it must always in fact pay great attention to the contentment of the governed. For example the fact that the German Government did not deign to reply to a certain Socialist interpellation in the Reichstag is no proof that the socialists were without influence on their government. (Contrary to Daily Mail's assertion, May 1918). Despite its lack of morality towards outsiders the German autocracy looked after its people.

#### IV5. Warlike Activity.

A nation is stirred up if it knows, or imagines, that a neighbouring nation has a warlike intention or "vigour-to-war" against it; but it is much more profoundly stirred up if its neighbour's intention is made evident by acts of invasion, bombardment, and the like. We may group all such acts under the general name of "Warlike Activity" and denote them by the letter A.

IV6. What then is the relation between the Vigour-to-War V and the Warlike Activity A of the same side? We had an instance of their relationship in the British Isles in 1914, September, for then the national Vigour to War was very large and yet business went on much as usual; so that A remained small in comparison with its value twelve months later. Evidently the relation between V and A depends on warming-up to new tasks, on the formation of habits by practice, and on the forgetfulness and fatigue which set a limit to the acquirement of skill. It may be said that, in so far as a thought or action is instinctive, it is not facilitated by practice, nor caused to be forgotten by disuse. The nervous connections which it involves are fairly permanently organized. Now the

general war-aim "to injure the enemy" is certainly partly instinctive; but its practical working out is by way of rifle-shooting, grenade throwing, gunnery, aviation and many other processes, which are not at all instinctive, and which can only be learnt by practice.

To such small extent as the Warlike Activity A is itself instinctive it may be taken as directly proportional to the Vigour-to-War V, so that

$$A = \alpha V, \text{ where } \alpha \text{ is a positive constant} \quad (5)$$

But to the much larger extent to which Warlike Activity is a matter of instruction and drill, we may reasonably say that what the Vigour to War does, is to cause the Warlike Activity to increase. To express this idea in symbols, bearing in mind the general principle that "Formulae are not to be complicated without necessity", we may put

$$\frac{dA}{dt} = \beta V, \text{ where } \beta \text{ is a positive constant.} \quad (6)$$

Then fatigue comes in to set a limit to A in the following manner. In the absence of any Vigour-to-War, fatigue would cause a diminution of Warlike Activity; not indeed a sudden cessation, for men would tend to carry on by mere habit; but a diminution more rapid according as A were larger and therefore more exhausting. This may be expressed by the formula

$$\frac{dA}{dt} = -\gamma A, \text{ where } \gamma \text{ is a positive constant} \quad (7)$$

However it may be noticed that a very little warlike activity, say  $\epsilon$  in amount, is not tiring but is rather pleasing to a nation as a whole; so that we ought to replace (7) by

$$\frac{dA}{dt} = -\gamma(A - \epsilon) \quad (8)$$

Now no one of equations (5), (6) and (8) is complete in itself. To describe any actual situation they must be combined. This can be done by differentiating (5) with respect to time and then writing in the additional terms from (6) and (8), with the result that

$$\frac{dA}{dt} = \alpha \frac{dV}{dt} + \beta V - \gamma(A - \epsilon) \quad (9)$$

Notice that this equation yields a credible deduction when applied to the conditions in the British Isles in the autumn and winter of 1914. Then V, the Vigour-to-War, was very large so that the term  $\beta V$  was also large. On the other hand the warlike activity A was already far too large to be pleasant, so that A was greater than  $\epsilon$ . The changes in V were slight so that we may neglect  $\alpha dV/dt$ , especially as the coefficient  $\alpha$  is small. Then, as A was increasing, it is



clear, from the equation (9), that  $\beta V$  must have exceeded  $\gamma(A - \epsilon)$ . Later on, in 1917 say, the warlike activity of the British Empire had risen to such a large value that  $\gamma(A - \epsilon)$  had become equal to  $\beta V$  and so, in accordance with equation (9),  $A$  had ceased to increase. And yet a stimulus to the vigour-to-war of the British, such as that caused by the great German offensive of March 1918, by increasing  $V$  and  $\beta V$ , set on foot a new increase of  $A$ , of which the visible sign was the raising of the military age-limit from 41 to 50 years.

Thus in many respects equation (9) correctly describes the relation of the vigour-to-war to the warlike activity; a relation direct as far as the term  $dV/dt$  is concerned, due to practice as far as the term  $\beta V$  is concerned, and due to fatigue (and perhaps also to forgetfulness) in respect to the term  $\gamma(A - \epsilon)$ .

Will the same equation also serve to describe the changes at the times of armistice, peace and demobilization? The signing of the armistice was the signal for a sudden drop in the popular vigour-to-war, on the Entente side, to a lower, but not negligible, level. On the same day fighting ceased, which we would represent by a sudden drop in  $A_E$  and  $A_G$ . The suddenness was due to the fact that it was done by order, carried out with a military habit of obedience. It cannot be expressed by an equation, such as (9), which slurs over the distinction between rulers and ruled, and which takes account only of a vigour to war common to both of them.

In the periods of peace which have followed the wars of the past, the warlike activities of nations have been represented by their armaments. If equation (9) is to apply to such a period, then as  $dA/dt$  was then small it may be said that armaments have been kept up at such a level  $A$  that  $\gamma(A - \epsilon)$  was just equal to  $\beta V$ , where  $V$  was the small vigour-to-war caused by lingering suspicion and distrust of their neighbours.

IV7. Freewill. It will now be shown how the equations may be generalized to admit of one of the current views about the freedom of the Will. As this is a subtle and controversial subject, the author is compelled to point out that there are other views. With this preface, it may be said that if Will were all-powerful, science would be impossible.

But a limited Will, such as we know, leaves room for a limited science, which may state what will happen according as the person in question tries to perform the action in question, tries not to perform it, or makes no effort. But whether he tries, has to be left indeterminate. How can we symbolize this? In what follows we have many equations expressing the effects of different influences on the vigour-to-war. Now these equations are mostly partial, in the sense that they suppose circumstances, other than the one immediately under consideration, to remain unchanged. In particular they suppose that the volitional efforts, of the individuals concerned, do not change. And so, after all the instinctive effects have been collected together to form the combined vigour-to-war  $V$ , we may add on a freely willed portion  $F$ , which may be either positive or negative; and then it will be  $(V + F)$  and not  $V$  alone which will be connected with the warlike activity, so that equation (9), as thus modified, will read

$$\frac{dA}{dt} = a \frac{d(V + F)}{dt} + \beta(V + F) - \gamma(A - \epsilon) \quad (10)$$

and  $F$  is left indeterminate, except that its magnitude will have to lie within a limited range.

#### IV8. Belief.

The remarkable contrast of belief between the two sides of the fighting line - obvious facts for us, appearing as deliberate lies to the Germans, and vice versa - is to be attributed (i) partly to different sources of information, emphasizing or repressing different aspects, (ii) but partly also to the unifying attribute of the human mind, which tends to make people believe statements which harmonize with the course of action in which they are engaged, and to disbelieve those which do not do so. (Vide Stout's "Manual of Psychology" page 678), (iii) the divergence of beliefs between the two combatants, when thus started, is further intensified by mass-suggestion, the gregarious tendency to say, and thence to think, the same as those with whom one has to do. This makes opinion on each side more uniform, by silencing the discordant voices.

An artist of my acquaintance, Mr. Eric Robertson, tells me that if you take a parti-coloured object, such as a map drawn with red roads, blue rivers, green woods and yellow boundaries upon a sheet of white paper, and if you place it



at a slight distance, so that it is possible to ignore its details, and if you ask a number of people to tint sheets of paper to match the average tint of the map, you will find that some of them match it as greenish, some as yellow, others as pink, others as of a blue tint. In a similar way, looking at Germany from a distance, the structure of her life appears blurred, and some see her as composed of perpetrators of atrocities, others as of deluded patriots, according to the temperament and associations of the one who sees. The truth exists nevertheless. "Too often is it said that there is no absolute truth, but only opinion and private judgment;..... Of such scepticism mathematics is a perpetual reproof."

An explanation is necessary about short intervals of time. The equations in this essay are written as if an act committed by the enemy - e.g. the sinking of the Lusitania - aroused immediately a state of feeling in the population as a whole. Actually of course half a day or so commonly elapses before the man in the street reads what those who control the newspapers see fit to tell him; and a considerably longer time passes before the man in the ploughed field comes to believe the same information. These intervals are, nevertheless, short compared to the duration of the war, and they have been neglected.

Mr. Bertrand Russell has sent me the following note about the making of opinion. It is expressed in a different notation to that used elsewhere in this essay.

"Problem: To produce in two nations a mutual will to war."

For the sake of mathematical simplicity, take two nations, say Tibet and Uruguay, which have never heard of each other. We will call the two  $x$  and  $y$ . Let each contain one individual, and only one, who is actuated by aggressiveness towards the other; call them  $I_x$  and  $I_y$ . Let us assume that each is able, makes a fortune, and invests it in newspapers.  $I_x$  fills his newspapers, to begin with, with unpleasant descriptions and a biased history of  $y$ ;  $I_y$  does likewise. Hence  $x$  and  $y$  feel moral reprobation towards each other: call these  $Mo_x$  and  $Mo_y$ .

1. Mr. Bertrand Russell, from whom this remark is quoted (Mysticism and Logic, p. 71) refers of course to strict deductive mathematics, which is of quite a different type from the symbolism used in this essay as an aid to memory and induction.

$I_x$  then writes articles which, by a slight misquotation, can be made to appear as if they urged  $x$  to attack  $y$ ;  $I_y$  makes the misquotation. Hence arises in  $y$  the fear of being attacked, rousing a defensive will-to-war  $D_y$ . By the same methods a defensive will to war  $D_x$  is produced in  $x$ . Since  $x$  does not know of the misquotation,  $D_y$  appears as an aggressive will-to-war, and thus heightens  $D_x$ , which in turn heightens  $D_y$ . And so on. In this way, in the absence of a positive will-to-peace, the two individuals  $I_x$  and  $I_y$  may suffice to drive their nations into war."

## V. VARIOUS INFLUENCES AFFECTING THE VIGOUR-TO-WAR.

### VI. Conquests, casualties and the destruction of wealth.

The loss of Alsace-Lorraine in 1871 bit much more deeply into the French popular mind than did the indemnity which France paid at the same time. Territory is an enduring and visible sign; and the first generation of those who have emigrated, in order to escape foreign rule, does not forget. Conquests have thus a large "value" in terms of Vigour-to-War, for both sides. Let  $P_x$  and  $P_y$  be the increases which the Vigour-to-War of the two sides would undergo if conquests could return to the pre-war frontiers. Then, as one side's joy, in this matter, is the other side's sorrow,  $P_x$  and  $P_y$  are, at any instant, of opposite signs. Whether the dissatisfaction of the side which loses is more or less than the satisfaction of the side which gains, depends on the special circumstances of the case. Only if the conquests are very large, one may expect from the principle of the satiability of wants, which has been elaborately studied by the economists, that the winning side, being glutted with territory, will probably derive less satisfaction from the last portion annexed, than the losing side will derive dissatisfaction, so that a very large annexation will probably decrease the sum of human happiness.

Next as to casualties, let  $B_x$  and  $B_y$  stand for the rates at which casualties were occurring. These rates cannot be negative. There is almost certainly a positive correlation between  $B_x$  and  $B_y$ , as they increased together wherever hard fighting was going on.

Also let  $L_x$  and  $L_y$  be the rates at which wealth was being destroyed. There is again almost certainly a positive correlation between  $L_x$  and  $L_y$ .



Now as to the relation of the warlike activity to the foregoing quantities. Since the object of the military leaders of the x-side is to cause casualties, destruction of wealth, and loss of territory to their opponents, it follows that  $B_y$  and  $L_y$  must increase with  $A_x$  and also with the military skill of the x side. But we must distinguish. Mere drilling and arming does not cause casualties to the enemy, so only that part of  $A_x$  should be counted, which represents actual fighting. Call this part  $J_x$ , and let activity in preparation for fighting be denoted by  $R$ ; so that

$$A_x = J_x + R_x \quad (10a)$$

Then  $B_y$  the rate of casualties on the y-side increases as  $J_x$  increases. Now if the x-side wants mainly to conquer or to reconquer territory, it may do so with small casualties on either side, if its enemies do not resist, that is to say if  $J_y$  is small. This suggests that the rates of casualties on the two sides are

$$B_y = \chi_x' J_x J_y \quad ; \quad B_x = \chi_y' J_y J_x$$

where  $\chi_x'$  and  $\chi_y'$  are positive constants. If this be so, the casualties on the one side would be a fixed multiple of those on the other, and the correlation between them would be unity. That is no doubt an over-simplified view of the affair, but it is perhaps the best that can be managed without going into much complicated detail. We shall require the casualties expressed in terms of  $A$  thus

$$B_y = \chi_x' (A_x - R_x)(A_y - R_y) \quad ; \quad B_x = \chi_y' (A_y - R_y)(A_x - R_x) \quad (11)$$

In armies that are not growing rapidly, the relation of training to fighting is more or less fixed, so that by a change of constants from  $\chi'$  to  $\chi$  we may put

$$B_y = \chi_x A_x A_y \quad ; \quad B_x = \chi_y A_y A_x \quad (13), (14)$$

as applicable to armies that have attained their full development.

Next as to rate of destruction of wealth. This is partly the loss of the services which men in the army would have rendered had they remained as civilians, partly the cost of the munitions exploded daily, and partly also the value of the buildings and land which are spoilt in the fighting. Altogether we shall probably not be far wrong in taking the rate of loss of wealth as given by  $L_x = aR + a'' J_x + b'' J_y$  (15)

Or, as before, in armies which have attained a steady condition,  $R$  is proportional to  $J$ ; so that, by a change of constants we may put:-

$$L_x = aA_x + bA_y \quad (16)$$

V2. War as a joyful adventure, the satisfaction of a long-suppressed instinct. Two brilliant essays on this subject come to mind, one by R. L. Stevenson on Sir Richard Grenfell, the other by William James on a Substitute for War. We may also refer to Edward Carpenter who says of voluntary recruits (The Healing of Nations, pp. 144-145): "The gay look on their faces, the blood in their cheeks, the upright carriage and quick, elate step - when compared with the hang-dog, sallow, dull creatures I knew before - all testify to the working of some magic influence.....It is simply escape from the hateful conditions of present-day commercialism and its hideous wage-slavery into something like the normal life of young manhood - a life in the open under the wide sky, blood-stirring enterprise, risk if you will, co-operation and camaraderie. These are the inviting, beckoning things.....". He might have mentioned also: the approving smiles of the fair sex. The love of adventure has, in most cases, been surfeited with excess of fighting. In this generation the appetite tends to sicken. What would have just satisfied it, would perhaps have been a nice short summer campaign, with "fighting from 2 to 4 on Tuesdays, Thursdays and Saturdays" and then England, Home and Glory.

Many who dislike campaigning and hate war are yet susceptible to the appeal of the ancient proverb "Dulce et decorum est pro patria mori".

With this youthful love of adventure may be grouped the opinion of older men that war "is salutary, necessary, and is the only national tonic that can be prescribed", as Lord Roberts said.<sup>1</sup> Similar statements may be found in the writings of the Prussian Bernhardt. Similar, at least in its relation to the war-like activity, is the "professional" desire to put design, training and preparation to the test. This also contributes to  $\epsilon$ .

### V3. The Defence of that which is held dear.

Let us consider some of the countries separately.  
GREAT BRITAIN. The universality and strength of the defensive disposition in Great Britain may be illustrated by the following facts, if it be necessary to illustrate anything

1. Loves Dickinson "The Choice Before Us", p. 74.



so obvious:-

(i) Britain's entry into the war was advocated by the Times newspaper in the critical days of August 1914 on the principle of: Our Turn Next if the Germans crush France.

(ii) Tribunals have very generally appealed to this disposition, in their efforts to persuade conscientious objectors to fight, by means of such questions as: "What would you do, if your mother were attacked by a bear?"

(iii) The German offensive of March and April 1918 caused the military age to be raised from 41 to 50.

FRANCE. To the vast majority of French people this war was obviously defensive: "Les Boches sont chez nous -" there was no more to be said.

ITALY. The German advance into Italy in November 1917 caused certain Italian politicians to change from being against the war, to being supporters of it. The Germans imagined that they "saw an Italy demanding a separate peace, and found that they had produced the very opposite effect". (Le Journal, Paris C.M.\*VII 131).

GERMANY. There is doubt in many English minds as to whether the same instinctive disposition exists in Germany, and so some evidence has been collected to show Firstly that the Germans often write or speak about it, Secondly that certain neutrals refer to this instinct in Germany. To those who are fixed in the belief that all these writings are merely a piece of organized hypocrisy, it is difficult to make any convincing answer, except to say that it would be very astonishing if our present enemies, alone among nations, should prove to be exempt from this very widespread instinct.

(i) On 1914, July 31st, at 7 a.m. von Moltke, chief of the German General Staff, speaking over the telephone to a General near the Russian frontier, is reported to have said "I must have tangible proofs if they (the Russians) are mobilizing against us in reality. Before I have them, I cannot get an order for mobilization through" (Vossische Zeitung 11 & 12 Sept. 1917, Camb. Mag. VII 10).

(ii) In Germany "on the outbreak of war every expression of jealousy was culled from the British Press, and every German knew by heart that monstrous leading article in which the Saturday Review called (11 September 1897) for a war with Germany, because "the destruction of her trade would add millions to our national income."" (Quoted from "A League of Nations", by H. N. Brailsford, p. 27). The object of the German press in publishing these instances of British

\* Note C.M. VII. 131 is a contraction for Cambridge Magazine Vol. VII, page 131.

jealousy, was presumably to satisfy a craving for justification for a war already begun, and also to stimulate the defensive instinct of the German people. What isolated and ancient instances sufficed to produce the effect they desired!

(iii) Kölnische Zeitung of 9 Sept. 1917 (Quoted in Camb. Mag. VII, 111) says "We entered the war, or rather we were swept into it, with the intention of defending our existence as a people and a state, and of creating preliminary conditions for a future in which we could work for the blessings of peace without interference from without.....It does not alter the character of a war of defence that we invaded the enemy's country in order to spare our own."

(iv) The rage of the German press over the Entente governments joint reply to President Wilson in 1917, January, especially about the clauses dealing with Austria, Constantinople and the German colonies.

(v) On 15 May 1917, in the Reichstag, Schiedemann, the leader of the Majority Social Democrats, declared "If the English and French Governments were, as the Russian Government has already done, to renounce annexations, and the German Government were then to desire to continue the war for conquest aims, then, you may rely upon it, you will have a revolution in the country". (Camb. Mag. VII 227).

(vi) In the "Volksstimme" (Frankfurt, Social Democrat) of 8 January 1918, Herr Quarck wrote: "Of course, nobody amongst us does think of striking. If for no other reason, then simply because we are threatened on all sides" (Camb. Mag. VII, 348).

(vii) Theodore Wolff in the Berliner Tageblatt of 24 Sept. 1917 wrote "Thus we were told that resistance to the lust of conquest might weaken the national spirit, and, to escape the reproach of luke-warmness, many, who might have spoken, remained dumb." (Camb. Mag. VII 39).

(viii) Bolo financed with German money the robustly patriotic French "Journal". Some entertain the view that the German Government wished him to make annexationist proposals appear in France, in order to stimulate the vigour-to-war of the German proletariat.

(ix) The German Majority Socialists, in conference at Würzburg in October, 1917, passed by an overwhelming majority a resolution which "enjoins the Socialist Reichstag deputies in future as in the past to make the voting of war credits contingent on whether such vote is demanded by the interests of national defence". (Camb. Mag. VII 136).

(x) An appeal published in Vorwaerts of 1 Nov. 1917 by the Executive of the German Social-Democrat Party contains this phrase: "Ready at any moment for peace, the German



people fights and suffers in order to protect the homeland but not for conquests or any other gains...." (Camb. Mag. VII 137).

(xi) See also Bevan's "Germany Social Democracy during the War", Allen and Unwin, 1918.

#### AUSTRIA-HUNGARY.

(i) Count Andrassy wrote in Nov. or Dec. 1917, "Our aim on entering the war was certainly not annexation, but purely self-defence...." (Camb. Mag. VII 267).

(ii) The following is an extract from a speech by Victor Adler at the close of the Austrian Social Democratic General Party meeting (Arbeiter Zeitung 25 Oct., 1917, quoted by Camb. Mag. VII 219). "It is said we adopted too credulously the catchword of the 'fight against Czarism' which was skillfully put forward by the Governments. That is not true. It is the truth that the danger of war with Czarism was a terrible one for us and for the Germans, which was thoroughly understood by us, and that we were determined to defend ourselves against it. That the Governments utilized it, of that there can be no doubt .... but that we ought to have made less of it because we recognized the fault of our Government, that is a false conclusion. If a house is on fire the first thing is to extinguish the flames."

NEUTRAL or ENTENTE comment on the defensive instinct of the Germanic alliance.

(i) National Zeitung (Basle, Switzerland) of 3 Oct. 1917 summarized in the Cambridge Magazine (VII 261) as follows: "The two trump cards in the present political game are victory and peace. The 'party of the Generals' in Germany promise 'victory'. The Reichstag majority put forward 'peace by reconciliation', but this party necessarily remains the weakest so long as the Entente on their side refuse to abjure annexations".

(ii) The Vaderland (Dutch, Liberal) referring to former French designs, now repudiated, on the Left Bank of the Rhine, writes of Ribot's speech: "In Germany itself this confirmation of Dr. Michaelis' revelations can naturally only form another reason for continuing the war. It will be all the same to the Germans whether the aforesaid German areas are to become French or autonomous. It was in any case intended to wrench them from Germany, which is hardly compatible with the principle of nationality. [Camb. Mag. VII 87] of 25 Aug. 1917].

(iii) "The World", New York, Radical Democratic 31 Aug. 1917. "An equally great mistake was the Paris Conference which planned a business boycott of Germany after the war

as a measure of continuing reprisals. .... The immediate consequence of it was to rally all the German people to more vigorous effort, and it gave to the German autocracy the chance to reassert its claim to be the defender of the German people against the plots to destroy them (Camb. Mag. VII 941)

(iv) That the rapid growth of the German Navy in the early years of this century was stimulated at least in part by the defensive disposition, is a conclusion which may be drawn from W. J. Ashley's pamphlet "The War and its Economic Aspects" (Oxford Pamphlets 1914), for instance on page 11 he quotes as credible the following statement from a German economist writing in 1900. "In one way or another, from 24 to 26 millions of Germans', out of a population, at the time of some 55, 'are dependent for their livelihood and work upon unrestricted import and export by water'".

The foregoing long series of extracts intended to show that the German Vigour to War was largely a determination to defend what they held dear, must not be taken to mean that it was exclusively of that nature. That they had some excuse in the later stages of the war for supposing that the Entente meant to destroy them, might be proved by numerous articles in Entente newspapers. A sample of one of the more serious and weighty of these may suffice. It is taken from a financial paper. [The Statist 11 May 1918, page 805]. "We do not ourselves doubt, in spite of the unsatisfactory state of things at present, that we shall win in the end, and that, in fact, Germany and Austria will be ruined. Before these ends are attained....."

Leaving now the discussion of the defensive disposition in particular countries, let us turn to the more general question: what is it that stimulates the defensive Vigour-to-War, is it the acts of the enemy or his intentions? Observation shows that it is both. For we form an opinion of a foreigner according to the same modes of thought by which we form an opinion of a neighbour; and by British law, or by the Code Napoleon, individuals are judged by their acts mainly, but also in part by their intentions as well. The same applies in non-legal matters, for instance Mr. Antony, in Galsworthy's "Strife" Act I, "What sort of mercy do you think you'd get, if no one stood between you and the continual demands of labour? This sort of mercy. [He puts his hand up to his throat and squeezes it]."



We have already denoted by  $-P_x$  the Vigour-to-War of the  $x$  side due to the loss of territory. This  $-P_x$  arises for the most part from the defensive disposition. In addition, almost everything we possess, is held more or less dear, when an outsider is seen to be destroying or threatening it; so that any Vigour-to-War or Warlike Activity of the one side immediately stimulates the defensive impulse of the other. Thus if  $V_x$  be the partial vigour to war due to the defensive disposition alone, we shall not be far wrong if we take  $V_x$  to be given by the following expression

$$V_x = -P_x + \zeta_x V_y + \eta_x A_y \quad (17)$$

where  $\zeta$  and  $\eta$  are positive constants. Of these  $\zeta$  may be called the "suspicion coefficient" because it depends largely on surmise rather than on reliable unbiased information.

#### V4. Vengeance.

Anger, the primary emotion which forms part of the Instinct of Pugnacity, is very prominent in war. It is stimulated by the warlike acts of the enemy. It forms one of the emotions which are potential in the defensive disposition, which has just been considered; it is also an essential part of the vengeful emotion, to which we now turn. There have been many expressions of vengefulness in the press on both sides.

A vengeful man reckons up his wrongs. The mere cessation of the warlike activity of the enemy would not appease him. Past debts must first be wiped out. This reckoning up, when expressed in mathematical language, would take the form of an integration. Thus if  $V_x''$  be the vigour-to-war of the  $x$  side, due to vengefulness alone, then  $V_x''$  depends mainly on casualties. But  $V_x''$  is certainly not proportional to  $B_x$ , the rate of casualties;  $V_x''$  is more probably proportional to  $\int B_x dt$  taken from the beginning of the war. This integral has already been denoted by  $C_x$ . The corresponding accumulated total of destroyed wealth has been denoted by  $M_x$ . It appears then that we may take the vigour-to-war, due to vengefulness alone, as of the form

$$V_x'' = \theta_x C_x + \lambda_x M_x \quad (18)$$

where  $\theta$  and  $\lambda$  are positive constants.

Now we have already discussed the relation of casualties and of the destruction of wealth to warlike activity, and have arrived at the expressions

$$C_x = \int B_x dt = \int \chi_y' (A_y - R_y)(A_x - R_x) dt \quad (1)$$

$$M_x = \int L_x dt = \int (aA_x + bA_y) dt \quad (2)$$

The integrals are to be reckoned from the beginning of the

These can be substituted in the expression for  $V_x''$  the partial vigour to war due to vengeance. When this is done it appears as a logical deduction that a side desires vengeance on its enemies for losses partly caused by warlike activity on its own. But that paradox is in accordance with experience. For instance if you hurt your knuckles by hitting another man on a hard button, you are apt to be annoyed with him, for wearing such buttons. This tendency has led, I suspect, to an exaggerated picture in Entente newspapers of the wanton destruction caused by the Germans in their retreat through France in October 1918. I refer to wanton destruction as distinct from the ordinary military operation of blowing up places of refuge such as cellars.



### V.5. RIVALRY or the Maintenance of National Prestige.

A determination not to be beaten, but to win through and thereby show that we are the better men. This emulation may safely be assumed to be a deep-rooted instinct, which no nation is without. As an indication of its distant ancestry, we have the fact that it may be observed even in dogs. Its influence on the present war may be illustrated by the views of Professor Krückmann, who, writing in the Berlin Junker paper "Kreuz Zeitung" on 1917 Sept. 20, said (Camb. Mag. VII.9) "Unless we keep Belgium no human being will believe that we have won - even the German people themselves will not believe it. We do not only want to win, we want everyone in the whole world unmistakably to recognize the fact.... the latter is as important as the former..... We are fighting for our future political prestige.... that prestige would be England's if we surrendered Belgium."

Similarly on the Entente side some say that the military pride of Germany will never be humbled until she has been forced to give up Alsace Lorraine. In contrast with these writers may be mentioned Mr. W. E. Gladstone, who was apparently scarcely at all influenced by the desire to win for winning's sake; for, after the British were defeated by the Boers at Majuba Hill, he was satisfied on obtaining, from the Boers, the terms which he had previously wanted, and did not consider it necessary, in addition, to inflict on them a military defeat. (See his Life by Morley, Book VIII, Ch. III).

How can the emulative impulse be expressed in symbols? It is not like the economic wants, discussed in section V.3. below, because its aim is not the gain of a statable quantity of any permanent thing. To quote Prof. Krückmann again (loc.cit.) "In the prolonged struggle between Germany and England that country will win which possesses one sack more of coal than the other". It is the difference which he regards, and coal may be taken as a picturesque illustration of war materials in general. But the imponderables cannot be neglected here any more than elsewhere, in fact less so, for emulation is essentially a question of morale, as is shown by Foch's proverb "Une bataille perdue, c'est une bataille qu'on a cru perdre". "A lost battle is a battle which one has believed to have lost." So that we reach the conclusion that the Germanic desire to win for winning's sake is a desire to keep  $(V_G - V_E)$  always positive and equal say to  $K_G$ , where  $K_G$  is the margin of superiority which is considered proper in Germany. Neither side however is directly aware of the vigour-to-war of its adversaries. The best it can do is to form an estimate from certain outward and visible signs

such as their resistance in battle, the tone of their newspapers, private reports of visitors to their country and the like. Let us represent the erroneousness of this estimate by writing  $V_G - \iota V_E$  in place of  $V_G - V_E$  where  $\iota$  would be unity if the estimate were perfect, then perhaps it would be correct to say that  $V_x'''$ , the partial vigour to war due to emulation alone, is given by

$$V_x''' = u(\iota V_y - V_x + K_x) \quad (21)$$

where  $u$  is a positive constant the "emulataunce".

Note that this equation implies that  $V_x'''$  would become negative if the total  $V_x$  were larger than  $\iota V_y + K_x$ ; that corresponds to a state of self-satisfied slackness.

### V.6. Business advantages.

The economic influences here discussed, do not include the privations of a population as a whole, which are grouped along with casualties under Pain. In the present section the discussion is confined to those wants of a powerful minority, which, before the war, created a situation which has been brilliantly described in Brailsford's "War of Steel and Gold".<sup>1</sup> As examples during the war may be mentioned a manifesto issued by the Six Associations of German Industrialists in May 1915, against which we may perhaps set the activities of the Anti-German League. Suffice it to say here, that these wants are much more thought-out than the instinctive impulse to defence, although they also may be rooted in the acquisitive, constructive and self-assertive instincts of business men. These wants also are satiable by concessions or indemnities that are clearly foreseen by those who want them; concessions which the enemy probably would not, but yet conceivably might, grant if he were undefeated. In this respect they are in contrast with anger which makes the defeat of the enemy the essential condition of its appeasement.

Most business men would regard with loathing the idea of making a war for their private gain. More subtle is the temptation to take advantage of a war already in progress. Thus Milyuhov, the Russian minister, speaking to French socialists in 1916 said:

1. "Ancient imperialism levied tribute, modern imperialism exports capital at interest".



"Yes, we are waging a war of defence ... Yes ... that's understood .... But .... as we are at war .... why not profit by it to attain certain results, practical, material....?" Camb. Mag. VII. 346 . Another well-known temptation is to camouflage war-aims. Those who feel the wants here discussed, are naturally conscious that they are a minority of the nation, and therefore they tend not to flaunt their desires too openly in front of the majority; but rather they are constantly tempted to stimulate the vigour-to-war of the majority, by such arguments as are likely to appeal to it. During the war, these special interests of a wealthy minority have largely been transformed into national interests, owing to the limitation of profits and the rationing of produce; that however may be only temporary.

Let us put  $Q_x$  for the material concession or indemnity acquired by the x-side. Since the y-side must have lost the same, it follows that  $Q_y = -Q_x$ .....(22) as  $Q$  is a material, not a mental, quantity. No such equality need exist between the utilities which the two sides attribute to  $Q_y$  and  $-Q_x$ . The desire to obtain  $Q_x$  no doubt existed, in a few minds, before the war, when  $Q_x$  was zero. Let its pre-war value in terms of vigour-to-war be  $W_x$ , and, more generally, let its value at any time be  $V_x^{iv}$ . Then if more and more  $Q_x$  is obtained,  $V_x^{iv}$  sinks from  $W_x$  towards zero, and might even become negative if acquisitions became embarrassingly large. The relation between the amount of a material thing, such as  $Q_x$ , and the sacrifices, measured by  $V_x^{iv}$ , which people are willing to undergo in order to obtain it, has been studied in a great variety of cases by the Economists, and they have found that in general the sacrifice which a person makes in order to acquire a definite increase of material, is less, the more of this material the person, who receives it, already possesses. Accordingly we may expect that  $-dV_x^{iv}/dQ_x$  will be less as  $Q_x$  increases. A function which would have these properties is

$$V_x^{iv} = W_x - p \log \left( \frac{Q_x}{q} + 1 \right) \dots \dots \dots (23)$$

where  $p$  and  $q$  are positive constants.

There are of course innumerable other functions which also have the aforementioned properties; but the function in the last equation, being perhaps the simplest, has been adopted for illustration.

### V.7. War as a Source of Income.

What has been said about business advantages refers to the possible results of victory. But there are other motives connected not so much with victory or defeat as with warlike activity. However much they may regret the occurrence of war, many people know that the cessation of preparation for it, would be financially inconvenient to themselves. Surely this knowledge must tend to produce some vigour, if not to war, at least to arming. As an extreme example there was a scandal in which a German armament firm was accused of starting rumours in France in order to provoke preparations in Germany. Ambassador Gerard in "My four years in Germany" (page 77) states that:- "Many of the larger newspapers are either owned or influenced by concerns like the Krupp". But the tendency is naturally not confined to Germans nor to capitalists. Away back in 1910 or 1911 an artisan said to me in Newcastle on Tyne: "Elswick will never be itself again, until we have a right good war".

Activity in preparation for war has been denoted by  $R$ . At any moment machinery and personnel are adapted to a certain value of  $R$ , say  $N$ . If  $R$  exceeds  $N$ , those engaged in preparations are overworked. If  $R$  is less than  $N$ , they are faced with the prospect of a diminished income. So manufacturers, who can control  $N$ , tend to adapt machinery and personnel somewhat in the sense of the equation

$$\frac{dN}{dt} = (R - N) \times (\text{a positive constant}) \dots \dots (23A)$$

At the same time it may be suspected that there is a tendency in certain quarters to bring pressure on the Government to adapt the scale of national preparation  $R$ , so that it shall not differ too much from  $N$ , and especially when  $R$  is less than  $N$ ; a pressure that is to say which, if it acted alone, as it never does, would perhaps affect  $R$  somewhat in the sense of

$$\frac{dR}{dt} = \frac{N - R}{h_1 + R} \times h_2 \dots \dots \dots (23B)$$

where  $h_1$  and  $h_2$  are positive constants.

Of course the Governmental control is far stronger than this pressure, as is well seen now in February 1919,  $dR/dt$  being negative although  $(N - R)$  is positive. To discuss further these internal relations would lead us too far away from the main theme, which is the external conflict of nations.



V.8. Security of Rulers in respect of the Avoidance of Internal trouble, as doctors sometimes blister the chest as a counter-irritation to pneumonia.

"Tsardom", as Baron Rosen says, was bent upon forcing war in order to stave off revolution" (Manchester Guardian 1918 Feb. 27, quoted by E. D. Morel). Again, one thing to be thankful for, in the British Isles, amid the general tragedy of the first month of the War, was that it had saved us from threatening trouble in Ireland.

Again as Avanti, the Milan socialist paper says 1917 Oct. 22, quoted Camb. Mag. VII. 106, "We will only remark generally, on a purely historical view of it, that no revolution ever took place in any country as long as its granaries were full, its civil life fully organized, and its armies victorious in the field."

However, though Tsardom may have wished to start a war to stave off revolution, it would hardly be likely to have wished long to continue a war for that reason. For war is a powerful medicine; quite a small dose would suffice to cure the internal malady in question; Schemers of this kind presumably wish to maintain the vigour-of-war  $V_x$  of their own side at a value, say  $f$ , sufficient to distract attention from internal grievances, and yet not so large as to be inconvenient. Their schemes, if they act on the nation, act then somewhat in the sense of the equation.

$$\frac{d(V_x + H_x + F_x)}{dt} = g \{ f - (V_x + H_x + F_x) \}. \quad (24)$$

where  $g$  is a positive constant. For this equation represents a tendency of the total vigour-to-war  $(V_x + H_x + F_x)$ , when for any reason it has been increased or diminished, always to return to the value  $f$ .

Some rulers, Julius Caesar or Napoleon I for example, apparently made war partly because they enjoyed it. That part of a ruler's ambition is similar to the same enjoyment among common people in so far as both can be represented by an increase in the value of  $e$  in equation (10).

### V.9. Fear.

Fear, according to Prof. McDougall, is the emotion which belongs to the Instinct of Flight. It has much to do with the losing of battles, but surprisingly little, at first sight, with the direction of policy. For even if part of the armies have to retreat rapidly, the leaders express themselves as calm, confident and full of courage; and the effect on the nation as a whole has usually been to rouse it, to come more vigorously to the aid of the army. For instance take the raising of the military age in Great Britain in April 1918, consequent upon the defeat of the 5th British army a few weeks before. In fact, fear of what the enemy may do, or is suspected of being likely to do, is generally a stimulant to the vigour-to-war, in the manner set forth in Bertrand Russell's pamphlet "War the Offspring of Fear" (Union of Democratic Control). The American Ambassador, Mr. Gerard, says in "My Four Years in Germany" 1917, (page 57), "To the outsider the Germans seem a fierce and martial nation. But in reality the mass of the Germans, in consenting to the great sacrifice entailed by their enormous preparations for war, have been actuated by fear." In the last stages of a war, however, when one of the sides is reduced to exhaustion and helplessness, then fear of what the enemy may do, makes it more ready to make peace. Thus, by the action of fear,  $V_y$  or  $A_y$  tends to increase  $V_x$  if the total casualties  $C_x$  and the total destruction of wealth  $M_x$  are small; but the same  $V_y$  or  $A_y$  tends to decrease  $V_x$  if the casualties and loss of wealth on the  $x$ -side exceed a certain amount. This may be represented by the following formula for  $V_x^v$ , the partial vigour-to-war due to fear.

$$V_x^v = (\mu A_y + \nu V_y)(1 - \xi C_x - \rho M_x) \quad (25)$$

in which  $\mu, \nu, \xi, \rho$  are positive constants which probably have different values for the two sides.



V.10. Pain.

When schoolboys struggle, the pain they inflict on each other has at first an exciting effect, stimulating them to struggle more fiercely. Only if pain is prolonged and exhausting does it make one of them cry "Pax". The course of this war indicates that the same is true of the pains, suffered by populations. The pain of a wound puts a wounded man into a dread of further shocks; but the knowledge that others are suffering, or the painful anxiety for their safety, on the whole has stimulated the vigour to war; and this indirect pain has much more than counterbalanced the direct pain, at least until a population has become so worn and starved as for example the Russians were when they made peace. Thus the effect of pain shows a reversal much like that of fear, and a very similar equation may be used to represent it, thus, if  $V_x^{vi}$  be the partial vigour-to-war due to pain

$$V_x^{vi} = (1 - \xi C_x - \rho M_x)(\sigma B_x + \tau L_x) \dots \dots \dots (26)$$

where  $\sigma$  and  $\tau$  are positive constants.

In this equation it is supposed that  $(\sigma B_x + \tau L_x)$  is proportional to the painful feeling, while  $(1 - \xi C_x - \rho M_x)$  changes from positive to negative when exhaustion is reached. Painful feeling is here made proportional to the rate at which casualties are occurring, by the term  $\sigma B_x$ , with an addition for the rate at which wealth is being destroyed. But there exists also pain depending on the memory of all the losses reckoned up. In addition both the fact that territory has been lost, or that it is being lost, are also painful to think upon. If all these sources of pain be included, the last equation changes into

$$V_x^{vi} = (\sigma B_x + \tau L_x + \nu C_x + \omega M_x - \pi S_x - \phi \frac{dS_x}{dt})(1 - \xi C_x - \rho M_x) \dots \dots \dots (27)$$

where  $S_x$  is the area of territory gained by the x side, and  $\sigma, \tau, \nu, \omega, \pi, \phi$  are positive constants. Note that all the subscripts in the last equation are alike.

I have supposed that the Vigour-to-War due to Fear or Pain passes gradually from positive to negative. Mr. Bertrand Russell sends me the following commentary which he regards "only as a suggestion, not as always or exactly correct". His notation differs from mine.

"Mild persecutions produce obstinacy, severe ones produce submission. Since Louis XIV, rulers have generally erred in making their persecutions too mild.

The transition from obstinacy to submission is sudden.

The will-to-war produced by a pain  $P$  may be represented by

$$\frac{1}{P_0 - P}$$

where  $P_0$  is the "critical pain" This will-to-war increases as  $P$  approaches  $P_0$ , tending towards infinity, and then, when  $P$  surpasses  $P_0$ , it passes suddenly into a very great will-to-peace. This transition represents the moment of defeat. As  $P$  increases beyond  $P_0$ , the will-to-peace diminishes, because the will generally, and all power of action, diminishes.

A formula, which avoids infinities, and yet exhibits the oscillation which Mr. Russell has observed, is in his notation

$$\frac{P_0 - P}{1 + (P_0 - P)^2}$$

V.11. Fatigue.

In the equation (10), connecting the Vigour-to-War with the Warlike Activity of the same side, an equation which runs

$$\frac{dA_x}{dt} = a_x \frac{d(V_x + F_x)}{dt} + \beta_x(V_x + F_x) - \gamma_x(A_x - e_x)$$

the term  $-\gamma(A - e)$  was inserted to represent the effect of the fatigue due to maintaining the activity  $A$ , due that is to say to drilling, fighting, munition-making and the like. There is also fatigue due to the enemy having cut off part of necessary food supplies, a fatigue which would perhaps best be represented by making  $\gamma_x$  increase with  $A_y$ , or say by replacing  $\gamma_x$  by  $\gamma_x(1 + rA_y)$  where  $r$  is a positive constant so that the equation reads

$$\frac{dA_x}{dt} = a_x \frac{d(V_x + F_x)}{dt} + \beta_x(V_x + F_x) - \gamma_x(1 + rA_y)(A_x - e_x) \dots (28)$$



V.12. Desire for Change.

Boredom on quiet parts of the front afflicted certain individuals with an intensity such that a minor wound would have been welcomed as a relief. This kind of boredom is a vague diffused discomfort due to unused capabilities, unpractised powers, skill lying idle, or, in Graham Wallas's phrase, due to a baulked disposition.<sup>1</sup> It prompts a man to seek change, even to fly to ills he knows not of. In war, boredom mainly takes the form of a yearning for a return to civil life, and with a slight admixture of fatigue, might go to the tune of "Three Blind Mice" thus: I, want-to-go, home; I, want-to-go, home; I, want-to-go, home; No more bloody war; no more bloody war; no more bloody war; I've stood, two years of this, blasted strife; I'm tired, of going, in fear of my, life; I want, to go home to, my children, and wife; I, want-to-go, home.... and so on, ceaselessly. But the excitement of a battle dispels boredom temporarily. Boredom arises, in other words, from the dim distracting effect of various activities, which are only potential, but which we should enjoy were they real. In times of prolonged peace, one of these beckoning activities is war. That effect has already been represented by the  $\epsilon$  in the equation (28) connecting warlike activity with vigour-to-war.

I suppose that one reason why some newspapers have so regularly denounced all German peace overtures as "traps", is because those who controlled these papers feared that thoughts of peace would "mind us of departed joys" and thereby intensify that boredom which distracted from the Vigour-to-War.

V.13. The Prospect of Success.

A side is cheered and heartened and has its vigour-to-war strengthened, if it believes that, beyond present difficulties, there lies ultimate success. For if this be not so, why have the leaders of the nations at war so frequently assured their peoples that they were confident of final victory? Again the tendency to enlarge war-aims after a victory, may be observed in the newspapers of both sides, and comes perhaps partly from an increase in the willingness to endure, although it may partly also come from the hope of obtaining more, without increased effort. Again Brailsford remarks ("A League of Nations" p.9) that "The cynical game of guessing when, and on which side a neutral would intervene, taught us that military success may avail more to win allies than community of race, political sympathies, or past services".

1. "The Great Society", by Graham Wallas.

Again the fear of being-thought-to-be-willing-to-discuss-peace, as exhibited in the Czernin-Clemenceau controversy, may perhaps arise from the knowledge that willingness to discuss peace is, in many people, a sign that they expect defeat.

The prospect of military success arouses a deep-rooted instinctive feeling which affects people in general. Those who have put their terms high naturally have to cut them down if their side is defeated. That has happened to the Pan-Germans. Those who have put their terms low, and who did not raise them when their side was victorious, may be accounted as unaffected by the prospect of success. For instance, the varying tide of great offensives to and fro across France did not greatly alter the war-aims of President Wilson, of the Union of Democratic Control, or the German Minority Social Democrats.

The question next arises: how does a nation, and especially how do its journalists, estimate this prospect? If the enemy shows signs of weakening, his ultimate defeat appears possible, but if at the moment he is very strong, that defeat may take a long time. Simultaneously each side is conscious that its own determination is liable to wear. In other words the prospect would appear to depend on the relative courses of the two vigours-to-war, regarded as functions of the time. We should here have to deal with the total vigour to war,  $(V + H + F)$  due to instinctive and higher influences and to freewill. That side is said to be beaten, whose total vigour-to-war first sinks to zero. Now the vigour-to-war of a side may suddenly change, owing to a sudden change in the peace terms offered by the opposing side. The prospect of success therefore depends not on the momentary vigour-to-war of the adversary, but on what his vigour to war would be, supposing the desired peace terms could be imposed upon him. Thus the prospect of success is a function of the peace terms and cannot be stated apart from them. There is however one condition of peace which will at least serve as an example, namely the return to the territorial boundaries which preceded the war. Now we have already denoted by  $P_E$  and  $P_G$  the increases which the vigours to war of the two sides would undergo, on a return to pre-war frontiers. On this definition  $P$  is positive for a side which has gained territory, negative for a side which has lost. So  $(P_E + V_E + H_E + F_E)$  and  $(P_G + V_G + H_G + F_G)$  are what the vigours-to-war would be, on a return to pre-war frontiers. For brevity denote these two expressions by  $U_E$  and  $U_G$  respectively. Suppose the courses in time of



$U_x$  and  $U_y$  are those represented by the thick lines on the adjoining diagram.

At the instant marked  $t_0$ , which side had the better prospect? Did people in general estimate  $x$ 's prospect to be better because the ordinate  $U_x$  was greater, or worse because the slope  $dU_x/dt$  was more steeply negative? Or did they combine the ordinates with the slope in judging that  $x$ 's prospects were worse, because

$U_x$  would sink to zero before  $U_y$  would, if both were continued along the tangents to the curves at the moment when the estimate was made. Or lastly were the courses carried forward in imagination along curved paths? In what follows it is assumed that the prospect of success is popularly estimated in some way equivalent to considering which tangent will first cut the line  $U = 0$ . Now, on this basis, the time, reckoned from  $t_0$ , which would elapse before  $U$  sank to zero, would be  $U \div dU/dt$ , both  $U$  and  $dU/dt$  being taken at  $t_0$ . The difference between these times for the two sides would be one measure of the prospect. Mathematically it is an inconvenient measure, because it has  $dU/dt$  in the denominator. So, bearing in mind that formulae are not to be complicated without necessity, I have taken instead the difference between the reciprocals of these times. Thus the "prospect of military success" for the Entente has been taken to be

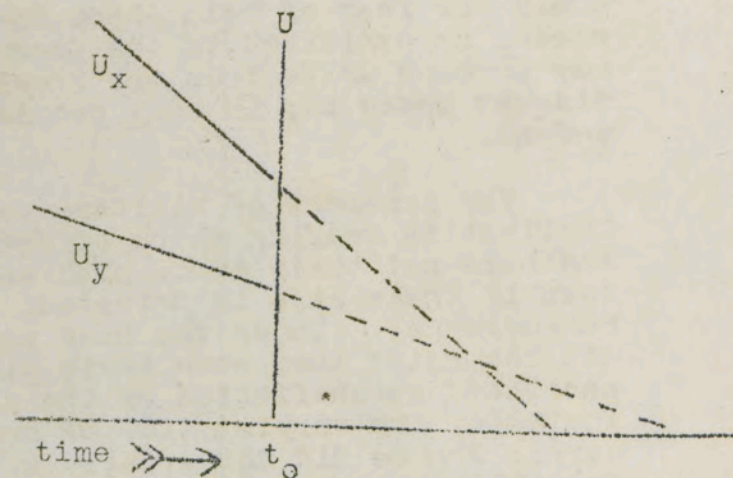
$$\frac{1}{U_G} \frac{dU_G}{dt} - \frac{1}{U_E} \frac{dU_E}{dt}$$

The prospect for the Germanic alliance was then the same quantity with its sign changed. Thus there appears to be a contribution  $V_{VII}$  to the vigour-to-war, given by

$$V_{xVII} = S_x \left\{ \frac{1}{U_y} \frac{dU_y}{dt} - \frac{1}{U_x} \frac{dU_x}{dt} \right\} \dots \dots \dots (29)$$

where  $S$  is a positive constant which may be called the "susceptibility to the prospect of military success".

We have heard much of the "swelled head" from which Germany suffered in the years just before the war. A temperate description of it is to be found in W.J. Ashley's



pamphlet "The War and its economic aspects" (Oxford 1914, pp. 4-6). "In academic circles" he says "the legitimate pride in German science seemed sometimes to have become almost an obsession, and to have the effect of shutting out of sight what was being done in other lands". Swelled head, in general, would seem to be analysable into (i) an error of judgment which we can represent by making the coefficient  $\epsilon$  in equation (21) much less than unity (ii) a resulting sense of unbounded power, which makes the prospect of success seem greater than that given by the "level headed" estimate of equation (29).

#### V.14. Duty to the Dead.

The greater losses a people has incurred in attempting to attain its war-aims, the more does their attainment appear to be worthy of further endurance, "Lest" in Mr. Asquith's phrase, "so many and willing sacrifices should have been in vain". If  $C_x$  be the total accumulated casualties, then, in the "drag" stage of a war, because casualties tend to decrease the vigour to war,  $\delta V_x / \delta C_x$  is negative; but it becomes less negative as  $C_x$  increases, on account of the feeling of duty to the dead.

#### V.15. Habituation.

Similar in form to the effect which has just been considered, although different in origin, is the mere deadening effect of habituation to continual casualties. Together they might be represented perhaps, by replacing the factor  $(1 - \epsilon C_x - \rho M_x)$  in the fear and pain equations, by  $(1 - \epsilon C_x - \rho M_x) / C_x$ . This  $C$  in the denominator would imply that, at the beginning of a war, when the numerator is positive, and  $C$  is small, very small casualties would produce an enormous inflammation of popular feeling. But that is just what happens when the news of the first encounter appears on the newspaper placards.



V.16. Pity for the Adversary.

In Galsworthy's wonderful study of "Strife", which contains so many analogies to the recent war, Pity for the Adversary is indicated as the best motive for making peace.

In the war such pity has been felt by scattered individuals, but it has scarcely had any outward effect, being almost entirely overwhelmed by the general anger at the war-like acts of the enemy. For it appears that only the great souled, for instance Abraham Lincoln, can have simultaneously two such opposing emotions as pity and anger towards the same object. In most people the stronger of these two emotions is not added to the weaker one; it displaces it. As an extreme instance of this kind of displacement there is the behaviour of King Lear in the first scene of the play.

It would not be easy to find expressions of pity for the adversary in the daily press. There probably are some in the publications of the Fellowship of Reconciliation, if they have not been suppressed by the censor, as being likely to prejudice recruiting. On the other hand the treatment for example of wounded Germans in French aidposts, as far as the author has seen it, has been practically identical with the treatment of French wounded, no sign of anger being mixed with a routine which has its origin largely in pity.

Now, if, before the war, there had been a disastrous earthquake in Germany, there would no doubt have been practical expressions of sympathy - Lord Mayor's funds and the like - in Entente countries. Again, as soon as the German menace was removed by their commencing disarmament in Nov. 1918, President Wilson agreed to supply them with food. So, to formulate, we may say that, in the absence of anger, the pity would depend upon, and increase with, the casualties and loss of wealth which the enemy has suffered or is suffering, that is to say on  $C_y$ ,  $M_y$ ,  $dC_y/dt$ ,  $dM_y/dt$ . So if  $V_x^{viii}$  be the, essentially negative, partial vigour-to-war due to pity, we may put

$$V_x^{viii} = -(jC_y + kM_y + l \frac{dC_y}{dt} + m \frac{dM_y}{dt}) \times (\text{a function of } V_x \text{ due to anger}). \dots (30)$$

in which  $j, k, l, m$ , are positive constants.

The function of  $V_x$  due to anger must be finite when anger is zero, and zero when anger is large. The simplest function of this kind is

$$\frac{1}{1 + (V_x \text{ due to anger})}$$

V.17. Racial Antipathy and Cohesion.

No marked racial differences occur in Great Britain, so that it is difficult for an Englishman to understand them in their Balkan intensity. Of course differences of clothing, of food, of manners, and especially of language, combine to make a man feel a stranger when he is in a foreign land. To feel a stranger is to feel depressed and out of touch with life, and is unpleasant. Thus many travellers tend to acquire a dislike of foreigners in general, coupled with a firm conviction that the ways of their own nation are better. But how can such comparatively mild feelings explain a story like the following, told by the Italian Premier, Signor Orlando, addressing in Rome in 1918 the leaders of the Congress of Oppressed Nationalities.<sup>1</sup>

"It was night, dark and gloomy, and our own and the enemy's first lines were plunged in that silence full of mystery and menace which broods over two armies confronting each other. In the Austrian advance posts there were at one point many Czechs.

"Suddenly in the darkness some one began to sing. Homer alone could have described the solemnity of the moment. It was the Czech national hymn. And then the sentinels were seen to change their positions, the soldiers in the trenches rose to their feet and stood bareheaded till the singer ended. Nothing more simple or more profound; in the night one felt the breath of epic poetry.

"These men, with an enemy in front, who might, in ignorance, fire upon them, with another worse enemy behind them, who at the sight of so bold and magnificent an assertion of national feeling might well fire on them treacherously from the rear - these men feared neither open nor hidden danger, and at the voice of the Fatherland sprang to their feet.

It has been suggested that these extreme cases of antipathy involve a hereditary difference of temperament, worked upon and aggravated by bad treatment (e.g. Ireland). Whereas the milder cases depend merely on custom and tradition, and as such are more easily healed. For instance no one now speaks, as they used to, of Frenchmen and Englishmen as "natural enemies".

Lest racial animosity be forgotten in counting up parts of the Vigour-to-War we might give it as symbol  $V_x^{ix}$ .

<sup>1</sup>. Here quoted from "Public Opinion" of 1918 May 10.



## VI. HIGHER INFLUENCES.

The types of behaviour discussed in the preceding pages have been mainly instinctive reactions of a regular quasi-mechanical type. It has been found to be possible to describe these types of behaviour, at least in their broad outlines, by means of mathematical expressions. These instinctive reactions are very prominent in war time. But they do not by any means make up the whole of behaviour.

We come now to a group of influences, powerful but elusive, including intuition, reason, justice and religion. Their relations are not obviously expressible by any kind of mathematical formulae. Their discussion lies outside the narrow scope of this essay. But, as a reminder of their great importance, it will be well to have a symbol for the vigour-to-war due to them. Let this symbol be  $H$ . Then  $H$  should appear in the equation connecting Warlike Activity with Vigour-to-war in the combination ( $V_x + H_x + F_x$ ) where  $V$  is the part of the vigour-to-war due to instinctive influences,  $H$  the part due to higher influences and  $F$  the part due to free will.

A few words about intuition, reason, justice and religion, in their relation to vigour-to-war, may not be out of place.

## VI.1. REASON and INTUITION.

A distinguished reasoner, Mr. Bertrand Russell, in contrasting reason with intuition, has written, "Reason is a harmonizing, controlling force rather than a creative one. Even in the most purely logical realm it is insight that first arrives at what is new" (Mysticism and Logic p. 13). In the midst of the passions of war, men find themselves reluctantly compelled to admit logical necessities, which desire cannot create, nor anger destroy.

VI.2. Justice.

The fundamental ideas of justice appear to be that human beings are, in limited and sometimes dimly realized respects, equal; and that the weak should therefore, in these respects, be protected from the strong. Of McDougall's list of primary instincts it is the protective parental instinct and also anger, which appear to be the instinctive mainsprings of justice; but these alone would not produce justice without a strong control by reason, which to continue the analogy, may be compared to the balance-wheel and escapement of the clock which indicates justice with its pointer.

As President Wilson has said (New York 1918 Sept. 27): "Impartial justice meted out must involve no discrimination between those to whom we wish to be just and those to whom we do not wish to be just. It must be a justice that knows no favorites and knows no standards but the equal rights of the several peoples concerned." To judge of the equality of rights, in a world where one man's food is occasionally another's poison, much sympathy and insight are needed; and for these qualities one would go to learn from the novelists of the several peoples concerned.

But some things have been very clear. Nearly all the world, including many Germans, has regarded the German attack on Belgium as a crying injustice. This attack was one of the principal causes of the enormous voluntary British enlistments in 1914.

Hear also "Vorwaerts" of 1917 Sept. 18 (summarized in Cambridge Magazine Vol. VII, p. 9): Whether the German Government can shut its ears to Belgium's claim for restitution is 'not merely a question of Power, and is not merely a question of political judgment, but it is a question of Right and Conscience'. The apostles of Force may sneer at morality and conscience, but "no policy seems to us really able which ignores these factors in international life."



VI.3. The aim to establish justice is closely allied to the aim TO DESTROY MILITARISM if by militarism be meant a National habit of doing things because the nation is strong enough to do them, regardless of the right or wrong of the matter.<sup>1</sup> The aim, to destroy militarism, was also a considerable cause of British enlistments in 1914, and still operates, although the net effect of the war, in all belligerent countries, up to the time of signing of the Armistice, has apparently been to increase militarism rather than to destroy it.

That however may possibly change in a later stage. The aim to destroy militarism tends to maintain the will-to-war of a side, until the other side shows signs of clearly understanding the meaning of right and wrong; as these are understood by the first side. In this matter each side is like the audience in a theatre, sitting in a dim light and watching, over its own heads, the other side brilliantly lit up on the stage.

#### VI.4. Religion.

All religious beliefs, which point to a God common to the opposing sides, in so far encourage that sympathy and insight which are necessary preliminaries to justice.

Certain religious systems encourage war with the infidel, for instance Mohammedanism has done so. Other religious groups or sects discountenance war with varying degrees of strictness, for instance the Buddhists, Quakers, Christadelphians and the Doukobors do so. It is sometimes said that war will forever be an institution, because it is instinctive in man, and because instincts are unchangeable. But as a small-scale sample of what can be done in the way of the atrophy, diversion or control of these particular instincts, it is interesting to notice the remarkable strength of the restraint which Quaker influence, whether of home or of school, has commonly exerted on the warlike proclivities of individuals.

The above refers largely to religious beliefs and practices, which are taught by one generation to the next. Of another order is that direct communication with Someone spiritually greater and better than ourselves, which persons of the most various religious beliefs, or of none, have testified to experiencing.

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1. See section V.13 above on the prospect of military success.

#### VI.5. A Sense of Guilt.

There having been such general agreement throughout the world that Germany's invasion of Belgium was a crime - even Bethmann-Hollweg the German Chancellor having admitted as much in a speech in August 1914 - many people on the Entente side spoke of the Entente army as a policeman and the German army as a burglar; and it was customary to say that the war ought to go on until Germany repented. But let us continue the analogy. The criminal struggles; the policeman also. They roll over and over, kicking, biting and tearing each other until both are half dead. Can one expect the burglar to repent of his crime at the moment when his face is being kicked? Such repentance is scarcely in human nature; the distractions are too great. Even so, in Germany signs of a knowledge of guilt, such for example as the book "I accuse" have been as faint as a whisper during the cheering of a crowd. The reason apparently is that the human mind has a limited span of attention. The instinctive disposition causes the attention to be focussed on defence, and thereby excludes almost entirely any thought about previous guilt. The latter is "inhibited by drainage" in Prof. McDougall's phrase. In 1918 October and early November, the German government was changed and the Kaiser made to abdicate. To continue the analogy it is as though the burglar cut off and cast away that one of his hands which had been guilty of the theft, while with the other hand he continued to struggle in self-defence against the policeman.



VII. A LIST OF SYMBOLS is here given for reference.

A. Warlike activity	a) See activity and	$\alpha$ Instinctiveness of
B. Rate of casualties	b) destruction	activity.
C. Total casualties	c. See business ad-	$\beta$ Practice coefficient
	vantages	$\gamma$ Fatigue coefficient
	d. Differentiator	
E. Subscript for		$\epsilon$ Activity which is
Entente		enjoyed.
F. Freely Willed	f.) See security of	$\zeta$ See Defensive disposi-
Vigour to War	) rulers	tions.
G. Subscript for Ger-	g.)	$\eta$
manic Allies		
H. Higher motivated		
Vigour to War	h. See Section V 7	$\theta$ See Vengefulness
	i. Influence coeffi-	$\iota$ See Emulation
	cient.	
J. Activity in	j )	$\kappa$ Used in Introduction
Fighting	)	
K. Margin of superi-	k ) See pity for	$\lambda$ See Vengefulness
ority	) the adversary	
L. Rate of destruc-	l )	$\mu$ )
tion of wealth	)	
M. Total wealth des-	m )	$\nu$ ) See Fear
troyed		
N. A value of R.	n. Number of peoples	$\xi$ )
P. Conquests as	p ) See business	$\pi$ See Pain
Vigour to War	) advantages	
Q. Business Advantage	q )	$\rho$ See Fear
R. Activity in Pre-	r See starvation and	
paration	fatigue	$\sigma$ )
S. Area of territory		$\tau$ )
transferred	s Susceptibility to	$\upsilon$ ) See Pain
	Success	
T. Tender Emotion,	t Time	$\phi$ )
Pity		
U = P + V + H + F	u Emulatace	
V. Vigour to-war	v Individual vigour-	$\chi$ Military skill co-
	to-war	efficient
W. See business	w See tender emotion	
advantages	and anger	
	x) Subscripts for	$\omega$ See Pain
	y) opposing sides	
Z. Anger	z See tender emotion	
	and anger	

VIII. SUMMARY.

## VIII.1. COLLECTED EQUATIONS.

For ease of reference the equations have been brought together. Here they are merely collected, no attention being paid to the fact that some overlap or replace others. Later an attempt will be made to assemble them.

First rough notion

$$\frac{dA_E}{dt} = \kappa_E A_G ; \frac{dA_G}{dt} = \kappa_G A_E \quad (1), (2)$$

The Collective Vigour to War of a State

$$V_x = n_{1x} i_{1x} v_{1x} + n_{2x} i_{2x} v_{2x} + n_{3x} i_{3x} v_{3x} \quad (3) \text{ and } (4)$$

The relation of the Warlike Activity to the Vigour-to-War of the same side.

$$\frac{dA_x}{dt} = \nu \frac{dV_x}{dt} + \beta V_x - \gamma (A_x - \epsilon) \quad (9)$$

Freewill Replace V in equation (9) by (V + F) (10)

Conquests alone  $V_x = -P_x$

Military activity and the resulting casualties and destruction of wealth

$$B_x = \chi_y' (A_y - R_y) (A_x - R_x) \quad (12)$$

$$= \chi_y A_y A_x \text{ for established armies} \quad (14)$$

$$L_x = a^I R_x + a^{II} J_x + b^{II} J_y \quad (15)$$

$$= a A_x + b A_y \text{ for established armies} \quad (16)$$

The Defensive disposition alone

$$V_x^I = -P_x + \zeta_x V_y + \eta_x A_y \quad (17)$$

Vengefulness alone

$$V_x^{II} = \epsilon_x C_x + \lambda_x M_x \quad (18)$$

Prestige alone

$$V_x^{III} = u(\iota V_y - V_x + K_x) \quad (21)$$

Business advantages alone

$$Q_x = -Q_y \quad (22)$$

$$V_x^{IV} = W_x - p \log \left( \frac{Q_x}{q} + 1 \right) \quad (23)$$



The avoidance of internal trouble

$$\frac{d(V_x + H_x + F_x)}{dt} = g \left\{ f - (V_x + H_x + F_x) \right\} \quad (24)$$

Fear

$$V_x^v = (\mu A_y + \nu V_y)(1 - \xi C_x - \rho M_x) \quad (25)$$

Pain

$$V_x^{vi} = (\sigma B_x + \tau L_x + \nu C_x + \omega M_x - \pi S_x - \phi \frac{dS_x}{dt})(1 - \xi C_x - \rho M_x) \quad (27)$$

Fatigue due to privations. Insert the term  $rA_y$  in (10) which then reads

$$\frac{dA_x}{dt} = \alpha_x \frac{d(V_x + F_x)}{dt} + \beta_x (V_x + F_x) - \gamma_x (1 + rA_y)(A_x - \epsilon_x) \quad (28)$$

The Prospect of military success

$$V_x^{vii} = s_x \left\{ \frac{1}{U_y} \frac{dU_y}{dt} - \frac{1}{U_x} \frac{dU_x}{dt} \right\} \quad (29)$$

where  $U_x = P_x + V_x + H_x + F_x$

Duty to the Dead and Habituation.

Replace the factor  $(1 - \xi C_x - \rho M_x)$  in the fear and pain equations by  $(1 - \xi C_x - \rho M_x)/C_x$ .

Pity for the Adversary

$$V_x^{viii} = -(jC_y + kM_y + l \frac{dC_y}{dt} + m \frac{dM_y}{dt}) \frac{1}{1 + (V_x \text{ due to anger})} \quad (30)$$

Racial Dislike V

Higher Influences Replace  $V_x$  or  $V_x + F_x$  by  $V_x + H_x + F_x$  wherever this has not already been done.

## VIII2. Assembling the Effects.

We have so far considered the changes in the vigour-to-war which would be produced by various influences acting singly, other things remaining the same. We now have to consider what happens when all these influences act jointly. Can we simply add algebraically the partial vigours-to-war due to the separate influences? It would seem natural to make such an addition for the effects of the defensive disposition, of vengefulness, of emulation, of business ambitions, of the prospect of military

success and of racial antipathy. It is not easy to say whether the effects of fear, and of pain, should also be added; when they increase the vigour-to-war they probably do so by way of anger; and as the defensive disposition and vengefulness also act by way of anger, there may be some overlapping.

The effects of a Sense of Duty to the Dead, and of Habituation to Casualties have already been incorporated in the Fear and Pain equations. The effects of Fatigue and of the Enjoyment of Warlike Activity appear in the equation connecting the total vigour-to-war with the warlike activity. The Desire to Avoid Internal Revolution has also been expressed as affecting the total vigour-to-war. As to Boredom, it appears to be impossible to express the effect of that in equations, without having symbols for activities which are rival to activity in war. There remain, of the instinctive vigours-to-war which have been mentioned above, only that due to Pity for the Adversary. Now the peculiarity of the latter is that it involves what the psychologists call "tender emotion", and that to have simultaneously tender emotion and anger towards the same person or nation requires a greatness of soul which is not common. Usually one of these two emotions displaces the other. Their relation is quite different to the coexistence, which envy, for instance, can have with either affection or dislike. Let T be the tender emotion towards a certain object excited by a stimulus I. Let Z be the anger with the same object excited by a stimulus II. Then if the stimuli I and II act together, as when, for example, a special friend of ours breaks one of our special treasures, it is often observed that the rival emotions of irritation and affection come and go alternately, both mingled with pain. They seem to enhance each other, because each makes us attend to the person, who is connected in our minds with the opposite emotion. On the other hand the duration of each is shortened, by the interruption caused by the other; and in the outward manifestations to which they prompt us, they tend to neutralize one another. The foregoing refers to the mild irritations of the drawing-room; the anger produced by war is often so much more intense as to fill the mind to the entire exclusion of pity. Now a formula which represents several of these facts in a simple manner, is that which makes the stimuli I and II, acting jointly, produce a vigour-to-war equal to  $\frac{Z}{1 + wT} - \frac{T}{1 + zZ}$  (31)

where w and z are positive constants. For, according to this formula, the vigour-to-war is Z-T, that is to say: (anger) - (tender emotion), if both anger and tender emotion



are small; whereas, if anger is large enough, it cuts out tender emotion altogether by the occurrence of  $Z$  in the denominator under  $T$  and vice versa.

May we next, in place of  $Z$  in the above formula (31), insert the sum of the partial vigours-to-war which work by way of anger, namely those connected with the defensive disposition, with vengeance and, except for an exhausted nation, with fear and pain? That would seem to be a passable expedient. Denote this sum by  $V$ . Then

$V_x = -P_x + \xi_x V_y + \eta_x A_y$  for the defensive disposition

$+ \theta_x C_x + \lambda_x M_x$  for vengeance

$$+ \frac{1 - \xi_x C_x - \rho_x M_x}{C_x} \left\{ \mu_x A_y + \nu_x V_y + \sigma_x B_x + \tau_x L_x + \upsilon_x C_x \right. \\ \left. + \omega_x M_x - \pi_x S_x - \phi_x \frac{dS_x}{dt} \right\} \quad (32)$$

The last term, representing fear and pain jointly, is only to be included if  $\xi_x C_x - \rho_x M_x$  is less than unity, that is to say only if the term as a whole is positive, as it is for a nation which is not exhausted.

Let  $T$  now stand for the Pity-for-the-Adversary so that according to section VI B we have  $T_x = j_x C_y + k_x M_y + l_x \frac{dC}{dt} + m_x \frac{dM}{dt}$ . Then the joint effect of all the instinctive influences which have been discussed is

$$\frac{V_x}{1 + w T_x} - \frac{T_x}{1 + z V_x} \quad \text{for anger and tender emotion}$$

$$+ \frac{1 - \xi_x C_x - \rho_x M_x}{C_x} \left\{ \mu_x A_y + \nu_x V_y + \sigma_x B_x + \tau_x L_x + \upsilon_x C_x + \omega_x M_x - \pi_x S_x \right. \\ \left. - \phi_x \frac{dS_x}{dt} \right\}$$

for fear and pain, to be included only if negative

$+ u(\nu V_y - V_x + K_x)$  for emulation

$+ W_x - p \log \left( \frac{Q_x}{q} + 1 \right)$  for business ambitions

$+ V^{1x}$  for racial antipathy

$+ S_x \left\{ \frac{1}{U_y} \frac{dU_y}{dt} - \frac{1}{U_x} \frac{dU_x}{dt} \right\}$  for the prospect of military success

$= V_x$

(33)

where  $V_x$  is the total vigour-to-war due to the ensemble of

instinctive influences. Now  $V_x$  appears on both sides of this equation, for  $-uV_x$  occurs in the emulation term in the first member. Collect the terms in  $V_x$  into the second member which then becomes  $(1+u)V_x$ , while the emulation term is cut down to  $(\nu V_y + K_x)$ .

Under the heading "The Security of Rulers" we arrived at an equation

$$\frac{d(V_x + H_x + F_x)}{dt} = g \left\{ f - (V_x + H_x + F_x) \right\}$$

intended to represent the effect of schemes to distract attention from internal grievances. Such schemes, by altering the beliefs of a nation, alter the "constants" in the long equation (33) for  $V_x$ .

Next, to the total instinctive vigour-to-war  $V_x$ , there must be added a part  $H_x$  representing the effect of reason, justice, religion and other higher influences. A further addition  $F_x$  has been made to represent the effect of freewill. This  $F_x$  is supposed to be indeterminate, except that it is confined to a limited range. Then the total  $(V_x + H_x + F_x)$  is related to the warlike activity  $A_x$  by an equation, which has already been incompletely formulated in number (28) above,

$$\frac{dA_x}{dt} = \alpha_x \frac{d(V_x + H_x + F_x)}{dt} + \beta_x (V_x + H_x + F_x) - \gamma_x (1 + r A_y) (A_x - \epsilon_x) \quad (34)$$

That completes a statement of the relations for the x-side. It has been assumed that the relations for the y-side are similar in form but with different values of the constants. There are cross-connections between the two sides wherever the x and y suffixes both appear in the same equation. And especially, in addition to the equations already reviewed, there are cross-connections through the following two equations:-

$$\text{Rate of Casualties } B_x = \chi'_x (A_y - R_y) (A_x - R_x) \quad (12)$$

$$\text{Rate of destruction of wealth } L_x = a' R_x + a'' J_x + b'' J_y \quad (15)$$

These equations containing both the suffixes x and y are one way of expressing in part the interdependence of nations, which, in diplomatic language, would be described as sovereign and independent. International influence powerfully exists, whether we like it or not; but it has not yet been fully organized.



What of the first rough notion with which this essay began? It was that  $\frac{dA_x}{dt} = \kappa_x A_y$ . Can it be deduced, as an approximation, from the very much more elaborate equations which have been evolved from it, by a long process of inductive reasoning? Well, in equation (34) which expresses the connection between the vigour-to-war and the warlike activity, we neglect  $\alpha$  which depends on the small part of the activity which is instinctive, and  $\epsilon$  which measures the small activity which is pleasurable and  $r$  which depends upon the privations due to the enemy having throttled supplies, there remains

$$\frac{dA_x}{dt} = \beta_x (V_x + H_x + F_x) - \gamma_x A_x \quad (35)$$

Next if we neglect higher influences and freewill we thereby omit  $H_x + F_x$ . And then if we regard the defensive vigour-to-war as so much the greatest of all the instinctive influences that we may put approximately

$$V_x = V_x^I = -P_x + \zeta_x V_y + \eta_x A_y \quad (36)$$

Then

$$\frac{dA_x}{dt} = \beta_x \{-P_x + \zeta_x V_y + \eta_x A_y\} - \gamma_x A_x \quad (37)$$

Finally if conquests were unimportant  $P_x$  might be neglected and in any case the "suspicion" term  $\zeta_x V_y$  is probably less than the "direct evidence" term  $\eta_x A_y$ , so that, neglecting also the former, the equation comes down to

$$\frac{dA_x}{dt} = \beta_x \eta_x A_y - \gamma_x A_x \quad (38)$$

which resembles the first rough notion  $\frac{dA_x}{dt} = \kappa_x A_y$  except that the term  $-\gamma_x A_x$ , representing the fatigue which sets a limit to all things human, cannot be neglected.

### IX. Conclusion.

The previous paragraph contains equations setting out the general form of the observed relationships. In conclusion some estimates will now be made of the relative size of the various terms. These estimates are admittedly more controversial than the general statements of relations.\* Previous to the war there existed, as causes tending to it, such purposes as: certain special business ambitions, racial jealousies, the desire for security on the part of dynastic rulers, the desire to regain Alsace Lorraine and the love of warlike activity in itself. These desires seem insignificant in comparison with the subsequent outburst of destructive fury. But they engendered suspicion between nations, and suspicion produced armaments, and armaments in turn increased suspicion, and so on alternately, distrust and preparedness-for-war mutually increasing each other; until war broke out. From the time of the first hostile act, the intensity of the war was increased by mutual reprisals; for all warlike measures appear almost as atrocities to those on whom they are inflicted, however justifiable and necessary they may seem to the inflicting party. In other words the largest part of the total vigour-to-war,  $(V + H + F)$ , was, on both sides, that due to the instinctive defensive disposition, and the second largest part, I should say, that due to vengefulness. A limit to the warlike activity was set by fatigue. This with fear, pain and boredom slowly wore down the vigour-to-war, until Germany practically confessed herself beaten. Now that fighting has ceased, the powerful distracting influence of the defensive and vengeful dispositions has abated, and Pity for the Adversary and what have been called the higher influences in all countries are beginning to reassert themselves.

\* Possibly numerical measures might be extracted from statistics of recruiting or of subscriptions to war-loans. But the analysis and interpretation would be beset with difficulties.



