MET. O.14

METEOROLOGICAL OFFICE
BOUNDARY LAYER RESEARCH BRANCH
TURBULENCE & DIFFUSION NOTE

T.D.N. No. 66

(up-dated version)

SOME SPECTRAL CHARACTERISTICS OF ATMOSPHERIC TURBULENCE ABOVE 50m

by

S.J. Caughey, R. Rayment & S.H. Moss

January 1976

Some spectral characteristics of atmospheric turbulence above 50 m

by

S.J. Caughey, R. Rayment≠ and S. H. Moss

Meteorological Research Unit, RAF Cardington, Bedford, England.

Abstract

The results of a series of turbulence measurements using Cardington turbulence probes at heights up to about 750 m in the convective atmospheric boundary layer are described. During all the runs well defined synoptic inversions were present and this permitted classification of some data in terms of the inversion height Z_i. Spectral shapes for the vertical and total horizontal velocities, temperature fluctuations and heat flux are compared with surface layer curves.

≠ Now at the Building Research Station, Garston, Watford.

1. Introduction:-

In recent years the structure of turbulence in the atmospheric surface layer has been studied extensively, especially in daytime convective conditions. As a result the variation with stability of the spectral distributions of the important turbulence statistics is fairly well documented in this region (see e.g. Kaimal et al 1972). However a general description of the characteristics of turbulence in the boundary layer as a whole has not yet emerged, due to the sparseness of data from the higher levels. This paper adds some further information on the region above 50 m in convective conditions.

The experiments described here were carried out at Earles Croome (52°05'N, 02°15'W), near Malvern, Worcestershire. Apart from the Malvern hills several miles to the West of the site the surrounding countryside consists mainly of low hills and is of the mixed agricultural/rural type. All the data were obtained using Cardington turbulence probes attached to the flying cable of a tethered balloon, (see Readings and Butler, 1972). The instruments measured the instantaneous temperature (T), the angle of the total wind vector to the horizontal plane (ø) and the total wind speed (V). These quantities were sampled at a rate of once/second and punched on paper tape.

The influence of balloon/cable motions on the turbulence statistics recorded by this instrumentation has been the subject of detailed experimental investigation (Haugen et al 1975). Their results indicated that balloon mounted turbulence probes overestimated σ_u^2 by about 20% relative to tower mounted instruments (sonic anemometers) and that this overestimation was closely correlated with the balloon's lateral motion. Spectral comparisons revealed contamination of $n S_u(n)$ centered near the natural frequency of the vane ($\approx$ .3Hz), however it was not clear how energy associated with the balloon/cable motions was transferred to those corresponding to the vane's natural period. Apart from a slight underestimation at middle and low frequencies the vertical velocity and temperature spectra were in generally good agreement with those from the tower-mounted sensors, as were the stress and heat flux cospectra.

The data set considered here can be divided into two groups

(a) a 5 hour run on 24 August 1972

(b) a series of shorter runs (typically 1 hour) on various other days between 14-25 August 1972.

Generally speaking information was available for up to three heights simultaneously, for example during run (a), the probes were located at 61, 91 and 152 m. For runs under (b) the probes were at heights between about 300m and 750m.

2. Derivation and presentation of the spectra:-

All the spectra presented here were derived from the one second data using a fast Fourier Transform (see Rayment, 1970). The differences between the spectral estimates when the data was extracted with respect to a linear time regression or mean were calculated and these are considered for the spectra presented.

It is common practice in atmospheric work to plot frequency spectra (either S(n) or n S(n)) against a reduced frequency $f(=\frac{nz}{\bar{u}})$ and in surface layer studies, to normalize the spectra by quantities such as U_*^2, T_*^2 etc, (see e.g. Kaimal et al 1972). However because of the irregular behaviour of these quantities at higher levels in the boundary layer this was not expected to be a useful procedure here and all spectra have been normalized by their total area.

A descriptive equation which can be fitted to some of the observed spectra has the form,

$$\frac{n\,S(n)}{\sigma^2} = \frac{A f}{F(f)} \qquad (1)$$

where A is a constant and F(f) is a function of f that tends to unity as f becomes small and some appropriate power of f as f becomes large. Hence in the low frequency limit the quantity $\frac{\bar{u}}{z}\frac{S(n)}{\sigma^2}$ tends to a constant A, which is related to the Eulerian integral scale, l_E, by the relation

$$A = \frac{4\,l_E}{z} \qquad (2)$$

Thus another useful way of presenting spectra is to plot,

$$\frac{\overline{U}\,S(n)}{Z\,\sigma^2} \quad \text{versus } f.$$

3. Vertical velocity spectra

The vertical component of turbulence is of great importance in the dispersal of atmospheric contaminants and the specification of the scale on which this mixing action occurs is a significant part of any general description of the boundary layer. Frequency spectra when plotted in the form n S(n) versus f generally show a maximum (f_m) at intermediate frequencies and this has been used to define a length scale λ_m which, if the spectra follow a universal form, is directly related to the Eulerian integral scale l_E. From recent work in the surface layer (see e.g. Kaimal et al 1972, Miyake et al 1970) it appears that λ_m/Z lies in the range 2-4 and this spread seems larger than could be attributed to any uncertainty in identifying the position of the peaks (see Pasquill, 1972). Above the surface layer the results are more scattered and no clearly defined variation of λ_m with height has emerged, furthermore the influence of thermal stratification remains unclear.

One would not expect λ_m to increase indefinitely with height since for example near a synoptic inversion new scales of mixing quite unrelated to Z will be introduced. This suggests that in interpreting W-spectra the height of the synoptic inversion should be a useful parameter to consider. The spectra presented here show how λ_m varies with height in conditions with clearly defined synoptic inversions. By investigation of the extreme low frequency end of some spectra, values for l_E/Z have also been derived and are compared with previous estimates (see Pasquill, 1972).

A study of the normalized spectra from the 3 probes at heights of 61, 91 and 152 metres (average $^{Z}/_{Z_i} \approx .3$) for the 5 hour run revealed no clear trends with height so all these spectra have been combined and are shown in Fig (1A) (for the W-component there was no significant change in the spectral estimates when calculated about the mean or a linear regression). This result tends to suggest that provided conditions are sufficiently unstable (average Ri $\approx$ - 14) a single universal curve may be applicable at these heights.

Some degree of subjectivity is involved when trying to locate the position of the maxima in power spectra and in this work it has been done by fitting smooth curves to the spectra and locating the peaks by eye. The maximum in this spectrum occurs at f $\approx$ 0.2 (i.e. $\lambda_m/Z \approx 5$), a value somewhat above that usually quoted for λ_m in neutral conditions.

Because the scatter in the spectral estimates is small an expression can be fitted which has the correct form and magnitude over the complete frequency range, this is,

$$\frac{n\,S_W(n)}{\sigma_W^2} = \frac{4f}{(1 + 7f + 49f^2)^{5/6}} \qquad (3)$$

and is drawn in Figs 1(a) and 1(b). In Fig 1(b) the spectra are plotted in the form $\frac{\bar{u}}{Z}\frac{S(n)}{\sigma_w^2}$ versus f and it is clear from this that the data shows a low frequency region in which S(n) becomes independent of f (i.e. n S(n) varies as n^{+1}). This region is shown in greater detail in Fig 2(a) (up to f $\approx$ .032 Hz) in which a height coding for the data has been introduced. It is clear that the data <u>from all levels</u> scatters around the line $\frac{\bar{u}}{Z}\frac{S_w(n)}{\sigma_w^2} \approx 4$. A linear regression gives an intercept at f=0 of 4.15 with a mean value is 4.3 and a standard error of 0.2. For convenience therefore the intercept will be taken as 4 and hence the Eulerian integral scale, given by,

$$l_E = \frac{S(0)\,\bar{u}}{4\,\sigma_W^2} \qquad (4)$$

becomes approximately equal to the height Z, in agreement with previous results of Panofsky and McCormick (1960) and Busch and Panofsky (1968).

The high frequency end of the W-spectra (i.e. above f = 1) is shown in Fig 2(b). A regression line through this data gives a slope of -1.62, very close to the inertial subrange value of -1.67 (some data from probe 1 has been ignored since it appears to have been contaminated).

Before considering the W-spectra from other heights and conditions one consequence of the spectra outlined above is worth considering. Hay and Pasquill (1959) suggested that the ratio of the Lagrangian and Eulerian time scales could be set equal to some constant β. Following Pasquill (1972)

$$K = \sigma_W^2 t_L = \beta \sigma_W^2 t_E = \beta i \sigma_W l_E$$

where K is the eddy diffusivity.

t_L and t_E are the Lagrangian and Eulerian time scales.

σ_W is the standard deviation of the vertical velocity.

i is $\sigma_W / \bar{u}$

and l_E is the Eulerian integral scale - identified with that for the W-spectrum.

An estimate for β is obtainable if we write for K

$$K = K_m = -\overline{u'w'} / \partial\bar{u}/\partial z$$

- averages over two hours were used to calculate the K's. Accepting the previous conclusion from this data that $l_E \approx z$ yields values for β of 0.9 and 1.2 (see Table 1). These results suggest the approximate equivalence of the Lagrangian and Eulerian parameters at these wind speeds and this result fits in well with the variation of β with wind speed for "rural" terrain described by Angell (1974).

All the information from these experiments on the position of the maximum of the vertical velocity spectrum (λ_m) has been plotted in Figure 3 in the form λ_m v. z with the data coded according to the value of z/z_i. This shows that the ratio $\frac{\lambda_m}{z}$ lies in the region 2-4 for the data from the lowest few hundred metres in agreement with most previous estimates for low heights in the boundary layer. Above about 500 metres λ_m appears to decrease substantially and this could be due to the influence of the synoptic inversion (most of the points in this region fall in the range $0.4 \leq \frac{z}{z_i} < 1.0$). A plot of λ_m/z is z/L_ℓ (where L_ℓ is the 'local' value of the Monin-Obukhov length - see Caughey and Readings (1975)) revealed no clear trends on the variation of λ_m with stability.

In view of the behaviour of λ_m with height it was thought useful to consider the data from each z/z_i range (i.e. ≤ 0.4 and > 0.4) separately. Five spectra from each range were averaged and these are shown in Fig. 4 with (for comparison purposes) the limits of the unstable surface layer spectra from Kaimal et al (1972) and the best fit line for the five hour average spectrum. The Kaimal et al near neutral curve fits the data rather better but it is noticeable that the fall off at the high frequency end (for the $z/z_i > .4$ data) is less than $-\frac{2}{3}$. This may be a consequence of entrainment of air through the synoptic inversion - i.e. the vertical velocity spectrum at these levels is determined by fluctuations from two different sources which combine in such a way to produce a single peaked spectrum of slope - 2/3. The five hour run spectrum (average Ri = -14) corresponds well to the spectral shape for the most unstable data ($z/L \approx -2$) from Kaimal et al (1972).

4. The temperature spectrum:-

The T spectra for the 5 hour data set, at the three heights, are shown in Fig. 5(a). Points calculated about the mean are shown as solid symbols and it is clear that the removal of a linear trend has a marked effect on the spectra at low frequencies. This is presumably a reflection of the diurnal variation of the temperature. At the low frequency end $nS_T(n)$ is still somewhat dependent on the frequency (see Figure 5(b)) hence the identification of the Eulerian integral scale for temperature fluctuations is difficult. The slope at the high frequency end is steeper than - 2/3 being about -.79, however the slope from the spectral estimates between f=1 and 10 is very nearly - 2/3 ($\approx$ -.68).

In an analogous manner to the W-spectra the remaining temperature spectra in convective conditions were averaged in sets of five for $z/z_i \leq$ and $>$ 0.4. These are shown in Figure (6) with the Kaimal et al unstable surface layer curves for shape comparison purposes. There is no obvious change in the spectral shape between the z/z_i ranges and close agreement with the surface layer curve is illustrated.

5. The horizontal wind speed spectrum:-

Figure 7 shows the average U-spectra (here U is the total horizontal component) for the 5 hour run and it appears that they are significantly contaminated for $f > 2$ (i.e. corresponding to natural frequencies of about 0.3 Hz). This is almost certainly due to some consequence of balloon movement. It is also clear that the effect of removing a linear trend is large and variable. This is in marked contrast to the W-spectrum case and agrees with the work of Berman (1965) and others which indicated that the low frequency part of the U-spectrum was inherently more scattered than the W-component (Panofsky (1973) related this to the sensitivity of the horizontal velocities to topographically induced eddies). This variability also precludes the estimation of an integral scale.

Average U-spectra for the two z/z_i ranges are shown in Figure (8) and these show much less correspondence with the Kaimal et al surface layer spectra - the peak (f_m) in the present data being apparently a decade in f higher. This does however agree with earlier work by Fichtl (1968) whose longitudinal wind speed spectra demonstrated a marked shift in f_m to higher values with increasing height.

6. The flux cospectra:-

6.1 Vertical heat flux:-

It was found that the five hour run $\overline{WT}$-cospectra were reasonably well defined, the removal of a linear trend having little effect. (See Fig.9). At the high frequency end the - 7/3 law (predicted by Wyngaard and Cote, 1972) fits better than the - 8/3 slope of Panofsky and Mares (1968). The average spectra from the two z/z_i categories show significant differences, (see Fig. 10). For $z/z_i \leqslant .4$ the spectra are quite regular and resemble closely those of Kaimal et al, however for z/z_i greater than about 0.4 the spectral estimates are very scattered and occasional negative values occur (even after gross averaging). This probably reflects the presence of downward heat flux regions associated with entrainment at the synoptic inversion.

6.2 Horizontal heat flux and stress cospectra:-

Both these sets of spectra were very scattered so they are not reproduced here. In the $\overline{UW}$ case even with gross averaging across large frequency ranges a reverse peak (i.e. UW positive) was found to persist around f $\approx$ .015, corresponding to a period of about 25 minutes. The origin of this peak is not understood. In general these cospectra suggested that $\overline{UW}$ is not a suitable parameter with which to describe turbulence at these heights and in these stability conditions.

The $\overline{UT}$ cospectra were also scattered but less than the $\overline{UW}$. In so far as anything could be said about the high frequency slope it tended to follow a − 3/2 power law rather than the − 2 law predicted by Wyngaard and Coté (1972) (i.e. in agreement with their own surface layer measurements).

7. Concluding comments:-

The spectral results described in this paper have tended to indicate the relevance of the inversion height (z_i) as a parameter in describing the characteristics of atmospheric turbulence in boundary layers with well defined capping inversions. Hence it would seem important to estimate this parameter in any future studies of the boundary layer. At the present time the most convenient way of monitoring the variation of z_i with time is with an acoustic sounder.

The spectral shapes were shown to be similar to these obtained in the surface layer by Kaimal et al (1972), except for the horizontal component of wind which showed a marked shift of the peak to higher reduced frequency. However it seems that much more data will be required, in a variety of stability classes, before any generalizations can be expected to emerge. The data should cover as wide a range of Z_i values as possible so that the degree of usefulness of this parameter for classifying and scaling boundary layer statistics can be established.

Acknowledgement

The authors wish to thank all their colleagues at the Meteorological Research Unit, Cardington for their help in the collection and analysis of the data.

TABLE 1

Hours	Hours (1 + 2)	Hours (2 + 3)
K_m (m^2s^{-1})	11.4	15.5
i	0.19	0.21
$\bar{u}$ (ms^{-1})	3.21	3.20
β	0.9	1.2

Values of the ratio of the Lagrangian and Eulerian time scales (β).

List of Figures

Figure 1:- (a) average W spectra for the 5 hour run plotted in the form

$$\frac{n\,S(n)}{\sigma_W^2} \quad \text{v.} \quad f$$

(b) as (a) but plotted in the form

$$\frac{\bar{u}}{z}\,\frac{S(n)}{W^2} \quad \text{v.} \quad f$$

The line drawn in the figure represents

$$\frac{n\,S(n)}{\sigma_W^2} = \frac{4f}{(1 + 7f + 49\,f^2)^{5/6}}$$

Figure 2:- (a) $\frac{\bar{u}}{z}\,\frac{S_W(n)}{\sigma_W^2}$ v. f for $f < .032$

with the data plotted on a height coding

Probe 1	61 m	●
Probe 2	91 m	■
Probe 3	152 m	▲

(b) $\frac{\bar{u}}{z}\,\frac{Sw(n)}{\sigma_W^2}$ v. f for f .6

coding as in (a).

Figure 3:- for the z/z_i categories as follows

$0 < z/z_i < .2$ (△), $.2 \leq z/z_i < .4$ (○), $.4 \leq z/z_i < .6$ (▲), $.6 \leq z/z_i < 1.0$ (●)

Figure 4:- Average of 5 W spectra for the following z/z_i, ranges.

		Slope
X	$0 < z/z_i < 0.4$	-.66
●	$0.4 \leq z/z_i < 1$	-.49

The dashed lines represent the limits of the W spectra obtained in unstable conditions in the surface layer by Kaimal et al (1972). These curves have been shifted to fit at the high frequency end so the comparison is one of shape only. The best fit line for the 4 hr run W spectrum is shown by the full curve.

Figure 5:- (a) average T-spectra for the 5 hour run in the form

$$\frac{n S_T(n)}{\sigma_T^2} \quad \text{v.} \quad f.$$

(b) as (a) but plotted in the form $\frac{\bar{u}}{z} \frac{S_T(n)}{\sigma_T^2}$ v. f.

The open circles represent spectral estimates calculated about the regression.

Figure 6:- Averages of 5 T-spectra with the same coding as Figure 4.

Slopes are × −.72 ● −.69

The dashed lines represent the unstable surface layer curves from Kaimal et al (1972), these curves have been shifted to fit at the high frequency end and so the comparison is one of shape only.

Figure 7:- (a) average U-spectra for the 5 hour run in the form

$$\frac{n S_u(n)}{\sigma_u^2} \quad \text{v.} \quad f.$$

(b) as (a) but plotted in the form $\frac{\bar{u}}{z} \frac{S_u(n)}{\sigma_u^2}$ v. f.

Figure 8:- Averages of 5 U-spectra with the same coding as Figure 4.

Slopes are × −.63 ● −.51

The dashed lines represent the unstable surface layer curves from Kaimal et al (1972), these curves have been shifted to fit at the high frequency end and so the comparison is one of shape only.

Figure 9:- (a) Average WT-cospectra for the 5 hour run in the form

$$\frac{n C_{WT}(n)}{\overline{WT}} \quad \text{v.} \quad f.$$

(b) as (a) but plotted in the form

$$\frac{\bar{u}}{z} \frac{C_{WT}(n)}{\overline{WT}} \quad \text{v.} \quad f.$$

Figure 10:- Averages of 5 $\overline{WT}$-cospectra with the same coding as Figure (4)

Slopes, × −.95 ● −.66

Bracketed values are negative.

The dashed lines represent the unstable surface layer curves from Kaimal et al (1972), these curves have been shifted to fit at the high frequency end and so the comparison is one of shape only.

REFERENCES

ANGELL J.K., 1974 "Lagrangian - Eulerian time-scale relationship estimated from constant volume balloon flights past a tall tower". Advances in Geophysics Vol. 18A 419-431.

BERMAN S, 1965 "Estimating the longitudinal wind spectrum near the ground". Quart. J.R. Met Soc., 91, 302-317.

BUSCH N.E. and PANOFSKY H.A., 1968 "Recent spectra of atmospheric turbulence". Quart. J.R. Met Soc., 94, 132-148.

CAUGHEY S.J. and READINGS C.J. 1974 "The vertical component of turbulence in convective conditions". Advances in Geophysics Vol. 18A pp 125-130.

FICHTL G.H., 1968 "Characteristics of turbulence observed at the NASA 150 m meteorological tower". J. App. Met, 7, 838-844.

HAUGEN D.A. KAIMAL J.C. READINGS C.J. and RAYMENT R. 1975 "A comparison of Balloon - Borne and Tower - Mounted Instrumentation for probing the Atmospheric Boundary Layer". J.App.Met.14, 540-555.

HAY J.S. and PASQUILL F. 1959 "Diffusion from a continuous source in relation to the spectrum and scale of turbulence". Advances in Geophysics 6, 345-365.

KAIMAL J.C., WYNGAARD J.C., IZUMI Y. and COTÉ O.R. 1972 "Spectral Characteristics of surface-layer turbulence". Quart. J.R. Met.Soc. 98, 563-589.

LUMLEY J.L. and PANOFSKY H.A., 1964 "The structure of atmospheric turbulence" Interscience Publishers, New York. 239 pp.

MIYAKE M., STEWART R.W. and BURLING R.W. 1970 "Spectra and cospectra of turbulence over water". Quart. J.R. Met Soc. 96, pp 138-143.

PANOFSKY H.A., 1973 "The boundary layer above 30 m" Boundary Layer Met. 4, 251-264.

PANOFSKY H.A. and McCORMICK R.A., 1960 "The spectrum of vertical velocity near the surface". Quart. J.R. Met. Soc. 86, 495-503.

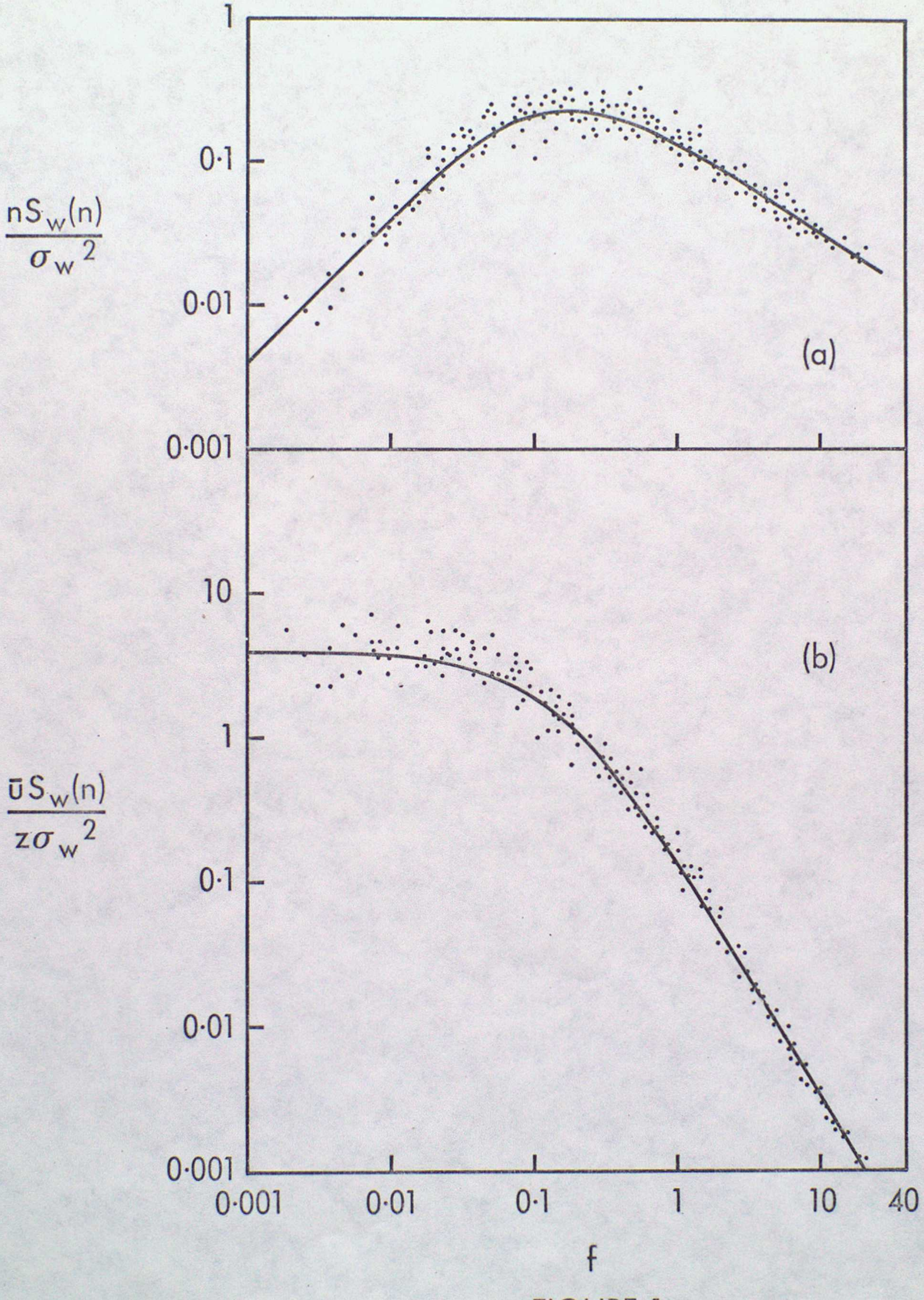
PANOFSKY H.A. and MARES E. 1968 "Recent measurements of cospectra for heat flux and stress". Quart. J.R. Met Soc. 94, 581-585.

PASQUILL F. 1972 "Some aspects of boundary layer description". Quart. J.R. Met Soc. 98, 469-494.

RAYMENT R., 1970 Introduction to the fast Fourier Transform (FFT) in the production of spectra" Met. Mag., 99, 261-270.

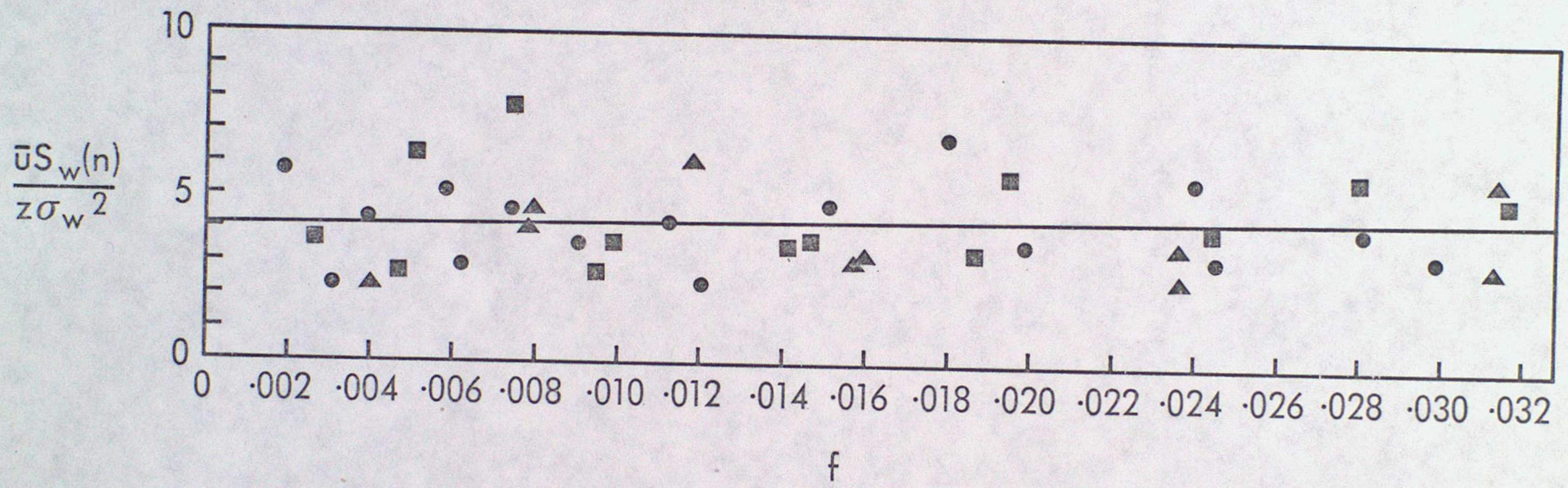
READINGS C.J. and BUTLER H.E. 1972 "The measurement of atmospheric turbulence from a captive balloon". Met. Mag., 101, 286-298.

WYNGAARD J.C. and COTÉ O.R. 1972 "Cospectral similarity in the Atmospheric Surface Layer". Quart. J.R. Met. Soc. 98, 590-603.



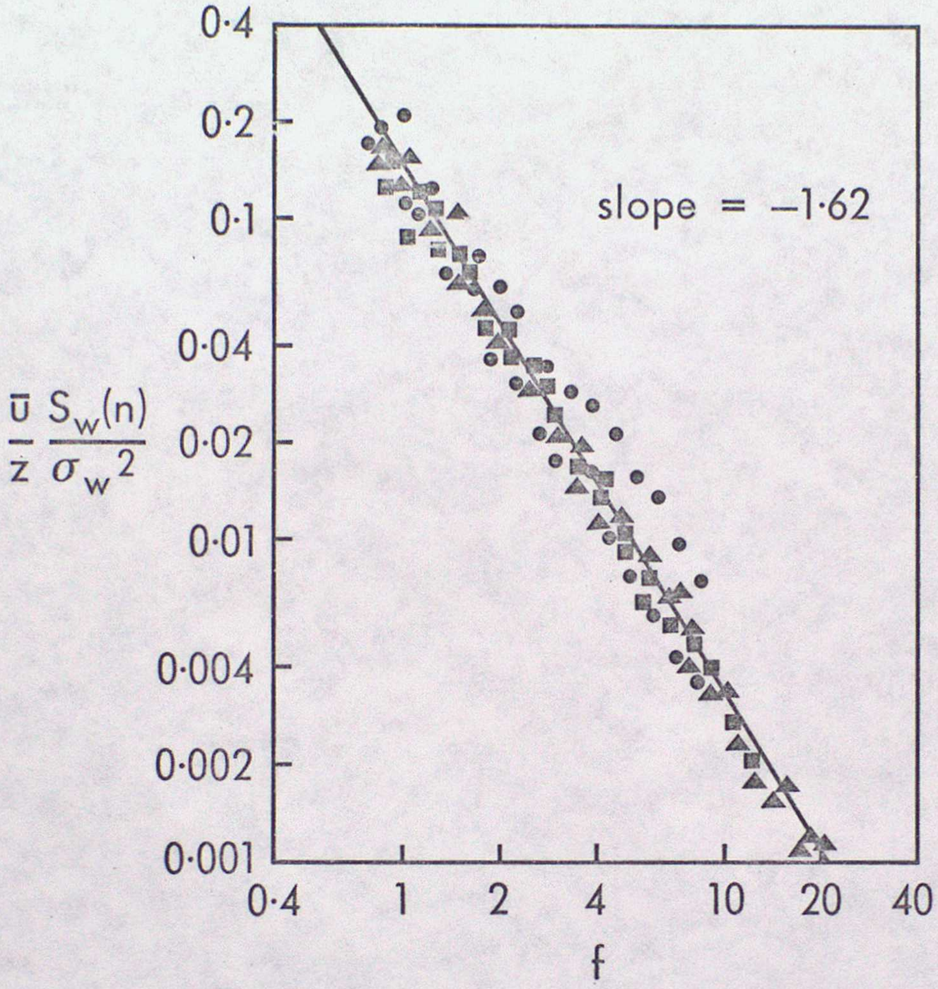
1
0·1
0·01
0·001
nSw(n) / σw²
(a)
10
1
0·1
0·01
0·001
ūSw(n) / zσw²
(b)
0·001
0·01
0·1
1
10
40
f

FIGURE 1.



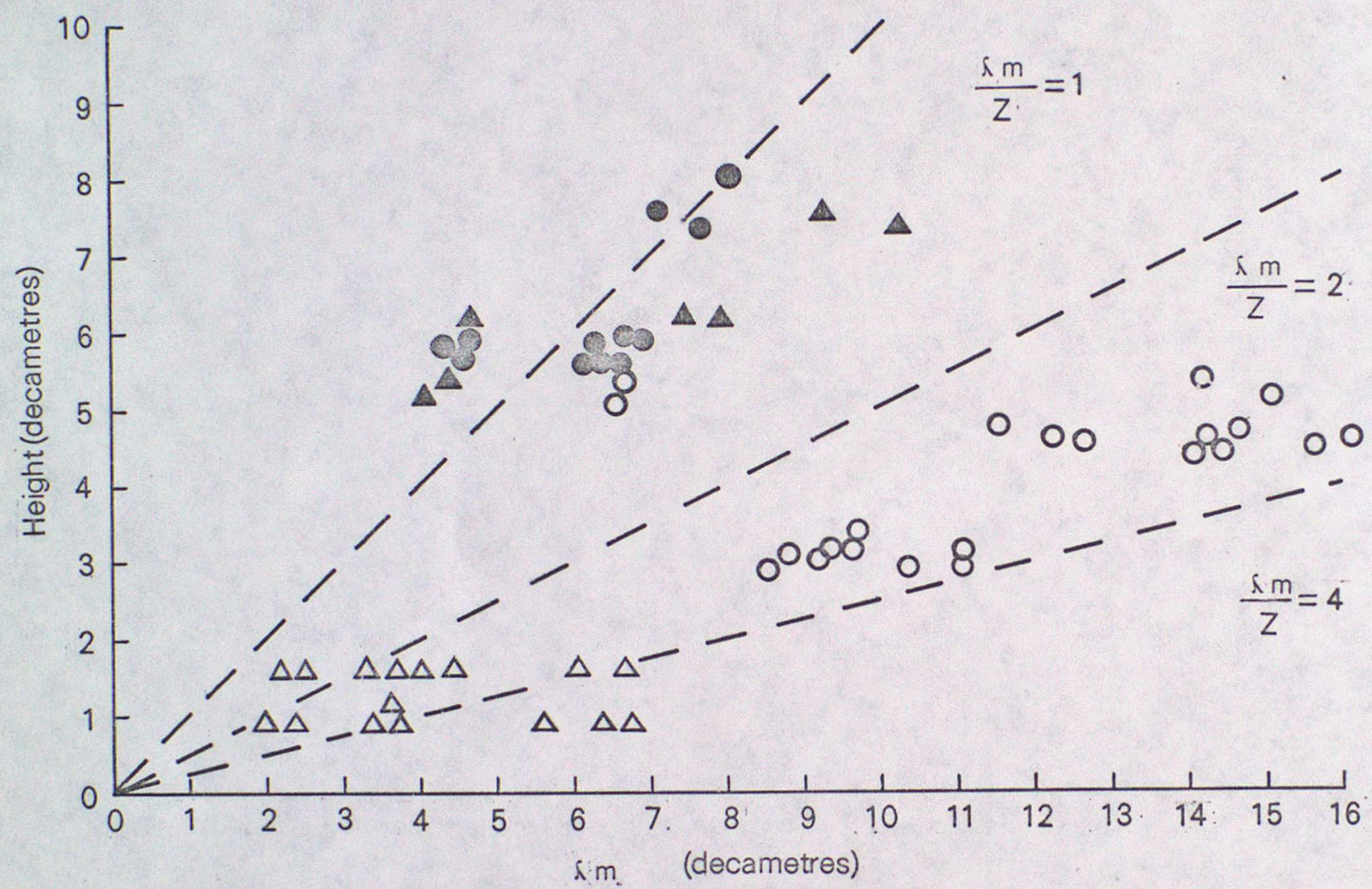
$\frac{\bar{u}S_w(n)}{z\sigma_w^2}$
10
5
0
0
·002
·004
·006
·008
·010
·012
·014
·016
·018
·020
·022
·024
·026
·028
·030
·032
f

FIGURE 2(a)



0·4
0·2
0·1
0·04
0·02
0·01
0·004
0·002
0·001
$\frac{\bar{u}}{z} \frac{S_w(n)}{\sigma_w^2}$
slope = −1·62
0·4
1
2
4
10
20
40
f

FIGURE 2(b)



Height (decametres)
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
ʎm (decametres)
ʎm/Z = 1
ʎm/Z = 2
ʎm/Z = 4

Figure 3

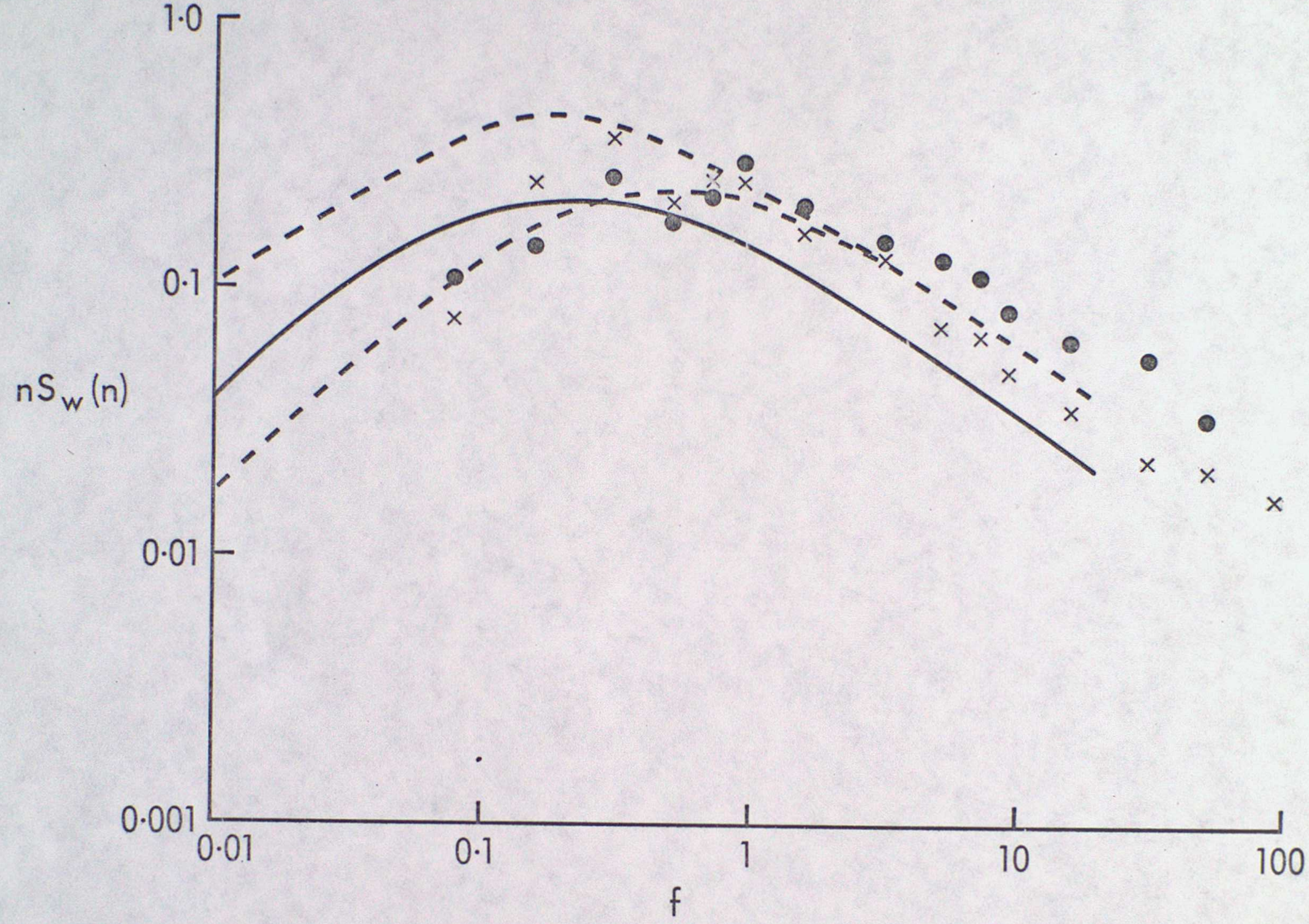


FIGURE 4.

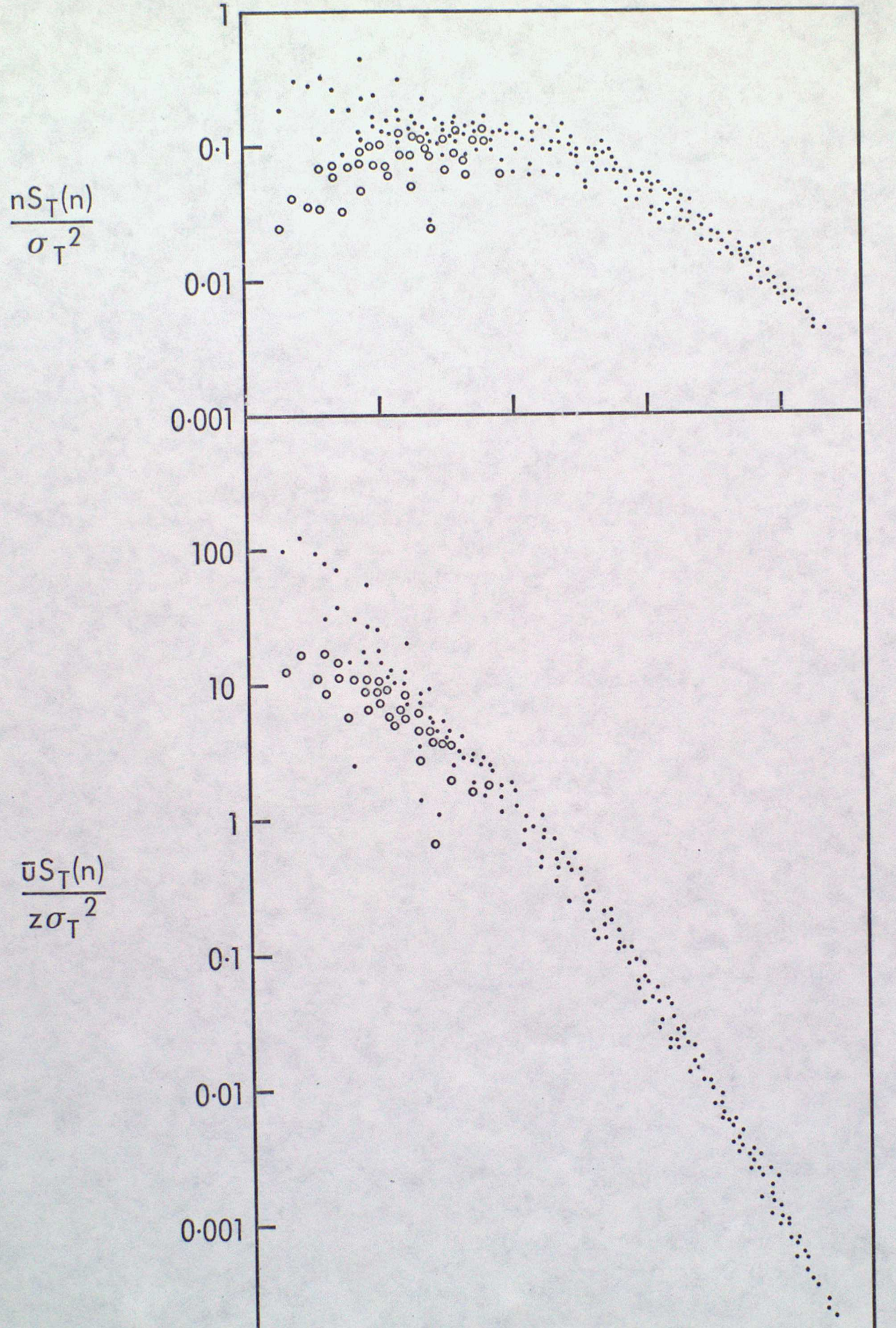

1
0·1
0·01
0·001
$\frac{nS_T(n)}{\sigma_T^2}$
100
10
1
0·1
0·01
0·001
0·0001
$\frac{\bar{u}S_T(n)}{z\sigma_T^2}$
0·001
0·01
0·1
1
10
40
f

FIGURE 5.

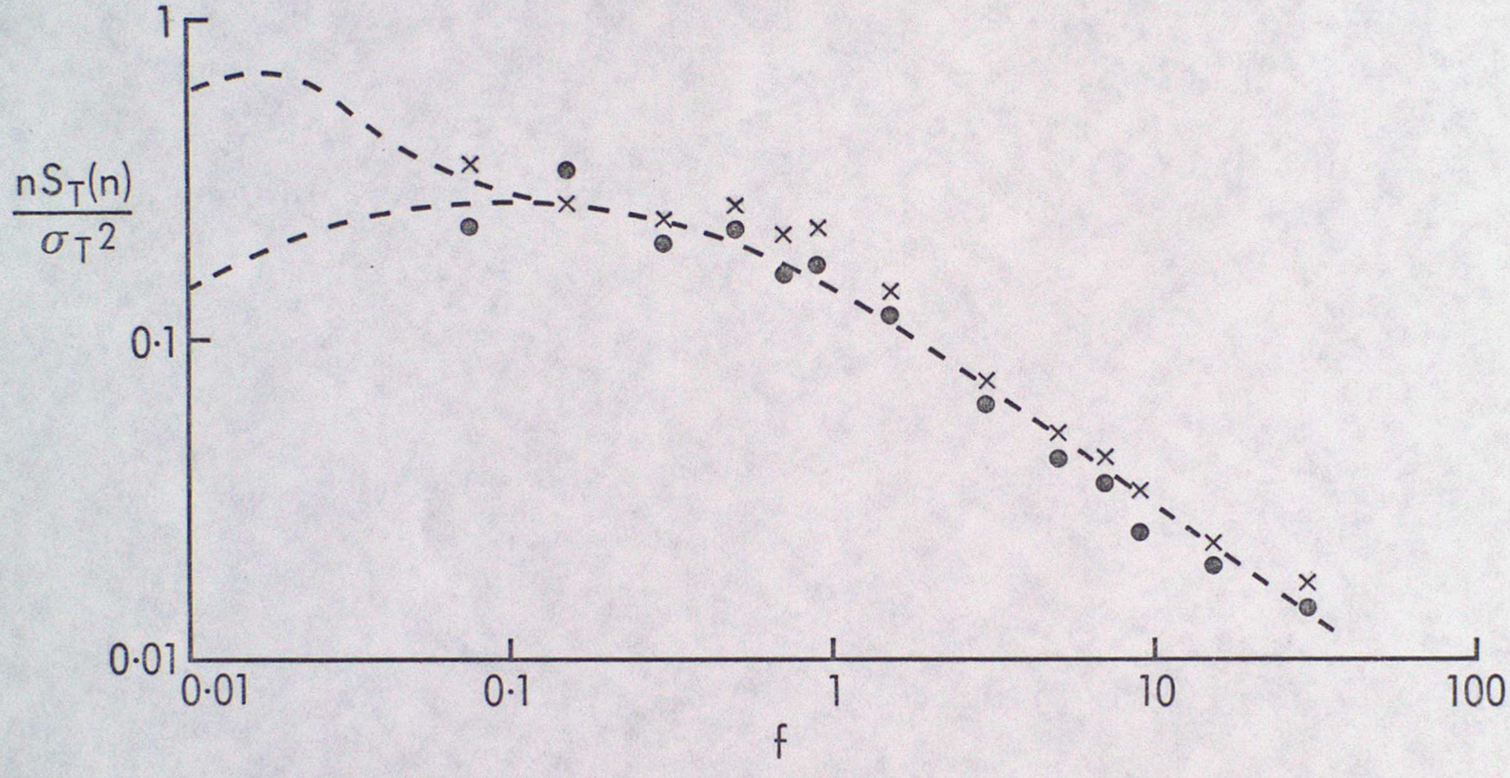


FIGURE 6.

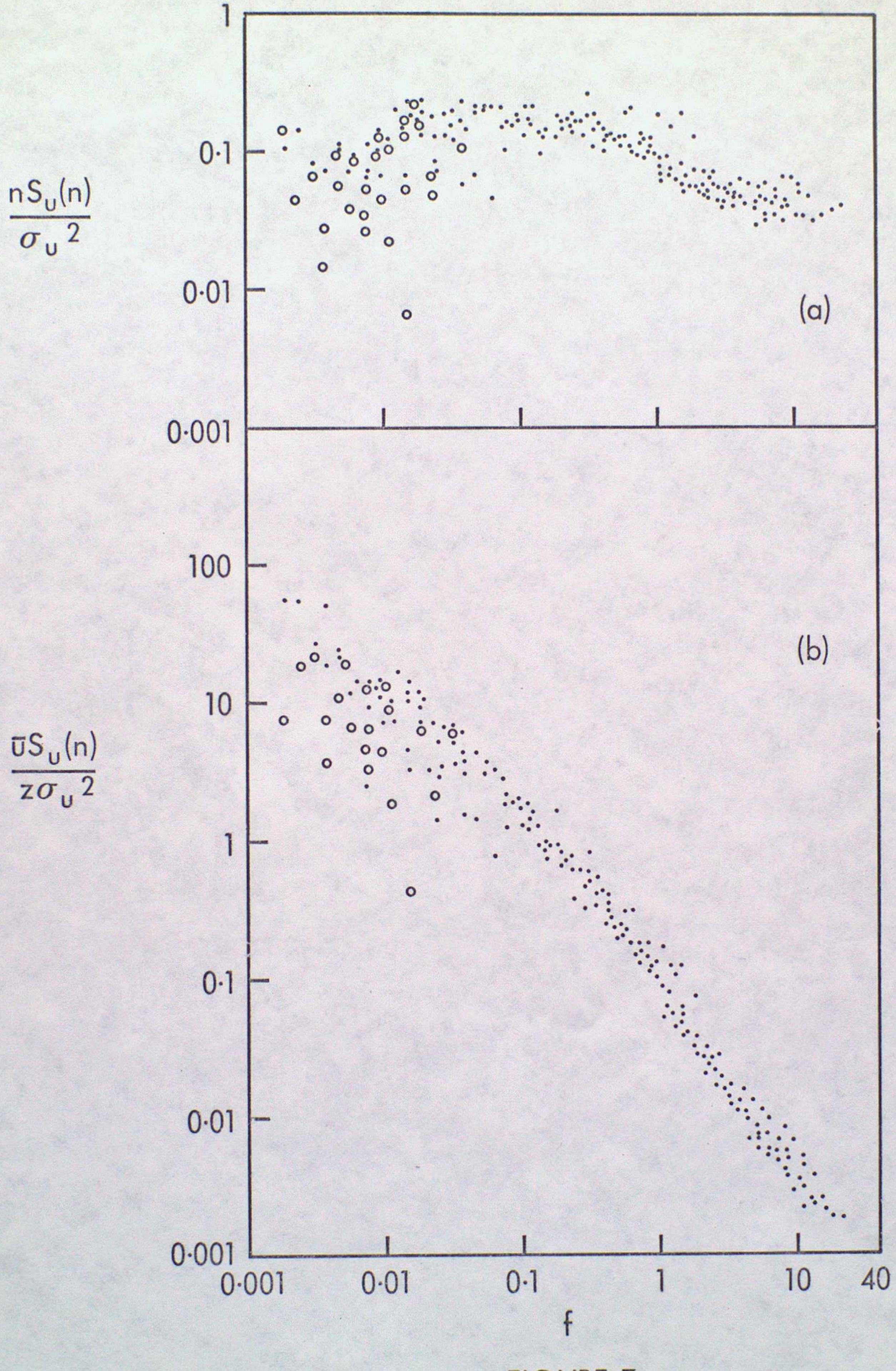

1
0·1
0·01
0·001
$\frac{nS_u(n)}{\sigma_u^2}$
(a)
100
10
1
0·1
0·01
0·001
$\frac{\bar{u}S_u(n)}{z\sigma_u^2}$
(b)
0·001
0·01
0·1
1
10
40
f

FIGURE 7.

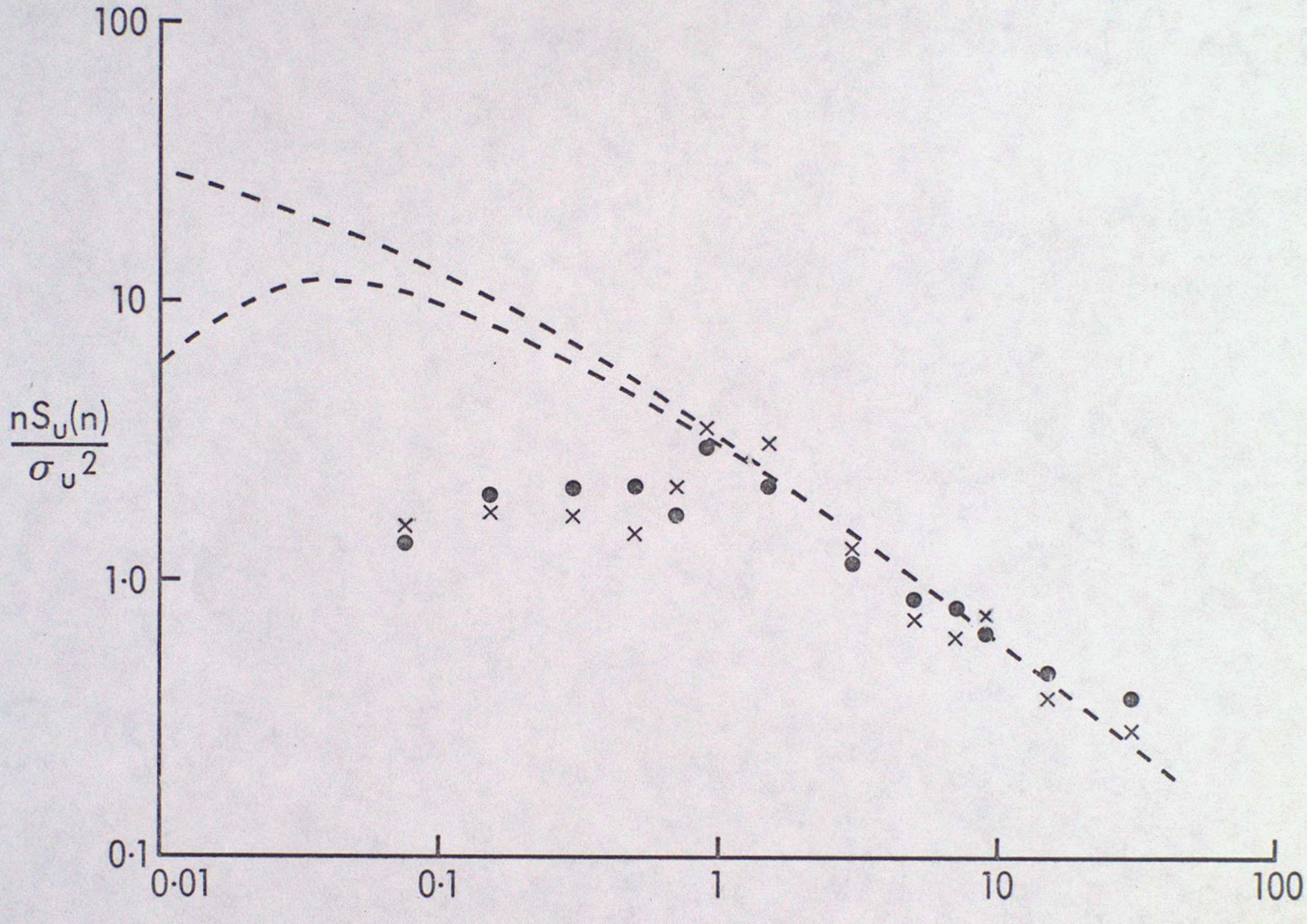
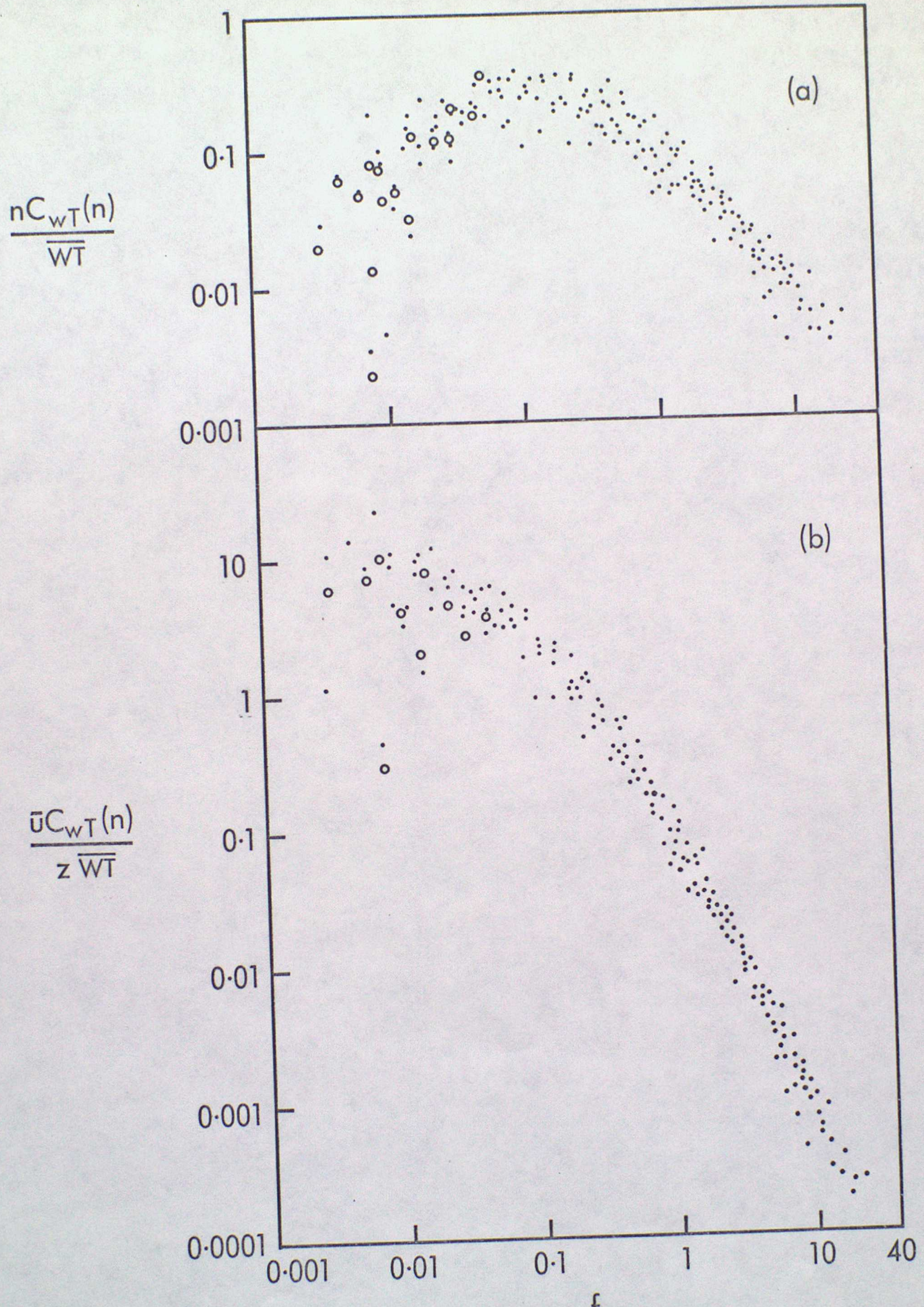


FIGURE 8.



1
0·1
0·01
0·001
(a)
$\frac{nC_{wT}(n)}{\overline{WT}}$
10
1
0·1
0·01
0·001
0·0001
(b)
$\frac{\bar{u}C_{wT}(n)}{z\,\overline{WT}}$
0·001
0·01
0·1
1
10
40
f

FIGURE 9.

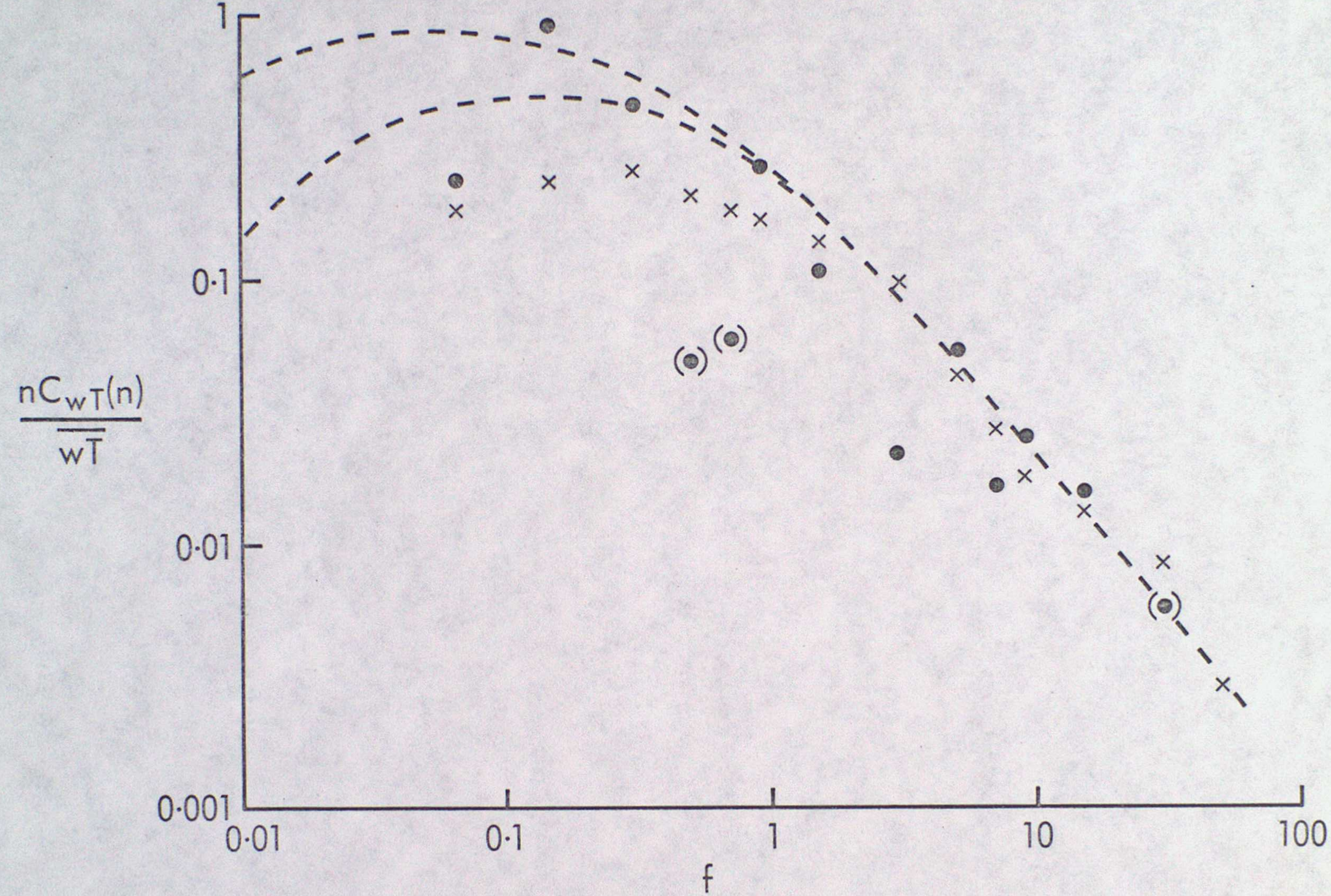


FIGURE 10.