

Meteorological Office

Boundary Layer Branch (Met O 14)

Turbulence and Diffusion Note No 14 (May 1971)

RELATIONS BETWEEN TURBULENT FLUXES OVER THE  
SEA AND NEAR-SURFACE MEAN PARAMETERS

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Note

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Deardorff (1968) discussed the variations with stability of the transfer coefficients which are often used to relate vertical fluxes of momentum, heat and moisture to near-surface mean parameters over the sea. In the course of the analysis of the data obtained during the Joint Air-Sea Interaction Experiment (JASIN), it has been necessary to estimate the stability dependence of these coefficients: Deardorff's formulations, which are based on analytical forms for the empirical functions in the Monin-Oboukhov relations for the constant flux layer and which give the results in terms of bulk Richardson numbers, have not been found convenient to use so alternatives have been derived which are described in this note. A brief discussion is also included of the likely quantitative relations between the three coefficients.

The present treatment is also based on similarity theory for the constant flux layer and makes use of the recent estimates of Dyer and Hicks (1970) for the necessary empirical functions. The governing equations are with the usual notation,

$$\frac{\partial U}{\partial z} = \frac{u_*}{kz} \phi_m(z/L) \quad (1)$$

$$\frac{\partial \theta}{\partial z} = \frac{T_*}{kz} \phi_h(z/L) \quad (2)$$

$$\frac{\partial q}{\partial z} = \frac{q_*}{kz} \phi_e(z/L) \quad (3)$$

where  $u_* = (\tau/\rho)^{1/2}$ ,  $T_* = -H/\rho c_p u_*$ ,  $q_* = -E/\rho u_*$

and  $L = -u_*^3 T_e c_p / k g (H + 175E)$ : in this last  $T$  has been taken as  $15^\circ\text{C}$ ,  $H$  is in Watts  $\text{cm}^{-2}$  and  $E$  is  $\text{gm cm}^{-2} \text{sec}^{-1}$  ( $c_p$  is  $1.00 \text{ J gm}^{-1} \text{C}^{-1}$ ). Integrating (1) - (3),

$$\frac{k}{u_*} (U - U_0) = \log z/z_0 - \psi_m(z/L) \quad (4)$$

$$\frac{k}{T_*} (\theta - \theta_0) = \log z/z_T - \psi_h(z/L) \quad (5)$$

$$\frac{k}{q_*} (q - q_0) = \log z/z_q - \psi_e(z/L) \quad (6)$$

where  $\psi = \int_0^{z/L} \frac{1-\phi}{z/L} dz/L$

Only the meteorologically important case of unstable density stratification will be considered here, and in these circumstances the tabulations of  $\psi$  given by Dyer and Hicks can be used.



Eq. (4) can be written

$$u_*^2 = C_D (U - U_0)^2$$

where  $C_D = \left\{ \kappa / (\log z/z_0 - \psi_m(z/L)) \right\}^2$  : in neutral stability  $C_D(N) = (\kappa / \log z/z_0)^2$

Conventionally  $U$  is measured at a height of 10m. Wind tunnel data of Keulegan (1951) suggest that  $U_0$  (the sea-surface drift velocity) is about 4% of  $U$ . Brocks and Krugermeier (1970) have analysed numerous wind profiles obtained over the sea in near-neutral conditions in winds up to  $12 \text{ m sec}^{-1}$  and concluded (assuming  $U_0 = 0$ ) that  $C_D(N) = 1.3 \times 10^{-3}$ , with  $z_0 = 1.2 \times 10^{-2} \text{ cm}$ : there was some evidence from their data for a few percent increase in  $C_D$  in going from low speeds to the highest observed. Smith (1970) found  $C_D = 1.35 \times 10^{-3}$  after averaging values found in both stable and unstable conditions in winds between 7 and  $16 \text{ m sec}^{-1}$  (he also assumed  $U_0 = 0$ ): there was again an indication of a very small increase with increasing speed. Other recent determinations of  $C_D$  (in wind speeds below  $12 \text{ m sec}^{-1}$ ) gave values of  $1.21 \times 10^{-3}$  (Hasse 1968) and  $1.09 \times 10^{-3}$  (Miyake et al 1970) - these authors also assumed  $U_0$  to be zero. All these data suggest that  $C_D$  varies with wind speed by no more than a few percent in winds up to about  $15 \text{ m sec}^{-1}$ , and it will be assumed therefore that  $C_D(N)$  is a constant, value around  $1.3 \times 10^{-3}$  (here assuming  $U_0 = 0.04U$ ): the uncertainty is probably less than about 20%.

The assumption is also made that  $z_0$ ,  $z_r$  and  $z_q$  are constants, independent of wind speed. This is likely to be the case because  $z_0$  at least is virtually independent of wind speed in neutral conditions, and a stability change for a given 10m wind speed results in a change of wind speed at lower heights - this is equivalent, as far as conditions near the surface are concerned, to a change of wind speed at 10m in neutral stability.

Equations (4) and (5) can be combined to give

$$\left\{ (U - U_0) / (\theta - \theta_0) \right\} T_* / u_* = \left\{ \log z/z_0 - \psi_m(z/L) \right\} / \left\{ \log z/z_r - \psi_m(z/L) \right\}$$

$$\text{or } H = \rho C_p (U - U_0) (\theta - \theta_0) C_D(N) \left\{ \log z/z_0 / (\log z/z_r - \psi_m(z/L)) \right\}^2 \left\{ \frac{\log z/z_0 - \psi_m(z/L)}{\log z/z_r - \psi_m(z/L)} \right\}$$

with a similar expression for  $E$ . If  $\theta_0$  (and  $q_0$ ) are equated to the surface values of



temperature and saturation specific humidity, as would be done conventionally, the variations of all the exchange coefficients with stability can be obtained. Thus

$$C_{H,E} / C_{H,E}(N) = \log z/z_0 \cdot \log z/z_{T,E} / \{ (\log z/z_0 - \psi_m(z/L)) (\log z/z_T - \psi_{H,E}(z/L)) \} \quad (8)$$

$$C_D / C_D(N) = \left\{ \log z/z_0 / (\log z/z_0 - \psi_m(z/L)) \right\}^2 \quad (9)$$

Table 1 calculates these variations with stability, assuming that  $z_0$ ,  $z_T$  and  $z_q$  are equal, and  $C_D(N) = 1.3 \times 10^{-3}$ . Selecting a different value for  $C_D(N)$

Table 1 Variation of transfer coefficients with stability (using values for  $\psi$  given by Dyer and Hicks (1970))

$-z/L$	$-L (z=10m)$ (m)	$C_D / C_D(N)$	$C_{H,E} / C_{H,E}(N)$
0.01	1000	1.00	1.00
0.02	500	1.00	1.01
0.05	200	1.01	1.03
0.10	100	1.03	1.06
0.20	50	1.07	1.11
0.50	20	1.14	1.21
1.00	10	1.21	1.31

makes little difference: for example, with  $C_D(N) = 1.1 \times 10^{-3}$  and  $z/L = -1.0$ , the ratio  $C_D/C_D(N)$  becomes 1.19, a change of only 2%.

Now,

$$L = -u_*^3 T_e c_p / k_g (H + 175E)$$

$$= -(U-U_0)^3 C_D^{3/2} T_e c_p / k_g \rho c_p C_{H,E} (U-U_0) (\theta_0 - \theta_{10} + 175(q_0 - q_{10}))$$

Multiplying top and bottom by  $(C_D(N))^{1/2} (\approx 3.6 \times 10^{-2})$  and putting  $T = 288^\circ K$ , this reduces to

$$L(m) = \left\{ -2.6 (U-U_0)^2 / (\theta_0 - \theta_{10} + 175(q_0 - q_{10})) \right\} \cdot \frac{C_D^{3/2}}{C_{H,E}(C_D(N))^{1/2}}$$

The maximum error in  $L$ , for  $-z/L \leq 1$ , which results from assuming  $C_D = C_{H,E} = C_D(N)$  is less than 3%, which produces less than 1% error in the estimates for  $C_D$  and  $C_{H,E}$ .



Hence we can write adequately

$$L = -2.4 U^2 / (\theta_0 - \theta_{10} + 175(q_0 - q_{10})) \quad (10)$$

If values for  $10/L$  are calculated for given  $U$  and  $(\theta_0 - \theta_{10} + 175(q_0 - q_{10}))$  and the corresponding  $C/C(N)$  are extracted from table 1, then simple nomograms showing the variations of  $C/C(N)$  with stability may be constructed (figure 1). Thus, for  $\theta_0 - \theta_{10} = 2^\circ\text{C}$ ,  $q_0 - q_{10} = 5 \times 10^{-3}$  and  $U = 5 \text{ m sec}^{-1}$ , then  $C_h/C_h(N) = 1.19$ . We see immediately that for winds above about  $10 \text{ m sec}^{-1}$  the effects of stability on the transport coefficients are small even when very large values are entered for air-sea temperature and humidity differences. In lighter winds the effects are significant but would not be easily discernible in measured fluxes unless winds are below about  $5 \text{ m sec}^{-1}$ .

It is of interest now to consider the likely relations between  $C_D$ ,  $C_h$  and  $C_E$  if it is no longer assumed that  $z_0$ ,  $z_T$  and  $z_E$  are equal, and if account is taken of the fact that for practical reasons  $q_0$ ,  $\theta_0$  and  $U_0$  are not necessarily measured (or estimated) at the same height. The published data are in some conflict. For example eddy correlation measurements by Hasse (1968) suggest that in near-neutral conditions  $C_h \approx 1.0 \times 10^{-3}$  but budget studies by Robinson (1966) give  $C_h \approx 1.3 \times 10^{-3}$  a value similar to that found by Chamberlain (1968) for  $C_E$  from wind-tunnel investigations of evaporation.

Owen and Thomson (1963) derived the empirical relation

$$1/B = \frac{1}{K} \log z_0/z_{T,q} = 0.52 \left( \frac{30 u_* z_0}{\nu} \right)^{0.45} \left( \frac{\nu}{kD} \right)^{0.8} \quad (11)$$

valid for  $\frac{u_* z_0}{\nu} > 3.3$ : here  $\nu$  is the kinematic viscosity and  $k$  and  $D$  are the thermometric conductivity, and the molecular diffusion coefficient for water vapour. Now  $z_0 \approx 1.2 \times 10^{-2}$  and it follows that the relation is not expected to hold for  $u_* < 40 \text{ cm sec}^{-1}$  or in winds less than about  $12 \text{ m sec}^{-1}$ . Chamberlain (1968) carried out wind tunnel experiments on the evaporation of water vapour from rough surfaces which gave results in approximate agreement with eq. (11) but also had no data corresponding to winds less than about  $15 \text{ m sec}^{-1}$  over the sea. However, using his



extrapolating curve to the value for  $1/B$  corresponding to flow over smooth surfaces, then for  $U = 10 \text{ m sec}^{-1}$  or  $\frac{U_* z_0}{\nu} \approx 3$ , we obtain a value for  $1/B$  around 2. In this case,  $z_0 \approx 2.2 z_v$  which in near-neutral conditions gives  $C_e/C_D \approx 0.94$ . A similar calculation for  $U = 20 \text{ m sec}^{-1}$  gives  $C_e/C_D \approx 0.91$ .

Because of the factor  $(\nu/k, D)^{0.8}$  in eq. (11) it follows that  $1/B$  for heat transport is 1.08 times that for water vapour, which suggests that  $C_h$  and  $C_e$  can be equated with negligible error. It appears therefore that the ratio  $C_{h,e}/C_D$ , in near-neutral conditions, decreases slowly with increasing wind speed but that the ratio departs by only about 10% from unity even in very strong winds (this is of course neglecting evaporation of spray droplets).

Stewart (1961) pointed out that significant momentum is likely to be transported to the sea in the process of wave generation and maintenance, and because the effective level at which this momentum is given up must be above that where wind speed is equal to the phase velocity of the waves, then the vertical flux of momentum must decrease as the surface is approached. Hence the turbulent mixing near the surface is less than would otherwise be expected and so in this region the transfer coefficients for heat and water vapour will also be less than the measurements at greater heights might suggest. As a result there will be an increase in gradients of temperature and moisture which will help to maintain fluxes close to those anticipated in a wave-free situation, but it is not clear that complete compensation will occur: the ratio  $C_{h,e}/C_D$  may thus be decreased.

Finally, in practical applications of the bulk formulations for heat and moisture transfers it is usual to equate  $\theta_0$  (and hence  $q_0$ ) to temperature measured at a depth of a few tens of centimetres. This introduces two sources of error. First, it is well known that in unstable conditions a cool "skin" forms at the surface where the temperature is typically around 0.2 deg C lower than the bulk water temperature. Thus the use of the bulk value should produce overestimates for both heat and moisture fluxes. Secondly, it is not correct to equate  $\theta_0$  and  $q_0$  to the surface (skin) values  $\theta_s$  and  $q_s$ , but if this is done a simple order of



magnitude argument suggests that overestimates of a few percent in the heat and moisture fluxes may result. Thus the thickness of the viscous boundary layer ( $\delta_v$ ) is of order  $\nu/k u_*$  (Kraus 1966), and this is approximately equal to  $\delta_T$  and  $\delta_v$ , the thermal and water vapour boundary layer thicknesses (Schlichting 1968). Taking  $z_0$  as  $1.2 \times 10^{-2}$  it then follows that for winds less than about  $10 \text{ m sec}^{-1}$ ,  $\delta_{T,q} > z_0$ . Hence except in strong winds we can assume that temperature varies linearly with height from surface up to  $z = z_0$ . Now,  $z_{T,q} \sim z_0 \sim 1 \times 10^{-2} \text{ cm}$ : hence,

$$H = \rho c_p C_H (\theta_s - \theta_{10})(U_{10} - U_0) = k (\theta_s - \theta_0) / 1 \times 10^{-2}$$

where  $k$  is the thermal conductivity. Putting  $C_H = 1.3 \times 10^{-2}$  and  $U_0 \sim 0$ ,

$$\theta_s - \theta_0 / (\theta_0 - \theta_{10}) \sim (\rho c_p U_{10} / k) \cdot 1.3 \times 10^{-5}$$

$$\sim 6 U_{10} \times 10^{-5}$$

The corresponding expression for  $q$  is

$$q_s - q_0 / (q_0 - q_{10}) \sim 5 U_{10} \times 10^{-5}$$



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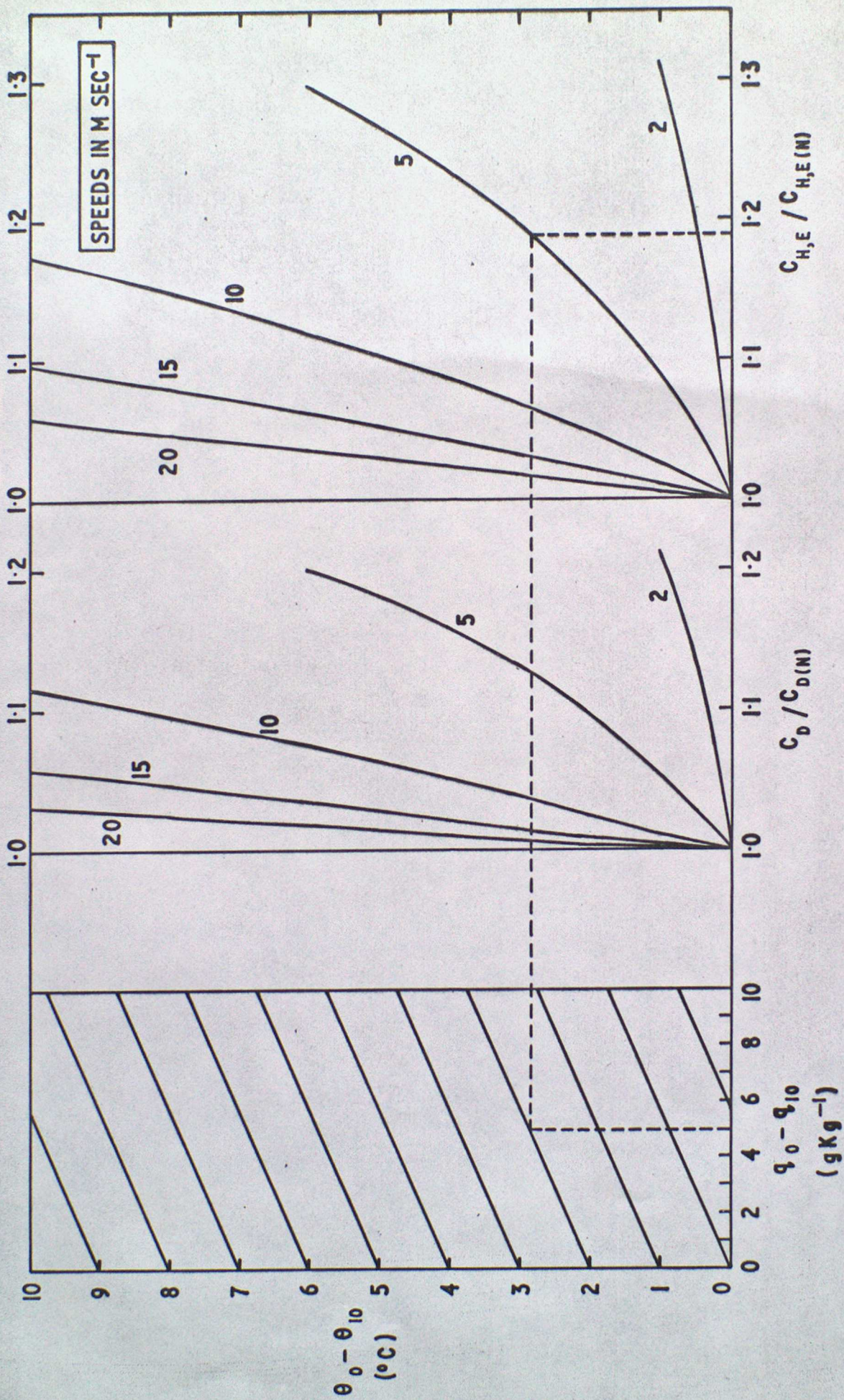


FIGURE 1. VARIATIONS OF TRANSFER COEFFICIENTS  
WITH WIND SPEED AND AIR-SEA  
TEMPERATURE AND HUMIDITY DIFFERENCES