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# WIND AND TEMPERATURE TO 50 KM. OVER ENGLAND

Anomalous sound propagation experiments, 1944-45

BY

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### Errata

Page 11, equation 19 ; for  $\int f \sqrt{n} . dh =$  read  $\int_0^H \sqrt{n} . dh = f$

Page 12, equation 27 ; for “ .... (27 ” read “ .... (27) ”.



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# WIND AND TEMPERATURE TO 50 KM. OVER ENGLAND

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## SUMMARY

This report describes the results of experiments made in England during 1944-45, when recordings of sound received by anomalous paths from large explosions were utilized in an attempt to obtain data on wind and temperature at great heights. The methods used and their limitations are outlined and the results of the calculations presented. The principal results are :—

- (i) In England the wind in winter at heights of 30-45 Km. is usually between SW. and NW. with speeds of 40-80 m./sec. In summer the directions are between NE. and SE. and the speeds less than 20 m./sec.
- (ii) The temperature between 35 and 50 Km. appears to increase to values approaching surface values. This increase is not as great in winter as in summer, and a 20-40°C. variation of temperature between summer and winter is likely at these levels.

## § 1—INTRODUCTION

The zones of audibility around a large explosion have been observed to be :—

- (i) An “inner zone” of audibility extending outwards from the explosion point with size and shape determined by the magnitude of the explosion and the wind and temperature distribution in the troposphere. The intensity of received sound usually decreases as the distance from the source increases.
- (ii) A “silent zone” (where the explosion is not heard) at greater distances from the explosion point.
- (iii) One or more “outer zones” of audibility at greater ranges still from the explosion point. On some occasions further zones of silence followed by other outer zones of audibility have been observed.

A sketch of a possible audibility distribution around a large explosion is given in Fig. 1. The outer zones of audibility result from the so-called “anomalous propagation”.

The observed characteristic features of this anomalous propagation in temperate latitudes in the northern hemisphere may be summarized :—

- (i) The inner boundary of the outer zone is usually 100-200 Km. from the explosion point and this distance is less in winter than in summer.
- (ii) The time taken by the sound to reach the outer zone is often one to two minutes longer than it would take if it travelled along the earth's surface from explosion point to outer zone.
- (iii) The outer zone may exist all around the explosion point, but it is most frequently to the west of it in summer and to the east in winter.
- (iv) It is very common in the outer zone to hear two or more sounds from a single explosion (multiple sound arrivals).
- (v) The sound waves observed in the outer zone are of low frequency and may be mainly below the audible range. Sounds of frequencies of the order of 1-10 c./sec. are commonly observed from the larger explosions.

The fact that sound travelling to the outer zone has a minimum range before it reaches the surface and takes a comparatively long time to reach this distance suggests that it must follow a path high in the upper air before returning to the earth's surface. It is unlikely that its trajectory will include a reflection at its highest point because a discontinuity sufficient to produce this reflection is improbable in the atmosphere. Hence it appears that the sound must be continuously refracted in its path. Such a path would imply that the maximum horizontal speed of the sound relative to the earth will occur at the top of its trajectory in order for it to be bent down to earth again. If the usual laws for the speed of sound in a gas are applied to this anomalous propagation of sound through the atmosphere, it appears that the speed of transmission of

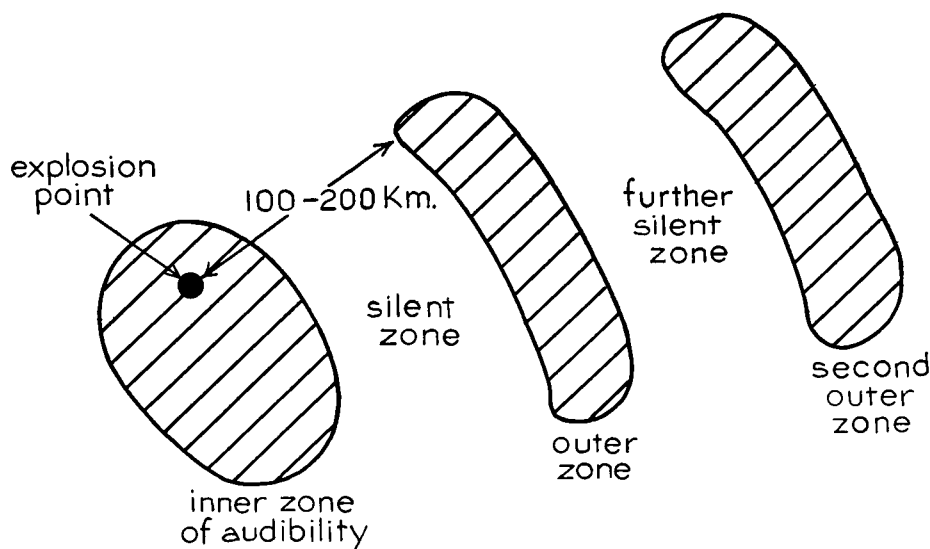


FIG. 1—ZONES OF AUDIBILITY

the sound relative to the earth depends only on the composition and temperature of the atmosphere and the wind at any level. It can be shown that, in general, neither a sufficiently great speed of sound nor a sufficiently large time delay is produced by sound paths with maximum heights below the present common limit of direct measurements of wind, temperature and composition of the atmosphere (*circa* 20 Km.). Hence the phenomena of anomalous sound propagation are likely to provide data about the atmosphere at greater heights. A comprehensive bibliography on this subject has been published<sup>1\*</sup>. The main results about which there is fairly general agreement are :—

(i) The maximum trajectory heights attained by these sound “rays” are usually 40–50 Km. above the earth's surface. Some writers<sup>2</sup> believe that occasionally heights of 100 Km. or more may be attained by very low frequency waves which are still of sufficient intensity to be detected in the outer zone. The intensity of the sound received depends on the size of the original explosion, the geometry of the path and the absorption along the path. The absorption is greater at high levels and also increases with the frequency, becoming very great when the mean free path of the molecules is no longer small compared with the wave-length of the sound wave<sup>3</sup>.

\* The index numbers refer to the bibliography on p. 20.

(ii) The most probable cause of the high sound speeds at 40–50 Km. is that the temperature there approaches surface values. The general temperature structure up to, say, 50 Km. in temperate latitudes will then comprise the troposphere up to about 10 Km., the mainly isothermal region of the stratosphere from 10 to 30 Km., and finally an inversion layer to 50 Km. or so. No substantial data on temperatures above 50 Km. have been obtained from this work. Opinion is not unanimous that high temperatures at 40–50 Km. are the sole cause of the high sound speeds there. It is very unlikely that there is any great change in the atmosphere's composition to account for them, but some suggestions have been made recently<sup>4,5</sup> that the speed of sound may increase at low densities, presumably because of a change in the nature of the propagation. If this is so it is doubtful how much the increase of sound speed may be due to a rise in temperature and how much to this other cause. In the present paper the assumption has been made that it is due entirely to a temperature change.

(iii) The outer zones of audibility are not usually symmetrical about the explosion point. In temperate latitudes they appear usually to the west of it in summer and to the east in winter although sometimes they are observed on all sides. This suggests that the wind at the highest levels traversed has predominantly easterly components in summer and westerly in winter.

During the period April 1944–April 1945 the opportunity arose of performing set experiments to use the phenomena of anomalous sound propagation as a means of obtaining further information about wind and temperature at high levels. The methods used and the results obtained are described below.

## § 2—EXPERIMENTAL ARRANGEMENTS

The locations of the various recording installations and explosion points were dictated by operational necessities, e.g. questions of communications, maintenance, proximity of buildings to explosion points, as well as by the most likely zones of audibility. Fig. 2 shows the locations of the various explosion points and recording installations. The design of the experiments was changed, usually by varying the explosion points, as more information about high-level winds and temperatures was obtained. In all, 16 experiments were carried out during the period April 1944–April 1945. A short description of individual experiments is given in Appendix III.

*Explosion points.*—These were located at

Friskney (Wash area)  
Larkhill (Salisbury Plain)  
St. Margaret's Bay (Dover)  
Trawsfynydd (North Wales)  
Okehampton (Devon)

Additional explosion points were used at Thetford, Spurn Head and Fylingdales Moor (Yorkshire Moors) during Experiment No. 7 only.

*Recording bases.*—

A lattice of at least 12 microphones arranged in squares or diamonds of 10-Km. side was constructed near Canterbury in south-east England  
A lattice of 12 microphones with an irregular layout was available at Larkhill (Salisbury Plain)  
A lattice of 4 and later 6 microphones in squares of 5-Km. side was installed at Friskney near the Wash  
A small lattice was installed near Malvern for the first experiment only.

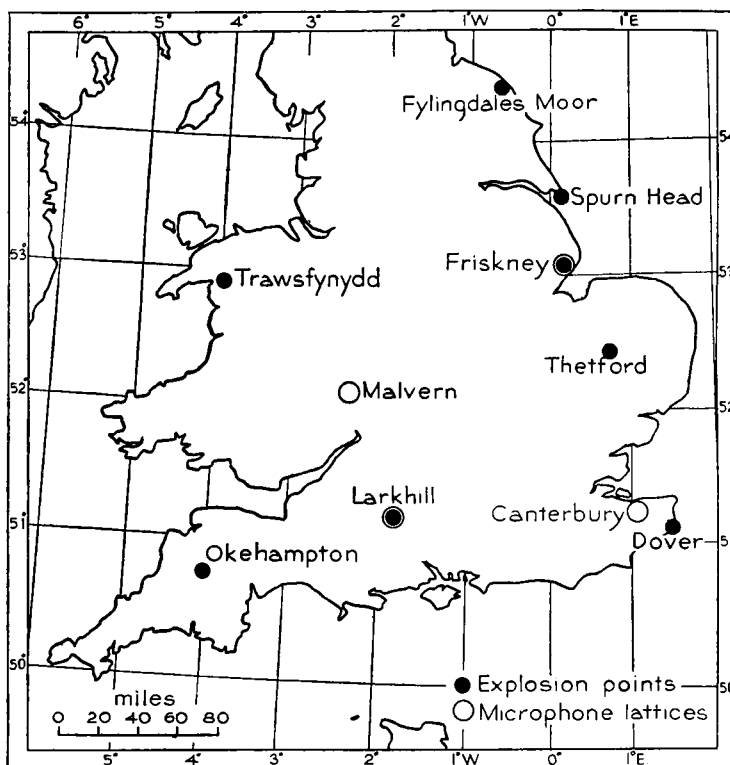


FIG. 2—LOCATION OF THE EXPLOSION POINTS AND RECORDING INSTALLATIONS

*Recording equipment.*—The microphones for recording the instant of arrival of sound waves were of the hot-wire type, and were placed in surveyed positions to form squares or triangles of sides 5–10 Km. They were connected by line to a sound-ranging recorder which included string galvanometers, with an optical projecting system and moving film for recording and a tuning-fork control for the timing. Synchronizing marks were arranged on the films either automatically from a chronometer or manually by producing electrically a sudden deflection of one of the strings. The timing motor produced vertical lines on the films at intervals of 10, 100 and 1,000 sound metres, a sound metre being defined as a unit of time equal to  $1/a_0$  sec., where  $a_0$ , the speed of sound in air, is taken as 337.6 m./sec. at 10°C. and 50 per cent. relative humidity. The time of sound inception at each microphone could be read to an accuracy of a few sound metres, the accuracy depending on the signal-to-noise ratio on the records and definition of the point of sound inception. Electrical types of interference and string deflections due to local effects, e.g. wind at the microphones, made film reading difficult at times. As is mentioned later, the accuracy with which the time of arrival of sound can be read from the film is an important limiting factor in the determination of the characteristics of the travel of sound.

### § 3—MEASUREMENTS OBTAINED OR AVAILABLE

$X_a$  the range and  $\alpha$  the azimuth (with respect to true north) from the explosion point to a microphone at a recording base were computed from accurately surveyed co-ordinates of the explosion points and microphones. The use of Cassini grid, instead of spherical, co-ordinates in these calculations introduced errors of less than 4 min. of angle and 10 m. of range, which are small in comparison with other errors.

The total time of travel,  $t_a$ , of the sound from the explosion point to a microphone, was given by the interval on the recording film between the arrival of the sound wave and the instant of the explosion as impressed on the record. The instant of explosion was transmitted by direct telephone link from explosion point to recorder point where an operator pressed a switch on the recorder. The error in  $t_a$  introduced by the operator's reaction time was probably between 0.5 and 1 sec. ;  $t_a$  was usually 10–15 min.

$A$ , the "characteristic speed" of the sound waves, and  $\beta$ , the apparent azimuth of travel, were derived from the times of arrival of a wave front at three or more microphones suitably deployed in lattice formation. The procedure and corrections involved are described in Appendix I.  $A$  is a basic parameter of sound travel and is equal to the speed at which the wave front travels horizontally at the apex of the trajectory. It is important that  $A$  and also  $\beta$  be determined with high accuracy. Great care was therefore taken to determine as accurately as possible the times of sound inception on the microphone film, though for various reasons determination was sometimes difficult. Some doubtful determinations were revealed by the fact that the resulting value of  $A$  was less than the value appropriate to some levels of the atmosphere for which normal measurements of temperature and wind were available. The use of a lattice of microphones suitably spaced enabled the most probable values of  $A$  and  $\beta$  to be deduced.

$\phi$  ( $= \alpha - \beta$ ) is a measure of the total sideways deflection  $Y_a$  caused by the cross component of wind throughout the sound trajectory and  $Y_a = X_a \sin \phi$ .

Wind and temperature measurements to heights of about 18 Km. were available from normal radio-sonde ascents at six-hourly intervals within the area on each of the experimental dates.

Details of the measurements made are given in Appendices IV and V.

#### § 4.—OUTLINE OF THE METHOD OF USING THE DATA

Three main assumptions were made in this work :—

- (i) That the travel of the sound from explosion to microphones through the atmosphere could be treated in terms of refraction only.
- (ii) That the general (Laplace) law for the speed of sound in a gas was valid throughout the path.
- (iii) That the atmosphere could be considered to be horizontally stratified, i.e. at any given level the wind and temperature would be the same all over the area of sound travel.

When the characteristic speed  $A$  and the angular twist  $\phi$  were known the path of the sound within the region of known wind and temperature (0–18 Km.) could be calculated uniquely to give the time  $t_b$ , range  $X_b$  and sideways deflection  $Y_b$  of the sound path below the level of 18 Km. Since the total time  $t_a$ , range  $X_a$  and deflection  $Y_a$  were measured, the values of the time  $t_a - t_b$ , range  $X_a - X_b$ , and deflection  $Y_a - Y_b$  to be ascribed to the portion of the sound path above 18 Km. were found.

The problem then became one of finding a sound path above the datum level of 18 Km. for a reception along each azimuth as illustrated in Fig. 3 to satisfy :—

- (i) A time  $t_a - t_b$
- (ii) A range  $X_a - X_b$
- (iii) A sideways deflection  $Y_a - Y_b$
- (iv) A horizontal speed of  $A$  at the top of the trajectory
- (v) Known values of the wind and temperature at 18 Km.



up to about 20 Km.<sup>6,7,8,9</sup>. Above that level there is a different type of régime with temperature increasing with height and stronger winds changing in direction from easterly in summer to westerly in winter. There was no special reason for choosing 35 Km. as the dividing level except that it usually divided the "unknown layer" in which the sound travelled from 18 Km. to about 50 Km. into two roughly equal sublayers. This division at 35 Km. into two layers, although somewhat arbitrary, probably has however a certain amount of physical reality, and variations of a few kilometres in this dividing level are not important in this work as all that can be expected from the results is a general picture of conditions above the limit of the radio-sonde measurements. A few calculations with the dividing level at different heights between 25 and 40 Km. showed no significant change in the general picture from that obtained by assuming it was always at 35 Km. By adopting this linear law of variation of  $n$  with height it also became a relatively straightforward matter to build up a system of equations with similar laws of variation of sound speed  $a$  through the air (proportional to the square root of the absolute temperature) and the two wind components  $w$  and  $w'$  with height. All the data including cross-wind deflections as well as times of travel on several different azimuths could then be utilized simultaneously. In some cases more data were available than were necessary to solve all the equations based on these simple assumptions, and it then became possible to obtain statistically the most probable values of wind and temperature to satisfy the data with this particular type of structure.

### § 5—BASIC EQUATIONS

The plane-wave propagation of sound through the atmosphere has been treated fully by Milne<sup>10</sup>, both for the case where the wind and temperature vary in three dimensions and for the simplified case of the horizontally stratified atmosphere which has been assumed here. Plane-wave propagation has also been assumed except in the derivation of the characteristic speed  $A$  and apparent azimuth of sound travel  $\beta$  at the microphones, when it was necessary to apply corrections for the curvature of the sound wave as described in Appendix I.

If the source of sound  $S$  and the microphone  $M$  are at the same level, the distance  $SM$  is denoted by  $X_a$ , the total time of travel by  $t_a$ , and the maximum height of the sound "ray" trajectory above the surface by  $Z$ ;  $x, z, y$  are the co-ordinates of a typical point  $P$  on the path with respect to the axes  $SM$ , the vertical and the mutual perpendicular through  $S$ , and  $t$  is the time the sound takes to travel from  $S$  to  $P$ . The wave-normals along the path are all parallel to a fixed vertical plane  $\Omega$  which is at an angle  $\phi$  to the  $xz$  plane. The perpendicular distance from the microphone  $M$  to the plane  $\Omega$  is called  $Y_a$ .

The horizontal wind has components  $w$  and  $w'$  parallel and perpendicular to this plane  $\Omega$  at any height  $z$ . The speed of sound

$$a = \text{constant} \times \sqrt{T}, \quad \dots (1)$$

where  $T$  is the temperature in degrees Absolute at height  $z$ . The inclination of the wave normal to the horizontal is denoted by  $\psi$ .

The general refraction law is

$$a \sec \psi + w = A, \quad \dots (2)$$

where  $A$  is the characteristic speed of the sound "ray" considered and is equal to the horizontal speed of the sound at the top of the trajectory. From the kinematical relations

$$\frac{dx}{dt} = (a \cos \psi + w) \cos \phi - w' \sin \phi \quad \dots (3)$$

$$\frac{dz}{dt} = a \sin \psi \quad \dots (4)$$

$$\frac{dy}{dt} = (a \cos \psi + w) \sin \phi + w' \cos \phi \quad \dots (5)$$

and noting that  $\phi$  is constant it follows that

$$X_a \cos \phi = 2 \int_0^z \frac{a \cos \psi + w}{a \sin \psi} dz \quad \dots (6)$$

$$\begin{aligned} Y_a &= X_a \sin \phi \\ &= -2 \int_0^z \frac{w'}{a \sin \psi} dz \quad \dots (7) \end{aligned}$$

$$t_a = 2 \int_0^z \frac{1}{a \sin \psi} dz. \quad \dots (8)$$

If  $n = A - (a + w) \quad \dots (9)$

it is also possible to deduce the relationship

$$X_a \cos \phi - A t_a = -2 \int_0^z \sqrt{\left\{ \frac{n}{a} \left( 2 + \frac{n}{a} \right) \right\}} dz \quad \dots (10)$$

and equations (6) and (7) may be rewritten as

$$X_a \cos \phi = 2 \int_0^z \frac{1 + \frac{w}{a} \left( 1 + \frac{n}{a} \right)}{\sqrt{\left\{ \frac{n}{a} \left( 2 + \frac{n}{a} \right) \right\}}} dz \quad \dots (11)$$

$$Y_a = -2 \int_0^z \frac{\frac{w'}{a} \left( 1 + \frac{n}{a} \right)}{\sqrt{\left\{ \frac{n}{a} \left( 2 + \frac{n}{a} \right) \right\}}} dz. \quad \dots (12)$$

The meteorological structure is known up to the maximum height of the radio-sonde measurements (*circa* 18 Km.), while values of range  $X_a$ , time of sound path  $t_a$ , angle of twist  $\phi$  and characteristic speed  $A$  are all measured. Using the above equations a unique path of the sound can be simply calculated from the surface to 18 Km. Above that level, however, the converse problem of determining the meteorological structure at all heights when the main parameters only of the sound path are known does not appear to be capable of a unique determination. It is therefore necessary to postulate some form of wind and temperature variation with height, e.g. linear, parabolic, exponential, and then solve for the constants of the laws of variation assumed. Moreover, the laws assumed will have to be comparatively simple in view of the small number of equations available in these experiments from which their constants will have to be deduced.

It thus appears that, in general, experiments of this type will not lead to unique solutions for the wind and temperature, but on the other hand may lead to some general conclusions about these meteorological elements at great altitudes. Bearing these facts in mind the simplified methods described below were developed.

*Approximations to the basic equations.*—Even if it is assumed that  $w$ ,  $w'$ , and  $a$  are simple functions of  $z$ , equations (10), (11) and (12) lead to complicated expressions, and hence approximations to these equations were first obtained. The following equations are correct to first order in  $n/a_0$  ( $a_0$  was taken to be the standard speed of sound 337.6 m./sec. in air at 10°C. and 50 per cent. relative humidity). As  $\phi$  is a small angle it is permissible to put  $\cos \phi = 1$  in these equations.

$$X_a = \sqrt{2a_0} \int_0^z \frac{1}{\sqrt{n}} \left( \frac{A}{a_0} - \frac{5n}{4a_0} - \frac{a - a_0}{2a_0} \right) dz \quad \dots (13)$$

$$t_a = \sqrt{2a_0} \int_0^z \frac{1}{\sqrt{n}} \left( 1 + \frac{3n}{4a_0} - \frac{a - a_0}{2a_0} \right) dz \quad \dots (14)$$

where  $t_a$  is measured in sound metres.

$$Y_a = \sqrt{2a_0} \int_0^z \frac{1}{\sqrt{n}} \left( \frac{w'}{a_0} \right) dz \quad \dots (15)$$

where  $n/a_0$  may be as much as 0.25 in the lower stratosphere but usually is much smaller than this. Moreover since the largest contributions to  $X_a$  and  $t_a$  occur at levels where  $n$  is small it is sufficiently accurate for calculations in the layer above 18 Km. to write

$$X_a \simeq t_a \simeq \sqrt{2a_0} \int_0^H \frac{dh}{\sqrt{n}} \quad \dots (16)$$

$$t_a - X_a \simeq \sqrt{2a_0} \int_0^H \frac{1}{\sqrt{n}} \left\{ \left( 1 - \frac{A}{a_0} \right) + \frac{2n}{a_0} \right\} dh. \quad \dots (17)$$

In these and subsequent equations referring only to the layer above the 18-Km. level,  $h$  denotes height above 18 Km. and  $H$  the maximum height of the sound trajectory above the same datum level.

Since total time  $t_a$ , total range  $X_a$ , and total sideways deflection  $Y_a$  were measured and time  $t_b$ , range  $X_b$ , and deflection  $Y_b$  were calculated for the portions of the sound paths between 0 and 18 Km., the remaining values of time  $t_a - t_b$ , range  $X_a - X_b$  and deflection  $Y_a - Y_b$  were known for the portion of the sound path above this level.

The equations used in this "remaining layer" were therefore

$$\frac{X_a - X_b}{\sqrt{2a_0}} \simeq \frac{t_a - t_b}{\sqrt{2a_0}} \simeq \int_0^H \frac{dh}{\sqrt{n}} = g \quad \dots (18)$$

$$\frac{(t_a - t_b) - (X_a - X_b) + \left( \frac{A}{a_0} - 1 \right) (X_a - X_b)}{\sqrt{2a_0}} \frac{a_0}{2} = \int_0^H f \sqrt{n} dh = \quad \dots (19)$$

$$\frac{Y_a - Y_b}{\sqrt{2a_0}} = \int_0^H \frac{1}{\sqrt{n}} \left( \frac{w'}{a_0} \right) dh = m. \quad \dots (20)$$

In each case  $g$ ,  $f$ , and  $m$  were calculated and  $n$ , which is a function of wind and temperature, was given a law of variation with  $h$  so that equations (18), (19) and (20) could be integrated directly.

*Linear variation of  $n$  with height above 18 Km.*—As has been pointed out above the assumption of a "two-straight-line" structure has considerable advantages over other systems, and methods based on this assumption have been used by investigators in recent American work<sup>11, 12</sup>.

In this section  $h$  and  $H$  indicate heights reckoned upwards from the 18-Km. level, subscripts 0 and 1 refer to values at 18 and 35 Km. above the earth's surface, and  $u$  and  $v$  are the components of wind towards east and north respectively. Generally,

$$\left. \begin{aligned} w &= u \sin \beta + v \cos \beta \\ w' &= u \cos \beta - v \sin \beta \\ n &= A - (a + w) \text{ (equation (9))} \\ &= A - a - u \sin \beta - v \cos \beta. \end{aligned} \right\} \dots (21)$$

Between  $h = 0$  and  $h = h_1$  the assumed linear variation of the wind components and the speed of sound may be represented by expressions of the type

$$w' = w_0' + \frac{h}{h_1} (w_1' - w_0') \quad \dots (22)$$

and it follows that

$$n = n_0 + \frac{h}{h_1} (n_1 - n_0). \quad \dots (23)$$

Similarly, between  $h_1$  and  $H$  ( $H$ , the maximum height of the trajectory, varies with the azimuth) and between  $h_1$  and a selected fixed height  $h_2$ :

$$\left\{ \begin{array}{l} w' = w_1' + \frac{h - h_1}{H - h_1} (w_H' - w_1') \\ w' = w_1' + \frac{h - h_1}{h_2 - h_1} (w_2' - w_1') \end{array} \right.$$

i.e.  $\left\{ \begin{array}{l} w' = w_1' + \frac{h - h_1}{H - h_1} \left\{ (u_H - u_1) \cos \beta - (v_H - v_1) \sin \beta \right\} \\ w' = w_1' + \frac{h - h_1}{h_2 - h_1} \left\{ (u_2 - u_1) \cos \beta - (v_2 - v_1) \sin \beta \right\} \end{array} \right\} \dots (24)$

also  $\left\{ \begin{array}{l} n = n_1 - \frac{h - h_1}{H - h_1} \left\{ a_H - a_1 + (v_H - v_1) \cos \beta + (u_H - u_1) \sin \beta \right\} \\ n = n_1 - \frac{h - h_1}{h_2 - h_1} \left\{ a_2 - a_1 + (v_2 - v_1) \cos \beta + (u_2 - u_1) \sin \beta \right\} \end{array} \right\}.$

Since  $n = 0$  at  $h = H$ , then

$$n = n_1 \left( 1 - \frac{h - h_1}{H - h_1} \right) \quad \dots (25)$$

$$n_1 = (a_H - a_1) + (v_H - v_1) \cos \beta + (u_H - u_1) \sin \beta \quad \dots (26)$$

$$n_1 = \frac{H - h_1}{h_2 - h_1} \left\{ (a_2 - a_1) + (v_2 - v_1) \cos \beta + (u_2 - u_1) \sin \beta \right\}. \quad \dots (27)$$

Equations (23) and (25) for  $n$  and (22) and (24) for  $w'$  in the two layers 18 to 35 Km. and 35 Km. to the maximum height of the trajectory enable equations (18), (19), (20) to be integrated, with the following results:

$$n_1 = \frac{a_0}{2(X_a - X_b)} \left\{ 3(t_a - t_b) - 3(X_a - X_b) + 3(X_a - X_b) \left( \frac{A}{a_0} - 1 \right) - h_1 \sqrt{32 \frac{n_0}{a_0}} \right\} \quad \dots (28)$$

$$H - h_1 = \sqrt{\frac{n_1}{a_0}} \left\{ \frac{X_a - X_b}{2\sqrt{2}} - \frac{h_1}{\sqrt{\frac{n_1}{a_0}} + \sqrt{\frac{n_0}{a_0}}} \right\} \quad \dots (29)$$

$$(Y_a - Y_b) \sqrt{\frac{a_0}{2}} = \frac{2}{3} \frac{h_1}{n_1 - n_0} \left\{ (w_1' \sqrt{n_1} - w_0' \sqrt{n_0}) + \frac{2(w_0' n_1 - w_1' n_0)}{\sqrt{n_1} + \sqrt{n_0}} \right\} \\ + \frac{2}{3} \frac{H - h_1}{\sqrt{n_1}} (2w_H' + w_1'). \quad \dots (30)$$

From equation (28)  $n_1$  is found, and then from equation (29)  $H$  is found.

If equations (28) and (29) are available on three azimuths,  $n_1$  and  $H$  are known for three different values of  $\beta$ , and hence  $a_1$ ,  $u_1$ , and  $v_1$  can be calculated from equation (21), and  $a_2$ ,  $u_2$ , and  $v_2$  from equation (27).

If equation (30) is used, however, it is necessary to have data on two azimuths only. The unknowns in equation (30) are  $w_1'$  and  $w_H'$  when  $n_1$  and  $H$  are known. From equation (24), it may be seen that  $w_H'$  can be replaced by a linear function of  $w_2'$  and  $w_1'$  or of  $u_1$ ,  $v_1$ ,  $u_2$ , and  $v_2$ . Hence equations of the form of equation (30) on each azimuth can be reduced to equations containing linear terms in  $u_1$ ,  $v_1$ ,  $u_2$ , and  $v_2$ .

Two more linear equations are available in  $a_1$ ,  $u_1$ , and  $v_1$  from equation (21) on the two azimuths.

Finally two linear equations in  $a_2$ ,  $u_2$  and  $v_2$  can be obtained from equation (27) on the two azimuths.

Hence from the results along two azimuths the six unknowns  $a_1$ ,  $u_1$ ,  $v_1$ ,  $a_2$ ,  $u_2$  and  $v_2$  may be obtained, the subscript 1 referring to a height of 35 Km. above the ground, and the subscript 2 to, say, 50 Km. (providing the values of  $H$  on the two azimuths extend approximately to that level). The values of  $a$  can readily be converted to temperature values from equation (1).

Theoretically therefore, when the cross-wind data are used in this manner, good receptions on two azimuths only are sufficient to produce a result from an experiment. In practice, however, the equations are very sensitive to the errors in observation because of difficulties in determining exactly the initial times of sound reception at the recording microphones, and it was decided to use the system only in experiments in which receptions on three or more azimuths were obtained. Thus for 3 azimuths, 9 linear equations in 6 unknowns were available; for 4 azimuths 12 equations and so on. The determination of the most probable values by least-squares methods was then carried out with consequent reduction of the effect of the errors of observation.

*Parabolic variation of  $n$  with height above 18 Km.*—Above the 18-Km. datum level the parabolic equation required will be of the form :—

$$n = \frac{n_0}{1 - \mu} \left( 1 - \frac{h}{H} \right) \left\{ 1 - \mu \left( 1 - \frac{h}{H} \right) \right\}, \quad \dots (31)$$

i.e. it has  $n = n_0$  when  $h = 0$  and  $n = 0$  when  $h = H$ ; and the constant  $\mu$  and maximum height  $H$  have to be determined.

The type of solution required will usually be that having  $d^2n/dh^2$  negative, i.e.  $\mu$  positive and less than unity. By integrating equations (18) and (19) with this condition

$$g = \frac{2\gamma H}{\tan\gamma\sqrt{n_0}} \quad \text{where } \mu = \sin^2\gamma \quad \dots (32)$$

$$\text{and} \quad \frac{2f}{gn_0} = \operatorname{cosec}^2 2\gamma - \frac{\cot 2\gamma}{2\gamma}. \quad \dots (33)$$

Hence  $H$  and  $\mu$  are determinable, and on a given azimuth a parabolic variation of  $n$  against  $h$  is found which will satisfy the range and time conditions of the sound travel.

If data on three azimuths are available equation (21) then allows  $a$ ,  $u$  and  $v$  to be calculated at any given height, e.g. 20, 30, 40 Km., etc., above the earth's surface.

Unfortunately equation (20) which represents important data is not used in this method, and so results cannot be obtained with data from two azimuths only as in the two-straight-line method. One way of obtaining some information from the sideways deflection data  $Y_a - Y_b$

in conjunction with the parabolic method is to postulate an "equivalent constant cross-wind" such that

$$\bar{w}' = \frac{\int_0^H \frac{w' dh}{\sqrt{n}}}{\int_0^H \frac{dh}{\sqrt{n}}} \quad \dots (34)$$

Then 
$$\bar{w}' = \frac{Y_a - Y_b}{t_a - t_b} \quad \dots (35)$$

and  $\bar{w}'$  is thus a wind component weighted according to the time spent by the sound in the various layers at different heights. If it is compounded with the equivalent constant cross-wind across a second azimuth it is possible to obtain "time" mean values  $\bar{u}$  and  $\bar{v}$  from two equations of the form

$$\bar{w}' = \bar{u} \cos \beta - \bar{v} \sin \beta. \quad \dots (36)$$

These values  $\bar{u}$ ,  $\bar{v}$  will have little quantitative significance as they are derived from two components referring to different trajectories and maximum heights. As such they will merely reveal the main wind trends at the higher levels. Further if

$$\frac{X_a - X_b}{t_a - t_b} = \bar{a} \cos \psi + \bar{w} \quad \dots (37)$$

$$A = \bar{a} \sec \psi + \bar{w} \quad \dots (38)$$

where  $\bar{a}$  and  $\bar{\psi}$  are weighted means of sound speed and inclinations of the wave normal to the horizontal and

$$\bar{w} = \bar{v} \cos \beta + \bar{u} \sin \beta \quad \dots (39)$$

$$\bar{a}^2 = (A - \bar{w}) \left( \frac{X_a - X_b}{t_a - t_b} - \bar{w} \right) \quad \dots (40)$$

which gives a certain amount of information about the temperature at the higher levels.

The system of using (time) mean values in this manner needs great care as it may lead to incorrect results when two trajectories of very different maximum heights are used to obtain the values of  $\bar{u}$  and  $\bar{v}$ . Any values of  $\bar{a}$ ,  $\bar{u}$  and  $\bar{v}$  obtained must of course be consistent with results depending on equations (32) and (33) or similar equations obtained from the original assumed laws of variation of  $n$  with  $h$ .

*Other laws of variation of  $n$  with height above 18 Km.*—Another possible law that was considered was an exponential variation of  $n$  with height of the form

$$\frac{n}{n_0} = \frac{1 - \exp \{ \lambda (h - H) \}}{1 - \exp (-\lambda H)}, \quad \dots (41)$$

where the constant  $\lambda$  and maximum height  $H$  have to be determined. The method of solution and use of the deflection  $Y_a - Y_b$  are similar in principle to that outlined above for the parabolic law and will not be described fully.

Other laws can be treated similarly but none of them appear to have any particular advantages over the others. The two-straight-line system was therefore regarded as the most useful, and the results it produced experimentally are described below.

*Computational stages—summary.*— $A$ ,  $\phi$ ,  $X_a$ ,  $Y_a$  and  $t_a$  were determined from the microphone films and survey data. Curves of  $a + w$  were plotted up to 18 Km. for each sound propagation

azimuth concerned, using the radio-sonde measurements of temperature and wind.  $X_b$ ,  $Y_b$  and  $t_b$  for the layer up to 18 Km. were obtained by numerical integration of equations (10), (11), (12) or more simply, but less accurately, equations (13), (14), (15); and thence  $X_a - X_b$ ,  $Y_a - Y_b$  and  $t_a - t_b$  were obtained. From this stage the computations depended on the data available from the individual experiments.

For experiments yielding data for only one azimuth, values of  $a + w$  above 18 Km. were determined along with a mean (time) wind component perpendicular to that azimuth.

From data along two azimuths, a mean (time) cross-wind and mean (time) temperature were determined.

From data along three or more azimuths, determination of wind and temperature were made up to about 50-Km. height. In addition the best mean (time) values of wind and temperature were obtained from the values of cross-wind.

These calculations were made along the lines indicated above, full computations being made for the two-straight-line and the parabolic laws of variation. The former method was regarded as the more satisfactory, and the results obtained from it are given below. Results obtained from the assumptions of a parabolic variation were given in previous papers<sup>13, 14</sup>.

## § 6—SUMMARY OF RESULTS

The results of the experiments are summarized in Tables I and II and Figs. 4–7. Table I gives the individual results of the determinations of vertical distribution of temperature and westerly and southerly components of wind up to 50 Km. obtained by the two-straight-line assumption in the 7 experiments where there was sound reception along 3 or more azimuths (Nos. 5, 6, 8, 11, 12, 15 and 16). Table II gives the mean (time) values above 18 Km. of wind and temperature obtained in all the experiments. The values shown are considered to be the most probable values of wind and temperature between heights of 18 Km. and about 50 Km. required to satisfy the observations of the sound travel. It is emphasized that these results are dependent on the physical assumptions made as well as on the simplifying mathematical assumptions, so that, while it is believed that they give a picture of the main features of the wind and temperature structure within these levels, their detailed accuracy is not likely to be great. This will be so in all similar work in which the meteorological parameters are calculated from the data of sound travel.

The diagrams (one for each experiment), in Fig. 4, show the degree of audibility in relation to the bearing and distance of the recording site from the explosion points which are all assumed to be located at the centre of the circles. In general it is seen that the points of good audibility tend to lie to the west of the explosion point in summer, to the east in winter and to be more uniformly distributed in the spring. The simplest explanation of this is that in these latitudes there is a seasonal change of winds at high levels from easterly in summer to westerly in winter and that the temperature is high at the level of the top of the sound trajectories in spring. Successful receptions in diametrically opposite directions suggest in general high temperatures. This is probably the case, although it must be remembered that the real criterion of audibility according to the refraction theory is that the function  $a + w$  shall have its maximum value at the top of the sound trajectory; and very large variations of wind could conceivably produce all-round audibility without the necessity of postulating an increase of  $a$  and hence of temperature at the greatest heights. Moreover large values of  $a + w$  in the troposphere can also have a large effect on the audibility diagram, because they may exceed  $a + w$  at the greater heights and so prevent refraction down from them even though  $a + w$  is comparatively great there. However,

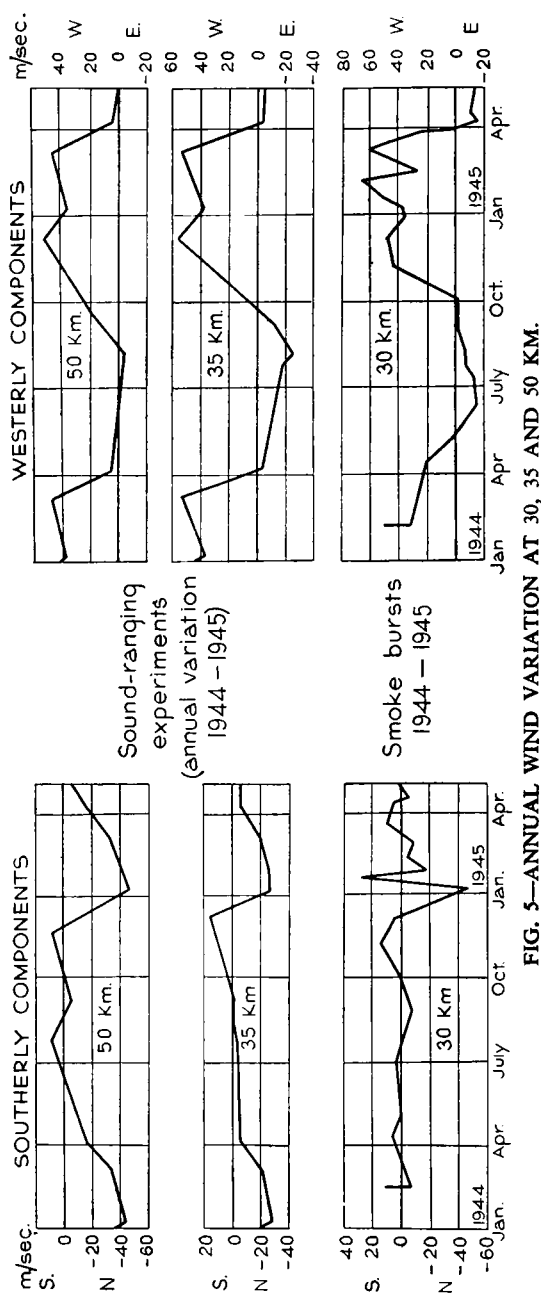


FIG. 5—ANNUAL WIND VARIATION AT 30, 35 AND 50 KM.

TABLE I—TEMPERATURE AND WESTERLY AND SOUTHERLY WIND COMPONENTS ON INDIVIDUAL EXPERIMENTS

Height	Series number and date															
	5 26.7.44		6 8.8.44		8 13.9.44		11 6.12.44		12 6.1.45		15 6.3.45		16 6.4.45			
	Temp- erature W. S.	Wind component W. S.	Temp- erature W. S.	Wind component W. S.	Temp- erature W. S.	Wind component W. S.	Temp- erature W. S.	Wind component W. S.	Temp- erature W. S.	Wind component W. S.	Temp- erature W. S.	Wind component W. S.	Temp- erature W. S.	Wind component W. S.		
Km.	°A.	m./sec.	°A.	m./sec.	°A.	m./sec.	°A.	m./sec.	°A.	m./sec.	°A.	m./sec.	°A.	m./sec.		
50	310	- 4	+ 10	- 4	+ 2	+ 15	- 5	+ 53	+ 9	+ 37	- 46	+ 46	- 33	+ 4		
45	278	- 9	+ 5	278	- 11	+ 6	- 4	242	+ 12	243	- 40	283	- 29	+ 1		
40	248	- 15	0	240	- 18	- 3	- 3	217	+ 14	223	- 33	224	- 23	+ 2		
35	220	- 20	5	208	- 25	- 13	- 2	193	+ 17	204	- 39	197	- 19	- 5		
30	222	- 12	3	212	- 17	- 8	- 1	199	+ 44	205	- 29	202	- 20	- 6		
25	222	- 5	1	216	- 8	- 4	0	206	+ 34	206	+ 19	206	- 20	- 3		

TABLE II—BEST MEAN VALUES (WITH RESPECT TO TIME) OF TEMPERATURE AND WESTERLY AND SOUTHERLY WIND COMPONENTS IN THE LAYER FROM 18 KM. TO THE TOP (ABOUT 50 KM.) OF THE TRAJECTORY

	Series number and date															
	1	2	3	5	6	7	8	9	10	11	12	13	15	16		
	16.4.44	6.5.44	13.6.44	26.7.44	8.8.44	23.8.44	13.9.44	11.10.44	8.11.44	6.12.44	6.1.45	6.2.45	6.3.45	6.4.45		
Temperature	..	226	241	244	238	..	degrees Absolute 236 227		..	177	178	133?	226	243		
Westerly component	+25	+11	-12	-14	-8	+2	metres per second +4 +9		+40?	+67	+53	+118?	+37	0		
Southerly component	..	-4	-2	-1	-8	..	-2 -6		+20?	-19	-57	-24?	-20	0		

These values, particularly the temperature, are all very approximate but they give consistent trends for the annual variation (see Fig. 7).

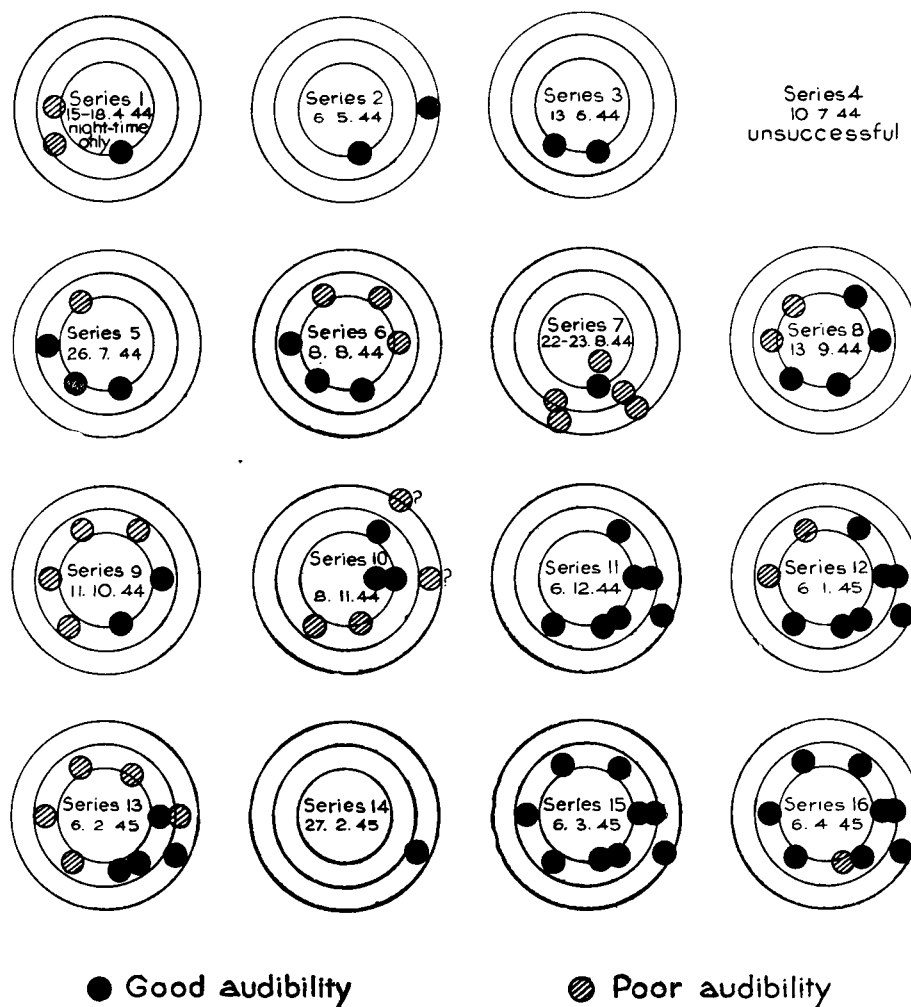


FIG. 4—AUDIBILITY DIAGRAM

The radii of the circles are 200, 300 and 400 Km.

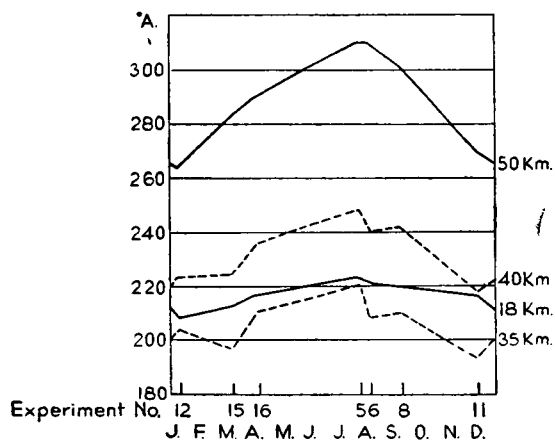


FIG. 6—ANNUAL TEMPERATURE VARIATION AT 18, 35, 40 AND 50 KM.

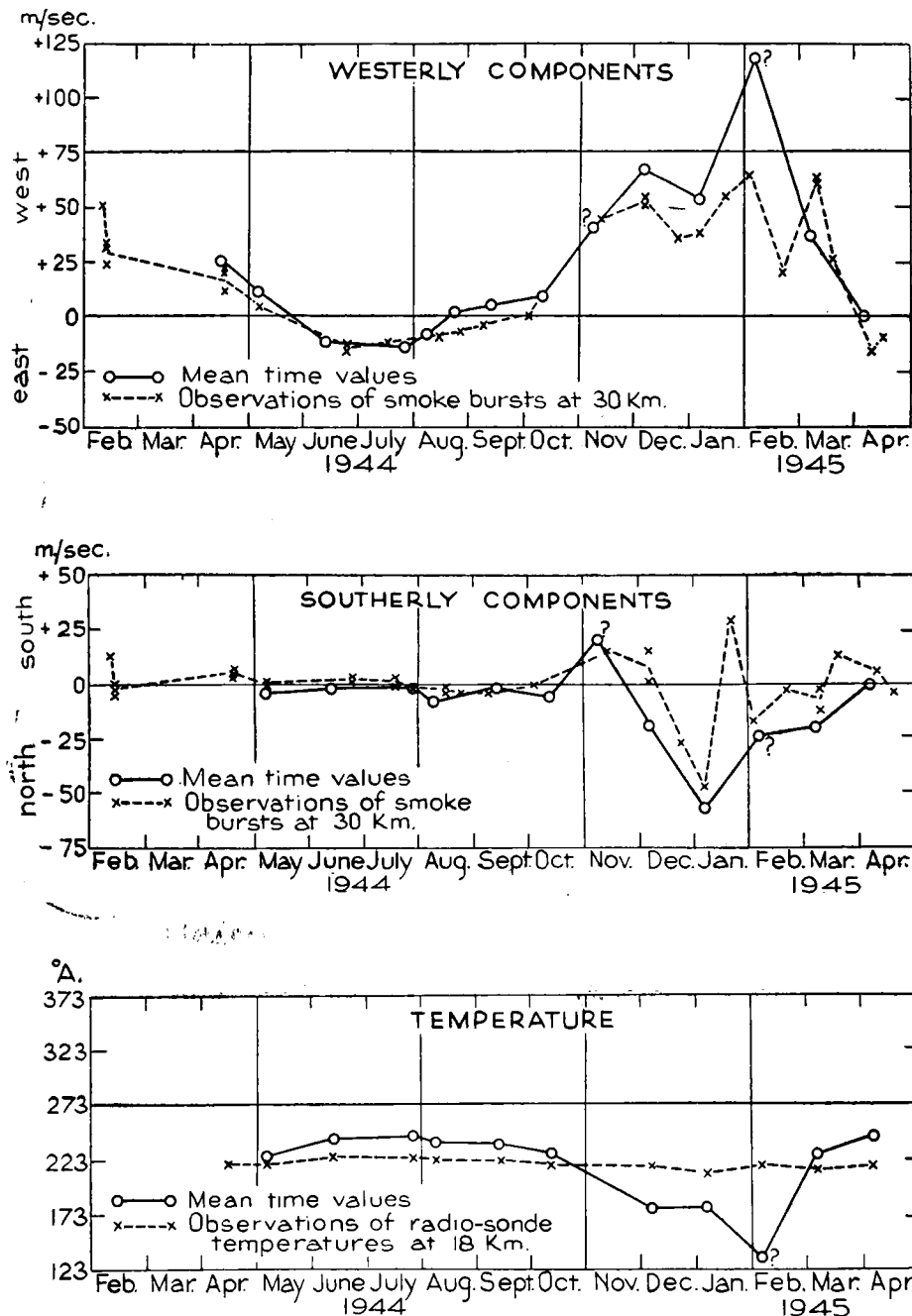


FIG. 7—BEST MEAN TIME VALUES OF WIND COMPONENTS AND TEMPERATURE

These values are very approximate and individual determinations showed considerable scatter from them

it will be seen that the other results tend to confirm the seasonal changes of wind and high temperatures at great heights inferred from the audibility diagram. More information on audibility diagrams is given by Murgatroyd<sup>15</sup>.

Figs. 5 and 6 give suggested annual variations of wind and temperature at high levels in these latitudes. The wind diagrams include comparisons with independent results obtained from

direct observations of wind from smoke bursts at 30 Km. over south-east England during the same period<sup>6</sup>. Since there is fair agreement between these and the results obtained from the calculations above they provide some confirmation of both wind and temperature results since they were both derived simultaneously from the same data. In these calculations the values of wind components and temperature are interdependent to a great extent—a change of 1 m./sec. in wind speed has approximately the same effect as a change of 1°C. in temperature on the horizontal speed of sound.

The final diagram (Fig. 7) shows the results of the mean (time) calculations. It has been explained above that this type of calculation must of necessity be “rough and ready”, but even so the agreement between the results and the smoke-bursts results is good. Furthermore the mean (time) temperature values obtained while not appearing very probable as regards absolute magnitude do give some support to the idea of a large annual variation of temperature at a height of 40–50 Km. as shown in Fig. 6.

### § 7—DISCUSSION OF RESULTS

The main conclusions are as follows :—

#### *Temperature.*—

(i) There is a large inversion of temperature in the 40–50-Km. levels in these latitudes. At 50 Km. values of temperature roughly equal to surface values are suggested.

(ii) From about 20 to 30 Km. the temperature is roughly isothermal, but there is some evidence, particularly in winter, that there may be a slight lapse of temperature there.

(iii) At any given level at 40 Km. or above there is a large annual variation of temperature. Approximately 40°A. is suggested for the amount of this variation at a height of 50 Km.

#### *Wind.*—

(i) The wind at 30–50 Km. in these latitudes is predominantly westerly in winter and easterly in summer. The winter westerly components may attain values of 50–60 m./sec. but the summer easterly components have maximum values of 20–30 m./sec. There is some evidence that at levels greater than 50 Km. the wind may be westerly during both winter and summer although the speeds are likely to be considerably greater in winter.

(ii) North-south components of wind are usually less than the east-west components at these high levels. Northerly components appear to be more frequent in winter and southerly components more frequent in summer, but the amount of data is too small to draw any more definite conclusions on the meridional circulation.

*General.*—Most of the above conclusions have been suggested in other work. Previous calculations based on anomalous sound receptions, meteor observations, calculation of direct absorption of solar radiation by ozone, work on atmospheric oscillations, and measurements in American rocket trials all lead to the conclusion that there must be a region of high temperature at about 50 Km. above the earth's surface, as found in this work.

As far as is known all other evidence so far available suggests that the stratosphere remains roughly isothermal from the tropopause until an inversion starts at about 30 Km. Recent radio-sonde measurements in Great Britain<sup>7</sup> show that in summer the inversion starts below 30 Km. but that in winter temperature falls continuously, though slightly, from near the tropopause to the limit of the ascents at about 30 Km.

The large annual variation in temperature at 40 Km. or above is in agreement with results given by Wexler<sup>9</sup> and Cray<sup>11</sup>. It is reasonable to suppose that if these high temperatures are caused by direct absorption of solar radiation the values attained would be less in winter than in

summer. This would however be complicated by the annual variation of ozone. If there is more ozone at higher levels in winter all the absorption could take place at higher levels in winter than in summer. In this case the maximum of temperature might occur at greater heights in winter than in summer. This result was also obtained by Crary<sup>11</sup>.

The annual change of easterly to westerly wind components in these latitudes at 30–50 Km. is now well known. It was discussed by Dr. F. J. W. Whipple in connexion with his pioneer work on anomalous sound reception<sup>16</sup>, and has since been supported by the work described by Murgatroyd and Clews<sup>6</sup>, Scrase<sup>7</sup>, Brasefield<sup>8</sup>, Crary<sup>11</sup>, and Richardson and Kennedy<sup>12</sup>. The reason for this annual change must be the differential heating between the poles and the equator at high levels, with the poles colder during the polar night but becoming warmer than the equator in the summer on account of increased absorption of solar radiation in high latitudes. The tendency for a return to westerly winds at all seasons about 50 Km. would indicate that 50–60 Km. may be the level of the effective top of the ozone layer.

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#### APPENDIX I—MEASUREMENT OF CHARACTERISTIC SPEED $A$ AND APPARENT AZIMUTH OF TRAVEL $\beta$ FROM TIMES OF ARRIVAL AT THE MICROPHONES

*Basic equations.*—First consider a microphone at M with a plane wave front of sound moving over the microphone lattice with the characteristic speed  $A$  as it descends. Denoting the inclination of the wave normal to the horizontal

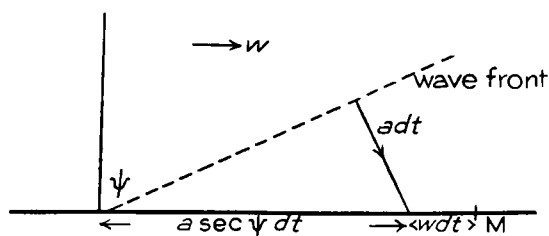


FIG. 8—MOVEMENT OF WAVE FRONT IN ELEVATION IN TIME  $dt$

by  $\psi$ , the speed of sound through air by  $a$  and the following wind component by  $w$ , it can be seen from Fig. 8, which shows in elevation the movement of the wave front in time  $dt$ , that

$$A = a \sec \psi + w. \quad \dots (42)$$

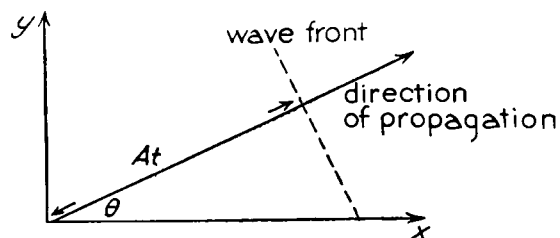


FIG. 9—MOVEMENT OF WAVE FRONT IN PLAN ACROSS LATTICE OF MICROPHONES

Fig. 9 shows, in plan, the plane wave front as it moves across a lattice of microphones with speed  $A$  in the direction  $\theta$ . Its equation at any time  $t$  referred to co-ordinates  $x$  and  $y$  is

$$rx + ly = Lt, \quad \dots (43)$$

where  $l/r = \tan \theta$  and  $(L/r) \cos \theta = A$ . If three microphones O, P and Q have co-ordinates  $(0, 0)$ ,  $(x_p, y_p)$  and  $(x_q, y_q)$  with respective time intervals between the arrival of the sound at O and P and O and Q of  $t_p$  and  $t_q$ , then

$$\left. \begin{aligned} rx_p + ly_p &= Lt_p \\ rx_q + ly_q &= Lt_q \end{aligned} \right\} \quad \dots (44)$$

Hence  $l/r$  and  $L/r$  can be found and thence  $A$  and  $\theta$  (and hence  $\beta$ ) can be calculated from a knowledge of the survey positions and arrival times at a minimum of three microphones. In practice squares or diamonds of four microphones 5–10 Km. apart were usually used, as the correction necessary in that case on account of the assumption of a plane wave was negligible. It was occasionally desirable to utilize irregular lay-outs however and it was then necessary to apply corrections for curvature of the wave front.

*Curvature correction.*—If, in Fig. 10, S is the explosion point, O and P are two microphones in plan view, PQ represents a curved wave front, and P'Q a plane wave front, then the correction to be applied to the position of P relative to O is PP', i.e. SP (sec  $\delta - 1$ ).

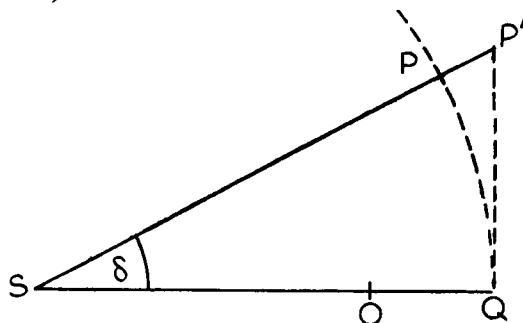


FIG. 10—CURVATURE CORRECTION DIAGRAM

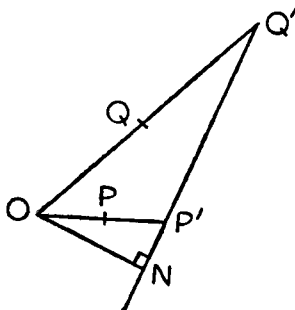
*Correction for height of microphone.*—From Fig. 8, if M is at a distance  $\delta h$  above the reference horizontal plane it will be necessary to apply a correction to the time of arrival of the sound at M. This correction can readily be seen to be  $\delta h \tan \psi / A$ .

When these corrections have been applied the source of sound S and all the microphones can be treated as if they were at one given level and the sound propagated as a plane wave which moves over the microphone lattice with speed  $A$  at an azimuth  $\beta$ . If the lattice comprises more than three microphones the most probable values of  $A$  and  $\beta$  may be obtained from equation (44) by least-squares methods or more simply by the graphical method described below.

*Graphical method of obtaining  $A$  and  $\beta$ .*—Let Fig. 11 represent in plan the three microphones O, P and Q, and suppose the sound arrives at P and Q at times  $t_p$  and  $t_q$  respectively after it arrives at O. Then if P' and Q' are constructed so that

$$OP' = \frac{OP}{t_p} \quad \text{and} \quad OQ' = \frac{OQ}{t_q}$$

the line P'Q' gives the position of the wave front after unit time. Hence the perpendicular distance ON to P'Q' is proportional to the speed  $A$  and the direction of ON gives the bearing  $\beta$  of the arriving sound.

FIG. 11—GRAPHICAL METHOD OF OBTAINING  $A$  AND  $\beta$ 

Using this principle a chart can be constructed for any lattice of microphones relative to any reference point O. A chart of this type is particularly useful when there are several microphones with poor inceptions of sound arrival. If all the readings are plotted on this chart the best straight line that can be fitted to the plotted points will give the most probable values of  $A$  and  $\beta$ . When this is done due weight can be given if necessary to the probable accuracy of each reading (which will depend on the sharpness of the sound inception on the record of the microphone considered). A refinement of this type could also be introduced of course in the least-squares solution mentioned above.

## APPENDIX II—LIST OF SYMBOLS

- $A$  = Characteristic speed of sound waves  
 $a$  = Speed of sound through air  
 $a_0$  = Standard speed of sound through air = 337.6 m./sec. at 10°C. and 50 per cent. relative humidity  
 $f, g$  = Parameters defined on p. 11  
 $H$  = Maximum height above 18 Km.  
 $h$  = Height above 18 Km.  
 $L, l, r$  = Equation parameters where  $l/r = \tan \theta$  and  $A = (L/r) \cos \theta$   
 $m$  = Parameter defined on p. 11  
 $n = A - (a + w)$  = a parameter  
 $T$  = Absolute temperature  
 $t$  = Time  
 $t_a$  = Total time of travel of the sound  
 $t_b$  = Time of travel of the sound below 18 Km.  
 $t_p, t_q$  = Time of interval between arrival of wave front at microphone at O and microphones at P and Q  
 $u$  = Component of wind velocity towards east  
 $v$  = Component of wind velocity towards north  
 $w$  = Wind component in apparent direction of propagation of sound  
 $w'$  = Wind component perpendicular to apparent direction of propagation of sound  
 $X_a$  = Range from explosion point to microphone  
 $X_b$  = Range of sound path below 18 Km.  
 $x$  = Horizontal co-ordinate in the direction from explosion point to microphone  
 $x_p, y_p, x_q, y_q$  = Co-ordinates of microphones at P and Q with respect to microphone at O  
 $Y_a$  = Sideways deflection = Distance from microphone to plane  $\Omega = X_a \sin \phi$   
 $Y_b$  = Sideways deflection of sound path below 18 Km.  
 $y$  = Horizontal co-ordinate at right angles to  $x$   
 $Z$  = Maximum height of sound wave  
 $z$  = Vertical co-ordinate  
 $\alpha$  = Azimuth (degrees true) from explosion point to microphone  
 $\beta$  = Apparent azimuth of travel  
 $\gamma$  = Equation parameter (parabolic variation with height) ( $\mu = \sin^2 \gamma$ )  
 $\delta$  = Angle subtended at the explosion point by two microphones  
 $\lambda$  = Equation parameter (exponential variation with height)  
 $\mu$  = Equation parameter (parabolic variation with height)  
 $\theta$  = Direction of motion of a wave front through a lattice of microphones  
 $\phi$  = Angular twist =  $\alpha - \beta$   
 $\psi$  = Inclination of the wave normal to the horizontal  
 $\Omega$  = Fixed vertical plane at an angle  $\phi$  to  $xz$ -plane  
 Subscript 0 = Values at 18 Km.  
 Subscript 1 = Values at 35 Km.  
 Subscript 2 = Values at 50 Km.  
 Subscript  $H$  = Values at maximum height  $H$

## APPENDIX III—GENERAL SUMMARY OF AUDIBILITY

Explosion Series	Date	Time	Explosion points	Recorders	Successful	Reception	Unsuccessful	Description of record	Remarks
1	15.4.44	{ 0412 1100 1700 2300 0404 1138 1708 2308 }	Friskney	Canterbury Malvern Larkhill	Friskney-Canterbury (1, 4, 8)	Friskney-Canterbury (2, 3, 5, 6, 7) Friskney-Malvern Friskney-Larkhill	Friskney-Canterbury explosions 1, 4, 8 all had reasonably good records with fair sound inceptions and signs of multiple arrivals	Successful recordings mainly at night (300 lb. gun-cotton)	
2	6.5.44 7.5.44	2100 0001	Friskney Okehampton	Canterbury	Friskney-Canterbury Okehampton-Canterbury	Nil	Both good inceptions with double arrivals	(300 lb. gun-cotton)	
3	13.6.44	{ 2100 2130 }	Friskney	Canterbury Larkhill	Friskney-Canterbury Friskney-Larkhill	Nil	Friskney-Canterbury, moderate amplitudes with signs of double arrivals both explosions. Friskney-Larkhill good record explosion 1, poor record explosion 2 with single arrivals	(300 lb. gun-cotton)	
4	10.7.44	{ 2100 2130 }	Friskney Dover	Canterbury Larkhill Friskney	Nil	Friskney-Canterbury Friskney-Larkhill Dover-Friskney Dover-Larkhill	Larkhill films unreadable due to local gunfire. Faint traces on Friskney records, but these could not be indentified definitely with Dover explosion	Sound of Friskney explosion received at Canterbury by tropopause path only (300 lb. gun-cotton)	
5	26.7.44	{ 2200 2230 }	Friskney Dover	Canterbury Larkhill Friskney	Friskney-Canterbury Friskney-Larkhill Dover-Larkhill	Dover-Friskney	Friskney-Canterbury moderate records with multiple arrivals. In one of the arrivals the measured characteristic speed appears to be exceeded by troposphere $a + w$ values along that azimuth. Friskney-Larkhill and Dover-Larkhill good records with single arrivals	Dover-Friskney received by troposphere paths but not by anomalous paths	
6	8.8.44	{ 2100 2130 }	Friskney Dover Larkhill	Canterbury Larkhill Friskney	Friskney-Canterbury(1) Friskney-Larkhill(2) Dover-Larkhill(2)	Friskney-Canterbury(2) Friskney-Larkhill(1) Larkhill-Friskney Larkhill-Canterbury Dover-Friskney Dover-Larkhill(1)	Friskney-Canterbury moderate to poor inceptions. Dover-Larkhill and Friskney-Larkhill good inceptions. Dover-Larkhill triple arrivals	Friskney-Canterbury (2) reception spoilt by electrical interference	
7	22.8.44	{ 2300 2330 }	Fylingdales Spurn Head	Canterbury Larkhill	Nil	Fylingdales-Canterbury Fylingdales-Larkhill Spurn Head-Canterbury Spurn Head-Larkhill	...	The explosions at Fylingdales, Spurn Head and Thetford were not recorded at Canterbury as clearly as the one made at Friskney and so these explosions points were not used again (600 lb. gun-cotton)	
3 } 4 }	23.8.44	{ 2300 2330 }	Friskney Thetford	Canterbury	Friskney-Canterbury	Thetford-Canterbury	Moderate record, signs of double arrival		

## APPENDIX III—continued

Explosion Series	Date	Time	Explosion points	Recorders	Successful	Reception	Unsuccessful	Description of record	Remarks
8	13.9.44	{ 0001 0030	Friskney Dover Larkhill	Canterbury Larkhill Friskney	Friskney-Canterbury Friskney-Larkhill Larkhill-Canterbury Larkhill-Friskney	Dover-Larkhill Dover-Friskney		All moderately good inceptions. Traces of double arrival on Friskney-Larkhill and Larkhill-Canterbury records	(600 lb. gun-cotton)
9	11.10.44	{ 0001 0030	Friskney Dover Larkhill	Canterbury Larkhill Friskney	Friskney-Canterbury Larkhill-Canterbury	Friskney-Larkhill Larkhill-Friskney Dover-Larkhill Dover-Friskney		Moderate to good inceptions. Trace of double arrival on Friskney-Canterbury	(600 lb. gun-cotton)
10	8.11.44	{ 0001 0030	Friskney Larkhill Okehampton*	Canterbury Larkhill Friskney	Larkhill-Canterbury Larkhill-Friskney Okehampton-Canterbury Okehampton-Larkhill Okehampton-Friskney	Friskney-Canterbury Friskney-Larkhill		Larkhill-Canterbury moderate to good inception Larkhill-Friskney moderate to poor inception Okehampton-Larkhill good inception. Multiple arrivals all cases	Receptions of normal troposphere paths Friskney-Canterbury, Larkhill-Canterbury, Okehampton-Larkhill (600 lb. gun-cotton)
11	6.12.44	{ 0001 0030	Friskney Larkhill Trawsfynydd	Canterbury Larkhill Friskney	Friskney-Canterbury Friskney-Larkhill Larkhill-Canterbury Larkhill-Friskney(2) Trawsfynydd-Canterbury Trawsfynydd-Larkhill(2) Trawsfynydd-Friskney	Larkhill-Friskney(1) Trawsfynydd-Larkhill(1)		Multiple arrivals Friskney-Canterbury, Larkhill-Canterbury, Larkhill-Friskney, Trawsfynydd-Larkhill, Trawsfynydd-Friskney Single arrivals Friskney-Larkhill, Trawsfynydd-Canterbury Good inceptions Larkhill-Canterbury Moderate inceptions Friskney-Canterbury, Larkhill-Friskney Poor inceptions, Friskney-Larkhill, Trawsfynydd-Canterbury, Trawsfynydd-Larkhill	Troposphere paths Friskney-Canterbury, Larkhill-Canterbury, Trawsfynydd-Larkhill, Trawsfynydd-Friskney (600 lb. gun-cotton)
12	6.1.45	{ 0001 0030	Friskney Larkhill Trawsfynydd Dover	Canterbury Larkhill Friskney	Friskney-Canterbury Friskney-Larkhill Larkhill-Canterbury Larkhill-Friskney(1) Trawsfynydd-Canterbury Trawsfynydd-Larkhill Trawsfynydd-Friskney	Larkhill-Friskney(2) Dover-Larkhill Dover-Friskney		Multiple arrivals Friskney-Canterbury, Larkhill-Canterbury, Trawsfynydd-Larkhill, Trawsfynydd-Friskney Single arrivals Friskney-Larkhill, Larkhill-Friskney Good inceptions Trawsfynydd-Larkhill Fair inceptions Friskney-Canterbury, Larkhill-Canterbury Rather poor inceptions Friskney-Larkhill, Larkhill-Friskney, Trawsfynydd-Canterbury, Trawsfynydd-Friskney	Troposphere paths Friskney-Canterbury Friskney-Larkhill Trawsfynydd-Larkhill (600 lb. gun-cotton)

\* Explosion 1 only

## APPENDIX III—continued

Series	Explo- sion	Date	Time	Explosion points	Recorders	Successful	Reception	Unsuccessful	Description of record	Remarks
13	1 } 2 }	6.2.45 {	0001 0030	Friskney Larkhill Trawsfynydd Dover	Canterbury Larkhill Friskney	Friskney-Canterbury(2) Larkhill-Canterbury Trawsfynydd-Canterbury Trawsfynydd-Larkhill	Friskney-Larkhill Larkhill-Friskney Trawsfynydd-Friskney Dover-Larkhill Dover-Friskney		Multiple arrivals on all films. Good inceptions, Friskney-Canterbury Fair inceptions, Larkhill-Canterbury, Trawsfynydd-Canterbury Rather poor inceptions, Trawsfynydd-Larkhill	Troposphere paths, Friskney-Canterbury, Larkhill-Canterbury. Recordings of anomalous path sound waves were obtained between azimuths of 90° and 163° only (600 lb. gun-cotton)
14	1 } 2 }	27.2.45 {	1500 1530	Trawsfynydd	Canterbury	Trawsfynydd-Canterbury	Nil		Moderate to good inceptions, single arrivals	(4,000 lb. ammunition)
15	1 } 2 }	6.3.45 {	0001 0030	Friskney Larkhill Trawsfynydd Dover	Canterbury Larkhill Friskney	Friskney-Canterbury Friskney-Larkhill Larkhill-Canterbury Larkhill-Friskney(1) Trawsfynydd-Canterbury Trawsfynydd-Larkhill Trawsfynydd-Friskney(1) Dover-Larkhill Dover-Friskney(1)	Larkhill-Friskney(2) Trawsfynydd-Friskney(2) Dover-Friskney(2)	Multiple arrivals on Friskney-Canterbury, Trawsfynydd-Friskney, and Dover-Friskney Single arrivals, Friskney-Larkhill, Larkhill-Canterbury, Larkhill-Friskney, Trawsfynydd-Canterbury, Trawsfynydd-Larkhill, Dover-Larkhill Good inceptions, Larkhill-Canterbury, Trawsfynydd-Larkhill Rather poor inceptions remainder although generally amplitudes good	Troposphere paths, Friskney-Canterbury. Friskney recorder out of action explosion (2). Recordings of anomalous path sound waves were obtained on all possible azimuths (600 lb. gun-cotton)	
16	1 } 2 }	6.4.45 {	0001 0030	Friskney Larkhill Trawsfynydd Dover	Canterbury Larkhill Friskney	Friskney-Larkhill Larkhill-Canterbury Larkhill-Friskney Trawsfynydd-Canterbury Trawsfynydd-Larkhill(1) Trawsfynydd-Friskney Dover-Larkhill(2) Dover-Friskney	Friskney-Canterbury Trawsfynydd-Larkhill(2) Dover-Larkhill(1)	Multiple arrivals on Larkhill-Canterbury, Trawsfynydd-Friskney, Trawsfynydd-Larkhill and Dover-Friskney Single arrivals, Friskney-Larkhill, Larkhill-Friskney, Trawsfynydd-Canterbury, and Dover-Larkhill Good inceptions, Larkhill-Canterbury, Trawsfynydd-Friskney and Dover-Friskney Moderate inceptions, Friskney-Larkhill, Larkhill-Friskney, Trawsfynydd-Larkhill, and Trawsfynydd-Canterbury Poor inceptions, Dover-Larkhill	Troposphere paths, Friskney-Canterbury. Troposphere values of $a + w$ appeared to exceed the measured values of characteristic velocity in 2 cases (600 lb. gun-cotton)	

APPENDIX IV—DETAILS OF SUCCESSFUL RECEPTIONS OF ANOMALOUS SOUND WAVES  
mean values (experiment 4 omitted)

Series No.	Explosion No.	Response No.	Azimuth of reception	Characteristic speed	Total time	Total range	Speed of total (time) mean cross-wind	Direction of total (time) mean cross-wind	Total delay time	No. of readings	Largest troposphere value of $a + w$	Remarks
			° true	m./sec.	sound-Km.	Km.	m./sec.	° true	sec.		m./sec.	
1	1	..	165	342	268	222	16.4	255	136	8	336	
	4	..	165	342	267	222	20.2	255	121	9	337	
	8	..	165	346	263	222	12.7	255	121	9	336	
2	1	..	165	345	252	221	1.4	75	92	4	340	* Small difference between troposphere $a + w$ and characteristic speed.
	1	..	90	341	348	312	3.4	360	106	5	333	
	2	..	165	346	253	222	1.3	75	92	5	340	
3	2	..	90	335	354	317	9.6	360	110	5	333*	
	1	..	165	347	261	222	0.3	225	116	9	339	
	1	..	214	339	282	244	1.2	124	113	4	332	
5	2	..	165	343	261	222	2.9	75	116	9	339	
	2	..	214	337	279	241	0.7	304	113	2	332	
	2	..	165	337	260	220	1.5	75	118	4	340	Several cases of troposphere $a + w$ greater than characteristic speed especially Response 1 165° both explosions.
	1	..	165	343	269	227	5.9	75	124	5	340	
	1	..	214	348	281	244	12.6	124	110	4	339	
6	1	..	270	349	258	224	0.8	360	101	4	341	
	1	..	165	337	259	220	2.5	75	116	4	340	
	2	..	165	345	265	223	2.5	75	124	4	340	
7	2	..	214	347	282	244	4.0	124	113	4	339	
	2	..	270	351	258	224	4.3	180	101	4	341	
	1	..	165	350	265	228	6.0	75	110	1	341	Small number of readings hence results not as reliable as in other cases.
	1	..	214	351	278	241	6.0	304	110	2	341	
	1	..	270	348	258	224	8.1	360	101	2	341	
8	2	..	214	345	277	245	4.7	124	95	3	341	
	2	..	270	350	257	224	4.8	360	98	2	341	
	1	..	165	344	270	230	0.0	..	118	9	340	
9	2	..	165	342	263	224	2.9	255	115	10	340	
	1	..	34	341	283	243	1.8	124	118	1	336	* One reading had characteristic speed less than troposphere $a + w$ .
	1	..	90	346	230	192	1.2	180	113	1	337	
10	1	..	214	347	287	244	2.1	304	127	4	340	
	1	..	165	341	260	220	2.6	255	118	8	338*	
	2	..	34	340	282	244	0.9	304	113	1	336	
9	2	..	90	342	281	202	1.8	360	116	5	337	
	2	..	214	349	286	244	8.8	304	124	4	340	
	2	..	165	342	258	220	3.2	255	113	8	338	
9	1	..	165	338	243	207	7.5	255	107	11	336	* One reading had characteristic speed less than troposphere $a + w$ .
	1	..	90	349	237	203	1.4	360	101	9	340	
	2	..	165	339	243	208	7.0	255	104	10	336*	
10	2	..	90	349	240	208	2.3	360	95	7	340	
	1	..	34	340	278	247	27.7	304	92	1	339	* Characteristic speed 34°, less than troposphere values of $a + w$ , and not much larger in other cases. Also small number of readings.
	1	..	70	358	200	164	5.5	360	107	3	353*	
	1	..	90	364	206	190	9.7	360	47	3	362*	
2	2	..	34	338	277	247	30.1	304	89	1	339	
	2	..	90	365	221	207	2.5	360	42	2	362	

## APPENDIX IV—continued

Series No.	Explosion No.	Response No.	Azimuth of reception	Characteristic speed	Total time	Total range	Speed of total (time) mean cross-wind	Detection of total (time) mean cross-wind	Total delay time	No. of readings	Largest troposphere value of $a + w$	Remarks
			° true	m./sec.	sound-Km.	Km.	m./sec.	° true	sec.		m./sec.	
11	1	1	163	336	247	208	26.1	253	116	4	337	Several cases of characteristic speed less than value of $a + w$ in the troposphere in individual cases.
	1	2	163	344	252	211	39.6	253	121	5	337	
	1	3	163	354	251	207	55.2	253	130	3	337	
	1	1	90	456	216	200	9.3	360	47	4	342	
	1	2	90	345	215	194	5.1	360	62	3	342	
	1	3	90	346	225	202	3.8	360	68	5	342	
	1	1	118	343	427	384	13.8	208	127	4	340	
	1	1	214	358	324	246	13.9	304	231	1	329	
	1	1	143	370	251	227	35.0	233	71	4	339	
	1	2	143	345	255	226	18.0	233	86	4	339	
	2	1	163	337	253	213	27.5	253	118	3	337	
	2	2	163	346	254	212	40.1	253	124	5	337	
	2	3	163	366	249	204	53.7	253	133	1	337	
	2	1	90	431	213	196	3.4	360	50	3	342	
	2	2	90	348	224	203	5.9	360	62	5	342	
	2	3	90	344	221	199	6.9	360	65	5	342	
	2	1	90	355	318	265	11.2	360	157	1	338	
	2	2	90	465	323	265	28.8	360	172	1	338	
	2	1	118	346	426	384	16.5	208	124	5	340	
	2	1	214	336	316	241	25.9	304	222	4	329	
	2	1	143	372	251	227	26.5	233	71	4	339	
	2	2	143	340	254	226	12.3	233	83	5	339	
	2	1	34	350	273	247	43.5	304	77	1	336	
	2	2	34	358	275	247	37.6	304	83	1	336	
12	1	1	90	391	225	201	40.6	360	71	6	332	(D) denotes direct path. (B) denotes path probably including one surface reflection. All figures quoted refer to calculations based on assumption of direct paths. Several doubtful readings where characteristic speed exceeded by troposphere $a + w$ .
	1	2	90	344	229	201	44.9	360	83	6	332	
	1	3	90	338	229	201	36.3	360	83	5	332	
	1(D)	1	90	350	293	267	53.7	360	77	2	334	
	1(D)	2	90	351	295	266	52.6	360	86	2	334	
	1(D)	3	90	325	298	267	44.8	360	92	2	334	
	1(B)	1	90	349	327	267	36.8	360	178	2	334	
	1(B)	2	90	359	333	267	34.6	360	195	2	334	
	1(D)	1	118	346	416	382	27.3	28	101	6	338	
	1(B)	2	143	372	452	387	33.3	28	193	4	338	
	1	1	143	350	238	228	15.5	53	30	3	345	
	1	2	163	346	225	212	1.4	73	39	3	343	
	1	1	163	380	227	207	26.7	253	60	6	343	
	1	1	214	353	284	246	19.7	304	113	3	337	
	1	1	34	338	293	241	58.9	304	155	2	331	
	2	1	90	390	225	202	31.5	360	68	6	332	
	2	2	90	339	229	202	37.8	360	80	6	332	
	2	3	90	337	241	212	14.0	360	86	1	332	
	2(D)	1	90	352	294	267	46.5	360	80	2	334	
	2(D)	2	90	348	295	267	47.3	360	83	2	334	
	2(B)	1	90	347	327	267	42.9	360	178	2	334	
	2(B)	2	90	377	332	268	34.6	360	190	1	334	
	2(B)	3	90	395	336	269	40.6	360	198	2	334	
	2(B)	4	90	377	339	267	44.9	360	213	2	334	
	2(D)	1	118	344	416	382	30.2	28	101	6	338	
	2(B)	2	143	376	451	387	30.6	28	190	4	338	
	2	1	143	355	234	224	13.1	53	30	4	345	
	2	1	163	345	225	210	3.4	73	45	3	343	
	2	2	163	379	229	207	21.5	253	66	6	343	
	2	2	214	353	283	245	16.9	304	113	4	337	

## APPENDIX IV—continued

Series No.	Explosion No.	Response No.	Azimuth of reception	Characteristic speed	Total time	Total range	Speed of total (time) mean cross-wind	Direction of total (time) mean cross-wind	Total delay time	No. of readings	Largest troposphere value of $a + w$	Remarks
			° true	m./sec.	sound-Km.	Km.	m./sec.	° true	sec.		m./sec.	
13	1	1	90	376	217	202	17.5	360	44	6	361	Several cases of characteristic speed less than value of $a + w$ in the troposphere in individual cases.
	1	2	118	373	404	384	10.3	208	16	6	368	
	1	3	118	369	407	382	9.4	208	74	4	368	
	1	4	143	373	242	227	58.4	233	44	2	364	
	2	1	90	373	213	201	18.9	360	36	6	361	
	2	2	163	363	237	207	52.4	253	89	4	360	
	2	3	163	381	241	206	59.4	253	104	6	360	
	2	4	118	390	400	374	8.5	208	77	4	368	
	2	5	143	376	242	226	61.1	233	47	5	364	
	1	6	120	357	439	382	25.6	210	169	6	343	
14	2	7	120	357	437	382	12.1	210	163	6	343	
	1	1	90	364	224	200	25.1	360	71	6	339	Several doubtful readings where characteristic speed exceeded by troposphere $a + w$ .
	1(B)	2	90	349	289	265	27.1	360	71	2	339	
	1(B)	3	90	373	322	265	28.1	360	169	2	338	
	1(B)	4	90	383	325	265	22.1	360	178	2	338	
	1	5	118	359	417	379	2.6	208	113	5	341	
	1	6	143	347	245	226	22.4	233	56	4	342	
	1	7	163	356	230	207	17.7	253	68	6	341	
	1	8	214	348	287	245	33.5	304	124	2	340	
	1	9	270	346	304	223	13.9	360	240	4	334	
	1	10	335	348	306	227	13.9	245	234	4	333	
15	1	1	34	341	297	241	31.0	304	166	3	334	Several doubtful readings where characteristic speed exceeded by troposphere $a + w$ .
	1	2	90	365	225	201	24.7	360	71	6	339	
	2(B)	3	118	361	420	382	7.1	208	113	6	341	
	2	4	143	351	245	224	7.8	233	62	3	342	
	2	5	163	356	233	211	17.0	253	65	5	341	
	2	6	163	360	232	207	15.9	253	74	6	341	
	2	7	270	345	302	223	13.8	360	234	4	333	
	1	8	90	344	236	200	8.1	360	107	5	339	
	1	9	90	364	242	200	7.8	360	124	5	339	
	1	10	90	342	206	267	5.9	360	116	5	338	
16	1	1	90	368	320	267	5.5	360	157	2	338	Several doubtful readings where characteristic speed exceeded by troposphere $a + w$ .
	1	2	90	355	412	337	0.5	208	104	4	342	
	1	3	118	352	260	227	7.7	233	98	4	342	
	1	4	143	338	278	242	4.2	304	107	3	335	
	1	5	214	341	282	244	6.4	304	113	4	335	
	1	6	335	331	276	227	0.9	65	145	2	334	
	1	7	34	341	288	241	7.8	304	139	2	336	
	1	8	90	357	241	200	6.7	360	121	5	339	
	2	9	90	360	242	200	9.3	360	124	4	339	
	2	10	90	343	303	263	11.0	360	118	2	338	
17	1	1	90	344	304	263	4.9	360	121	2	338	Several doubtful readings where characteristic speed exceeded by troposphere $a + w$ .
	1	2	118	355	412	377	0.6	28	104	4	342	
	1	3	143	355	412	377	0.6	28	104	4	342	
	1	4	214	346	281	246	2.8	304	104	3	335	
	1	5	270	340	267	224	14.2	360	127	4	334	
	1	6	335	332	274	226	1.1	65	142	1	334	
	1	7	34	338	290	244	2.5	304	136	2	336	
	1	8	90	344	236	200	8.1	360	107	5	339	
	1	9	90	364	242	200	7.8	360	124	5	339	
	1	10	90	342	206	267	5.9	360	116	5	338	

## APPENDIX V—TEMPERATURE AND WIND CONDITIONS UP TO 18 KM. (RADIO-SONDE MEASUREMENTS)

Values are means over the area  
18-Km. values were often extrapolated when measurements up to 15 Km. only were available

Series No.	Explosion No.	Time	Date	Surface	1.5 Km.	3 Km.	6 Km.	9 Km.	12 Km.	15 Km.	18 Km.
				Temperature °A.	Wind ° m./sec.	Temperature °A.	Wind ° m./sec.	Temperature °A.	Wind ° m./sec.	Temperature °A.	Wind ° m./sec.
1	1	0412	15.4.44	280	Calm	265	200 10-1	221	210 10-7	219	230 5-8
	4	2300		282	Calm	266	200 15-2	220	230 21-6	218	260 12-2
	8	2308	17.4.44	280	Calm	266	248 340 6-7	227	340 6-7	215	280 7-3
	1	2100	6.5.44	277	Calm	262	242 10 28-7	223	20 41-5	220	350 4-3
	2	0001	7.5.44							219	Calm
3	1	2100	13.6.44	285	250 7-0	266	260 15-6	225	260 42-3	224	260 16-2
	2	2130									
4	1	2100	10.7.44					not used			
5	1	2200	26.7.44	290	200 3-0	271	210 15-2	229	190 16-8	224	230 9-1
	2	2230									
6	1	2100	8.8.44	291	220 1-8	272	280 5-5	231	330 18-3	219	330 18-3
	2	2130									
7	1	2300	23.8.44	289	230 2-1	274	230 9-1	234	230 11-3	220	190 16-1
	2	2330									
8	1	0001	13.9.44	283	80 3-1	274	140 7-6	230	270 12-5	216	260 4-6
	2	0030									
9	1	0001	11.10.44	284	190 2-7	270	260 8-8	226	240 9-8	216	230 9-1
	2	0030									
10	1	0001	8.11.44	279	270 5-8	266	290 28-7	229	290 38-6	210	300 28-7
	2	0030									
11	1	0001	6.12.44	275	270 5-8	258	290 11-9	222	300 34-1	217	290 20-4
	2	0030									
12	1	0001	6.1.45	272	280 0-9	260	360 15-2	213	350 45-7	210	360 32-0
	2	0030									
13	1	0001	6.2.45	279	260 4-6	264	290 27-4	218	310 64-0	213	300 24-4
	2	0030									
14	1	1500	27.2.45	286	250 6-1	269	260 14-9	225	240 38-6	220	240 19-2
	2	1530									
15	1	0001	6.3.45	290	320 4-3	265	350 16-8	223	345 30-2	215	350 28-9
	2	0030									
16	1	0001	6.4.45	290	300 2-4	267	315 12-2	227	320 33-5	219	335 9-1
	2	0030									