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Turbulence and Diffusion Note No 1.

THE TURBULENT STRUCTURE OF THE PLANETARY BOUNDARY LAYER

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The turbulent structure of the planetary boundary layer*

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The classical view of the atmospheric boundary layer recognises a fairly deep layer in which the flow driven by pressure gradient is controlled and modified by surface friction, by buoyancy forces generated by surface heating and cooling, and by the rotation of the earth. Throughout the depth of the planetary boundary layer or friction layer, which turns out to be about 1000 m, there is a horizontal shearing stress which in general falls from the value appropriate to the friction at the surface to zero at the top of the layer. For some shallow region near the ground the fall in shearing stress is a small fraction of the whole and for many purposes may be neglected. This region of effectively constant shearing stress, in which also the effects of the rotation of the earth are of minor importance, is usually referred to as the surface boundary layer.

Much of the work on the detailed structure and diffusive action of atmospheric flow has been concentrated on the surface boundary layer, for the obvious combination of reasons - accessibility and practical interest. To an ever increasing degree interest in these features is being extended into the higher reaches of the planetary boundary layer, for which observational description is still in a relatively primitive stage, and for which the one simplicity of the surface layer - constant shearing stress - no longer holds. In any review of the position it is difficult to avoid giving the major attention to the relatively well-known features of the surface layer, but in the present discussion the intention is to bring out as much as possible the less well-known features which call for clarification in the higher regions of the boundary layer.

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<u>The wind profile and vertical transfer in the whole friction layer</u>

It is appropriate to begin with the long-standing problem of the change of mean wind velocity with height above ground, and the relation to the horizontal shearing stress, in which is reflected the capacity of the small-scale atmospheric motion for transferring momentum vertically. Considering axes fixed in the earth, with x and y horizontal and z vertical, and with the components of motion u, v, w respectively, the equations of horizontal mean motion may be written as follows (with overbars signifying mean values):

$$\underset{(a)}{\frac{d\bar{u}}{dt}} \quad - \quad \underset{(b)}{f\,\bar{v}} \quad = \quad \underset{(c)}{-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}} \quad + \quad \underset{(d)}{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\tau_{zx}\right)} \tag{1}$$

$$\frac{d\bar{v}}{dt} \quad + \quad f\,\bar{u} \quad = \quad -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} \quad + \quad \frac{1}{\rho}\frac{\partial}{\partial z}\left(\tau_{zy}\right) \tag{2}$$

Here ρ is air density, p pressure, τ_{zx} and τ_{zy} the components of the horizontal shearing stress (or in other words the vertical fluxes of u - momentum and v - momentum $\overline{\rho\, w'u'}$ and $\overline{\rho\, w'v'}$ respectively), and f the Coriolis parameter ($=2\omega$ Sin ϕ where ω is the angular velocity of rotation of the earth and ϕ is latitude). The term containing this parameter is an apparent deviating force which is purely a consequence of the reference to axes fixed in the earth.

The first simplification of the above equations is the restriction to steady homogeneous flow, in which case term (a) is zero. Near the ground orders of magnitude make term (b) negligible, and now taking the x-axis along the surface wind direction the first equation integrates to:

$$\tau\ (z) \ = \ \tau\ (o) \ + \ z\ \frac{\partial p}{\partial x} \tag{3}$$

and again for sufficiently small values of z (tens of metres) and normal values of $\frac{\partial p}{\partial x}$ this reduces to:

$$\tau\ (z) \ \simeq \ \tau\ (o) \tag{4}$$

i.e. the condition of effectively constant stress. On the other hand at a

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sufficiently great height, $\tau = 0$ and term (d) disappears, leaving the so-called geostrophic balance between terms (b) and (c), giving the components of geostrophic wind in the form

$$\bar{v}_g = \frac{1}{\rho f} \frac{\partial p}{\partial x}, \quad \bar{u}_g = -\frac{1}{\rho f} \frac{\partial p}{\partial y} \tag{5}$$

At intermediate heights term (d) must be retained and then, for example, the first equation integrates to:

$$\tau_x(z_2) - \tau_x(z_1) = f \int_{z_1}^{z_2} \rho\, (\bar{v}_g - \bar{v})\, dz \tag{6}$$

The above system of equations provides for three separate ways of specifying the relation between the shearing stress and the wind profile.

A. For the layer of constant stress, in the absence of buoyancy forces, the only parameters which can be of importance are u_* ($= \sqrt{\tau_o / \rho}$) and z, and on dimensional grounds

$$\frac{d\bar{u}}{dz} = \frac{u_*}{kz} \tag{7}$$

In general, with buoyancy forces, the effects of which are represented by the length parameter L ($= -\frac{\rho c_p T}{kg} \frac{u_*^3}{H}$ where H is the vertical flux of heat, k von Karman's constant, C_p specific heat at constant pressure, T absolute temperature, g acceleration due to gravity)

$$\frac{d\bar{u}}{dz} = \frac{u_*}{kz} \phi\left(\frac{z}{L}\right) \tag{8}$$

Typically L has values ranging from $\pm$ 100 m in near-neutral conditions to $\pm$ 10 m in conditions of marked thermal stratification (L is negative with an upward heat flux). In the simple neutral case, H = 0, $L = \infty$, $\phi\left(\frac{z}{L}\right) = 1$, and Eq. (7) applies, yielding the wind profile

$$\frac{\bar{u}}{u_*} = \frac{1}{k} \ln \frac{z}{zo} \quad (\bar{u} = 0 \text{ at } z = zo) \tag{9}$$

Thus in neutral conditions u_* is in principle determinable from observations of the wind speed at two heights and the surface is

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characterised in roughness by the parameter z_o. In the general case the function ϕ must be known, and its precise nature has for some time been a matter of contention. However, intensive field studies of the vertical profiles of wind and temperature over the last decade are now yielding tolerably well-defined empirical laws for ideally exposed sites.

B. For the friction layer as a whole Eqs. (1) and (2) can be further developed by writing

$$\tau_{zx} = \rho K \frac{\partial \bar{u}}{\partial z}, \quad \tau_{zy} = \rho K \frac{\partial \bar{v}}{\partial z} \qquad (10)$$

where K is an eddy diffusivity, for vertical transfer of momentum. Still neglecting term (a) (i.e. assuming steady homogeneous conditions) and assuming tractable forms for K the equations can be solved analytically. The cases of K constant (leading to the classical Eskman spiral solution), $K \propto z^{0.843}$ (Prandtl and Tollmein), $K \propto z^m$, $m < 1$ (Kohler), $K \propto z$ to some intermediate height h, then decreasing to a residual constant value at the top of the boundary layer (Rossby and Montgomery) have been summarized by Sutton (1953). Since then a solution for $K \propto z$ indefinitely has been given by Ellison (1955). Also, for certain more complex forms of K numerical solutions have been obtained by Buagitti and Blackadar (1957) and Blackadar (1962, 1965), the former allowing finite values to $\frac{\partial \bar{u}}{\partial t}$ and $\frac{\partial \bar{v}}{\partial t}$. Briefly, the upshot of all these treatments is that the broad features of the wind profile can be reproduced but there is a failure in detail which probably reflects the inadequacies of the assumptions made about K.

C. An alternative, which avoids assumptions about the form of K, is provided by Eq. (6), which gives the <u>change</u> of τ with height from measurement of the departure of the v-component of the wind from the geostrophic value. In the classical application v_g (and hence $\frac{\partial p}{\partial x}$) was taken to be constant with height. Even then there is a considerable problem in assigning the direction of v_g accurately enough, and this has led to attempts to avoid the difficulty by a trial and error method (Lettau 1950) in which a range of values of the angle between the surface wind and the geostrophic wind is tried, correct choice being indicated by consistency in the derived values

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of $\tau_{zx}/\frac{d\bar{u}}{dz}$ and $\tau_{zy}/\frac{d\bar{v}}{dz}$. In general, however, there is a further difficulty in that allowance must be made for a variation with height of the pressure gradient as a consequence of the horizontal gradient of temperature. This may lead to important complications such as those encountered by Sheppard et al. (1952) in their studies of the westerlies in the N.E. Atlantic.

The result for the constant stress layer in neutral conditions (Eq. (9)) implies that the vertical diffusivity for momentum is proportional to height and to wind speed. In terms of the classical expression of diffusivity as a product of a characteristic turbulent velocity and a characteristic length scale, this is consistent with the combination of a characteristic velocity which is constant with height and a scale which increases linearly with height.

Outside the constant stress layer one of the most enlightening results emerges from the application of method C by Lettau (1950) to the analysis of an intensive series of pilot balloon ascents carried out over Leipzig by Mildner. The measurements were made in a 'uniform warm air mass' with 'no indication of convectional processes occurring from 0915-1615 hours during which period the 28 pibal ascents were carried out'. With the wind strengths involved (near 10 m/s at a height of 50 m) the conditions in the constant stress layer must have been effectively neutral. At greater heights the conditions were undoubtedly slightly stable, for Lettau points out 'In the same air mass the 1400 hour sounding at the aerological station Lindenberg (less than 100 miles from Leipzig) showed a rather uniform lapse rate of - 0.65°/100m in the layer under consideration .....'. Lettau's results give values of ρK rising to a maximum at a height near 250m and thereafter falling off to about one-third of this maximum value at a height of 1000m.

The precise significance of this reduction in K with height as regards the characteristic velocity and length scales of the turbulence is debatable, but some indication is provided by Blackadar's analyses using method B. These analyses yield a relation between the surface stress (expressed as a drag

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coefficient $C_g = \tau/\rho V_g^2$ where V_g is the geostrophic wind speed) and the 'surface Rossby number' V_g/fz_o. Data in conditions of neutral flow near the surface have been collected by Lettau (1959) to show a systematic relation between the above quantities, the value of C_g falling from about $2.3\text{x}10^{-3}$ to $0.5\text{x}10^{-3}$ as V_g/fz_o increases from about $2.2\text{x}10^5$ to $5.6\text{x}10^8$. Of this large range in V_g/fz_o a change of 400 to 1 is accounted for by the change in z_o, 0.05 to 20 cm, provided by surfaces ranging from the sea to forests. It is noteworthy that such a change in z_o would cause the C defined in terms of wind speed at a height of 10 metres to change by a factor of more than 30. It is also noteworthy that Blackadar has added three more values of C, at values of V_g/fz_o near 10^5 and 10^6, which fit in well with the trend of Lettau's collection. In his most recent analysis Blackadar (1965) finds that this trend is fairly well explained by taking the following form for K

$$K = l^2 s, \qquad l = kz \Big/ \left(1 + \frac{kz}{\lambda}\right) \tag{11}$$

where l is the characteristic scale and s is the magnitude of the wind shear vector (i.e. $\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{dv}{dz}\right)^2\right]^{1/2}$). In this formulation the K near the surface is consistent with the constant stress law, reaches a maximum around 150m, and thereafter falls off to about one-third of the maximum value at a height of 800 m.

The above variation of K has been achieved by taking the characteristic scale to increase linearly with height at first and then to approach a constant value (λ) monotonically, and by assuming (implicitly) that the characteristic turbulent velocity is ls (as in the classical mixing length approach). The apparent ultimate decrease of K with height thus requires an appropriate reduction with height of the turbulent velocities. It seems unlikely, however, that this type of analysis can provide a unique specification of the separate variations with height of the characteristic velocity and length scale. Also, the effect of buoyancy has so far been omitted and its satisfactory incorporation in the present type of analysis has yet to be achieved. With these difficulties still outstanding further progress in specifying and understanding the properties of turbulence above the constant stress layer is likely to arise mostly from more direct considerations of the turbulent fluctuations.

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Turbulent energy balance and related features

Turbulent kinetic energy, as evident in the variance of the various eddy components, arises from two basic sources:

(a). There is a transfer from the bulk kinetic energy of the mean motion, through the action of the eddy shearing stresses which arise from the aerodynamic drag of the surface. This is simply the turbulent equivalent of the conversion of bulk kinetic energy to molecular agitation by the action of a viscous stress. The magnitude of the transfer is the rate of working of the horizontal shearing stresses, i.e. $-\overline{u'w'}\ \frac{d\bar{u}}{dz}$, $-\ \overline{v'w'}\ \frac{d\bar{v}}{dz}$ (per unit mass of air). Near the ground the second term is negligible.

(b) There is an actual generation (as distinct from a transfer) by the action of buoyancy forces set up in an unstable thermal stratification of the air, which may arise from solar heating of the surface or from the over-running of a surface by relatively cool air. The magnitude is $\frac{gH}{\rho c_p T}$ or $g\ \frac{\overline{w'T'}}{T}$, where T is the mean absolute temperature and T' the departure from the mean. This energy then appears locally as an increase in the variance of the eddy components, or is transferred or disposed of in various ways as follows.

(c) There is a transfer between eddy components by the action of pressure forces, and here it may be noted that source (a) contributes originally to the u-component, source (b) to the w-component.

(d) If the stratification is thermally stable (in which case the only source is (a)), the vertical motion requires work to be done against gravity, with the result that energy is withdrawn, originally from the vertical component, and at the rate of $-\ g\ \frac{\overline{w'T'}}{T}$.

(e) Turbulent kinetic energy is handed down in a cascade process, by local shearing stresses acting on progressively decreasing scales, ultimately to be dissipated as heat by viscous stresses.

(f) Before the above processes take effect the energy may be diffused away and become effective in another region of the flow, the magnitude being the divergence of the flux of total turbulent kinetic energy.

According to the well-known Kolmogorov theory for the small-scale structure of turbulence, the energy density at high values of the wave-number κ should be

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a function only of the rate of viscous dissipation ϵ and the kinematic viscosity, and possibly of ϵ alone over a more restricted range of wavenumbers. In the latter case (the inertial sub-range) the one-dimensional spectral density S (κ) is given by the relation

$$S(\kappa) = C\epsilon^{2/3}\kappa^{-5/3} \qquad (12)$$

where $\int_0^\infty S(\kappa)\,d\kappa = \overline{u'^2}$ etc.

For the one-dimensional spectra with which we necessarily deal in practice it is a consequence of continuity and incompressibility that the constant C differs according as the spectrum is <u>longitudinal</u> in sense (e.g. describing the u-component variations along the x-axis or the w-component variations along the z-axis) or <u>transverse</u> (e.g. describing the u-component variations along the y or z axes, or the w-component variation along the x-axis). The ratio of the constants in the longitudinal and transverse senses is $\frac{3}{4}$. In practice spectra are derived from measurements at a fixed point in space and are expressed in terms of frequency n, and the assumption is made that frequency and wavenumber characteristics are equivalent with $\kappa = \bar{u}n$. For spectra in the longitudinal sense the best estimate of the universal constant C appears to be 0.47 (with κ in radians per cm).

It is convenient also at this point to note the corresponding relation for temperature fluctuation

$$S_T(\kappa) = C_T\epsilon^{-\frac{1}{3}}\chi\kappa^{-\frac{5}{3}} \qquad (13)$$

where $\int_0^\infty S_T(\kappa)\,d\kappa = \overline{T'^2}$

and χ is the rate of reduction of the quantity $\overline{T'^2}$ by the smoothing effect of molecular mixing. For C_T the present estimate (unpublished) is 0.37, and we note that in this case, temperature being a scalar, no 'longitudinal' or 'transverse' distinction is involved. In the budget of mean square temperature fluctuation there is a production term, corresponding to (a) above, arising from the vertical flux of heat down the vertical gradient of potential temperature θ, of magnitude $-\overline{w'T'}\,\frac{d\bar{\theta}}{dz}$. There may also be a diffusion term corresponding to (f). Analogous relations will apply to fluctuations of specific humidity, q, with q' replacing T', $\bar{q}$ replacing $\bar{\theta}$.

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The interest of these relations (12) and (13) is of course in the respect that measurement of $S(\kappa)$ at suitable values of κ provide ϵ, while a combination of both $S(\kappa)$ and $S_T(\kappa)$ (or $S_q(\kappa)$) provides χ as well. Alternatively, estimates of χ and ε provide a prediction of the spectral densities of the velocity, temperature or humidity fluctuations at high wavenumber. In the simplest application of these principles, it is assumed that the turbulent fluctuations are dissipated locally (i.e. process (f) above negligible), and that conditions are steady and uniform in the horizontal). Accordingly, there must be a balance between (a), (b) or (d) and (e), which is expressed in the relations

$$\epsilon = -\overline{u'w'}\,\frac{d\bar{u}}{dz} + \frac{g}{T}\,\overline{w'T'} \qquad (14)$$

$$\chi = -\overline{w'T'}\,\frac{d\bar{T}}{dz} \text{ or } -\overline{w'q'}\,\frac{d\bar{q}}{dz} \qquad (15)$$

Accordingly, the addition of measurements of wind velocity and temperature (or humidity) gradients to those of wind and temperature (or humidity) fluctuations provides through Eqs. (14) and (15) estimates of the shearing stress and heat (or vapour) flux. (In the special case of neutral conditions the second term on the right-hand side of Eq. (14) is zero and measurements of wind fluctuation and mean gradient alone are sufficient to give the shearing stress). Alternatively, independent estimates of the vertical fluxes provide estimates of the magnitude of the fluctuation at high wavenumber.

Although the existence of an inertial sub-range in atmospheric turbulence appears now to be widely accepted, the position seems far from settled as regards the validity of assuming local dissipation. Panofsky (1962b, see also Lumley and Panofsky, 1964) has listed results demonstrating that the divergence of the vertical flux of turbulent kinetic energy may be an important term in the energy budget at heights in the range 10-100 metres. An interesting feature of these results is that in unstable conditions this term appears roughly to balance the rate of production by buoyancy, leaving the mechanical and dissipation terms also in balance, as in the simple case of neutral flow. Record and Cramer (1966) have also concluded that the mechanical and dissipation terms balance at 16 and 40 m. irrespective of thermal stratification. However, their results failed to show an overall balance in unstable conditions, the divergence and buoyancy terms being

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very different. The authors suggest that this discrepancy may be partly a consequence of lack of homogeneity in the airflow, their measurements having been made over a field with a wooded area only 300-400 m upwind. More recently, Busch and Panofsky (1967) have concluded tentatively that total production from mechanical and thermal sources is balanced by dissipation, over homogenous terrain.

The simplification suggested by Lumley and Panofsky (1964) for the 10-100 m layer in unstable conditions, namely that the divergence and buoyancy terms are dominant, appears not to be supported by existing data. For somewhat higher levels, at which they suggest that the buoyancy and dissipation terms are dominant, no firm observational data are yet available. However, some preliminary observations at a height of 600 m over Cardington, Bedfordshire, using instruments carried on a captive balloon, indicate that in convective conditions the diffusion term may be important. For the case of temperature fluctuation, it has recently been pointed out by Deardorff (1966) that some measurements made by Telford and Warner at heights of 350, 578 and 997 m show the diffusion terms to be in approximate balance with the production term. Furthermore, Record and Cramer's (1966) result that Eq. (15) under-estimates the heat flux in unstable conditions could well be a consequence of neglecting this divergence term at 16 and 40 m. Indeed a positive (upward) divergence term equal to about half the production term would bring their results into better consistency. At greater heights again, the preliminary data from Cardington point to the conclusion that the diffusion term is significant. Here, as in Telford and Warner's results, the action of the heat flux, being directed <u>against</u> the gradient of potential temperature, is actually to <u>suppress</u> temperature fluctuation. To sum up, it seems that the budget of turbulent fluctuation is often very complex and that more critical measurement of the various quantities is required before any convincing generalisation can be made. In the meantime it would seem advisable to be cautious about accepting simplified balances represented in Eqs. (14) and (15).

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The intensity and scale of the planetary boundary layer turbulence

It is useful now to review the position reached in the statistical description of the typical magnitudes and scales of planetary boundary layer turbulence. The information available is mainly for the more accessible constant stress layer, and it is in this layer that the greatest progress has been made in providing a general theoretical framework. A simplified summary is given in Table 1.

For the vertical component in the constant stress layer much useful generalization has been provided by the Monin-Obukhov similarity theory, which starts with the hypothesis that this property, as well as the wind and temperature profiles, is uniquely determined by the parameters u_* and L. Dimensionally, this immediately leads to a very simple result for neutral conditions, namely that the r.m.s. magnitude σ_w is proportional to u_*, and is therefore independent of height and proportional to mean wind speed at a fixed reference height. Similarity also implies that the wavenumber spectrum is a universal function of κ z and hence that the characteristic scale is proportional to z. Both relations are fairly well supported (see Lumley and Panofsky (1964) for a detailed discussion). There has been some doubt about the exact magnitude of σ_w/u_* with several estimates in the range 1.2-1.3 and with a much lower value of 0.8 from the Russian literature. The evidence has recently been extended by Busch and Panofsky and now seems in favour of a value near 1.3. The frequency spectra characteristically show a single main peak in the magnitude of the product of spectral density S(n) and frequency n, at an equivalent wavelength $\lambda_p = \bar{u}/n_p$) equal to 4z according to Panofsky and McCormick (1960) and approximately 3z according to Gurvic (1960) (whose results are quoted by Monin (1962)) and Busch and Panofsky's latest review. The ratio of λ_p and the integral scale ℓ_s (in practice $\bar{u}t_s$, where t_s is the integral time scale) depends on the shape of the spectrum. The form which according to Panofsky and McCormick showed the best fit to earlier data implies $\lambda_p/\ell_s = 4$, whereas a form now advocated by Busch and Panofsky implies a value near 6. This assumes that spectral forms fitting relatively high frequency estimates are reliable for the relatively low frequencies, which are not usually adequately specified by the measurements. Unfortunately, from some tests by

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Pasquill and Butler (1964), in which the low-frequency contributions at a height of 2m were adequately evaluated, it appears that a much lower value (near 2) may be required for the ratio λ_p/ℓ_s.

The position is even less satisfactory for thermally stratified flow. According to similarity theory σ_w/u_* should be a universal function of $\frac{z}{L}$, increasing for unstable values of the parameter and vice versa. Correspondingly, we expect σ_w/u_* to increase or decrease with height according as conditions are unstable or stable. These features are broadly indicated by the data, but there is considerable scatter and also a rather puzzling discrepancy between figures in Gurvic's (1960) original paper and Monin's (1962) summary of measurements ascribed to Gurvic. Monin's summary, and data collected by Panofsky and Prasad (1964), are reasonably consistent in showing a relatively slight dependence on $\frac{z}{L}$, σ_w/u_* being decreased or increased by only roughly 50% of its neutral value when $\frac{z}{L}$ is $\pm$ 1. Monin's summary also appears in Monin and Jaglom's book. Gurvic's data, in terms of Richardson Number, show σ_w/u_* decreasing or increasing by a factor of nearly 3, whereas Monin's summary shows a change of only roughtly 50% above or below the neutral value. More recent summaries by Panofsky and Prasad (1964), Klug (1965), Busch and Panofsky (1967) all indicate a slight dependence on $\frac{z}{L}$ and indeed the last review concludes that σ_w/u_* is essentially independent of stability for $\frac{z}{L}$ in the range of +0.5 to -1.0. (It should be noted however that the variation of $\sigma_w/\bar{u}$ will be more marked, since $\frac{u_*}{\bar{u}}$ increases with increasing instability). The last review also indicates that the spectrum scale represented by λ_p is the same in neutral and unstable air, the only obvious variation being a decrease in stable air, in which case also the spectra no longer show a universal dependence on $nz/\bar{u}$ (assumed equivalent to κz).

In Russian work the above similarity theory is also applied to the properties of the horizontal components, though here the basis for so doing seems open to serious question. Although in neutral conditions both σ_v and σ_u are closely proportional to wind speed on a given site, the ratios σ_v and σ_u/u_* are considered by Lumley and Panofsky to differ significantly between one site and another, suggesting that the small-scale roughness, which determines the wind profile, is not the only feature determining the horizontal components. Furthermore, in

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unstable conditions, while σ_v does not change appreciably with height it is noticeably sensitive to changes in L. The dependence of σ_u on thermal stratification appears to be intermediate between that of σ_v and σ_w .

For σ_v in stable conditions an interesting feature was noticed by Smith and Abbott (1961). They found that in moderate stability the magnitude of σ_v was insensitive to both stability (as indicated a parameter of the Richardson Number type) and wind speed. At first sight this may seem to be in conflict with Lumley and Panofsky's statement that in stable air σ_v is proportional to $\bar{u}^{3/2}$. The explanation probably lies in the fact that their statement refers to results which are not classified according to Richardson Number and that large values of $\bar{u}$ will normally be associated with small values of Richardson Number and vice versa.

Scale characteristics of the lateral component do not show any obvious variation with height, in contrast to the vertical component. For the longitudinal component the behaviour is intermediate i.e. a slight increase with height, the latest analysis (Berman 1965) giving a variation of λ_p with $z^{0.25}$ in neutral conditions. With thermal stratification there is on the whole a tendency for instability to strengthen the low-frequency contributions to the spectrum, and for stability to weaken them, but the effects on λ_p and ℓ_s have not been well determined. The influence on the v-component seems to be greater than on the u-component, a notable feature being the tendency for the reduction of the v-component in stable conditions to be most noticeable over an intermediate range of frequencies, so leaving peaks in the nS(n) spectrum at both high and low frequency.

Finally, the greatest uncertainty of all still surrounds the properties at the more inaccessible heights above the constant stress layer. In this case published information is confined to the vertical component. Results from measurements made with instruments on the cable of a captive balloon at Cardington (Smith (1961), Pasquill (1967)) show that $\sigma_w/\bar{u}$ varies widely with stability in light winds but tends to very small values irrespective of stability in strong winds. In effect the plot of $\sigma_w/\bar{u}$ against $\bar{u}$ indicates extreme values which are roughly inversely proportional to $\bar{u}$, implying extremes of σ_w which are roughtly independent of $\bar{u}$, and these are of magnitude about 1m/sec. This pattern is consistent with the idea that the generation of turbulent energy at these higher

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levels (in the region of 200-1000 m) is dependent primarily on the vertical flux of heat, and only to a slight extent on the shearing stress.

The variation of the magnitude of the turbulent fluctuation over the depth of the planetary boundary layer has not yet been prescribed in any detail, largely owing to the practical difficulty of obtaining simultaneous measurements at appropriate heights. At low heights (<100m) there is some indication (see Lumley and Panofsky) of an increase of σ_w with height in unstable conditions and a decrease in stable conditions. For greater heights Moore (1967b) has considered the evidence available from a combination of separate observations from a tower at heights of 38, 114 and 187 m (Moore 1967a) and from the captive balloon at Cardington at heights of 300 and 1200 m (Pasquill 1967). In the absence of stable layers the r.m.s. angular fluctuation (roughly equivalent to $\sigma_w/\bar{u}$) shows little or no variation with height in light to moderate winds (2-10 m/sec), but an appreciable fall-off (by a factor of about 3 over the whole height range) is indicated by the small sample of data in strong winds (>10 m/sec). However, these features require examination in much greater detail before any serious generalisation can be attempted.

The rapid (near linear) increase of scale with height is confined to the first 200 m or so. The results quoted by Busch and Panofsky (1967) for Cedar Hill indicate a λ_p around 550 m between 200 and 300 m above ground in unstable conditions. Lappe et al's (1959) earlier aircraft results show λ_p continuing to increase with height, though more slowly, reaching 1000 m at a height of 500 m. Unpublished analysis of data obtained at Cardington shows a wide range of λ_p at heights of 300 and 1200 m, with a median value near 750 m and a most frequent value about 500 m. There seems little doubt that well above the surface the spectrum scale as observed is a very variable quantity.

Observed spectra usually show a close approach to a power law decay of spectral density with increasing wavenumber for χ greater than roughly $4/\lambda_p$. There seems to be a preference for exponents in the neigh-bourhood of -5/3, as required by the Kolmogorov law, though here again the scatter is considerable (see for example the wide distribution of exponents evaluated for heights of 300m and 1200 m at Cardington - Pasquill, 1967). Busch and Panofsky's review draws attention to considerably larger values of the exponent in stable conditions at heights up to 200 m, but there is no indication of this tendency in the

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Cardington results.

It is relevant also at this point to note the information available on the rate of viscous dissipation of turbulent kinetic energy. When buoyancy forces are relatively unimportant, and ϵ presumably balances the rate of mechanical production of energy, we expect $\epsilon \propto \bar{u}^3/z$ near the ground. At greater heights this simple relation must be complicated by the fall of stress with height, and by the increase in the relative importance of buoyancy production or extraction of energy and possibly of the divergence of the vertical flux of energy. Nevertheless, an early review of the information on ϵ from various sources (Ball 1961) did show the linear fall-off with height to be maintained in a broad sense even up to 1000 m and more. However, in detail the estimates of ϵ at any one height are very widely scattered. The estimates obtained recently from measurements of the high-frequency component of turbulence at Cardington at a height of 300m and 1200m show a spread from below 1 to above 100 cm^2 sec^{-3}, without any obvious connection with wind speed, though there is some indication of a reduction in ϵ between 300 and 1200 m. A recent Russian review (Zilitinkevich et al, 1967) indicates an even wider scatter at such heights, over the range 10^{-1} to 10^3.

One of the most interesting results which has recently appeared concerns the relation between ϵ, σ_w and n_p. On dimensional grounds one would expect

$$\sigma_w^3 \propto \bar{u}\epsilon/n_p \qquad (16)$$

Furthermore, in the special case of neutral conditions in the constant stress layer, with the following relations expected and observed

$$\begin{aligned} \epsilon &= u_*^3/kz \\ \sigma_w &= au_* \\ \bar{u}/n_p &= bz \end{aligned} \qquad (17)$$

it follows that the coefficient on the R.H.S. of Eq. 16 should be a^3k/b. Observations reported by Kaimal and Haugen (1967) at heights up to about 300 m, and by Pasquill (1967) at heights of 300 and 1200 m, show agreement, not only with the functional form of Eq. 16 but also, <u>irrespective of stability</u>, with the magnitude of the coefficient which is provided by the latest estimates of a (1.3) and b (≃ 3).

/Regarding

Regarding the thermal stratification in the upper part of the boundary layer it is perhaps worth noting the frequency of incidence of elevated inversions. These arise particularly in association with anticyclonic conditions and also with frontal situations. An analysis of radiosonde ascents in the British Isles in 1949, 1950 and 1951 indicates that such inversions (that is, excluding the radiation-type inversion starting at the ground) occur within the classical friction layer on a significant number of occasions. As an example, some of the data for Larkhill are summarized in Table 2.

Table 2

No. of days in year when the lowest overhead inversion at Larkhill was below height z, at 1500 GMT (1949-1951)

Approx. z (m)	No. of days
520	9
950	49
1410	105
1890	154

(from Table IV of Meteorological Office Investigation Division Memorandum No. 74)

A current unpublished analysis of the more detailed information on variation of temperature with height at Cardington also shows a substantial number of lowest inversions with base between 500 and 1000 m, though the frequency of occurrence appears to be rather less than that indicated by the Larkhill data.

The few measurements of turbulence which have been made in the vicinity of elevated inversions leave no doubt that turbulence is damped out in the stable layer, though so far no direct information is available on the details of the turbulent structure in such regions. However, we see that the planetary boundary layer may occasionally be effectively confined below 500 m, while on a substantial number of occasions it may extend above 1500 m. During summer, with convection initiated by surface heating, a more organised system of vertical motion arises in the form of 'thermals'. These obviously have an important effect on the wind and temperature fluctuations, but much remains to be clarified in this respect. During winter the overhead inversions frequently mark the top of extensive sheets of stratocumulus cloud, in which case the structure in the top section of the boundary layer is also affected by condensation and release of latent heat.

/Size

Size and Shape of Eddies

The length scales given in the previous section (mostly deduced from time scales and assuming Taylor's hypothesis) can be looked upon as representing a statistical 'size' of eddies in the direction of the mean wind.

Corresponding estimates for the other directions require actual space correlations, or the close approach to these which can be achieved by carrying the instrument rapidly through the air on an aeroplane. Using low-level (2m) data obtained in special turbulence measurements in Nebraska, Panofsky (1962) has evaluated scale properties of the horizontal components both along and across wind. These showed the scales alongwind to be the greater than those across, especially for the u-component in stable and neutral conditions, in which case the ratio was 6.5:1. In convective conditions the difference is slight, and to a reasonable approximation they could be considered equal. However, as Panofsky points out, this does not imply isotropy in the horizontal, especially as the scale of v alongwind is about twice that of u acrosswind. It is noteworthy also that the scale of both components, in both senses, were an order of magnitude larger in unstable air than they were in stable air.

For scales in the vertical direction a few estimates have been made from measurements on towers at heights up to about 100 m and these have been summarized by Panofsky and Singer (1965). In air which was neutral or only slightly unstable the correlation between different levels was found to be a function of the difference in $z^{1/3}$, for all three components. This implies length scales proportional to $z^{2/3}$. Also in these conditions the magnitudes of the correlation for the v and w components were not very different and were generally smaller than those for the u component. In more extreme conditions the results were much more erratic. In convective conditions for example the u and v data implied little or no variation of scale with height whereas the w data indicated a linear variation.

From the phase lag required to give the best correlation between different levels the 'slope' of the eddies can be estimated. The above results showed that the horizontal/vertical slope was generally 1-2 for the u and v-components in neutral or slightly unstable air and also for the v-component in very unstable air.

In the latter conditions the u-component showed a structure much closer to vertical, with the above ratio near 0.2.

In general statistics of the above kind provide little insight into the actual geometry of the turbulent motion and much more penetrating study is required. This aspect has attracted much attention in convective conditions, in which the observed fluctuations of temperature and wind leave little doubt that a significant evolution of the flow structure occurs with increasing distance from the ground. At heights greater than L a systematic pattern of vertical motion is evident, in which distinct upward streams of relatively warm air alternate with downward streams of relatively cool air. By their very nature and origin these 'plumes' are presumably very effective in transferring heat vertically, relative to their action in transferring momentum say, or any other property which is not closely correlated with the heat content of the air. However, the magnitudes of the effective diffusivities in such conditions have not been well determined and this represents one of the more serious gaps in the knowledge required for understanding transfer problems in the overall depth of the boundary layer.

Table 1. Simplified summary of the magnitudes and scales of turbulence

(a) Vertical component

Height	Thermal Stratification	σ	λ_p
<100m	neutral unstable stable	$1.3u_*$) slight increase) slight decrease	$\propto 3z$ Increase with height not maintained
>200m	neutral unstable stable	$\approx 0.1u$ extremes ≈ 1m/sec independent of u mainly 0 - 0.05u	Large range about 500m

(b) Horizontal components at heights up to 100m

	Thermal Stratification	σ	λ_p
Lateral	neutral unstable stable	$C_v u_*$ (C_v 1.3 - 2.6 according to terrain)) $\propto u$ Up to 4 times neutral value) Roughly 0.2m/s in moderate stability, independent of u	300m no obvious variation with height Marked reduction in scale, but also separate low-frequency peak
Longitudinal	neutral unstable) stable)	$C_u u_*$ (C_u 2.1 - 2.4 according to terrain) Weaker dependence on stability than σ_v	Slow increase with height ($\propto z^{0.25}$) about 600m at 100m Weaker dependence than for v component

σ = standard deviation

λ_p = equivalent wavelength at which spectral density x frequency is a maximum.

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