# Hadley Centre Technical Note 93

## A review of lateral diffusion for Met Office ocean shelf modelling

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## 1. Introduction

Within all fluid motion there exists a component of the motion which is diffusive; the motion of individual fluid particles, or of small parcels of fluid, independent from the averaged motion of the fluid, and the resulting transport of the properties of these individual particles and fluid parcels, acting independently from the mean transport of the water mass properties.

Diffusive processes are of particular importance in ocean modelling. Numerical geophysical fluid dynamic models are limited by resolution, and so small scale processes, from molecular diffusion up to processes which occur at the grid scale cannot be explicitly modelled and must be parameterised to ensure proper inclusion of the effects of sub-grid scale processes. For ocean models the horizontal grid size is frequently of the order of kilometres, and so diffusive processes include those from molecular scales up to the sub-mesoscale and often the mesoscale itself as well.

For deep-ocean *Z*-coordinate models the best approach for diffusion of both tracers and momentum is generally well understood with much relevant literature available (e.g. Cox 1987 and Griffies 1998). The majority of basin-scale *Z*-coordinate models diffuse tracers along isoneutral surfaces, and apply momentum diffusion, predominantly as a numerical closure, along the geopotential model coordinate surfaces. More consideration is needed concerning the most suitable way to apply lateral diffusion in an *S*-coordinate coastal-ocean model. The change of coordinate system itself introduces complexities as the equations used in *Z*-coordinate models generally cannot be used in an *S*-coordinate system. Also the physical regimes in coastal areas are different to those in ocean basins and tend to be modelled at higher horizontal and vertical resolution. Therefore the assumptions which are made when applying lateral diffusion in a *Z*-coordinate model are often not valid in the modelling of coastal regions.

The current implementation of lateral diffusion in the Met Office 7km Forecasting Ocean Assimilation Model Atlantic Margin Model (FOAM AMM7) has little scientific justification, and was chosen under considerable time constraints, with the initial aim of removing noise whilst maintaining acceptable model results. As such it is hoped that a more comprehensive consideration of diffusive processes, and the way they can be parameterised and applied within coastal *S*-coordinate models will result in improvements to FOAM-AMM7.

This report aims to summarise the various ways to apply lateral diffusion within coastal *S*-coordinate models, assess their suitability, and make recommendations for improvements to the way lateral diffusion of both momentum and tracers can applied in the Met Office FOAM AMM7 model. The report begins with a brief introduction to diffusive processes. There is then a discussion on the diffusion operators, the most suitable surfaces for applying diffusion, the different ways to apply diffusion within an ocean model, and the different diffusion coefficients. Various *S*-coordinate ocean models are considered, and recommendations for the FOAM AMM7 model are proposed. The Gent McWilliams parameterisation of the effects of mesoscale eddies, whilst strictly not a diffusive process, has been considered alongside model diffusion. As such the Appendix gives a brief discussion on this topic. In ocean modelling the sub-grid scale processes are divided into the lateral and vertical directions. Whilst vertical sub-grid scale processes, including dianeutral diffusion, are an important aspect of ocean modelling this report will focus only on the application of lateral diffusion.

## 2. Diffusion

Diffusive mixing is a result of motion which occurs on smaller scales and independently to the mean flow. As such the scales on which diffusive processes are defined is dependant upon the scales of interest, and diffusive scales can range from the molecular scale to many kilometres. Generally diffusive processes are separated into the molecular, the sub-mesoscale and the mesoscale.

Molecular diffusion is the random motion of molecules of fluid, which results in some molecules acting against the mean flow. This motion results in mixing of tracer properties and smoothing of concentration gradients on extremely small scales, however tracer mixing on this scale is negligible when looking at oceanic processes. More importantly the frictional processes at the molecular level are responsible for creating a kinematic energy sink and closing the ocean momentum budget. Sub-mesoscale processes such as internal and surface wave breaking have a more important effect on the mixing of ocean tracers. These processes act against concentration gradients, stirring tracers such as temperature and salinity, and reducing tracer gradients. This sub-mesoscale diffusive motion also acts to slow the large scale flow and to lessen shear, transferring momentum from the large scale to smaller scales. Mesoscale diffusive processes are dominated by eddy mixing, which again acts to reduce momentum, cascading energy to smaller scales, and to mix tracer properties, smoothing ocean fronts. Eddy motion can also act to shallow isopycnals and restratify the ocean, as described in Gent and McWilliams (1990).

Diffusion occurs in all directions, however many diffusive processes, particularly mesoscale and sub-mesoscale, are anisotropic. Fluid particles move laterally along neutral surfaces easily, without any change to the gravitational potential energy of the particle. However for a fluid particle to move across a neutral surface, some energy input must be supplied to balance the change in potential energy of the water particle. Because of this, movement along neutral surfaces is several orders of magnitude bigger than movement across them. Diffusion across neutral surfaces is primarily though turbulent diffusive processes, which act on small scales, and are generally isotropic. Eddy diffusivity is highly anisotropic, with eddy diffusion along neutral surfaces being approximately $10^8$ times larger than eddy diffusivity across neutral surfaces (Griffies, 2004). Therefore, eddy diffusivity in the dianeutral direction is very small, and far less significant than the dianeutral component of the turbulent diffusive processes. However the large scale eddy diffusivity is a far more significant process than turbulent diffusion for mixing along neutral surfaces.

In numerical modelling of fluid dynamics, on all scales, averaging is applied; when we speak of the velocity of a fluid we are not referring to the velocity of each individual fluid particle, but the velocity of the water mass averaged over the grid cell in question. Similarly when we discuss the transport of the properties of a water mass, we are discussing the average transport of properties, not the transport of properties of individual particles. Numerical diffusion in an ocean model is intended to represent the motion of small parcels of the fluid, which can be large but still sub-grid scale, moving independently of the large scale resolved dynamics. Because of the different scales of the dominant processes in the isoneutral and dianeutral directions, in ocean models dianeutral diffusion is parameterised separately to isoneutral diffusion, and lateral diffusion terms aim to exclude all dianeutral diffusive motion.

To calculate the transport of ocean properties we must consider not only the average movement of a parcel of fluid, but also the random movement of individual particles into and out of this fluid parcel. This is the distinction between advective processes (ocean properties moving with a fluid parcel) and diffusive processes (ocean properties changing as a result of individual particles moving into or out of a fluid parcel). It is worth noting that in ocean models the numerical distinction between advection and diffusion is imprecise, as advection schemes are often not numerically exact, and include varying amounts of implicit diffusion. As such the application of numerical diffusion should take into consideration the advection scheme that is being used.

The general advection-diffusion equation in one dimension is;

$$\frac{\partial \Phi}{\partial t} + u\frac{\partial \Phi}{\partial x} - D\frac{\partial^2 \Phi}{\partial x^2} = 0 \qquad (1)$$

Where $\Phi$ is the property being advected and diffused, and $D$ is a diffusion coefficient. In this equation the second term represents advective processes, and the third term represents diffusive processes. If the advection term is removed from equation 1, the one-dimensional diffusion equation remains, which is better known as Fick's 2$^{nd}$ law. This relates the random motion of particles of a substance, to a large scale spread of the substance which acts against the concentration gradient. The diffusion equation can be solved in ocean modelling in a number of different ways. Some differences arise from different ways to discretise the equation, but the main differences and the most important things to consider when applying diffusion in an ocean model, are the surfaces along which the diffusion should be applied, and the choice of the diffusion coefficient, $D$.

By applying Fick's 2nd law within an ocean model we include the non-random mixing effects of sub-grid scale 'random' processes. This mixing acts against concentration gradients, thus smoothing model fields. When applying diffusion in an ocean model we are attempting to represent both truly random molecular scale motion and, often more significantly, sub-grid scale motions which cannot be resolved explicitly. These sub-grid scale motions are not random but are considered as statistically random, and often modelled as equally likely to happen in any grid box, and in any lateral direction. Although more sophisticated implementations of diffusion, such as that described in Smagorinsky (1965), apply some variation in the diffusion coefficient, resulting in increased diffusion being applied in some areas of the modelled domain. Eddy induced mixing is an example of a non-random process, which in many ocean models is modelled as pseudo random, in that the mixing due to eddies is applied constantly across the model domain as part of the diffusion terms. However, it is worth noting that some models instead apply parameterisations, such as Gent et al (1995), which attempt to implicitly respect the non-random effects of eddy motion.

In numerical ocean models diffusion can be applied to both the tracer fields and the momentum fields. The transport of tracers via the motion of sub-grid scale fluid parcels acting independently to the mean flow must be considered alongside transport due to motion with the average velocities of the fluid, as both can lead to changes to the mean state. If the effects of these diffusive sub-grid scale processes are not included an important element of mixing is ignored, leading to excessively strong concentration gradients, and overly sharp fronts. When diffusion operators are applied to the tracer fields they provide a parameterisation of the mixing of tracers through the sub-grid scale motions of the fluid. Similarly, when calculating momentum, the speed and direction of sub grid scale sections of the fluid cannot be ignored, particularly where these differ from the mean velocities. Diffusion is often essential for numerical reasons, to parameterise frictional processes and energy dissipation in order to close the horizontal momentum equation. In the real ocean frictional dissipation is small, however in numerical models a much larger amount of frictional energy dissipation, included through the momentum diffusion terms, is needed to ensure model stability. This difference is due to the scales of ocean models being of the order of kilometres, whilst the Kolmogorov scale in the ocean is of the order of millimetres. On small scales frictional processes occur within the ocean, which remove kinetic energy from the system; these effects are parameterised by the momentum diffusion, resulting in a reduction of kinetic energy and smoothing of the velocity fields.

## 3. Applying Diffusion within Numerical models

### 3.1 Diffusion operators

Diffusion can be applied using either a laplacian or a bilaplacian operator (sometimes referred to as harmonic and biharmonic operators).

The laplacian diffusion operator is a 2$^{nd}$ order operator, based on Fick's 2$^{nd}$ law;

$$\frac{\partial \Phi}{\partial t} = \nabla \cdot \left[ D \nabla \Phi \right]$$

Where the operator $\nabla$ acts along the lateral diffusion surfaces (discussed later). $D$ is a diffusion coefficient as in equation 1 (the choices for diffusion coefficients are also discussed later in this paper), and $\Phi$ is the variable being diffused (u and v for momentum diffusion, and tracers such as temperature and salinity for the tracer diffusion). As a second order operator, the laplacian operator impacts the resolved features of the model, causing some differences in large scale patterns. It is often used to parameterise large scale mixing, such as eddy diffusion.

Griffies and Hallberg (2000) introduced a 4$^{th}$ order operator, known as the bilaplacian, or biharmonic operator;

$$\frac{\partial \Phi}{\partial t} = \Delta \cdot \left[ \Delta \Phi \right] \quad \text{where} \quad \Delta(*) = \nabla \cdot \left[ \sqrt{D} \nabla \Phi \right]$$

With $\nabla$, $\Phi$ and $D$ as before. The bilaplacian operator is simply the laplacian operator applied twice. The development of the bilaplacian operator was motivated by a need to include frictional dissipation in numerical ocean models in a way which limited changes to the model fields. Use of a laplacian diffusion operator for the application of friction is often not motivated by physical principles, and so can result in unwanted impacts on the resolved scales of motion due to the large scales of the laplacian operator. The fourth-order bilaplacian diffusion-operator acts more strongly on small scales and more weakly on large scales in comparison to the laplacian operator. As such it can be used to effectively parameterise small processes such as frictional dissipation whilst minimising changes to the models resolved scales, and can be used to remove grid scale noise and provide a numerical closure to the momentum scheme, whilst preventing undesired changes to model features.

### 3.2 Diffusion coefficients

When implementing numerical diffusion in an ocean model the amount of diffusion required needs to be carefully considered. If insufficient amounts of diffusion are included, sub-grid scale processes will not be properly taken account of, and the model can become unstable. However excessive amounts of diffusion can result in over mixing, and features such as fronts can become over smoothed. The amount of explicit diffusion needed by an ocean model is highly dependant upon the grid size, the dynamic features of the region and the amount of diffusion implicitly included through the advection scheme. Very high resolution models can be run with no explicit tracer diffusion, as processes such as eddy diffusion are resolved by the model scale, and implicit diffusion in the advection scheme is adequate for implicit inclusion of small scale processes which remain sub grid scale. However in coarser model grids, eliminating tracer diffusion leads to unrealistic results as the mixing effects of the mesoscale activity are not included, and the models can become unstable. Regions with strong velocity gradients generally contain large amounts of eddy motion and turbulent motion, meaning mixing through sub-grid scale processes is substantial and much energy needs to be dissipated. However in areas with low velocity shears, the effects of eddy motion and turbulent motion is reduced. As such greater amounts of numerical diffusion are recommended for areas with high velocities and high velocity shears. The amount of diffusion added is controlled by the diffusion coefficient, $D$.

The simplest option is to apply a constant amount of diffusion across the entire model domain, however this approach does not take into account differences in grid size and physical regimes within the modelled region. Models which run on non-uniform grids can account for the change in resolution over the model domain by implementing diffusion coefficients which are scaled with grid size. This has clear benefits in models where the resolution varies largely. Smagorinsky (1965) suggests the use of diffusion coefficients which vary with both the local resolution and local dynamics, resulting in a diffusion scheme where large amounts of diffusion are applied where they are likely to be needed, with lower levels of diffusion elsewhere in the domain. Smagorinsky's (1965) formulation was designed for atmospheric modelling, however a very similar approach is taken by many ocean models. Both the Regional Ocean Modelling system (ROMS) and the Princeton Ocean Model (POM) define the momentum viscosity, $D_M$ , and the tracer diffusivity, $D_H$ , according to the equation;

$$\left(D_M, D_H\right) = \left(C_{vis}, C_{diff}\right)\Delta x \Delta y \left[\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{1}{2}}$$

Where $C_{vis}$ and $C_{diff}$ are user defined values for calculation of the viscosity and tracer diffusion.

Ezer and Mellor (2000) investigates the effects of varying the values of $C_{vis}$ and $C_{diff}$ in both a high resolution and low resolution version of a North Atlantic simulation using POM. It finds that *S*-coordinate models can tolerate lower levels of tracer and momentum diffusion than *Z*-coordinate models with similar horizontal resolution (when using the diffusion scheme described in Mellor and Blumberg, 1985). It also finds that it is numerically possible to run with viscosity only (no tracer diffusion), but that this results in noisy tracer fields. The models discussed in Ezer and Mellor (2000) use curvilinear grids, giving large variations in grid size within each model. The low resolution model varies in grid size from approximately 25km to 60km, and the high resolution model varies between 15km and 40km. Both configurations are run with values of $C_{vis}$ and $C_{diff}$ varied between 0.01 and 0.1, however due to the Smagorinsky style formula used this gives very different amounts of diffusion applied both between the two models, and across each model. The mesoscale activity in the models, in particular eddy shedding from the loop current in the Gulf of Mexico, and Gulf Stream separation, varied with both the diffusion parameters used and the model resolution. When using the same values for $C_{vis}$ and $C_{diff}$ (note that due to the grid dependence of the Smagorinsky-style formula this means different absolute values for diffusion), the higher resolution runs gave greater variability than the low resolution runs, particularly around the region of the western boundary current. It was found that reducing the horizontal diffusion by a factor of 10 had little effect on the variability in both the high and low resolution runs. However, changing the diffusion affected the location of the gulf stream separation.

Wakelin et al (2008) briefly describes the lateral diffusion coefficients used in a ~12km configuration of the Proudman Oceanographic Laboratory Coastal Ocean Modelling System (POLCOMS, described in Holt & James, 2001, and Holt et al, 2001) configuration of the North East Atlantic. It was found that in the shallow shelf-sea region (depths of less than 200m) no explicit diffusion or viscosity is needed. Their work showed that applying a depth dependant diffusion coefficient in deeper regions ($D_M, D_H$ =0.2H m$^2$s$^{-1}$, for 200<H<3000, and $D_M, D_H$ =600m$^2$s$^{-1}$ for H>3000, where H is the water depth in metres) ensures stability in deep water, whilst minimising over smoothing of features on shelf.

Holt and James (2006) describes a high resolution POLCOMS (~1.8km) model with Smagorinsky style diffusion coefficients (using the Mellor and Blumberg (1985) operator). The effects of varying $C_{diff}$ and $C_{vis}$ between 0.0 to 0.4 were investigated. It was found that if no tracer or momentum diffusion is included large amounts of eddy activity forms within the model, which were generally found to be too long lived and too energetic. Increasing the diffusion coefficients reduced the amount of eddy activity. The runs were compared to SST data; $C_{diff}$ and $C_{vis}$ equal to 0.2 was found to give the closest fit to the observations. $C_{diff}$ and $C_{vis}$ equal to 0.4 resulted in over smoothing of features and the benefits of running with high resolution were removed, with results being worse than a 12km resolution POLCOMS model of the same region. Generally these experiments showed very high sensitivity to the value of the diffusion coefficient.

Storkey et al (2010) describes the system used for global and ocean basin modelling at the Met Office. This suite of models includes a North Atlantic model which runs at 1/12$^o$ resolution. The tracer diffusion applied in this model uses a laplacian operator with a coefficient, $D$, of 100m$^2$s$^{-1}$. The momentum diffusion is applied using a combination of laplacian diffusion with a coefficient of 50m$^2$s$^{-1}$ and bilaplacian diffusion with a coefficient of 1.0x10$^{10}$m$^2$s$^{-1}$. The tracer diffusion in particular is far less than that used in Wakelin at al (2008). These differences are most likely due to the different advection schemes used, Storkey et al. applies advection using the TVD scheme, whilst Wakelin et al uses a PPM advection scheme. These different order advection schemes mean the two models have large differences in the amounts of implicit diffusion in the models, and so explicit diffusion coefficients are not comparable.

### 3.3 Diffusion Surfaces

In the ocean the majority of diffusive processes, particularly large scale eddy diffusion, occur along neutral surfaces (McDougall, 1987). A neutral surface is defined as a surface such that 'small isentropic and adiabatic displacements of a fluid parcel in a neutral surface do not produce a buoyant restoring force on the parcel' (McDougall, 1987). Neutral surfaces are not the same as potential density or isopycnal surfaces (the neutral surface is tangential to the potential density surface only at the reference pressure of the potential density surface), but are instead tangential to the in-situ density everywhere. To optimally model the lateral tracer and momentum diffusion, the lateral diffusion should be applied along the model neutral surfaces, or along a surface which is a close approximation to these isoneutral surfaces.

The simplest way to apply the diffusion equations is directly along the model's vertical coordinate levels, using the unchanged model variables (in *Z*-coordinates this gives diffusion along geopotential surfaces, which is discussed separately below). This results in very simple equations which can be easily implemented, however in an *S*-coordinate model the coordinate surfaces can be markedly different from isoneutral surfaces, particularly in regions of steeply sloping bathymetry. Therefore large amounts of undesired diapycnal mixing can be introduced. This diapycnal mixing is a particular issue for tracer diffusion, as it changes the density profile within the model, which leads to further unphysical mixing.

Often it is assumed the slopes of isoneutral surfaces are small, and so geopotential surfaces are assumed to be a close approximation to the isoneutral surfaces. ROMS (Mellor, 2004) and POM (Ezer & Mellor, 2000) both take this approach and apply diffusion along geopotential surfaces. In *S*-coordinate models diffusing along geopotential surfaces simplifies the transformation methods used to apply the diffusion (these are discussed later in this report) in comparison to applying diffusion along isoneutral surfaces. However, applying lateral diffusion along geopotential surfaces introduces undesired diapycnal mixing wherever the isopycnals are not a close approximation to the geopotential surfaces, with levels of diapycnal mixing increasing as the slope between the two surfaces steepens. The assumption that isoneutral surfaces are shallow sloping is a reasonable approximation for deep-ocean regions, but is less valid in coastal-ocean modelling. Different physical regimes operate in coastal regions meaning isoneutral surfaces can become steeply sloped. In addition to this coastal-ocean models generally have higher vertical resolution than deep ocean models, and so are capable of better resolving the vertical stratification. As such the models isoneutral surfaces are likely to be considerably more displaced relative to geopotential surfaces. Any discrepancy between isoneutral surfaces and geopotential surfaces in the model leads to undesired diapycnal mixing if diffusion is applied along geopotential surfaces.

To limit undesired diapycnal mixing many global ocean and ocean basin models, such as the non-tidal UK Met Office Forecasting Ocean Assimilation Models (Storkey et al, 2010), diffuse tracers along isopycnal surfaces using a rotated diffusion operator. Isopycnal surfaces are used as these are a close approximation to isoneutral surfaces in all ocean regions. To simplify the diffusion operator it is assumed that the slope between isopycnal surfaces and the model's coordinate surfaces are small. For example in the NEMO model the Cox diffusion operator limits the slope between the model surfaces and the isopycnal surfaces to 1/100 (Madec, 2008). For deep ocean, *Z*-coordinate models it is valid to assume that the geopotential surfaces will rarely diverge from isopycnal surfaces by more than this. However, in an *S*-coordinate model this assumption is again less valid; the model surfaces may be inclined in the opposite direction to the isopycnal surfaces, giving large differences between the two slopes. This means to diffuse along isopycnal surfaces the full equations would need to be used, giving complicated and computationally expensive equations to solve.

### 3.4 Numerical implementation of the diffusion equations

If diffusion is applied along surfaces other than the model's coordinate surfaces the equations must be transformed in some way. The most common method is to use a rotated operator, these operators are applied directly to model variables in such a way that the diffusion acts along geopotential or isoneutral surfaces.

Redi (1982) discusses the need to correctly model the anisotropy of some turbulent motions by resolving the diffusion equations in directions parallel and perpendicular to the isopycnal surfaces instead of the geopotential surfaces. This is commonly realized in *Z*-coordinate models, and Redi (1982) proposes a continuous form of a transformed mixing tensor for use in diffusing tracers along isopycnal surfaces in *Z*-coordinate models.

Cox (1987) (hereafter referred to as C87) implements a discretised version of the technique described by Redi (1982), and run a simple test; A *Z*-coordinate model is run with only diffusive processes being modelled, initialised with an anomaly imposed on an otherwise homogenous passive tracer field, with three different density fields. Firstly the density is purely depth dependent, secondly the isopycnal slopes exactly follow the diagonal of the grid spacing, finally the isopycnal slopes are increasingly curved upwards. The ambient density field is held constant throughout the run. Were the geopotential diffusion scheme applied in Bryan (1969) used, the 3 test cases would all show identical results (the same as that achieved for the first geopotential isopycnal test case), however implementing the Redi (1982) scheme resulted in the passive tracer spreading predominantly along the isopycnals in each case. A perfect solution would give no diapycnal diffusion, and this is the case for the first run, where the isopycnals are parallel to the *Z*-coordinate surfaces. For the other cases averaging is required to deal with the isopycnals crossing the model surfaces, and this averaging results in numerical effects, which cause small errors in both the isopycnal and diapycnal diffusion. In the isopycnal directions these errors are negligible compared to the large amounts of true isopycnal diffusion, however in the diapycnal direction they are more significant, as they cause a non-zero amount of diapycnal diffusion. In addition to this a series of ripples are shed in the diapycnal direction in both the second and third tests, leading to negative tracer anomalies. C87 states that the numerics of the algorithm cause small amounts of negative diffusion in the diapycnal direction, with the extent of this being dependent on the shape of the initial anomaly, with respect to the grid spacing. Tests were carried out which indicated that features which are well resolved by the grid scale produce acceptably small numerical errors, but still result in some negative diffusion. It is also noted that applying the diffusion in this isopycnal coordinate system may not be efficient enough to remove grid scale noise which is inherent in most ocean models. As such C87 recommends that a background mixing is applied along the model coordinate surfaces, with a smaller coefficient than that used for the isopycnal mixing.

Griffies (1998) critiques the method proposed in C87, stating it to be both unphysical and unstable. The requirements for producing physically based, stable, isoneutral diffusion schemes for *Z*-coordinate models are considered and the need for 'downgradient orientation of the diffusive fluxes along the neutral directions' (property 1) and 'zero isoneutral diffusive flux of locally referenced potential density' (property 2) are emphasised. The paper argues that the need for background horizontal diffusion when using the C87 method is not due to grid noise occurring from other processes, as suggested by C87, but is due to inherent issues with the method itself, arising from the above properties not being satisfied. It is noted that whilst individual flux components may be upgradient, the overall flux vector must remain downgradient. It is then shown that C87 is susceptible to computational modes, which can cause components of the flux to unrealistically vanish under certain conditions, leaving only the upgradient components resulting in upgradient tracer diffusion, and an increase in tracer variance. Further to this, in C87 the computed neutral surfaces differ to true isoneutral surfaces due to the calculation of the density gradients not explicitly including the thermal expansion coefficients. This combined with the nonlinearity of the diffusion tensor (itself being a function of the active tracers, whilst also operating on them) can lead to an imbalance in the fluxes of active tracers thus providing a non-zero isoneutral flux of potential density, which can cause grid scale noise even in situations when tracer variance is being reduced. An alternative discretisation of the Redi (1982) scheme is proposed (hereafter referred to as G98), which eliminates the need for background diffusion and is designed to respect properties 1 and 2. The C87 scheme and the G98 method were tested by running a model with diffusion only, with both one and two active tracers. It was shown that the model runs were more stable when running with the G98, and that it is possible to run a stable model with all background horizontal diffusion and explicit dianeutral diffusion removed. Tests show the computational mode in the C87 method considerably increases the temperature extremes, giving values outside of the initial range. This is not seen in runs using G98, however there are small increases in the salinity extremes. It is explained that the G98 method does not completely fulfil the stated properties, in particular the scheme satisfies purely downgradient diffusion over a finite volume of more than one grid cell, but not for each individual tracer cell. This is thought to be the cause of the small increases in the salinity extremes. Another source of error in the G98 method arises from the need to split the vertical fluxes into an explicit and implicit part. This separation is not numerically exact, meaning the active tracer fluxes are not precisely balanced. However, the errors resulting from the G98 scheme are shown to be much less than the errors resulting from C87.

S-coordinate models generally implement diffusion along geopotential surfaces. Mellor and Blumberg (1985) (hereafter referred to as MB85) showed that taking the non-rotated operators used to apply geopotential diffusion in a *Z*-coordinate model and simply transforming them for use in an *S*-coordinate system results in unrealistic bottom boundary layer features in regions with sloping bathymetry. An alternative rotated diffusion operator was proposed which is both mathematically simpler and computationally more efficient, and which preserves bottom boundary features in regions of steep bathymetry. However, in the case of horizontally stratified temperature and salinity with no initial motion and a constant diffusion term, the MB85 method produces an unrealistic cross slope heat transfer, as the diffusion term does not reduce sufficiently in regimes of low or no motion.  This can be overcome by using a diffusion coefficient which is dependant on the velocities (such as in Smagorinsky, 1965), or the effects can be minimised by applying the diffusion to the anomalies only.

Huang and Spaulding (1996) discussed the errors related to using the conventional diffusion equations transformed to *S*-coordinates, and the limitations of the MB85 formula.  An alternative rotated diffusion operator was proposed, and assessed against the conventional method, and the MB85 method.  Each method was tested for a pure horizontal diffusion problem, in a two-dimensional model with sloping bathymetry, with no external forces, where the only process modelled was horizontal diffusion.  The simulations were initialised with a vertically stratified salinity field with no initial motion. The results were assessed after a 30 day simulation by comparing the final salinity fields and by calculating salinity changes at two points in the domain.  A numerically exact geopotential diffusion scheme would give no change.  As expected the conventional method performed badly.  The run using this method did not reach a steady state at any point in the 30 day run, and at the end of the simulation the salinity change was much as 4 psu at the locations measured.  The MB85 method also performed badly in this test, however these are the conditions for which it is known that the MB85 method has issues.  The run with the MB85 method did reach a steady state, but only once the salinity stratification had aligned itself to the coordinate system, and salinities changed by over 2 psu at the measured locations.  The method proposed by Huang and Spaulding (1996) gave far better results; a steady state was maintained throughout the simulation, at the 2 locations examined the salinities changed by less than 0.1 psu, and the vertical stratification pattern was preserved.

The alternative to rotated diffusion operators is to use an interpolation method. Fortunato and Baptista (1994) (hereafter referred to as FB94) investigated the application of lateral diffusion using a ‘localised sigma coordinates’ approach.  Instead of applying a rotated diffusion operator, all horizontal processes were calculated by interpolating the surrounding coordinates to the same geopotential level, then horizontal processes were applied along these new local coordinates, in a true horizontal sense. The method was implemented for the baroclinic pressure gradient calculations (where errors are reduced by an order of magnitude), but its use for dealing with horizontal diffusion is also mentioned.  A similar approach has been applied in NEMO for use in calculating horizontal pressure gradients in *S*-coordinates.

Huang and Spalding (2002) compared the two main methods for applying horizontal diffusion in *S*-coordinate models: the MB85 method of using a rotated operator, and the FB94 method of interpolating into horizontal space before applying diffusion, as well as introducing a new rotated diffusion operator (hereafter referred to as HS2002). Unlike the majority of rotated diffusion operators which use Taylor Series expansion to calculate the variables in the discretised equation, that developed in Huang and Spaulding (2002) is based on the use of a second order Lagrangian interpolation scheme. Each method was tested by setting up a diffusion only simulation exactly as was done in Huang and Spaulding (1996) with the results after one month of simulation assessed. Both the MB85 and FB94 methods showed considerable errors. As was found in the Huang and Spaulding (1996) paper, the MB85 diffusion operator caused artificial diffusion, particularly where the bathymetry is steep. This paper goes further and attributes these errors to numerical errors in the horizontal diffusion terms which lead to incorrect salinity distributions. It was noted that in a full ocean model these errors would lead to an incorrect pressure gradient force, causing unrealistic flows and amplifying errors in the salinity field. It was found that the FB94 interpolation method maintains the vertical stratification pattern, but incorrectly changes the salinity values at each vertical level, giving falsely decreased salinity at the surface and falsely increased salinity near the sea bed. It is suggested that the majority of the errors in the interpolation method are due to errors in the interpolation itself, indicating that use of a higher order interpolation scheme would reduce these errors. The HS2002 method showed considerably smaller errors. When used in conjunction with a stepwise bottom boundary condition on the diffusion equations the vertical stratification pattern was reasonably well conserved. Maximum errors across the domain are given for the end of a 30 day run. These show the HS2002 method to give a maximum error of 0.3ppt, compared with 5.7 from MB85, and 5.8 from the linear interpolation method. However, the HS2002 method is dependent upon the model coordinate system satisfying the hydrostatic condition (Haney, 1991), and all tests are carried out in a domain which meets this criteria. In reality many ocean models, including the FOAM AMM7 model do not fully meet this restriction.

## 4. Application of lateral diffusion in *S*-Coordinate numerical models

POLCOMS was run operationally at the Met Office for many years, to model various regional coastal domains, and is also used by other oceanography institutions. The predecessor to the current FOAM AMM7 model was a 12km resolution POLCOMS model, described in Wakelin et al (2008). In this model lateral diffusion was applied in a geopotential direction, using the FB94 method of applying a spline interpolation to the model variables before and after diffusing them. The diffusion coefficients used in this model are dependent on the depth of the water column, with no diffusion applied in areas shallower than 200m, and diffusion amounts increasing with depth in regions deeper than 200m (as described earlier in this report). Holt and James (2006) describe a high resolution (~1.8km) POLCOMS model. In this high-resolution configuration, diffusion is applied along geopotential surfaces using the Blumberg and Mellor (1985) laplacian operator, with Smagorinsky-style diffusion coefficients.

ROMS is widely regarded as one of the best regional-ocean numerical models, and is extensively used in a variety of organisations. ROMS applies lateral diffusion along geopotential surfaces using the MB85 formulation. Mellor (2004) notes that whilst this gives valid bottom boundary simulations, inherent errors in the MB85 formulation give unrealistic cross slope mixing, and undesired diapycnal mixing occurs wherever isoneutral surfaces are not parallel to geopotential surfaces. In ROMS these errors are limited by use of both of the solutions suggested by MB85; Smagorinsky coefficients are used to limit diffusion in regions of low velocities, and diapycnal mixing is limited by diffusing anomalies only, which considerably limits the vertical component of the diffusion. These can be either anomalies compared with a climatology, or a mean model state. By using this method the model gradually drifts toward the climatology or mean values at time scales of approximately 20 years.

Ezer and Mellor (2000) discussed the application of diffusion in POM in a domain which is eddy permitting. Lateral diffusion is also applied in POM along geopotential surfaces using the MB85 calculation with Smagorinsky style coefficients. In Ezer and Mellor (2000) the diffusion of entire model fields is compared with diffusion of anomalies only. The model is started from climatological values, and run with identical inputs (surface heat flux, wind stress, evaporation minus precipitation fluxes) and identical boundary conditions, with one run diffusing anomalies only, and one run diffusing entire model fields. The run diffusing entire fields showed greater undesired diapycnal mixing and far larger drift from the initial climatological values than the run which diffused anomalies only. Generally the thermocline and halocline were weakened in the run which diffused entire model fields, with surface values of temperature and salinity decreasing slightly. Ezer and Mellor (2000) therefore recommends that anomalies only are diffused in POM.

### 4.1 Application in FOAM AMM7

The FOAM AMM7 model based on the NEMO code (Madec, 2008) is described in O'Dea et al. (2012). It is a free surface, tidal, *S*-coordinate model which runs operationally at the UK Met Office, for a rectangular domain ranging from 40$^{o}$ North to

65$^o$ North, and from 20$^o$ West to 13$^o$ East, with a resolution of 1/9$^o$ by 1/15$^o$. The open boundaries are forced by data from the Met Office Forecasting Ocean Assimilation Model - North Atlantic 12$^{th}$ degree (FOAM NATL12), described in Storkey et al. (2010), and surface forcing is provided from various Met Office atmospheric models. The vertical coordinates used are a modified version of the Song and Haidvogel (1994) coordinates; evenly spaced sigma coordinates are used in depths of less than 150m, with stretching applied as water depth increases, using the Song and Haidvogel formulation. The stretching parameters are set to focus the increased resolution around the surface, and at 4/5 of the water column depth. FOAM AMM7 currently assimilates sea surface temperature data, and in future will be developed to assimilate both temperature and salinity profile data, and sea surface heights. Temperature and salinity are advected using a Total Variance Dissipation (TVD) scheme, and a Generic Length Scale (GLS) scheme is used for the vertical mixing.

Being located around 50$^o$N with a model resolution of ~7km, FOAM AMM7 is an eddy permitting, but not fully eddy resolving, model. Therefore the mixing effects of eddies and other mesoscale processes are not fully resolved by the model. Small amounts of laplacian tracer diffusion could be used to parameterise this missing mesoscale mixing, as well as to include the effects of sub-mesoscale mixing processes. Conversely, some large scale eddy mixing is implicitly included in the resolvable model dynamics, and so too much laplacian mixing could result in over smoothing of realistic model features. Bilaplacian diffusion can be applied either instead of or as well as laplacian diffusion. Bilaplacian diffusion alone is unlikely to fully parameterise the unresolved large scale eddy mixing, however if grid scale noise in the tracer fields was an issue, bilaplacian diffusion could be used alongside laplacian diffusion, thus ensuring stable, smooth fields, whilst limiting large changes to resolved model features.

As tracer diffusion should be applied in FOAM AMM7 primarily to parameterise large scale anisotropic mixing, unphysical diapycnal mixing should be limited. The diffusion of tracers changes the density profile of the model, and so errors in the tracer diffusion will lead to erroneous density driven flows. Both ROMS and POM apply tracer diffusion along geopotential surfaces, and limit unrealistic diapycnal diffusion by diffusing anomalies only. Both models use the MB85 rotated diffusion operator and resolve the known issues with this by using Smagorinsky style coefficients to limit the diffusion in regions of small velocities. This is an option available for use in FOAM AMM7. Minimal coding would be required as rotated diffusion operators for geopotential diffusion in *S*-coordinate models already exist, small amendments would be needed to calculate the anomalies and apply the diffusion to these. However, the weak relaxation to climatological values is undesired, and if this approach was to be taken careful consideration would need to be given to what should be used as the climatology. Generally climatologies do not have accurate, high resolution data in coastal regions, and so use of a standard climatology could lead to over smoothing of features. A model field, such as a reanalysis product, or a model climatology could be used, however this may perpetuate model biases. Coding would also be needed to implement Smagorinsky coefficients, (see below) although work is being done on this at the National Oceanography Centre (NOC).

The FB94 approach used in POLCOMS applies diffusion along geopotential surfaces and diffuses the entire model fields, leading to large amounts of diapycnal diffusion. To implement the FB94 method within the current NEMO framework would require code changes, but by utilising existing pieces of code, this can be minimised. Code to interpolate the model variables to geopotential levels already exists as part of the horizontal pressure gradient correction code, and once the variables have been interpolated, the *Z*-coordinate geopotential diffusion operator, which already exists within the code, can be applied. The FB94 method which was assessed by Haung and Spaulding (2002) used linear interpolation, however the code used for calculating the horizontal pressure gradients uses quadratic interpolation. As such it would be simpler to apply a quadratic interpolation scheme, and this would also reduce errors. However undesired diapycnal mixing will still be introduced as a consequence of applying the diffusion along geopotential levels and not isoneutral levels. Interpolating to isoneutral surfaces, instead of geopotential surfaces, and applying the diffusion equations directly on the new isoneutral coordinate system would resolve this issue. However, when using an *S*-coordinate system and the full equation of state in NEMO it is not possible to calculate isoneutral surfaces. An alternative approach is to diffuse along isopycnal surfaces, which can be easily computed and interpolated to. Isopycnal surfaces are always a very close approximation to isoneutral surfaces, and so any errors resulting from this difference would be negligible. This method would limit diapycnal diffusion, whilst allowing the diffusion to be applied to the entire model field without any relaxation to other fields, but would include additional coding and testing.

The scheme described in Griffies (1998) has been implemented in the latest NEMO release. It has been done in such a way as to allow the rotated diffusion operator to work within an *S*-coordinate framework. This means that it is possible to implement isoneutral diffusion within the AMM7. Early tests show some issues with the scheme, thought to be due to the bugs in the coding. As such work is needed to identify the cause of the errors and fix them. Currently the AMM7 model runs at vn3.2 and so will need to be updated to run using the vn3.4 code to enable use of the Griffies (1998) diffusion option, however this work is already underway, and should not be a significant undertaking. It is uncommon for isoneutral diffusion to be applied in *S*-coordinate models, with both ROMS and POM applying geopotential diffusion. However there is little scientific justification for this, and the decision seems to be based on a lack of robust, numerically accurate rotated operators for applying isoneutral diffusion in *S*-coordinate models.

When run with low levels of momentum diffusion FOAM AMM7 fields show large amounts of grid scale noise. As such FOAM AMM7 requires viscosity to be included primarily as a numerical closure scheme, and to parameterise small scale frictional processes. Laplacian momentum diffusion is likely to result in changes to the resolved velocity patterns, which is unnecessary and undesired. Bilaplacian momentum diffusion would ensure model stability and remove grid scale noise, whilst minimising changes to the resolved fields, and is therefore preferable.

If momentum diffusion is being applied as a numerical closure, to remove grid scale noise and parameterise very small scale processes this should be applied along model levels; the grid scale noise is a numerical artefact which appears on model levels, and the small scale frictional processes being parameterised can be assumed to be isotropic. Applying the momentum diffusion within FOAM AMM7 along geopotential or isoneutral surfaces is more computationally expensive, and the benefits are limited. Dianeutral mixing of momentum would be limited, but considering the isotropy of processes being parameterised by the momentum diffusion this is less of an issue, and a geopotential or isoneutral operator may be less effective at removing grid scale noise, as the noise occurs along model levels.

The Met Office North Atlantic 1/12$^{th}$ degree model (FOAM NATL12) covers the North Atlantic region, including that modelled by FOAM AMM7, and provides boundary conditions for FOAM AMM7. Whilst the two models have a number of differences, they are both based on the NEMO code, and run using a number of similar options, and importantly implement the same tracer advection scheme. The resolution between the two models differs, but is of a similar order, with both models being eddy permitting. As such levels of implicit diffusion within the models, and necessary levels of diffusion for stability and accurate results are likely to be similar, and so the diffusion coefficients used within FOAM NATL12 provide a useful starting point for the choice of diffusion coefficients for use in FOAM AMM7.

Although the grid box sizes do vary within the FOAM AMM7 domain, the variation is limited due to the domain size. As such the benefits of using grid size dependant diffusion coefficients are limited. Code to apply diffusion using Smagorinsky coefficients does not yet exist within NEMO. This could be done in a relatively simplistic way, where the derivatives are taken purely in the horizontal model directions. This would be easy to implement, and is a reasonable, but not exactly correct, way to apply the Smagorinsky style coefficients. To calculate the Smagorinsky coefficients exactly the derivatives should be taken in the same direction as the diffusion is being applied, and so in *S*-coordinates the derivatives should often be taken across different vertical model levels. In the majority of cases there will be little difference between the two methods. Both will give diffusion coefficients which vary with grid size, however the way the diffusion coefficients scale with velocity will vary in regions of highly baroclinic flow patterns, giving possible small differences.

## 5. Conclusions

Diffusion is an essential part of numerical ocean models. It is required to parameterise the effects of sub-grid scale processes. These processes cannot be explicitly resolved, yet inclusion of the mixing effects of these processes is necessary for both scientific accuracy and numerical stability. Diffusive processes include truly random processes such as frictional dissipation of energy, and processes which are modelled as being pseudo random, such as mixing of tracers through eddy motion.

Two different diffusion operators are commonly used. The laplacian operator is a $2^{nd}$ order operator, which impacts the resolved model features. The bilaplacian operator is a $4^{th}$ order operator, which impacts smaller scales than the laplacian operator.

In the lateral direction large scale diffusive processes, such as eddy diffusion dominate. However, in the vertical direction mixing due to eddies is limited, and small scale processes such as turbulence provide the majority of mixing. As such ocean models parameterise lateral diffusion separately to vertical diffusion. Lateral diffusion occurs along neutral surfaces, however applying lateral diffusion along neutral surfaces in an *S*-coordinate model is computationally difficult. The simplest method is to apply lateral diffusion along model levels, however in *S*-coordinate models this results in large amounts of physically unjustified dianeutral mixing. Often it is assumed that neutral surfaces are shallow sloping, and can be approximated by geopotential surfaces, with diffusion being applied along these.  This assumption, although valid in ocean basins, is inaccurate in coastal regions, and again can lead to physically unjustified dianeutral mixing.

When applying diffusion along surfaces other than the model's vertical coordinate system, the equations must be amended in some way.  Frequently this is done through the use of a rotated diffusion operator, whereby the diffusion is applied directly to the model variables in such a way that the diffusion acts along a separate surface.  There are a number of rotated diffusion operators.  Those suggested in Mellor and Blumberg (1985) and Huang and Spaulding (1996) which are used to apply diffusion along geopotential surfaces in *S*-coordinate models, and those described by Cox (1987) and Griffies (1998) for applying isoneutral diffusion in *Z*-coordinate models are discussed in this report.  An alternative scheme is to interpolate the model variables to the diffusion slopes, apply the diffusion on these interpolated variables, and then interpolate back to the original model levels.  This method is discussed in Fortunato and Baptista (1994).  All methods have errors associated with them.  The errors in the rotated diffusion operators are implicit in the operators and are amplified in regions of steep bathymetry.  The errors in the interpolation method come mainly from errors in the interpolation, and can therefore be limited by use of high order interpolation schemes.

The amount of diffusion which should be explicitly included within an ocean model depends on the model resolution, the dynamics of the region being modelled, and the amount of diffusion implicitly included in the advection routine.  The diffusion can be applied using a constant value, can vary depending on grid size, or can vary with the grid size and the regions dynamics by use of a Smagorinsky style coefficient.

ROMS and POM are both *S*-coordinate models.  They both apply diffusion in the geopotential direction using the Blumberg and Mellor (1985) method.  Errors associated with this method are limited by diffusing anomalies only (which weakly constrains the model to a climatological state) and by use of Smagorinsky style diffusion coefficients to limit the amount of diffusion applied.

POLCOMS is also an *S*-coordinate model.  This has been run with a Fortunato and Baptista (1994) style diffusion, with depth dependant diffusion coefficients, applying no diffusion at all in shallow waters (less than 200m), and increasing diffusion amounts with water depth, using the Blumberg and Mellor (1985) method.

These findings in the context of the Met Office FOAM AMM7 model are discussed. The horizontal resolution and geographical location of this model mean it is eddy permitting, but unable to fully resolve mesoscale eddy motion. As such small amounts of laplacian tracer diffusion are recommended to parameterise the missing effects of eddy mixing. However as some eddy mixing effects are explicitly included in the resolved model features it is recommended that low levels of laplacian tracer diffusion are applied to avoid over smoothing these features. Laplacian diffusion alone is expected to be adequate for the FOAM AMM7 model. However if grid scale noise exists in the tracer fields when low levels of laplacian diffusion are used, then it is recommended that bilaplacian diffusion be applied alongside the laplacian diffusion to remove this.

Tracer diffusion is being applied to parameterise eddy diffusive processes which occur along neutral surfaces, therefore the numerical diffusion should ideally be applied to isopycnal or isoneutral surfaces. However if this is not feasible then geopotential diffusion should be applied. The system used in ROMS and POM is unattractive due to the weak relaxation to a climatological state. An approach similar to that used in Fortunato and Baptista (1994) could be implemented within NEMO, where the variables are interpolated to isopycnal surfaces instead of geopotential surfaces, allowing diffusion along isopycnal surfaces. Use of a quadratic interpolation scheme, as is currently used in the calculation of horizontal pressure gradients, would further reduce the errors associated with this method. However, this approach would require additional coding and testing. It is instead recommended that further assessment of the new Griffies (1998) diffusion scheme is carried out to investigate its suitability for use in an *S*-coordinate model. It is hoped that the current issues with this option can be resolved, and that this scheme can to be used to apply isopycnal tracer diffusion. As this option has already been implemented in NEMO it should be easier to implement than the FB94 option. Also this code is highly likely to be used in other models and by other institutions leading to bigger efficiencies if further developments are needed in this area and reducing the divergence of coastal applications from the open ocean.

Momentum diffusion is necessary in FOAM AMM7 to parameterise small scale processes such as frictional dissipation, and to ensure stability and remove grid scale noise. The processes being parameterised are small scale processes, and no changes to the resolved velocity features of the model are required. Therefore the bilaplacian operator is the most suitable to use here. The isotropic characteristics of the processes being parameterised reduces the importance of the surfaces on which the diffusion is applied. As diffusion is being used partly to remove grid scale noise which occurs along model levels it is recommended that the bilaplacian momentum diffusion operator is applied along model levels, which also aids computational efficiency.

The amount of diffusion applied needs to be carefully tested. Regarding momentum diffusion the minimum amount of diffusion which successfully removes grid scale noise and ensures stability should be used. For tracer diffusion more careful consideration is required. Again a minimum amount may be necessary to ensure smooth model fields, although in some parts of the domain it may be possible to run without explicit tracer diffusion, as enough diffusion may be supplied implicitly by the advection scheme. Runs should be carried out with varying levels of tracer diffusion. The model variability, and the ability for the model to correctly capture features such as fronts should be assessed, and the tracer diffusion coefficient decided based on the results of these tests. If code for Smagorinsky style diffusion coefficients is made available within NEMO then this option should also be tested for both momentum and tracer diffusion, however it is recommended that constant coefficients are used initially.

Recommendations;

- It is recommended that tracer diffusion is applied along isopycnal surfaces, using a laplacian Griffies (1998) diffusion operator.
- Sensitivity studies to determine optimal values for the tracer diffusion coefficients should be undertaken.
- If large levels of laplacian diffusion are required to remove grid scale noise, such that model features appear over smoothed, then bilaplacian diffusion should be applied alongside low levels of laplacian diffusion.
- Momentum diffusion should be applied predominantly as a numerical closure scheme, to ensure stability.
- Momentum diffusion should be applied along model levels using the bilaplacian operator.
- The minimum amount of bilaplacian diffusion needed to remove grid scale noise and ensure stability should be used.
- If Smagorinsky style diffusion coefficients become available in NEMO these should be tested for use in both the tracer and momentum diffusion

## Appendix – Gent McWilliams mixing

Gent and McWilliams (1990) introduces a parameterisation (hereafter referred to as GM) for the effects of mesoscale eddy mixing. Whilst originally formulated as a diffusive term more recent work considers the GM mixing term as an advection term.  However the interactions between model diffusion and GM mixing are important, and so GM mixing has been briefly considered in the scope if this work.  The GM mixing term acts to shallow isoneutral slopes.  This is done in the original implementation by diffusing the thickness of the isopycnal surfaces, however more recent coding applies this effect by advecting the isopycnal layer thicknesses.

At the time of the Gent and McWilliams (1990) paper being published the majority of *Z*-coordinate global ocean models applied lateral diffusion along isoneutral surfaces using the scheme described in Cox (1987) with additional background horizontal diffusion required for model stability.  The implementation of GM removed the need for additional horizontal background diffusion, allowing stable runs with purely isoneutral diffusion.  As such many global ocean models began running with GM and applying diffusion in the isoneutral direction only, and saw large benefits.  False diapycnal diffusion was reduced, resulting in colder water below 1km depth, and there were large improvements to deep water formation, with deep water formation in models more closely matching observed locations.  However Gent (2011) acknowledges that many of these improvements are due to the removal of the background horizontal diffusion, which can also be achieved by implementing the Redi (1982) equations in the way described in Griffies (1998), rather than the benefits being directly attributable to the addition of the GM scheme.  Tests with GM also showed a large reduction in transport in the upper ocean close to the ACC, due to changes in the overturning in this region.  This is attributed to the GM parameterisation flattening isopycnals due to baroclinic instability.

Gent (2011) discusses diffusion and GM mixing within the mixed layer, stating that GM is designed for the nearly adiabatic ocean interior, and so its use in the mixed layer, and in regions with steeply sloping isopycnals needs to be carefully considered.  Tapering of the coefficients has been commonly used to reduce GM effects when isopycnals became steep.  Ferrari et all (2008) and Treguier et al (1997) suggest that in the mixed layer eddy induced velocity exists in the horizontal direction with no vertical shear, and therefore diffusion should be applied parallel to the ocean surface in this region.  However it is noted that implementation of GM within the ocean mixed layer remains an active area of research.

The use of GM mixing in models of eddy permitting and eddy resolving models is also discussed in Gent (2011).  However, again the focus is on the use of GM mixing as an alternative to geopotential diffusion, and so although it is suggested that GM should be used even in eddy resolving models this is predominantly due to the benefits of removing false diapycnal mixing associated with geopotential diffusion.

The vast majority of literature surrounding GM mixing focuses on its use in global ocean models, running with either *Z*-coordinates or isopycnal coordinates, with a particular emphasis on its use in climate models. This is due to the importance of maintaining ocean stratification and distinct water masses in the deep ocean, especially during long climate simulations. By contrast there appears to be no literature examples of GM mixing being applied in coastal ocean *S*-coordinate models. In FOAM AMM7 much of the on-shelf region of the domain is seasonally well mixed, and the majority of deep water is at eddy permitting resolution. As such consideration of issues surrounding GM in the mixed layer and GM in eddy resolving models become important. Whilst it is noted that within an eddy permitting model which covers a large area of well mixed water it is important to remove diapycnal tracer mixing, this is hoped to be achieved by implementation of the Griffies (1998) diffusion operator. The author has failed to find indication in current literature that the shallowing of isopycnals achievable by implementing GM mixing is of importance in a model such as FOAM-AMM7, and so whilst it is unlikely that it would be detrimental to model performance, there is little justification for implementing it in FOAM-AMM7 at present. However, it should remain a consideration, and if following further model improvements and assessments the stratification, and water mass sturcture within the model is seen to be poor then implementation of GM mixing should be reconsidered.

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