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Abstract

We devise a family of kinematically possible motions in shallow water versions of semi-geostrophic theory, with variable Coriolis parameter, for which two distinct measures of potential vorticity take the same value. We describe conditions for which this value is conserved following the actual motion of a particle. We relate the results to the existing literature, in particular for a constant Coriolis parameter.

1 Introduction

Shallow water theory can serve as a simplified basis for the study of so-called semi-geostrophic theories in geophysical fluid dynamics. We examine shallow water theory and two semi-geostrophic models of that type, with particular reference to a definition of potential vorticity associated with each, and allowing for the Coriolis parameter f to be spatially varying wherever possible. We describe a class of kinematically possible motions under which the expressions for two of the potential vorticities are the same. We give hypotheses under which that value is conserved following the particle.

The plan of the paper is as follows. In §2 we recall the shallow water equations, and the associated measure (q in (6)) of potential vorticity attributed to Rossby (1940, equation (9)). We describe a variational principle for these equations, prompted by Salmon's (1983) work. We state the semi-geostrophic approximation to the shallow water equations for constant f, and we recall (in (12)) the measure of potential vorticity, analogous to Hoskins' (1975, §3(*iii*)) definition, which is conserved by motions which satisfy that approximation. In §3 we describe a generalization, for variable f, of the geostrophic momentum transformation. This leads naturally to the third model, which is a pair of pseudo-hamiltonian

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equations, also with variable f, studied by Salmon (1985, equations (3.22)), and their associated potential vorticity (Q in (20)).

We are then in a position to exhibit, in §4, a class of kinematically possible motions for which $q = Q$. This our main result. In §5 we establish hypotheses which are sufficient to ensure that both q and Q are conserved following the motion of the particle. This applies, in particular, to the semi-geostrophic equations. Specializing to constant f in §6, we make some specific connections with the previous literature, including Salmon's (1988, equation (5.18)) constraints.

We remark that these results, which stem largely from equation (27), were discovered, in the first instance, using the properties of covariant skew-symmetric tensors, often referred to in the mathematical physics literature as forms, but we have rephrased the proofs in the more elementary form described here. (The calculation about the conservation of q on pp. 70-71 of Rossby (1940) can be understood in terms of these quantities.)

2 Governing equations

The motion of a typical particle in shallow water theory can be described by expressing the current cartesian horizontal coordinates

$$x = x(a, b, t), \quad y = y(a, b, t) \tag{1}$$

as functions, on the right, of the particle labels a, b and the time t. Throughout this paper we shall use the same symbol to denote a function and its generic values, as in this example (1). If $t = 0$ is the reference time, the functions in (1) have the properties $x(a, b, 0) = a$, $y(a, b, 0) = b$.

The incompressibility hypothesis requires the current depth h to be a function $h(a, b, t)$ with the property

$$\frac{h(a, b, 0)}{h(a, b, t)} = \frac{\partial(x, y)}{\partial(a, b)}, \tag{2}$$

where the jacobian on the right is that of the mapping (1). The time derivative of (2) following the particle gives the differential equation of continuity. We regard (2) as describing the explicit solution $h(a, b, t)$ of that continuity equation in terms of (1), and of $h(a, b, 0)$. The latter function of a and b only needs to be given. It is not uncommon to simplify and normalize by choosing $h(a,b,0)=1$ in (2), and we make this choice forthwith, but we note that this does constrain the free surface to be parallel to the bed at $t=0$. In compensation one can take the view that events to be studied take place at much later times.

The equations of horizontal momentum balance for flows over a flat bed which is rotating with position dependent Coriolis parameter $f(x,y)$ are

$$\ddot{x}+g\frac{\partial h}{\partial x}-\dot{y}f=0,\ \ \ddot{y}+g\frac{\partial h}{\partial y}+\dot{x}f=0\,. \tag{3}$$

Here g is a given constant, representing the combined effect of the acceleration due to gravity and a centrifugal component due to the Earth's rotation. Common choices of the Coriolis function are $f=$ a constant or βy, as approximations to $2\Omega\sin\phi$, depending on purpose. Here β and Ω are constants, and ϕ is latitude, which depends on y, so that x is absent. The superposed dot signifies time differentiation following the particle, i.e. partial differentiation with respect to t when a and b are held constant. The inverse

$$a=a(x,y,t),\ \ b=b(x,y,t) \tag{4}$$

of the mapping (1) has been used, with $h(a,b,0)=1$ in (2), to express $h(a,b,t)$ as another function

$$h(x,y,t)=\frac{\partial(a,b)}{\partial(x,y)}, \tag{5}$$

whose derivatives appear in (3). The basic problem is therefore to solve (3) with (5) for (1), which will then deliver $h(a,b,t)$ from (2).

Let

$$q=\frac{1}{h}\left(\frac{\partial\dot{y}}{\partial x}-\frac{\partial\dot{x}}{\partial y}+f\right) \tag{6}$$

denote a measure of potential vorticity. This name was introduced by Rossby (1940). We shall use q for velocity fields $\dot{x}$, $\dot{y}$ which are *not* solutions of (3) in §§4 and 6.

A result of Salmon (1983, equation (2.20)) can be adapted to show under what conditions (3) are the natural conditions of a variational principle. Let $p(x,y)$ and $r(x,y)$ be any two functions which satisfy the partial differential equation

$$\frac{\partial p}{\partial x}+\frac{\partial r}{\partial y}=f(x,y). \tag{7}$$

The motion (1) can be used to define a functional

$$F[x,y] = \int\int_{t_1}^{t_2} \left[\frac{1}{2}(\dot{x}^2+\dot{y}^2) - r\dot{x} + p\dot{y} - \frac{1}{2}gh\right] \mathrm{d}A_0\mathrm{d}t \tag{8}$$

where the integral is over the area A_0 of label space occupied by the fluid particles under consideration, and over an arbitrary time interval $t_1 \le t \le t_2$. The first variation of (8) for small variations $\delta x(a,b,t)$ and $\delta y(a,b,t)$, taking due account of (2) and (7), is

$$\begin{aligned}\delta F &= \left[\int((\dot{x}-r)\delta x + (\dot{y}+p)\delta y)\mathrm{d}A_0\right]_{t_1}^{t_2} + \int\int_{t_1}^{t_2} \frac{1}{2}gh^2(l\delta x + m\delta y)\mathrm{d}S\mathrm{d}t \\ &\quad - \int\int_{t_1}^{t_2}\left[(\ddot{x} + g\frac{\partial h}{\partial x} - \dot{y}f)\delta x + (\ddot{y} + g\frac{\partial h}{\partial y} + \dot{x}f)\delta y\right]\mathrm{d}A_0\mathrm{d}t \end{aligned} \tag{9}$$

where S, l, m denote the current boundary of the fluid and its outward unit normal components. Thus whenever the first two terms are zero in (9), we see that (3) implies $\delta F = 0$ and stationary F. It is necessary that, for example, either $h = 0$ or $\delta x = \delta y = 0$ on S.

The semi-geostrophic approximation to equations (3), in the case when f is a constant, is the replacement of the true acceleration by the time derivative of another vector

$$u_g = -\frac{g}{f}\frac{\partial h}{\partial y}, \quad v_g = \frac{g}{f}\frac{\partial h}{\partial x} \tag{10}$$

following the particle. The vector (10) is a notional velocity, called the geostrophic velocity. The semi-geostrophic approximation therefore seeks to find motions (1) satisfying

$$\dot{u}_g + g\frac{\partial h}{\partial x} - \dot{y}f = 0, \quad \dot{v}_g + g\frac{\partial h}{\partial y} + \dot{x}f = 0 \tag{11}$$

with (5) and constant f. Associated with these equations, the expression

$$\frac{1}{h}\left[f + \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} + \frac{1}{f}\frac{\partial(u_g,v_g)}{\partial(x,y)}\right] \tag{12}$$

is conserved. This is the shallow water version of Hoskins' (1975) potential vorticity. When f is variable, no measure of potential vorticity associated with (11) is known to be conserved.

3 Transformation to momentum coordinates

The definition (10) of geostrophic velocity suggests a transformation of coordinates

$$X = x + \frac{g}{f^2}\frac{\partial h}{\partial x}, \quad Y = y + \frac{g}{f^2}\frac{\partial h}{\partial y} \tag{13}$$

where (5) is used. However, in this definition we regard f as a *given* function $f(x,y)$ of the *spatial* coordinates in the first instance. Equations (13) are a transformation

$$X = X(x,y,t), \;\; Y = Y(x,y,t) \tag{14}$$

which, at each t, has an inverse

$$x = x(X,Y,t), \;\; y = y(X,Y,t) \tag{15}$$

certainly if

$$\frac{\partial(X,Y)}{\partial(x,y)} \neq 0. \tag{16}$$

Then (15) can be used to write $f(x,y) = f(X,Y,t)$, and the latter will *not* be a *given* function if the motion is unknown in advance. The explicit presence of t in this last expression becomes important in Theorem 2 below. There are some reasons from the literature about constant f (e.g. Eliassen 1948, Hoskins 1975), to call X and Y momentum or geostrophic momentum coordinates.

It is known that (13) is a Legendre (i.e. gradient) transformation when f is constant. This is also true for variable f if and only if

$$\frac{\partial(h,f)}{\partial(x,y)} = 0, \tag{17}$$

i.e. only for certain special motions, and evidently not all motions. Such Legendre transformations have been studied systematically by Chynoweth and Sewell (1989, 1991) for constant f.

Salmon (1985, equations (3.22)) studied certain *generalized* semi-geostrophic equations with pseudo-hamiltonian form in X, Y space, namely

$$\dot{X} = -\frac{1}{f}\frac{\partial H}{\partial Y}, \;\; \dot{Y} = \frac{1}{f}\frac{\partial H}{\partial X} \tag{18}$$

where

$$H(X,Y,t) = \frac{1}{2}(u_g^2 + v_g^2) + gh. \tag{19}$$

Associated with these equations is a *generalized* potential vorticity defined by

$$Q = \frac{f}{h}\frac{\partial(X,Y)}{\partial(x,y)}. \tag{20}$$

Salmon's version (1985, equations (3.12)) of (13) has $f(X,Y)$ written in. Thus (16) is implicitly assumed, but the t dependence in $f(X,Y,t)$ is omitted. When f is a constant, (20) is equivalent to (12).

4 Identification of potential vorticities

We are now in a position to establish the main results of this paper, as follows.

Theorem 1 *There is a class of kinematically possible motions, described in the following proof, and permitting f to vary in physical space, for which*

$$q = Q, \quad \text{and therefore} \quad \dot{q} = \dot{Q}. \tag{21}$$

Proof

We use the variables X, Y defined by the momentum transformation (13) to construct another transformation

$$\tilde{X} = \tilde{X}(X, Y, t), \quad \tilde{Y} = \tilde{Y}(X, Y, t) \tag{22}$$

to new coordinates $\tilde{X}, \tilde{Y}$. The functions on the right of (22) are required to be any which satisfy the partial differential equation

$$\frac{\partial(\tilde{X}, \tilde{Y})}{\partial(X, Y)} = f(X, Y, t). \tag{23}$$

This is one equation, from which to determine two functions, so it will have many solutions. For example, in the special case when f is a non-zero constant, (22) may be any canonical transformation (cf. Sewell and Roulstone, 1993), with t absent.

The momentum transformation (14) can be inserted into the transformation (22) to define a pair of functions $\tilde{X}(x, y, t), \tilde{Y}(x, y, t)$. Let $\lambda(x, y, t)$ be any function, and construct the family of velocity fields $\dot{x}(x, y, t)$, $\dot{y}(x, y, t)$ defined by

$$\left.\begin{aligned} \dot{x} &= r + \frac{\partial \lambda}{\partial x} + \frac{1}{2}\left[\tilde{X}\frac{\partial \tilde{Y}}{\partial x} - \tilde{Y}\frac{\partial \tilde{X}}{\partial x}\right], \\ \dot{y} &= -p + \frac{\partial \lambda}{\partial y} + \frac{1}{2}\left[\tilde{X}\frac{\partial \tilde{Y}}{\partial y} - \tilde{Y}\frac{\partial \tilde{X}}{\partial y}\right], \end{aligned}\right\} \tag{24}$$

By differentiation we see that these satisfy

$$\frac{\partial \dot{y}}{\partial x} - \frac{\partial \dot{x}}{\partial y} = -f + \frac{\partial(\tilde{X}, \tilde{Y})}{\partial(x, y)}, \tag{25}$$

using (7). The chain rule for jacobians gives

$$\frac{\partial(\tilde{X}, \tilde{Y})}{\partial(x, y)} = f\frac{\partial(X, Y)}{\partial(x, y)} = hQ \tag{26}$$

using (23) and (20).

By (6), (25), and (26) we obtain $(21)_1$, and differentiating it following the particle gives $(21)_2$.

□

Summarizing thus far, the two potential vorticities (6) and (20) are the same for any velocity field in the family (24). In other words, (24) is a solution of

$$\frac{\partial \dot{y}}{\partial x} - \frac{\partial \dot{x}}{\partial y} = \left(\frac{\partial(X,Y)}{\partial(x,y)} - 1 \right) f. \tag{27}$$

5 Rate of change of potential vorticity

It follows that either both potential vorticities are conserved, or neither is, during the motions (24).

Theorem 2 *Any solution $X = X(t)$, $Y = Y(t)$ of (18), for* any *hamiltonian function $H(X,Y,t)$ (not only (19)) and for variable $f(X,Y,t)$ $(= f(x,y)$ via (14)), has the property*

$$\dot{Q} = \frac{\partial(X,Y)}{\partial(a,b)} \frac{\partial f}{\partial t}. \tag{28}$$

Here $\partial f/\partial t$ is the partial t derivative of $f(X,Y,t)$, and the jacobian is that of the transformation

$$X = X(a,b,t), \;\; Y = Y(a,b,t) \tag{29}$$

obtained by substituting (1) into (14).

Proof

Combining (5) and (20) gives

$$Q = f \frac{\partial(X,Y)}{\partial(a,b)}. \tag{30}$$

Differentiating following the particle gives

$$\dot{Q} = \dot{f} \frac{\partial(X,Y)}{\partial(a,b)} + f \frac{\partial(\dot{X},Y)}{\partial(a,b)} + f \frac{\partial(X,\dot{Y})}{\partial(a,b)}. \tag{31}$$

The chain rule, with the pseudo-Hamilton equations (18), implies

$$\dot{f} = \frac{\partial f}{\partial X}\dot{X} + \frac{\partial f}{\partial Y}\dot{Y} + \frac{\partial f}{\partial t} = \frac{1}{f}\frac{\partial(H,f)}{\partial(X,Y)} + \frac{\partial f}{\partial t} \tag{32}$$

and

$$\frac{\partial(\dot{X},Y)}{\partial(a,b)} + \frac{\partial(X,\dot{Y})}{\partial(a,b)} = \frac{1}{f^2}\frac{\partial(X,Y)}{\partial(a,b)}\frac{\partial(f,H)}{\partial(X,Y)}. \tag{33}$$

The result (28) follows.

□

When f and Q are constants, (30) shows that (29) is canonical.

Theorem 3

$$\dot{Q} = \dot{q} = 0 \tag{34}$$

for any motion (24) which satisfies the dynamical equations (18), and which also has the property that $\partial f/\partial t = 0$, i.e. for which t is absent from f when expressed as $f(x(X,Y,t), y(X,Y,t))$ in terms of X and Y via (15).

The latter requirement is satisfied a fortiori when f is a constant.

Proof

This is a consequence of Theorems 1 and 2.

□

6 Specializations to f-plane semi-geostrophic theory

Schubert and Magnusdottir (1994) prove that, when f is constant, (27) can be rearranged as a zero curl condition which implies the representation

$$\left.\begin{aligned} \dot{x} &= \frac{\partial \chi}{\partial x} + \frac{1}{2}f\left[(X-x)\frac{\partial Y}{\partial x} - (Y-y)\left(\frac{\partial X}{\partial x}+1\right)\right], \\ \dot{y} &= \frac{\partial \chi}{\partial y} + \frac{1}{2}f\left[(X-x)\left(\frac{\partial Y}{\partial y}+1\right) - (Y-y)\frac{\partial X}{\partial y}\right], \end{aligned}\right\} \tag{35}$$

for arbitrary $\chi(x,y,t)$. It can be verified directly that (35) implies (24) when f is constant. It can also be seen that (35) is included within (24) by making the particular choices

$$p = \frac{1}{2}fx,\;\; r = \frac{1}{2}fy,\;\; \tilde{X} = |f|^{\frac{1}{2}}X,\;\; \tilde{Y} = |f|^{\frac{1}{2}}Y,\;\; \lambda = \chi - \frac{1}{2}(xY - yX) \tag{36}$$

when f is a constant.

A brief and elementary calculation using (10) and (13) allows (35), when χ = constant, to be rewritten as

$$\left.\begin{array}{rcl} \dot{x} & = & u_g + \dfrac{1}{2f}\left(u_g\dfrac{\partial}{\partial x} + v_g\dfrac{\partial}{\partial y}\right) v_g \ , \\ \dot{y} & = & v_g - \dfrac{1}{2f}\left(u_g\dfrac{\partial}{\partial x} + v_g\dfrac{\partial}{\partial y}\right) u_g \ , \end{array}\right\} \tag{37}$$

which are precisely Salmon's (1988, equation (5.18)) constraints. The latter *define* his Dirac bracket formulation of f-plane semi-geostrophic theory. When the velocities in (37) are substituted into (6) we obtain (12) identically. Therefore $q = Q$. The potential vorticity Q is conserved and consequently so is q, i.e. $\dot{q} = 0$.

We note that when f is a constant, (13) can be written

$$X = \frac{\partial P}{\partial x}, \ \ Y = \frac{\partial P}{\partial y}, \ \ \text{with} \ \ P(x,y,t) = \frac{1}{2}(x^2+y^2) + \frac{g}{f^2}h(x,y,t). \tag{38}$$

The transformation (14) is then of Legendre type, having inverse (15) of the form

$$x = \frac{\partial R}{\partial X}, \ \ y = \frac{\partial R}{\partial Y}, \ \ \text{with} \ \ R(X,Y,t) = Xx + Yy - P. \tag{39}$$

This duality leads to a Monge-Ampère equation for P, from (20) and (38), namely

$$Q = \frac{f}{h}\begin{vmatrix} \dfrac{\partial^2 P}{\partial x^2} & \dfrac{\partial^2 P}{\partial x \partial y} \\ \dfrac{\partial^2 P}{\partial y \partial x} & \dfrac{\partial^2 P}{\partial y^2} \end{vmatrix} . \tag{40}$$

In practice, although we have identified Q with the Rossby expression (6), one still has to solve (40) (with suitable boundary conditions) for P, given Q, f and h, in order to calculate X and Y from $(38)_{1,2}$.

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