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THE VARIATION OF WIND VELOCITY WITH HEIGHT.

BY

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THE VARIATION OF WIND VELOCITY WITH HEIGHT.

CAPTAIN E. H. CHAPMAN, R.E.

§ 1.—INTRODUCTION.

Various formulæ have been given in which an approximate relationship between wind velocity and height has been stated empirically. Stevenson[1] in 1880 gave a formula for approximate wind velocity at any level up to 51 feet. His formula was

$$V = v\sqrt{\frac{H+72}{h+72}}$$

where V = wind velocity at height H to be found,
and v = known wind velocity at height h,
H, h are measured in feet.

For wind velocity at great heights above sea level Stevenson gave the formula

$$V = v\,\frac{H}{h}$$

Archibald[2] in 1885 gave a formula for wind velocities between 300 feet and 2,000 feet. His formula was

$$V = v\left(\frac{H}{h}\right)^{\frac{1}{4}}$$

where V, v, H, h, have the same meaning as before.

Within more recent years Sir Napier Shaw[3] has suggested

$$V_H = \frac{H+a}{a}\,V_o$$

as a likely formula
where V_H = wind velocity at height H above anemometer,
V_o = anemometer velocity.
and a is a constant depending on exposure, etc.

Captain C. J. P. Cave[4] gave a = 167 metres in Shaw's formula for Ditcham, the place from which most of his pilot balloon ascents were made.

Shaw supposed his formula to apply until the geostrophic wind velocity was reached.

Cave states in his book that of 174 ascents 61 per cent. showed increase of velocity according to Shaw's linear formula.

The three formulæ mentioned, Stevenson's, Archibald's, and Shaw's are all the result of examination of observations. It

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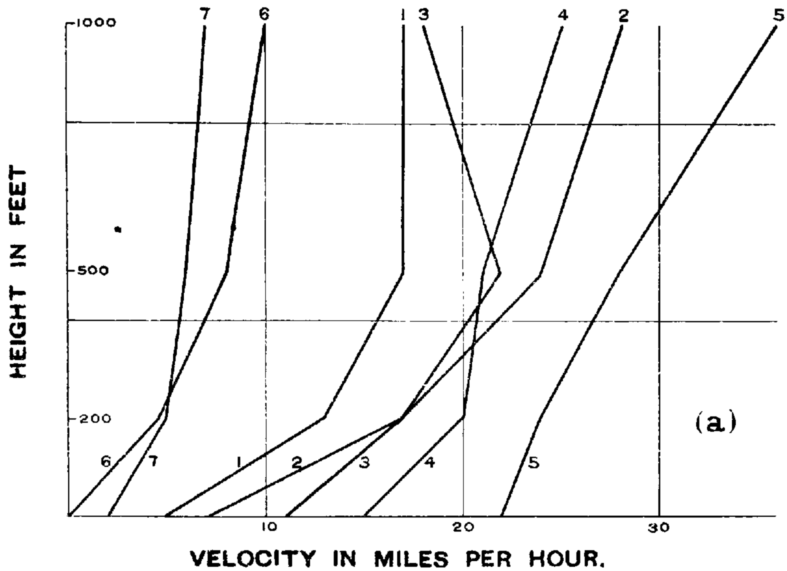
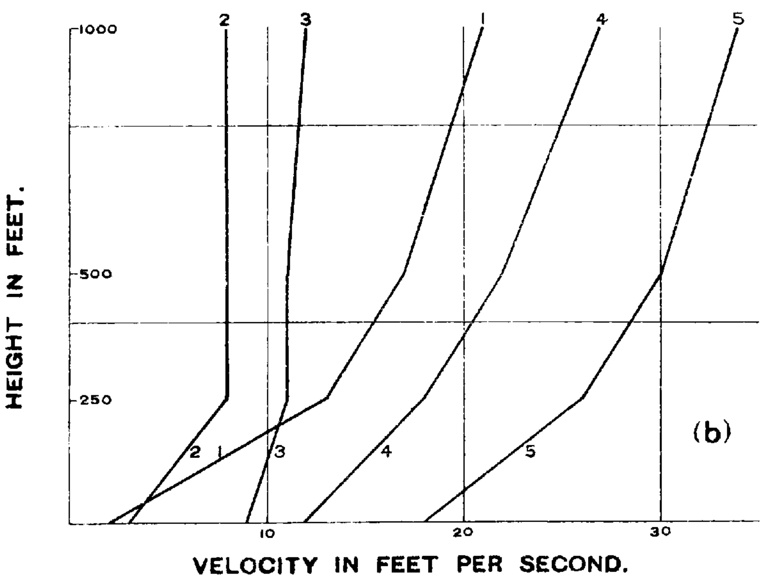
[1] Journal Roy. Scot. Met. Soc., 1880, p. 348.
[2] "Nature," 1885.
[3] Advisory Committee for Aeronautics, Reports and Memoranda, 1909. No. 9, p. 8.
[4] "The Structure of the Atmosphere in Clear Weather." Camb. Univ. Press.

Variation of Wind Velocity with Height. Individual Observations from Pilot Balloon Ascents.





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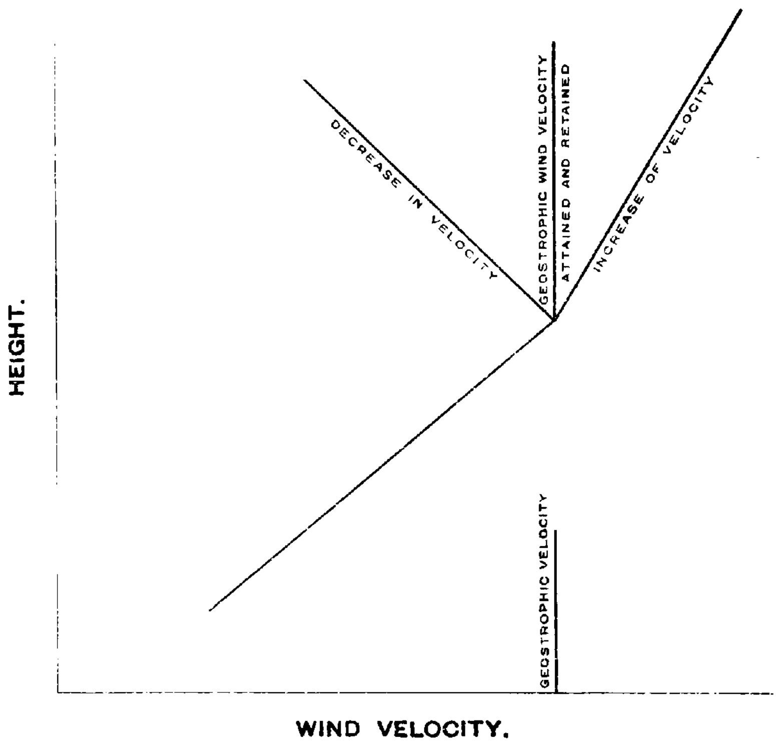
Captain Cave's Diagram of the Relation between Wind Velocity and Height.



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is interesting to note that the observations were made under different conditions and with different instruments. Stevenson used anemometers fixed on poles and took his observations over a field of growing oats. Archibald worked with anemometers suspended from kites, while Cave obtained his results, from which Shaw suggested his linear formula, by means of pilot balloons.

§ 2.—OBSERVATIONS TREATED SINGLY.

The way in which wind velocity changes with height varies from one observation to another. Figure I. will serve to show this. The upper diagram (a) illustrates seven observations of wind velocity taken from a set of pilot balloon ascents referred to later in the paper as Set 1. The lower diagram (b) illustrates five observations from a different set, referred to as Set 2 later in the paper. In each observation the wind velocity near the surface, at 200 or 250 feet, at 500 feet, and at 1,000 feet is shown.

Shaw[5] gave a diagram which was reproduced by Cave[6] in a simple form as in Figure II. of this paper. In this diagram it is assumed that wind velocity increases linearly with height until the geostrophic wind velocity is reached. After the geostrophic velocity has been attained the wind velocity either decreases, remains constant, or increases further with height. From the few ascents illustrated in Figure I. it will be seen that this simple diagram of Cave's is correct. Ascents numbered 2, 4, 5, 6, 7 in Figure I. (a), and 1, 4, 5, in Figure I. (b) show increase of wind velocity with height at varying rates. Ascents numbered 1 in Figure I. (a), and 2, 3, in Figure II. (a) show a certain velocity attained and retained. The ascent numbered 3 in Figure I. (a) shows an increase of velocity up to a certain point followed by a decrease in velocity It must be pointed out, however, that the heights at which the changes take place from an increasing velocity to a constant or decreasing velocity in ascents 1, 3, Figure I. (a), and 2, 3, Figure I. (b) are lower than would be expected from Cave's diagram as reproduced in Figure II. Ordinarily speaking, the geostrophic velocity would not be attained at a height of 200 or 500 feet.

Further illustrations of change of wind velocity with height may be seen in a paper by G. M. B. Dobson[7] describing the results obtained from 97 pilot balloon ascents made at Upavon during the year 1913.

Cave[8] in his book gives diagrams illustrating 35 of his pilot balloon ascents. In these diagrams the very varied way in which wind velocity changes with height is clearly shown.

Although for individual ascents wind velocity does not change with height according to any very definite law, it is quite possible that some definite law exists for change of mean wind velocity with height. For the remainder of this paper mean

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[5] loc cit., p. 8.
[6] loc. cit., p. 35.
[7] Quarterly Journal Roy. Met. Soc., 1914, p. 123.
[8] loc. cit.

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wind velocities at various heights will be considered instead of wind velocities for individual observations.

§ 3.—MEAN WIND VELOCITY NEAR SURFACE.

Reference has been made to a formula given by Stevenson. At the end of the paper in which the formula was given, Stevenson gave tables of mean wind velocities up to a height of 51 feet. These tables giving mean wind velocities for nine sets of observations are reproduced in Table I. Figure III.

TABLE I.

MEAN WIND VELOCITIES in miles per hour for heights up to 51 feet, nine sets of observations.

From Stevenson's paper.

Height in feet.	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9
0	*8·1								
½		10·2	6·8	9·8					
1⅓					*22·2	3·3			
2⅓			8·7						
3		12·1							
3½				12·4	25·6	4·4	*13·9	*6·8	*19·7
4⅓			9·8						
9				13·8	31·9	5·7	22·7	16·5	28·5
9½			10·4						
10	18·4								
12		15·9							
14			10·5	14·3	33·7	5·7	24·9	18·6	31·0
20	19·4								
24		16·9							
25			11·5	15·0	37·1	6·4	28·9	20·6	35·4
30	20·1								
40	20·3								
44		18·0							
50	21·0		12·1						
51				16·3	42·7	7·4	32·2	23·8	39·9

has been drawn to illustrate these mean wind velocities graphically. Stevenson's observations were taken over a field cropped in oats. Sets 1-4 were taken before the grain appeared, sets 5-6 were taken when the grain was a foot high, and sets 7-9 were taken when the grain was 3½ feet high. For the lighter winds (under 25 m.p.h. at 51 feet) there is no great increase of wind from 10 to 50 feet. For the stronger winds (over 25 m.p.h. at 50 feet) mean wind velocity increases rapidly up to 9 feet, and then less rapidly from 9 to 51 feet. It would be possible to give a linear equation for increase of wind velocity with height for sets 1-4, 6, from about 10 to 50 feet, but the increase of wind velocity with height for sets 5-8, 9 is not linear. Thus of Stevenson's nine sets of observations 5 show increase of wind velocity fairly linearly with height from 10 to 50 feet, but below 10 feet the rate of increase is greater. The remaining four sets do not show linear increase of wind velocity with height.

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2⅓			8·7						
3		12·1							
3½				12·4	25·6	4·4	*13·9	*6·8	*19·7
4⅓			9·8						
9				13·8	31·9	5·7	22·7	16·5	28·5
9½			10·4						
10	18·4								
12		15·9							
14			10·5	14·3	33·7	5·7	24·9	18·6	31·0
20	19·4								
24		16·9							
25			11·5	15·0	37·1	6·4	28·9	20·6	35·4
30	20·1								
40	20·3								
44		18·0							
50	21·0		12·1						
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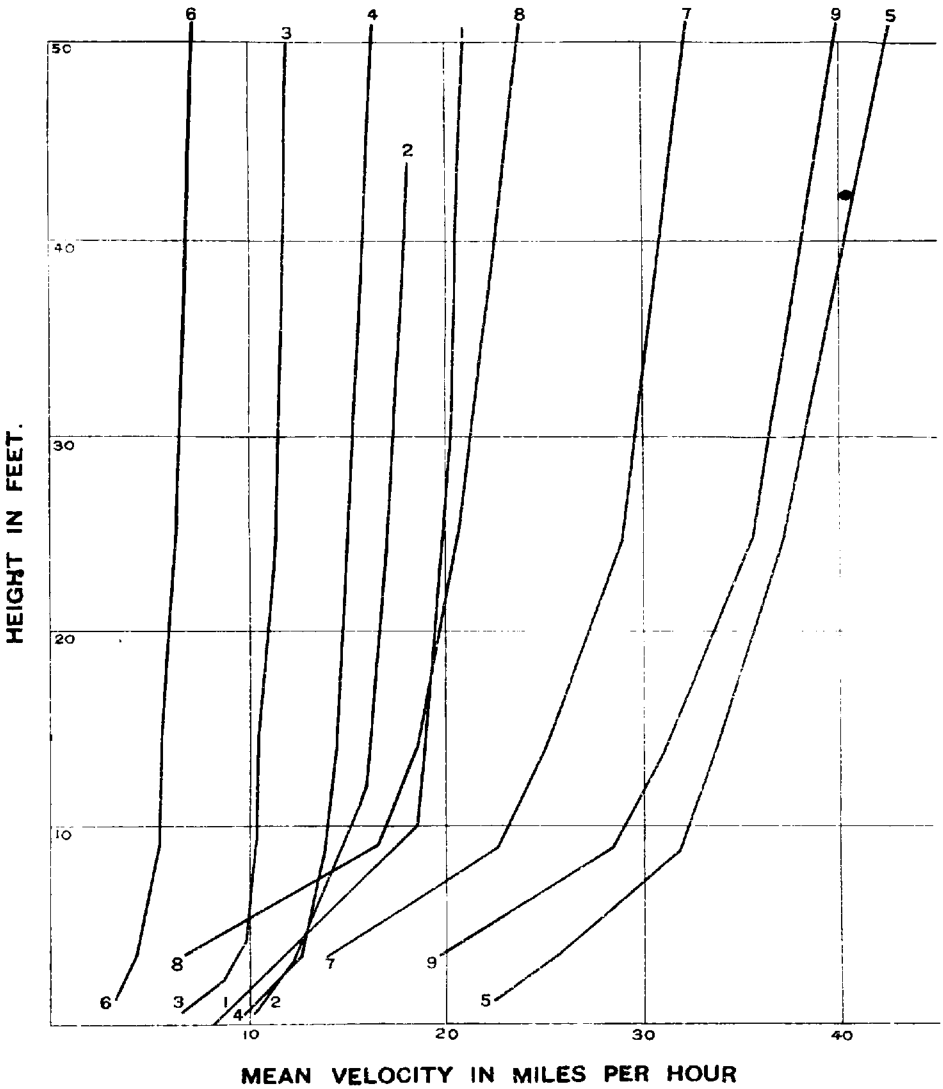
Diagram of Mean Wind Velocities for various heights np to 51 feet, nine sets of observations numbered 1 to 9.

From Stevenson's paper.



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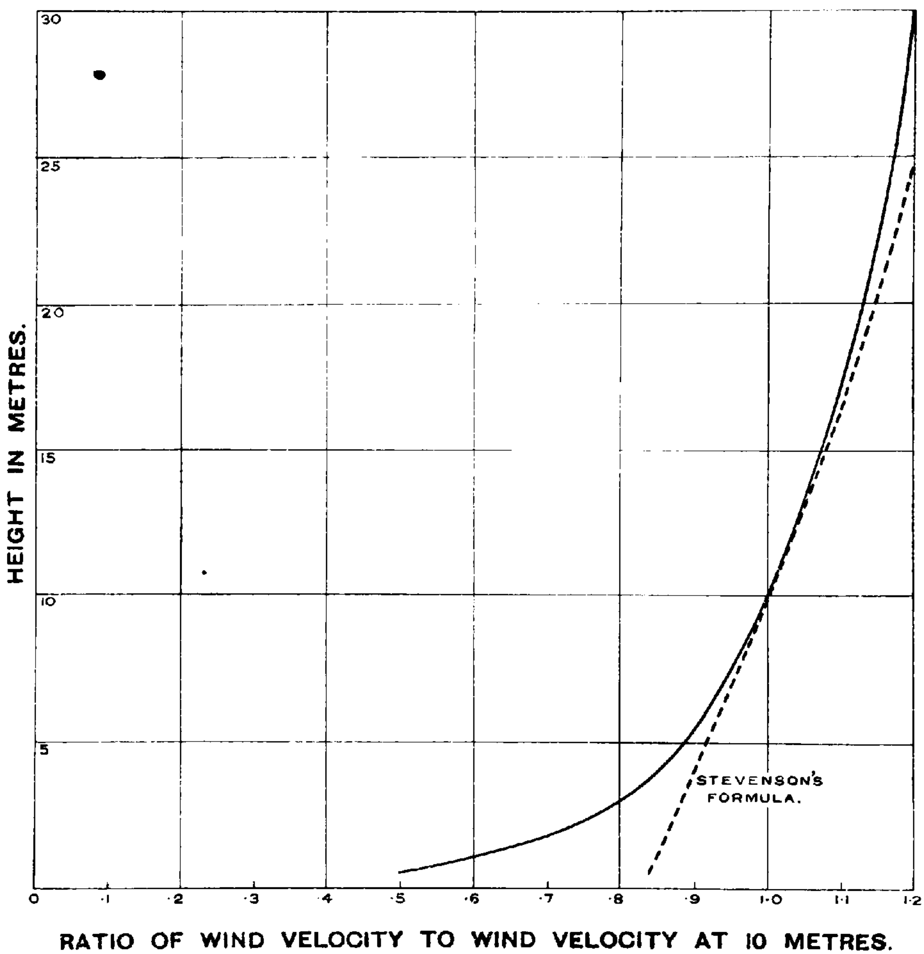
From Annual Wind Summary, 1916 (Meteorological Office, London).
Made by comparison of records from various anemographs in different experiments.



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In the Annual Wind Summary of the Meteorological Office for 1916 a table is given showing mean wind velocity from $\frac{1}{2}$ metre to 30 metres above open grass-land as a fraction of the mean wind velocity at 10 metres. The table was constructed from the comparison of records from various anemographs taken during different experiments. The table is given here:—

Height in metres.	$\frac{1}{2}$	1	2	3	4	5	10	15	20	25	30
Ratio of mean wind velocity to that at 10 ms....	·50	·59	·73	·80	·85	·89	1·00	1·07	1·13	1·17	1·20

The following note is added:—

"It is to be understood that the ratios shown in this table are only approximate. The increase of wind with height is more rapid in proportion when the air is not much disturbed by convection, *i.e.*, in cold weather and at night; it is less rapid under the opposite conditions."

This Meteorological Office table is illustrated graphically in Figure IV. The ratio figures give a smooth curve. There is a good deal of similarity between this smooth curve and curves numbered 5-8, and 9 of Figure III. The diagrams for change of wind velocity with height for ascents numbered 1, 2, 6 in Figure I. (a), and those numbered 1, 4, 5 in Figure I. (b) are also similar to the smooth curve of Figure IV.

Stevenson's formula, which in metres becomes

$$V = v\sqrt{\frac{H + 22}{h + 22}}$$

does not apply to the ratios of mean wind velocities given by the Meteorological Office. Taking the wind velocity at 10 metres as standard we get for the values of the ratio (r) of wind velocity at height H to wind velocity at 10 metres the formula

$$r = \sqrt{\frac{H + 22}{32}} \qquad H \text{ is in metres.}$$

The values of this ratio r, from $H = \frac{1}{2}$ to $H = 25$ are shown graphically in Figure IV. by a broken curve. The agreement between Stevenson's formula and the Meteorological Office figures is fairly close from 7 to 20 metres. Below 7 metres the wind velocity values given by Stevenson's formula are much too high according to the Meteorological Office figures.

If we assume an anemometer height of 10 metres in Shaw's formula, we have for the ratio (r) of wind velocity at height H (above 10 metres) to the wind velocity at 10 metres.

$$r = \frac{H + a}{a}$$

Applying this formula to heights of 15, 20, 25, and 30 metres and using the values of r from the Meteorological Office table we get in succession the following values for the constant "*a*" in Shaw's formula:—71, 77, 88, 100. This constant "*a*" should, of course, have the same value at each height, so that Shaw's linear formula cannot be applied to the Meteorological Office figures. A different linear law could be applied roughly to the portion of the curve from 10 metres to 30 metres, or from ½ metre to 2 metres. The point remains, however, that neither Stevenson's nor Shaw's formula can be applied to the curve of Figure IV.

§ 4. MEAN WIND UP TO 500 METRES.

In the Fourth Report on Wind Structure,[9] J. S. Dines gives the results of the analysis of a number of pilot balloon ascents made at South Farnborough during the months of October and November, 1912. The observations in this paper are most conveniently arranged for the study of change of wind velocity with height up to 500 metres. In the first table of the paper over 50 ascents are tabulated according to the velocity of the wind at 500 metres. Four groups are given according as the wind at 500 metres was—

(i) 4 metres per second or less—very light winds.
(ii) 4·1-5·0 metres per second—light winds.
(iii) 4·1-10·0 metres per second—light or moderate winds.
(iv) Over 10·0 metres per second—strong winds.

Mean values are given for heights of 50, 100, 150, 200, 250, 300, 400, 500 metres. The numbers of observations in the groups are 7, 5, 22, 25 respectively The author gives a diagram illustrating these mean values. The mean wind velocity curves of this diagram are reproduced in Figure V. of this paper.

The author then arranges his ascents according to the time of the day at which they were made. He gives three groups, 7 a.m., 1 p.m., 4 p.m., the number of observations being 14, 17, 16, respectively. Mean values of wind velocity are given for the same heights as before, and a diagram is given. The curves from this diagram are reproduced in Figure V

The author next groups his ascents as "clouded" or "unclouded" at both 7 a.m., 1 p.m., and 6 p.m. Mean velocities are again given and illustrated graphically. The curves, up to a height of 500 metres, are reproduced in Figure VI. The number of observations on which the mean values of wind velocity are calculated are given in the diagram.

Lastly, the author re-arranges certain specially selected ascents into groups with—

(i) strong vertical circulation,
(ii) weak vertical circulation,
(iii) lightly clouded sky,
(iv) overcast sky,

mean values are given for the same heights as before. These mean values are illustrated in Figure VII. of this paper.

Glancing through the diagrams taken from J S. Dines' paper

[9] Aeronautics Reports and Memoranda No. 92.

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Glancing through the diagrams taken from J. S. Dines' paper

[9] Aeronautics Reports and Memoranda No. 92.

Taken from a paper by J. S. Dines in Fourth Report on Wind Structure.

		Wind at 500 m s.	Number of Observations.
WIND STRENGTH	Very light winds	4 m/sec. or less ..	7
	Light winds..	4·1—5·0 m/sec. ..	5
	Light or moderate winds	4·1—10·0 ,, ..	22
	Strong winds	over 10·0 ,, ..	25
Diurnal Effect		7 h. ..	14
		13h.	17
		16h.	16

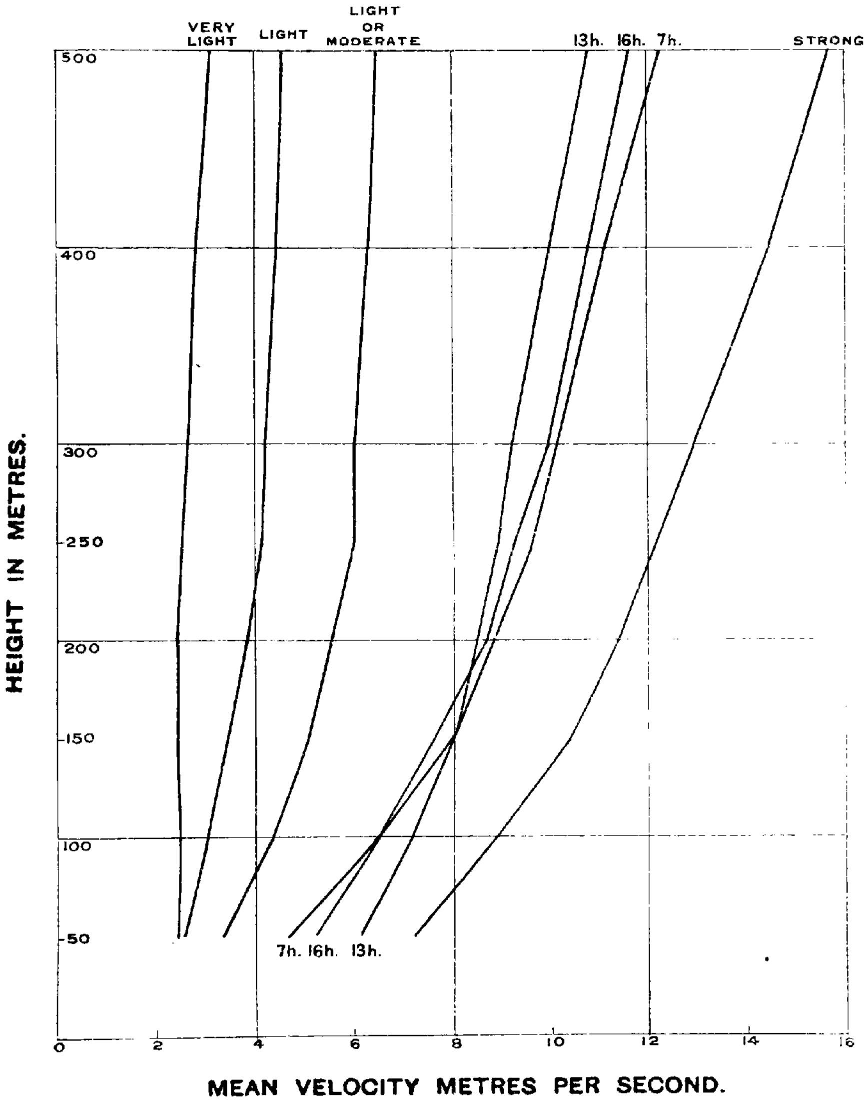

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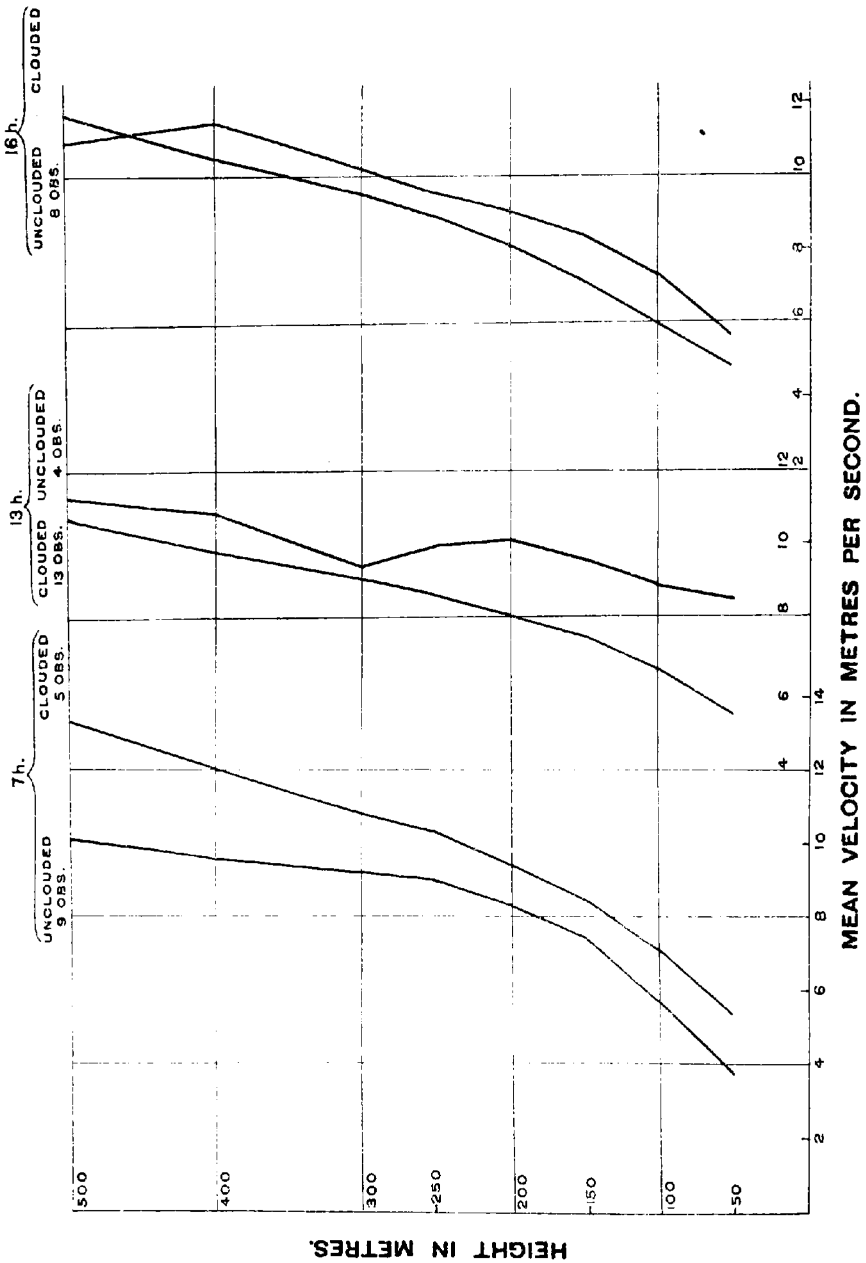
Figure VI.

Taken from a paper by J. S. Dines in the Fourth Report on Wind Structure.



Taken from a paper by J. S. Dines in Fourth Report on Wind Structure.

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Figure VII.

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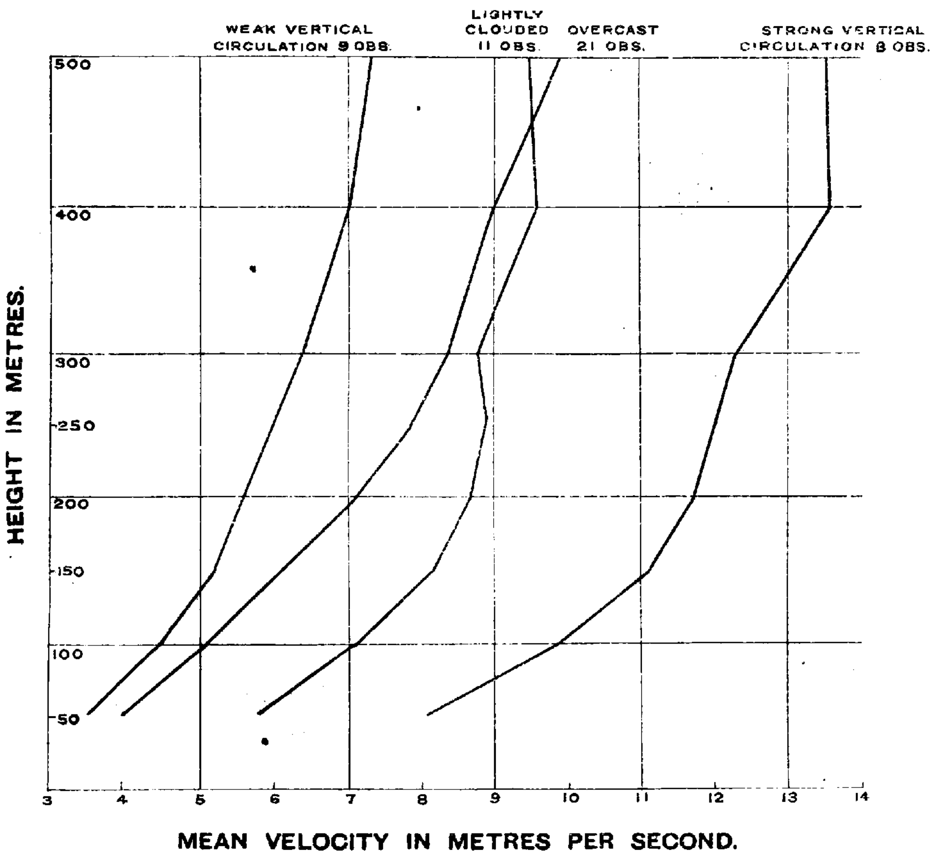

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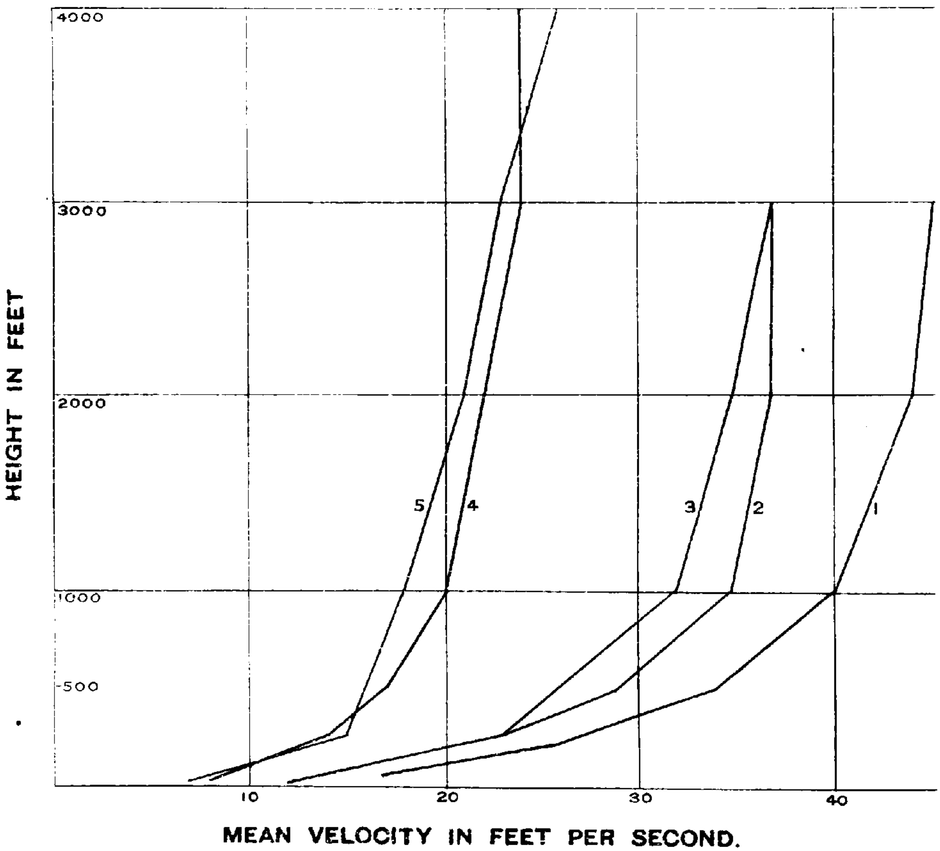
From five sets of Observations made in N.E. France.



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we see that, no matter whether mean wind velocities up to 500 metres are given for pilot balloon ascents grouped according to—

(i) strength of wind,
(ii) time of day,
(iii) amount of cloud, or
(iv) vertical circulation,

the curve showing change of mean wind velocity with height is of the same type. The only exception is the curve for very light winds. The curves are sufficiently flat for straight lines to be fitted to them for certain portions of their length, but in no case, except for very light winds showing little change in velocity up to 500 metres, could a straight line be fitted at all accurately to the curve from 50 to 500 metres.

It is a point of great interest to compare the curves of Figures V., VI., VII., with the curves of Figures III. and IV. Figure III. gives curves showing mean wind velocities from the surface of the ground up to 16 metres. Figure IV. shows the variation in mean wind velocity from ½ metre to 30 metres. Figures V., VI., VII. show the variation in mean wind velocity from 50 to 500 metres. There is a certain family likeness about these curves drawn for different heights and for different and varied groups of observations. An attempt will be made to express this likeness by a mathematical formula in a later part of the paper.

§ 5. MEAN WIND UP TO 1·5 KILOMETRES.

The origin of the present paper lay in a consideration of the five sets of observations illustrated in Figure VIII. The observations were made in N.E. France at three stations, which may be referred to as A, B, and C. The five sets were as follows:—

Set.	*Station.*	*Observations by pilot balloon*		
1.	A	67 morning ascents,	winter,	1915-16.
2.	B	31 ,, ,,	,,	1916-17.
3.	B	32 afternoon ,,	,,	,,
4.	C	30 morning ,,	summer,	1916.
5.	C	30 afternoon ,,	,,	,,

In Figure VIII. mean wind velocities for these five sets of observations are given from the surface up to 3,000 or 4,000 feet. The curves are again similar to those of previous diagrams. The curves of Sets 1, 2 and 4 should be compared with the curve of Figure IV., and the curves numbered 7, 9, 5 in Figure III.

Further values of mean wind velocities at various heights up to 2,440 metres are provided in the paper by G. M. B. Dobson,[10] previously referred to. The results given in this paper are of more than usual interest, since the author gives mean values of geostrophic winds, and since the results were used by G. I. Taylor[11] as illustrations of his theoretical considerations of wind velocity in the atmosphere.

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[10] loc. cit.
[11] Eddy Motion in the Atmosphere. (London Phil. Trans. A. Vol. 215. 1914.)

Dobson first gives mean values of wind velocity for ascents arranged in groups according to direction. These mean values are illustrated graphically in Figure IX. of this paper. In each of the four wind groups, NE., SE., SW., and NW., it will be noticed that the mean wind velocity increases rapidly until a height is reached at which the mean geostrophic velocity is either attained or exceeded. For the NE. and SE. groups this height is roughly 300 metres. For the SW. and NW groups this height is about 440 metres. Above this height mean wind velocity varies considerably.

The author next gives mean wind velocities, up to 2,440 metres, for his ascents arranged according to wind strength in three groups, light, moderate and strong winds. These mean values are illustrated in Figure X. For light winds, Figure X. shows that mean wind velocity increases from the surface up to 300 metres. After this height mean wind velocity decreases, remains constant, and increases again before the geostrophic velocity is reached at 1,000 metres. In the moderate wind group, mean wind velocity increases from the ground up to 300 metres, when the geostrophic velocity is almost attained. For strong winds there is a rapid increase of mean wind velocity from the surface up to 450 metres, when the mean geostrophic wind velocity is attained.

Dobson further rearranges his observations into early morning and middle of the day ascents for both Spring and Summer. The mean values of wind velocity for the resulting four groups are shown in Figure XI. In each case mean wind velocity is shown to increase from the surface upwards to 300 or 400 metres. In three cases the mean geostrophic wind velocity is passed before this height is reached, but in the fourth case, Spring mid-day ascents, the mean geostrophic velocity is not reached until 900 metres.

The general result to be drawn from Dobson's paper is that mean wind velocity increases from the surface upwards to a height of 300 or 400 metres. In most cases the mean gradient wind velocity has been attained before this height is reached. Within this region of rapid increase of mean wind velocity with height the curve showing the increase of mean wind velocity with height is not linear. It is similar to previous curves. Even on the small height scale of Figures IX–XI the similarity can be seen.

One other set of mean wind velocities is that illustrated in Figure XII. The diagram is taken from a paper by W. J. Humphreys[12] entitled Wind Velocity and Elevation. The observations illustrated in the diagram are from pilot balloon ascents made by Dr. Cesare Fabris at Vigna di Valle (an aerological station 40 kilometres N.W. of Rome) during the year June, 1910—May, 1911. The general features of the diagram are given by Humphreys as follows:—

1. A region of rapid linear increase of mean wind velocity with height extending from the surface 272 metres above sea-level where the mean velocity is least, to a height of 600 or 700 metres.

[12] Monthly Weather Review, U.S.A., Jan., 1916, p. 14.

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The author next gives mean wind velocities, up to 2,440 metres, for his ascents arranged according to wind strength in three groups, light, moderate and strong winds. These mean values are illustrated in Figure X. For light winds, Figure X. shows that mean wind velocity increases from the surface up to 300 metres. After this height mean wind velocity decreases, remains constant, and increases again before the geostrophic velocity is reached at 1,000 metres. In the moderate wind group, mean wind velocity increases from the ground up to 300 metres, when the geostrophic velocity is almost attained. For strong winds there is a rapid increase of mean wind velocity from the surface up to 450 metres, when the mean geostrophic wind velocity is attained.

Dobson further rearranges his observations into early morning and middle of the day ascents for both Spring and Summer. The mean values of wind velocity for the resulting four groups are shown in Figure XI. In each case mean wind velocity is shown to increase from the surface upwards to 300 or 400 metres. In three cases the mean geostrophic wind velocity is passed before this height is reached, but in the fourth case, Spring mid-day ascents, the mean geostrophic velocity is not reached until 900 metres.

The general result to be drawn from Dobson's paper is that mean wind velocity increases from the surface upwards to a height of 300 or 400 metres. In most cases the mean gradient wind velocity has been attained before this height is reached. Within this region of rapid increase of mean wind velocity with height the curve showing the increase of mean wind velocity with height is not linear. It is similar to previous curves. Even on the small height scale of Figures IX–XI the similarity can be seen.

One other set of mean wind velocities is that illustrated in Figure XII. The diagram is taken from a paper by W. J. Humphreys[12] entitled Wind Velocity and Elevation. The observations illustrated in the diagram are from pilot balloon ascents made by Dr. Cesare Fabris at Vigna di Valle (an aerological station 40 kilometres N.W. of Rome) during the year June, 1910—May, 1911. The general features of the diagram are given by Humphreys as follows:—

1. A region of rapid linear increase of mean wind velocity with height extending from the surface 272 metres above sea-level where the mean velocity is least, to a height of 600 or 700 metres.

[12] Monthly Weather Review, U.S.A., Jan., 1916, p. 14.

Taken from a paper by G. M. B. Dobson, Journal Royal Met. Soc., April, 1914.

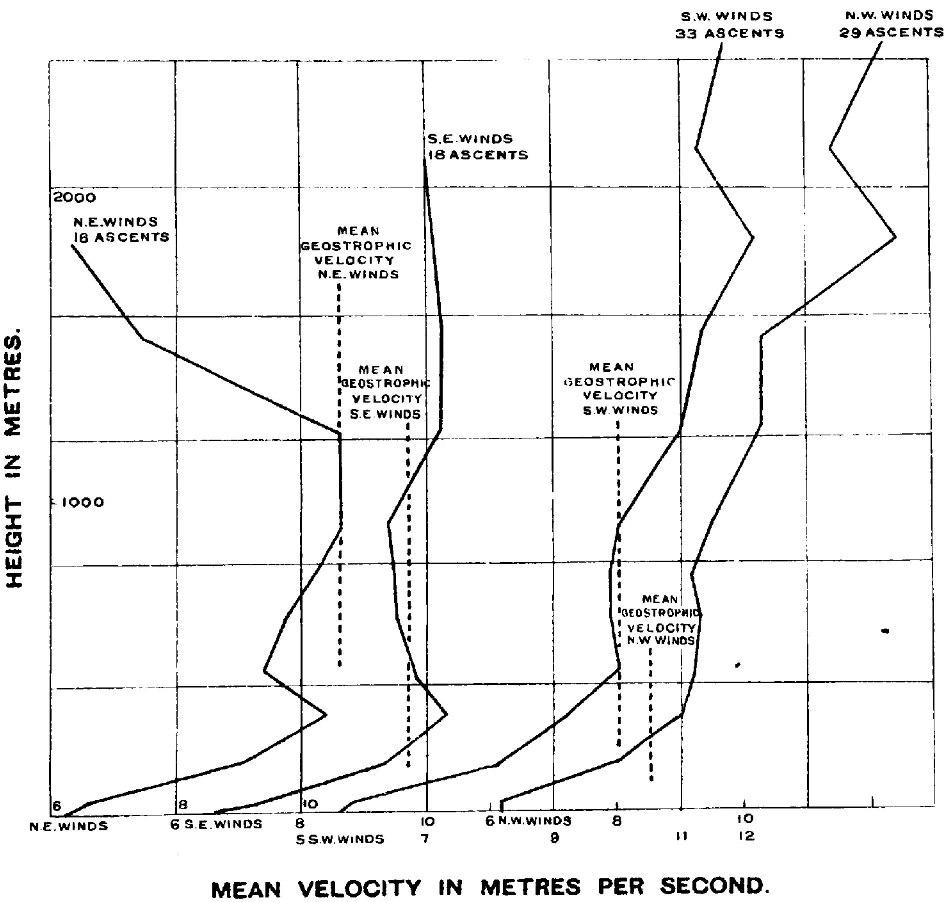

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Figure X.

Taken from a paper by G. M. B. Dobson, Journal Royal Met. Soc., April, 1914.

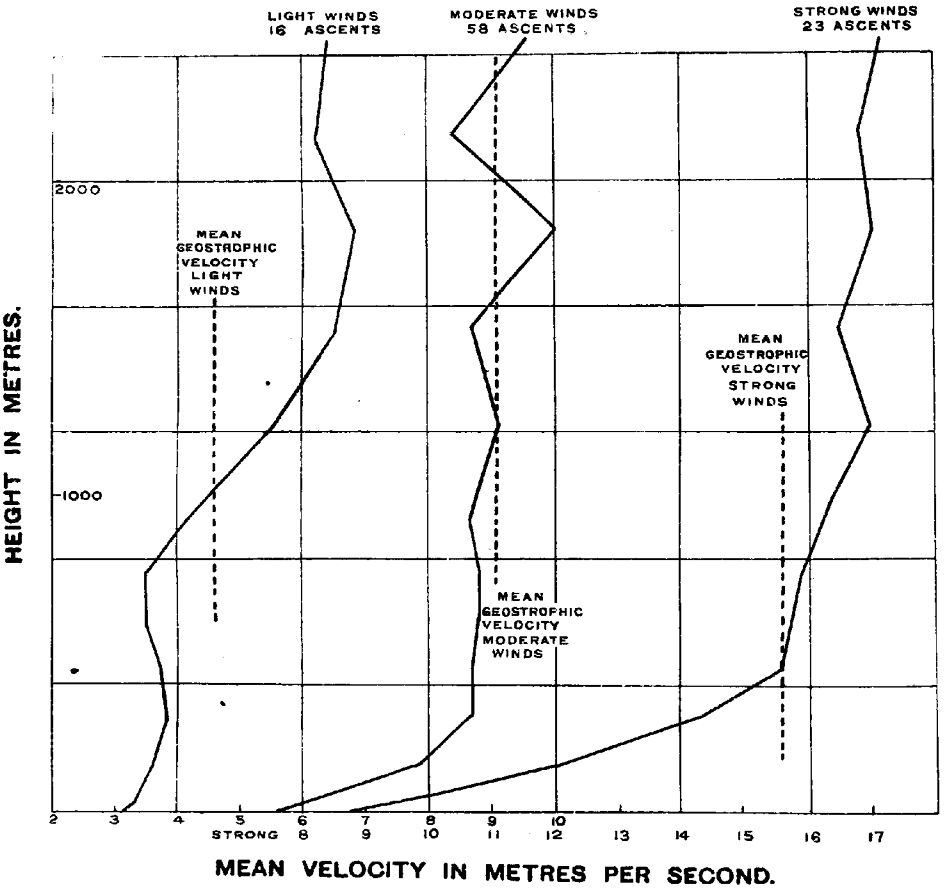

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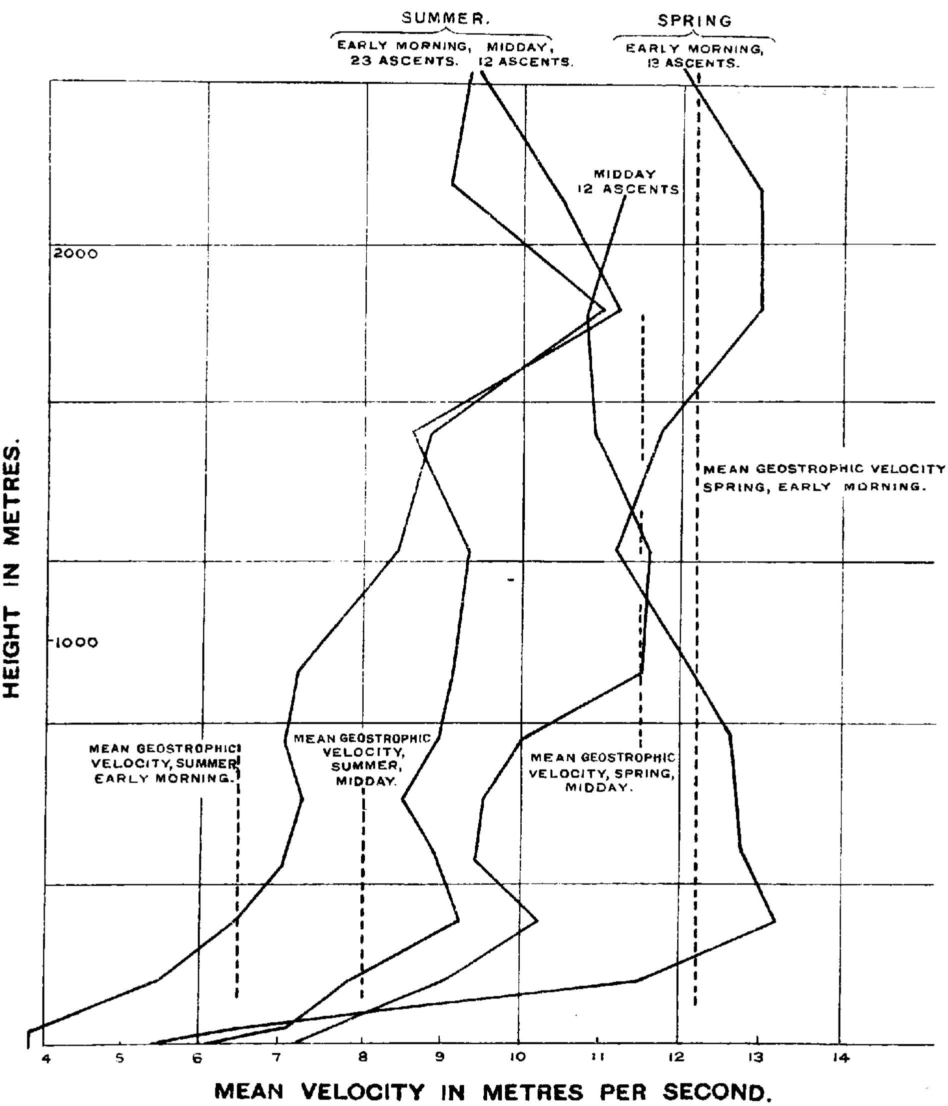
Figure XI.

Taken from a paper by G. M. B. Dobson, Journal Royal Met. Soc., April, 1914.



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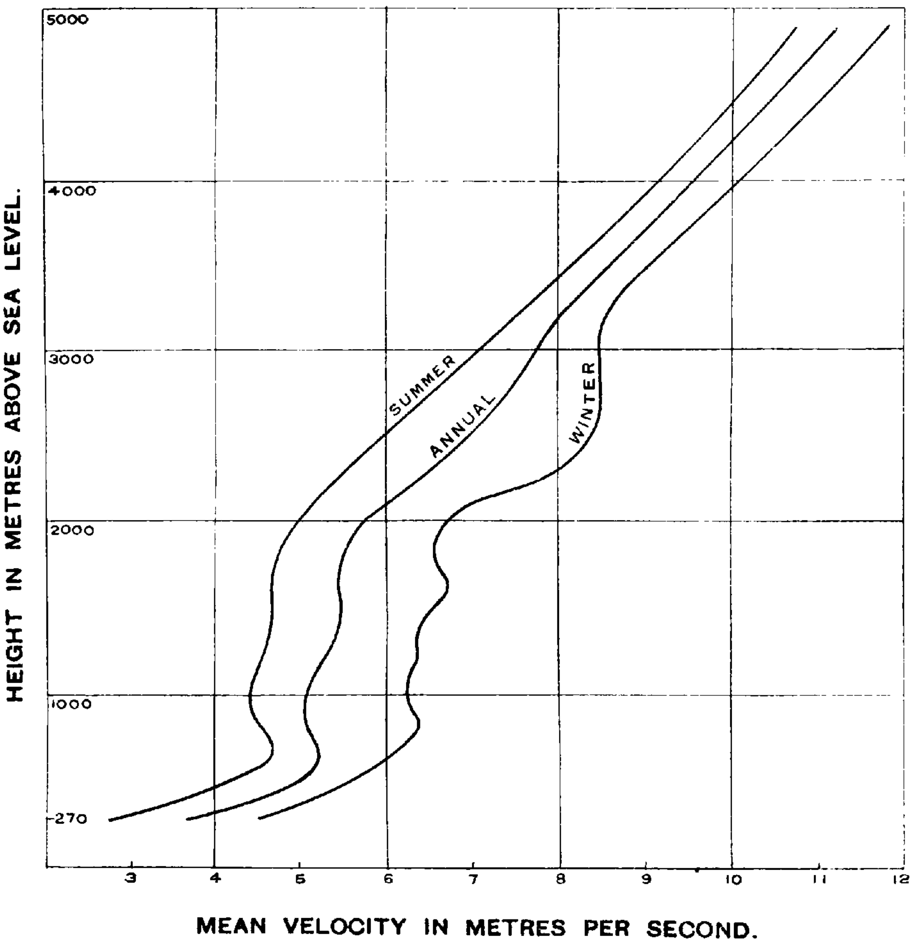
Taken from a paper entitled Wind Velocity and Elevation by W. J. Humphreys, Monthly Weather Review, Washington, January, 1916, page 14.

The diagram illustrates mean wind velocities based on 200 pilot balloon ascents made by Dr. Cesare Fabris at Vigna di Valle, 40 kms. N.W. of Rome, during the year June, 1910—May, 1911.



2. A region of mean wind velocity decreasing with increase of height about 100 metres thick, and coming immediately above 1.

3. A region of irregular winds where mean velocity slowly increases with height. This region extends roughly from 500 to 1,500 metres above the surface.

4. A region of approximately constant increase of velocity with increase of height beginning at about 1,500 metres above the surface and extending to at least 5,000 metres.

This completes the data chosen to illustrate the change of mean wind velocity with height.

It has been shown that the formula $V = v\sqrt{\frac{H+72}{h+72}}$ given by Stevenson does not apply to the figures given in the Meteorological Office Wind Summary for 1916 (*see* Figure IV). There is an insufficient falling off in wind velocity near the ground in Stevenson's formula. It will be seen on reference to the original paper how this formula was constructed, and how it is that it fails near the ground. The formula $V = v\,\frac{H}{h}$ proposed by Stevenson for wind velocity at great heights does not apply below 2,500 metres judging from Dobson's observations. It may apply roughly to Fabris's ascents at Vigna di Valle from about 3,000 to 5,000 metres.

With regard to wind velocities at great heights there is the formula deduced by Egnell[13]

$V = \frac{V_o\,\rho_o}{\rho}$ where V, ρ are respectively velocity of wind and density at a certain height, and V_o ρ_o are the corresponding values near the surface.

Archibald's formula $V = v\left(\frac{H}{h}\right)^{\frac{1}{4}}$ which was intended to apply from 100 to 600 metres, can be very conveniently tested by J. S. Dines' observations. Dines gives mean wind velocities expressed in percentages of the mean wind velocity at 100 metres. If we put $h = 100$ in Archibald's formula we have $\frac{V}{v} = \left(\frac{H}{100}\right)^{\frac{1}{4}}$. Giving H the values 100, 150, 200, 250, 300, 400, 500 we have in succession the following values for $\frac{V}{v}$:—

$$1{\cdot}00,\ 1{\cdot}11,\ 1{\cdot}19,\ 1{\cdot}23,\ 1{\cdot}32,\ 1{\cdot}42,\ 1{\cdot}49.$$

If these numbers are multiplied by 100 they are comparable with Tables II. and VI of Dines' paper[9] in which mean winds at the above heights are expressed as percentages of the mean wind at 100 metres. The comparison can be made from Table II. of this

[13] Comptes Rendus, 1903. International Cloud Observations Trappes.

2. A region of mean wind velocity decreasing with increase of height about 100 metres thick, and coming immediately above 1.

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1·00, 1·11, 1·19, 1·23, 1·32, 1·42, 1·49.

If these numbers are multiplied by 100 they are comparable with Tables II. and VI. of Dines' paper[9] in which mean winds at the above heights are expressed as percentages of the mean wind at 100 metres. The comparison can be made from Table II. of this

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[13] Comptes Rendus, 1903. International Cloud Observations Trappes.

TABLE II.

MEAN WINDS at heights up to 500 metres expressed as percentages of the Mean Wind at 100 metres.

Height in metres	100	150	200	250	300	400	500
$\left(\frac{H}{100}\right)^{\frac{1}{4}}$ in Archibald's formula	100	111	119	123	132	142	149
$\frac{V}{v}$ from Dines' results—							
Very light winds	100	96	96	100	104	112	124
Light winds	100	117	127	137	140	147	153
Light or moderate winds ...	100	118	126	135	138	145	151
Strong winds	100	116	128	137	145	163	175
Strong vertical circulation	100	111	117	121	124	137	136
Weak vertical circulation ...	100	116	125	134	143	155	162
Lightly clouded	100	115	123	126	124	135	134
Overcast	100	122	140	155	165	177	194
Average of above	100	114	123	130	135	146	154

paper. The second row of this table gives the values 100, 111, etc., calculated as above. Underneath are given corresponding values from Dines' paper. Although Archibald's formula does not accommodate itself to the different sets of observations as grouped by Dines, it is in fairly close agreement with the average of Dines' observations.

Shaw's formula $V = \frac{H+a}{a} V_0$ assumes a linear increase of wind velocity with height. Although a linear increase of wind velocity with height is noticeable in the case of some individual ascents, it does not appear that mean wind velocity increases linearly with height. Figure III. indicates that the increase of mean wind velocity with height is curvilinear from the surface up to 51 feet. Figure IV. confirms this from ½ metre to 30 metres above the surface. Figures V.-VII. show the same thing from 50 to 500 metres, Figure VIII. to 3,000 feet. Figures IX.-XI. show that mean wind velocity increases with height at a decreasing rate from near the surface until the gradient wind velocity is approximately attained, or is exceeded, the height being 300 or 400 metres. Above this height mean wind velocity does not appear to increase with height according to any very definite law. It is claimed from Figure XII. that mean wind velocity increases linearly with height from the surface up to 600 or 700 metres above sea-level (*i.e.*, 300 or 400 metres above the surface, *cf.* Dobson's results). The smallness of the height scale may account for this statement. If the actual figures were available, and were plotted on a larger height scale, it is possible that the apparent linearity would disappear, and the curve become similar to previous curves.

§ 6.—SUMMARY.

Summing up the preceding part of the paper we have the following:—

(i) Mean wind velocity increases from the surface up to a height of 300 or 400 metres. Above this height changes in mean wind velocity are irregular.

(ii) From the surface up to 51 feet (Figure III.), from ½ metre to 30 metres (Figure IV.), from 50 metres to 500 metres (Figures V.-VII.), from the surface to 3,000 feet (Figure VIII.), and from the surface to 300 or 400 metres (Figures IX.-XI.) mean wind velocity plotted against height gives a curve of a particular type. The same type of curve is also shown in some individual ascents (Figure I.).

§ 7.—SUGGESTED FORMULA FOR INCREASE OF MEAN WIND VELOCITY WITH HEIGHT.

The observations first considered by the present writer were those illustrated in Figure VIII. The actual values of the mean wind velocities are given in Table III. The curves of

TABLE III.

MEAN WIND VELOCITIES for 5 sets of observations in NE. France.

Height in feet.	Mean Velocities in feet per second				
	1	2	3	4	5
10,000					35
9,000					32
8,000					32
7,000					31
6,000					29
5,000					28
4,000				24	26
3,000	45	37	37	24	23
2,000	44	37	35	22	21
1,000	40	35	32	20	18
500	34	29	26	17	16
250		23	23	14	15
200	26				
40	17				
Surface		12	12	8	7

Set.	Station.	Number of Pilot Balloon Ascents.	Time of Day.	Season.
1	A	67	Morning	Winter 1915–16
2	B	31	"	" 1916–17
3	B	32	Afternoon	" "
4	C	30	Morning	Summer 1916
5	C	30	Afternoon	" "

increase in mean wind velocity with height in Figure VIII, especially the curve marked 1, suggested an exponential curve of the type $H = e^{aV}$ where V was mean wind velocity at height H. To see whether such a type of curve could be applied mean wind velocity was plotted against $\log H$ The result is shown in Figures XIII and XIV The mean surface winds for Sets 2–5 have been omitted since they were estimations made by different observers. In Set 1 the surface winds were taken from an anemometer 40 feet high and are therefore consistent. It will be seen that with the exception of the 3,000 ft. value of Set 2 the approach to linearity between log(Height) and mean wind velocity at that height is quite close. An interesting feature of these diagrams is that in Set 5 the 250, 500, 1,000-ft. means suggest one straight line, while the mean velocities from 2,000 to 10,000 feet suggest a different straight line.

For the first three points of Set 5 in Figure XIV we get the best fitting straight line to be

$$V = 5{\cdot}26\log H + 2{\cdot}00.$$

For the next nine points we get

$$V = 20\log H - 46{\cdot}4.$$

The logarithms in these equations are to base 10, and the mean wind velocities are in feet per second. The corresponding equations for Sets 1–4 are roughly

$$\begin{array}{ll} \text{Set } 1 & V = 15\log H - 9 \\ \quad ,, \ 2 & V = 14\log H - 9 \\ \quad ,, \ 3 & V = 15\log H - 14 \\ \quad ,, \ 4 & V = 9\log H - 7 \end{array}$$

There appeared to be reason therefore to suppose from these five sets of observations that mean wind velocity V was connected with height H by an equation of the type

$$V = a\log H + b$$

where a, b are constants which not only vary from place to place, but also vary for different sets of observations taken at different times of the day at the same place.

No doubt a more accurate mathematical formula could be found to fit the values illustrated in Figure VIII, but such a formula would probably be complicated, and the gain in accuracy would not balance the loss in simplicity of formula.

If the curve of increase of mean wind velocity with height is an exponential curve it would explain why the assumption is usually made that wind velocity increases linearly with height, for at the extreme left of an exponential curve the curve becomes almost indistinguishable from a straight line. The smaller the height scale the more likely it would be that the curve would be taken to be a straight line (*see* Figure XII).

We shall now see how an equation of the type $V = a\log H + b$ fits other sets of observations used earlier in this paper.

Mean wind velocities calculated for various heights up to 51 feet by Stevenson are given in Table I, and are illustrated graphically in Figure III. The same mean wind velocities plotted against $\log H$ are shown in Figure XV It will be at increase in mean wind velocity with height in Figure VIII, especially the curve marked 1, suggested an exponential curve of the type $H = e^{aV}$ where V was mean wind velocity at height H. To see whether such a type of curve could be applied mean wind velocity was plotted against $\log H$. The result is shown in Figures XIII and XIV. The mean surface winds for Sets 2–5 have been omitted since they were estimations made by different observers. In Set 1 the surface winds were taken from an anemometer 40 feet high and are therefore consistent. It will be seen that with the exception of the 3,000 ft. value of Set 2 the approach to linearity between log(Height) and mean wind velocity at that height is quite close. An interesting feature of these diagrams is that in Set 5 the 250, 500, 1,000-ft. means suggest one straight line, while the mean velocities from 2,000 to 10,000 feet suggest a different straight line.

For the first three points of Set 5 in Figure XIV we get the best fitting straight line to be

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The logarithms in these equations are to base 10, and the mean wind velocities are in feet per second. The corresponding equations for Sets 1–4 are roughly

Set	1	$V = 15\log H - 9$
,,	2	$V = 14\log H - 9$
,,	3	$V = 15\log H - 14$
,,	4	$V = 9\log H - 7$

There appeared to be reason therefore to suppose from these five sets of observations that mean wind velocity V was connected with height H by an equation of the type

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where a, b are constants which not only vary from place to place, but also vary for different sets of observations taken at different times of the day at the same place.

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From five sets of Observations made in N.E. France.

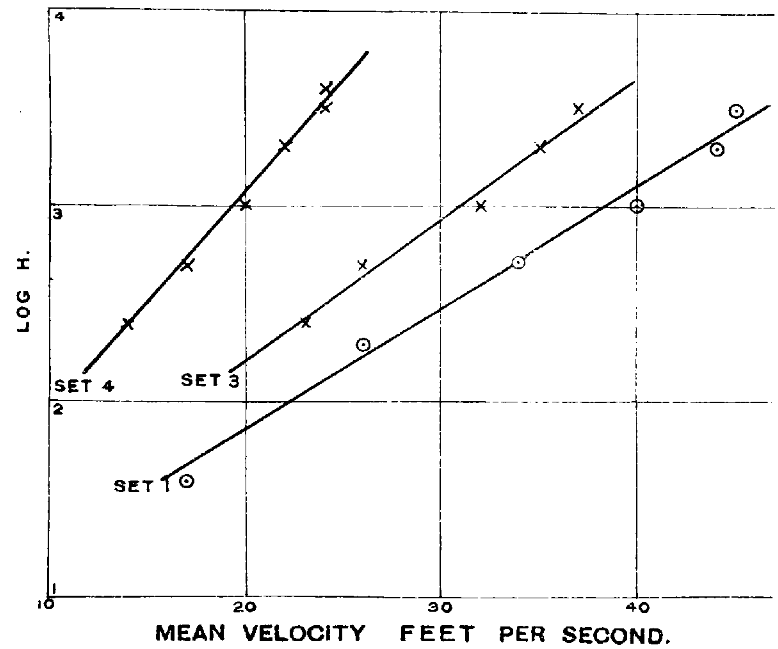


FIGURE XIII.

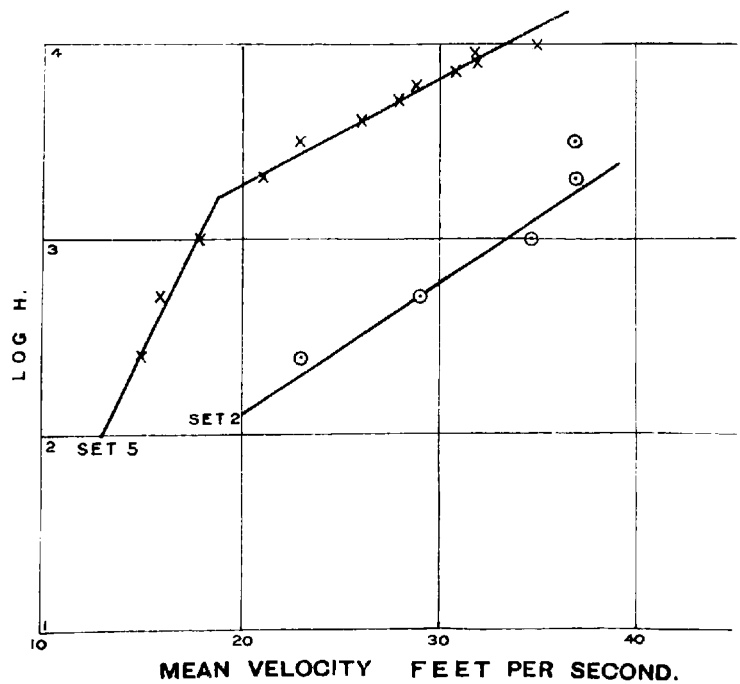
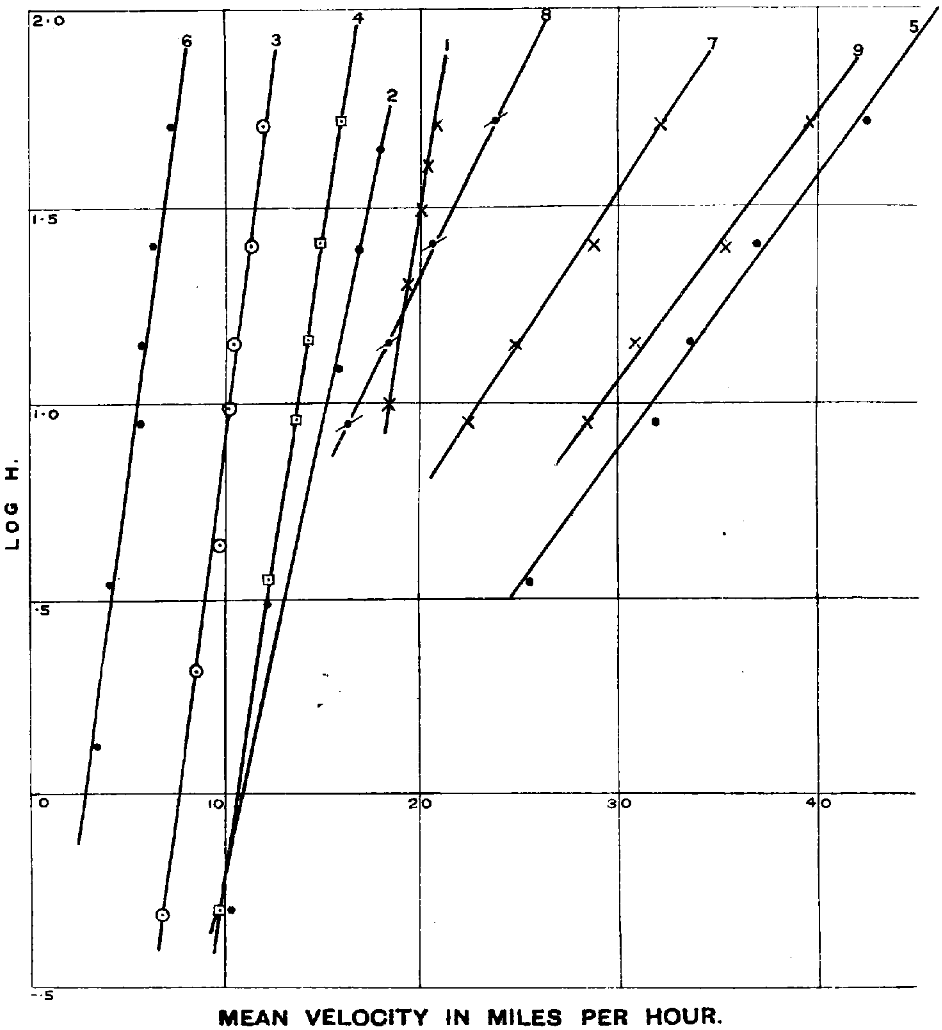


FIGURE XIV.

From Stevenson's Observations.



once evident that an equation of the type $V = a\log H + b$ can be fitted to each set of mean values. For each set four or more points lie along a well-marked straight line. The five points which do not fit in with the suggested straight lines are marked with an asterisk in Table I. These five points represent mean wind velocities near the ground. The first (height given as 0) was actually on the ground then free from crops. The second was only 4 inches above the tops of growing oats which were then 12 inches high. The remaining three were actually at the level of the tops of the grown oats $3\frac{1}{2}$ feet high. It is worth noting that in sets 2, 3, 4 of Stevenson's observations the equations of the type $V = a\log H + b$, as shown by the straight lines in Figure XV. hold down to within six inches of the ground. Another point to which attention may be drawn is the parallelism of lines 1, 2, 3, 4, 6. The mean winds at 50 feet for these five sets of observations were between 7 and 22 miles per hour. Lines 5, 7, 9 are also nearly parallel. The mean winds at 50 feet in these three sets of observations were from 32 to 43 miles per hour.

Giving the numerical coefficients to the nearest whole number the following equations result:

Observations	Set	1	$V = 4\log H + 14$
,,	,,	2	$V = 4\log H + 11$
,,	,,	3	$V = 3\log H + 7$
,,	,,	4	$V = 3\log H + 11$
,,	,,	5	$V = 15\log H + 17$
,,	,,	6	$V = 3\log H + 3$
,,	,,	7	$V = 17\log H + 5$
,,	,,	8	$V = 10\log H + 7$
,,	,,	9	$V = 15\log H + 14$

In these equations V is in m.p.h., H is in feet, and the logarithms are to base 10.

Figure XVI shows the ratio of mean wind at heights from $\frac{1}{2}$ to 30 metres to the mean wind at 10 metres plotted against the logarithm of the height. The ratios taken from the Meteorological Office publication are plotted against height in Figure IV. The linearity in Figure XVI is very evident. The equation of the straight line fitting the points on the diagram is

$$R = {\cdot}40\log H + {\cdot}60$$

where R is the ratio of mean wind velocity at height H to mean wind velocity at 10 metres. H is measured here in metres, and $\log H$ is given to base 10.

If an equation of the type $V = a \log H + b$ holds for mean wind velocity at height H we have for mean wind velocity at height 10 metres

$$V_{10} = a + b.$$

$$\text{Hence } \frac{V}{V_{10}} = \frac{a}{a+b} \log H + \frac{b}{a+b}$$

$$\text{or } R = \frac{a}{a+b} \log H + \frac{a}{a+b}.$$

i.e., if the mean wind velocity at height H be expressed as a ratio of the mean wind velocity at 10 metres the coefficient of $\log H$ and the numerical term in the equation together make unity. The equation found for Figure XVI satisfies this since $\cdot 40 + \cdot 60 = 1$.

Thus from Stevenson's observations and from the Meteorological Office figures we have the result that from near the ground upwards to 30 metres mean wind velocity V increases with height H very approximately according to the law $V = a\log H + b$.

In order to compare the equations from Stevenson's observations with the equation from the ratios of mean wind velocities of the Meteorological Office it is necessary to change the equations given for the former into a type comparable with the equation given for the latter.

The linear equation from Figure XVI was

$$R = \cdot 40\log H + \cdot 60.$$

The corresponding equations from Stevenson's observations are—

Observations,	Set 1.—	Equation	$R = \cdot 2\log H + \cdot 8$
,,	,, 2.—	,,	$R = \cdot 2\log H + \cdot 8$
,,	,, 3.—	,,	$R = \cdot 3\log H + 7$
,,	,, 4.—	,,	$R = \cdot 2\log H + \cdot 8$
,,	,, 5.—	,,	$R = \cdot 7\log H + \cdot 3$
,,	,, 6.—	,,	$R = \cdot 4\log H + \cdot 6$
,,	,, 7.—	,,	$R = \cdot 6\log H + \cdot 4$
,,	,, 8.—	,,	$R = \cdot 5\log H + \cdot 5$
,,	,, 9.—	,,	$R = \cdot 4\log H + \cdot 6$

The coefficients of $\log H$ and the numerical terms in these equations are given correct to one place of decimals.

The equations just given are comparable in every way with that found for Figure XVI. The height H in each case is in metres. It will be seen that there is no advantage to be gained by expressing mean wind velocities at various heights as ratios of the mean wind velocity at 10 metres. Such a method does not result in a constant coefficient for $\log H$, or a constant numerical term in the equation of the type $R = p\log H + q$.

Considering mean wind velocity up to 500 metres we have from J. S. Dines' observations results of varied character. Figure XVII shows the linearity of mean wind velocity V with log(Height) for observations grouped according to wind strength of 500 metres. The very light winds do not show any definite increase of mean wind velocity with height. The light winds, 4·1–5·0 metres per second at 500 metres, show a linear increase of mean velocity with $\log H$ from 50 to 500 metres. The light or moderate winds show a fairly linear increase of V with $\log H$ from 50 to 500 metres. The strong winds show a linear increase of V with $\log H$ from 100 to 300 metres. Figure XVIII corresponds to three of the curves in Figure V. The observations are grouped according to the time of the day at which they were made. Figure XVIII shows the increase in mean wind velocity with log (Height) for observations made at 7 a.m., 1 p.m. and 4 p.m. The 7 a.m. set shows marked linearity between V and

From the Meteorological Office Wind Summary.

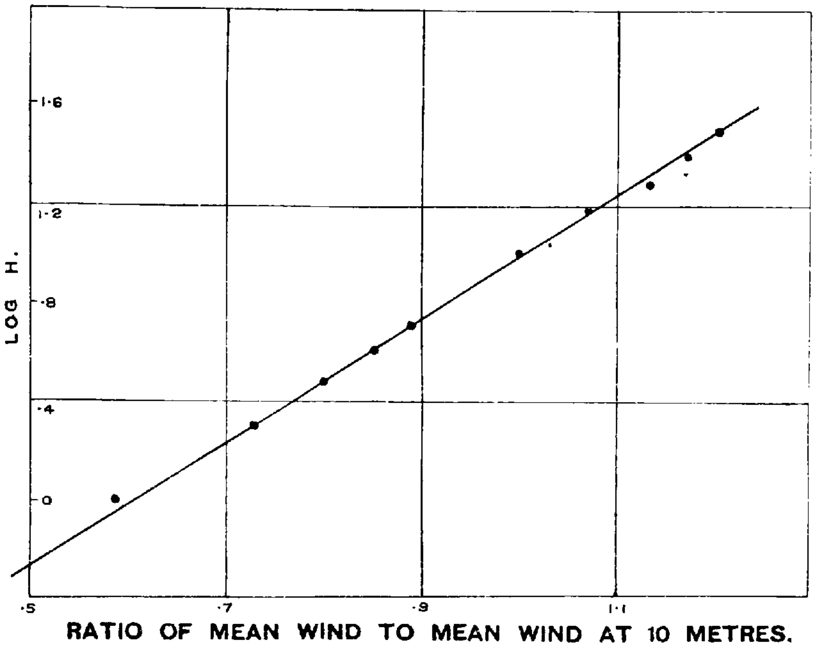


FIGURE XVI.

From J. S. Dines' paper.

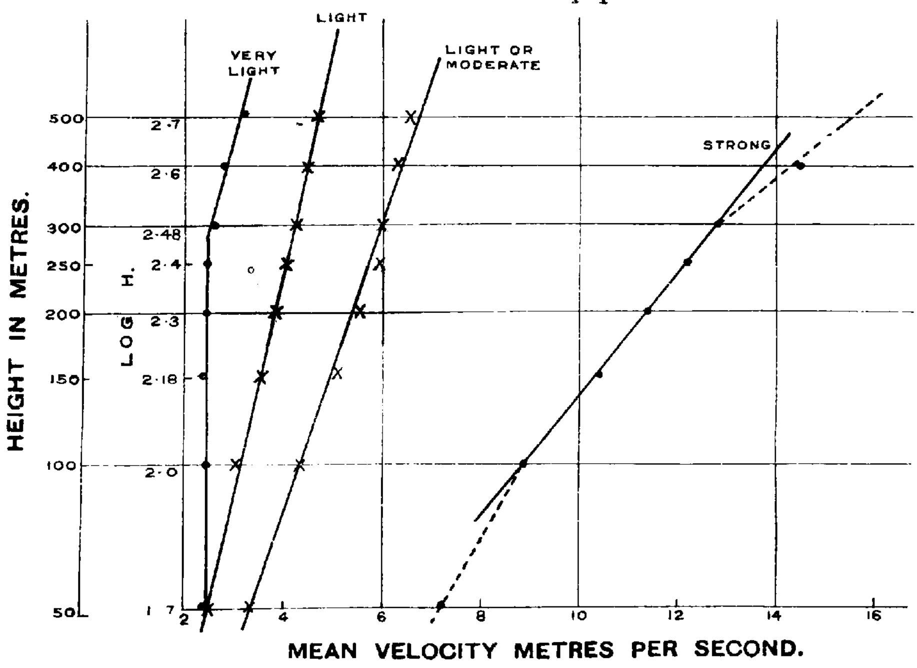


FIGURE XVII.

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From J. S. Dines' paper.

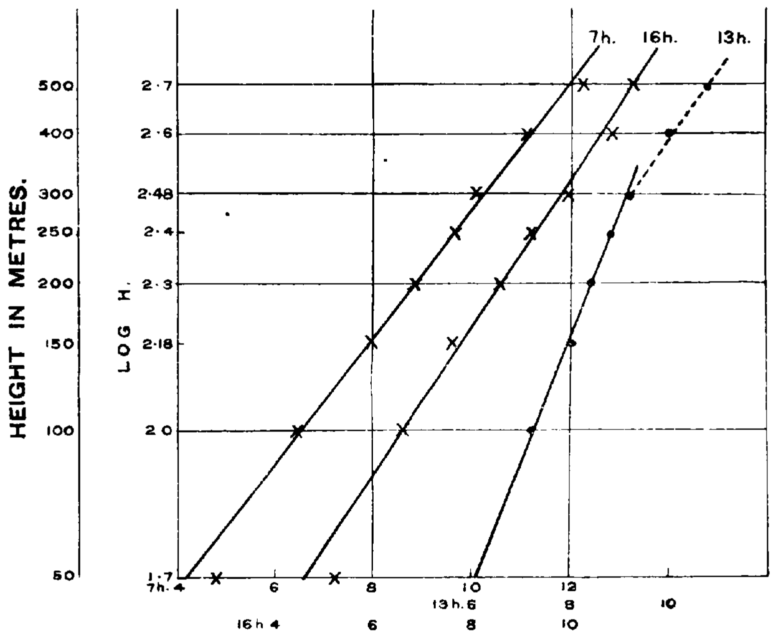

FIGURE XVIII.

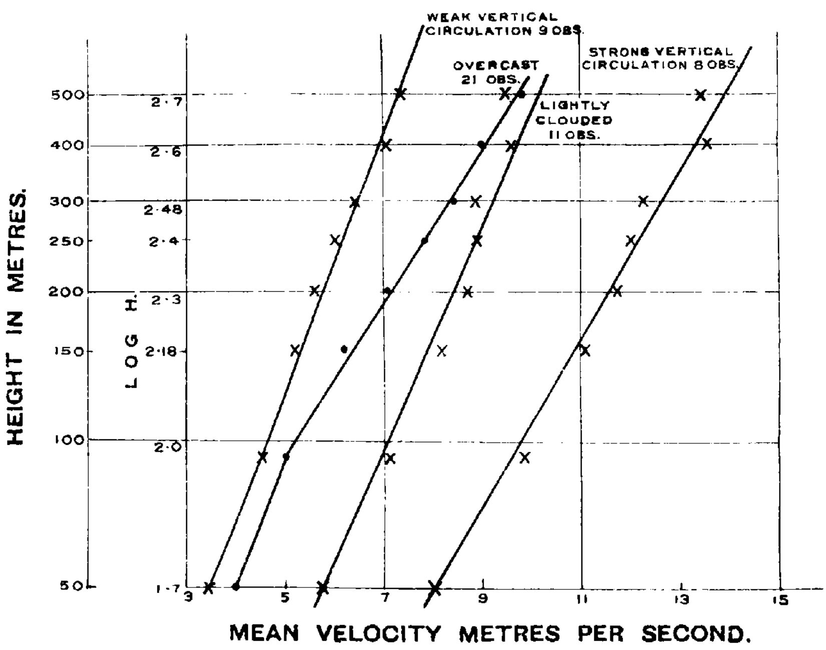

FIGURE XIX.

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From J. S. Dines' paper.

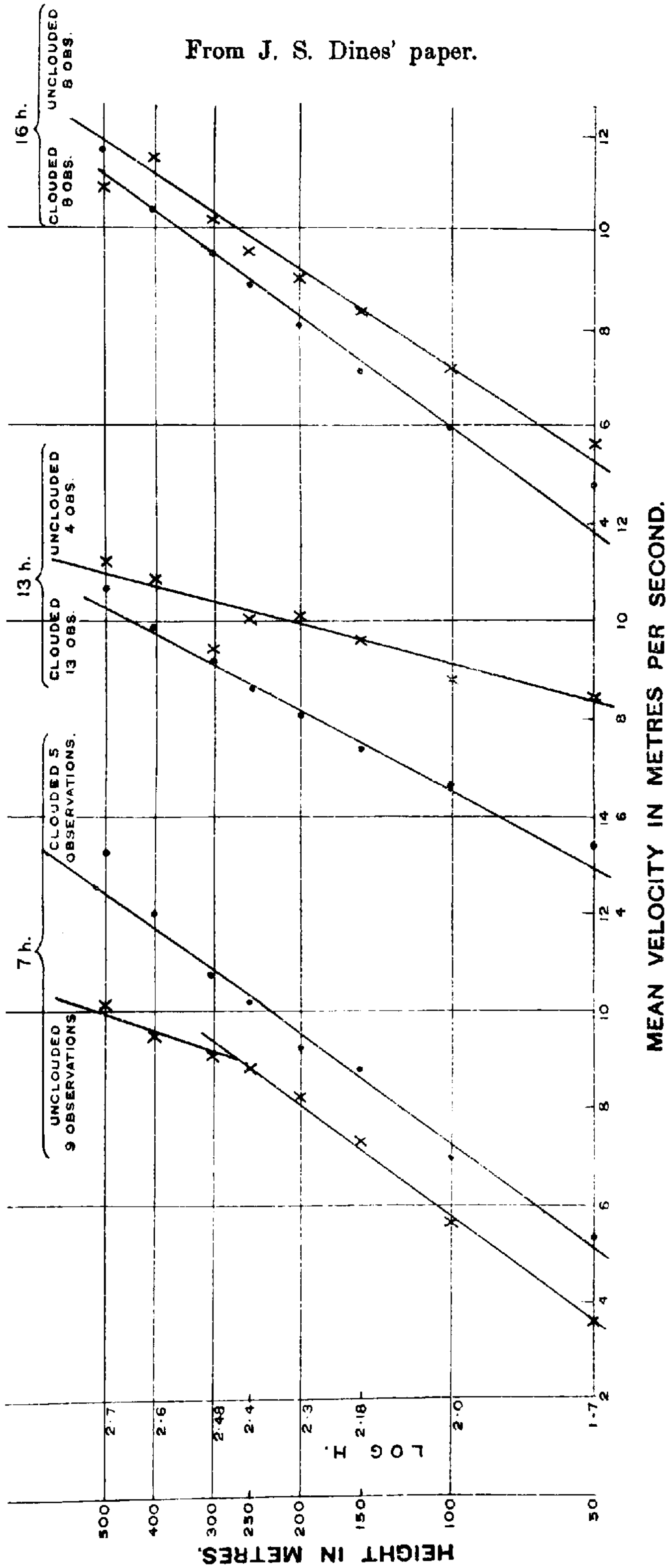

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Figure XXI.

From Dobson's paper.

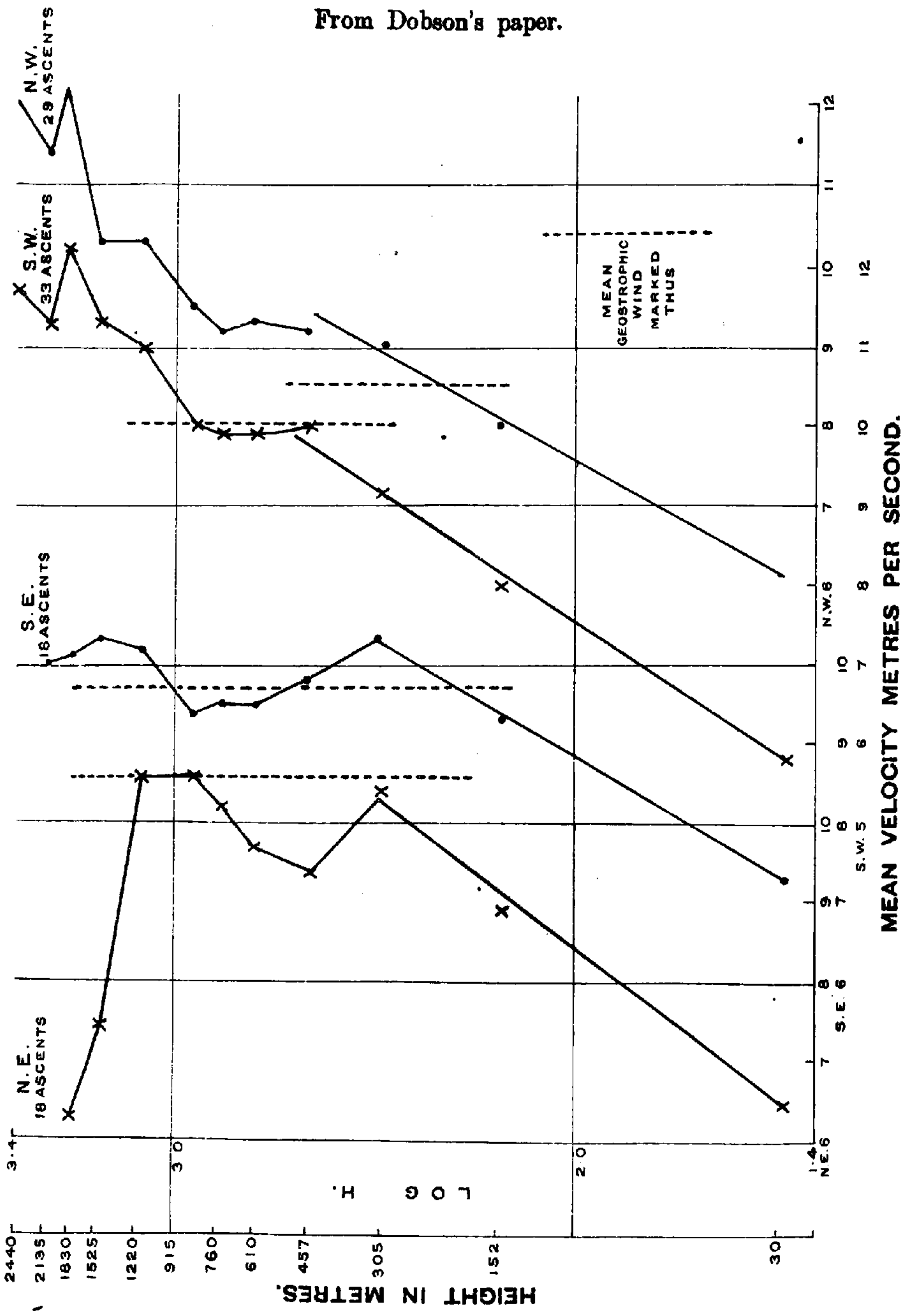

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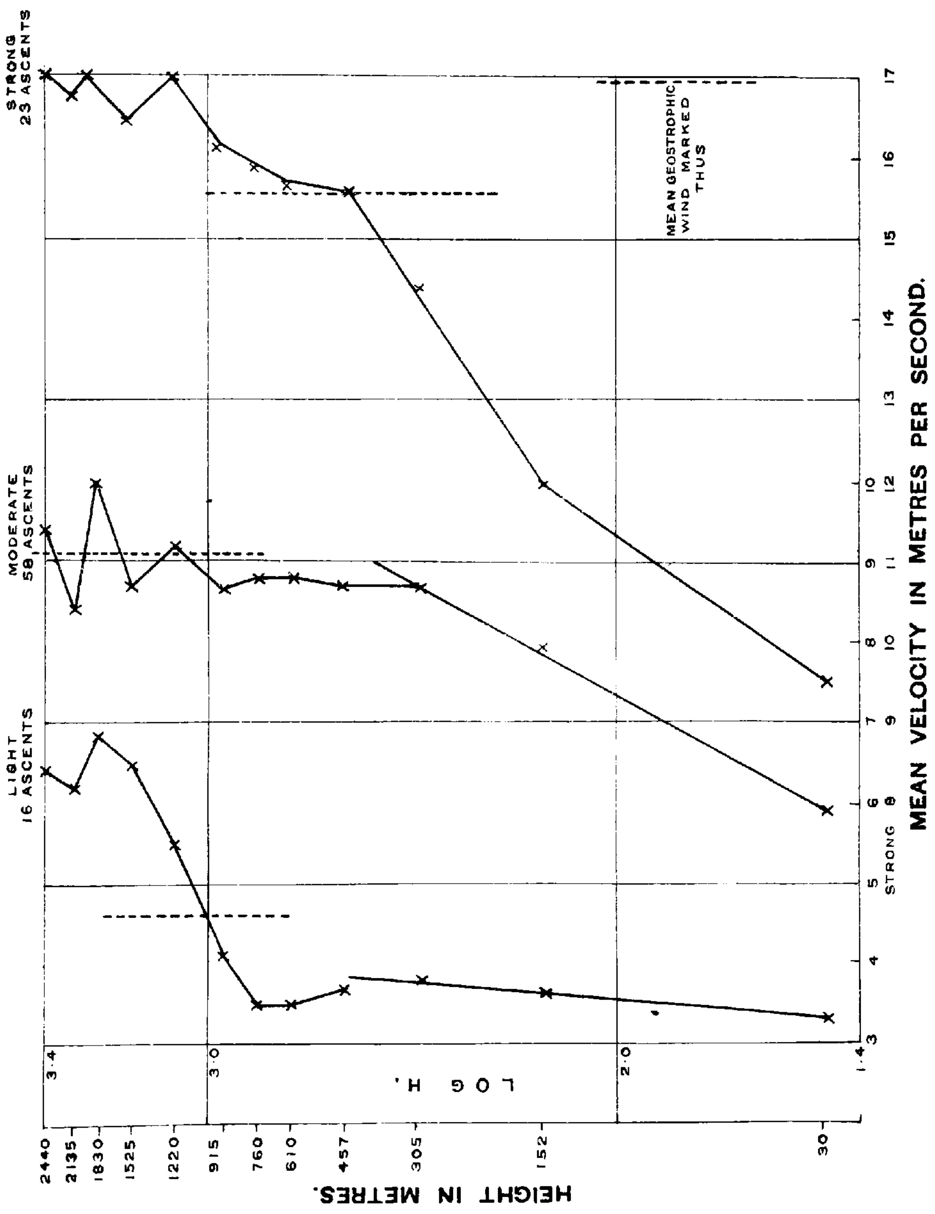
Figure XXII

From Dobson's paper.



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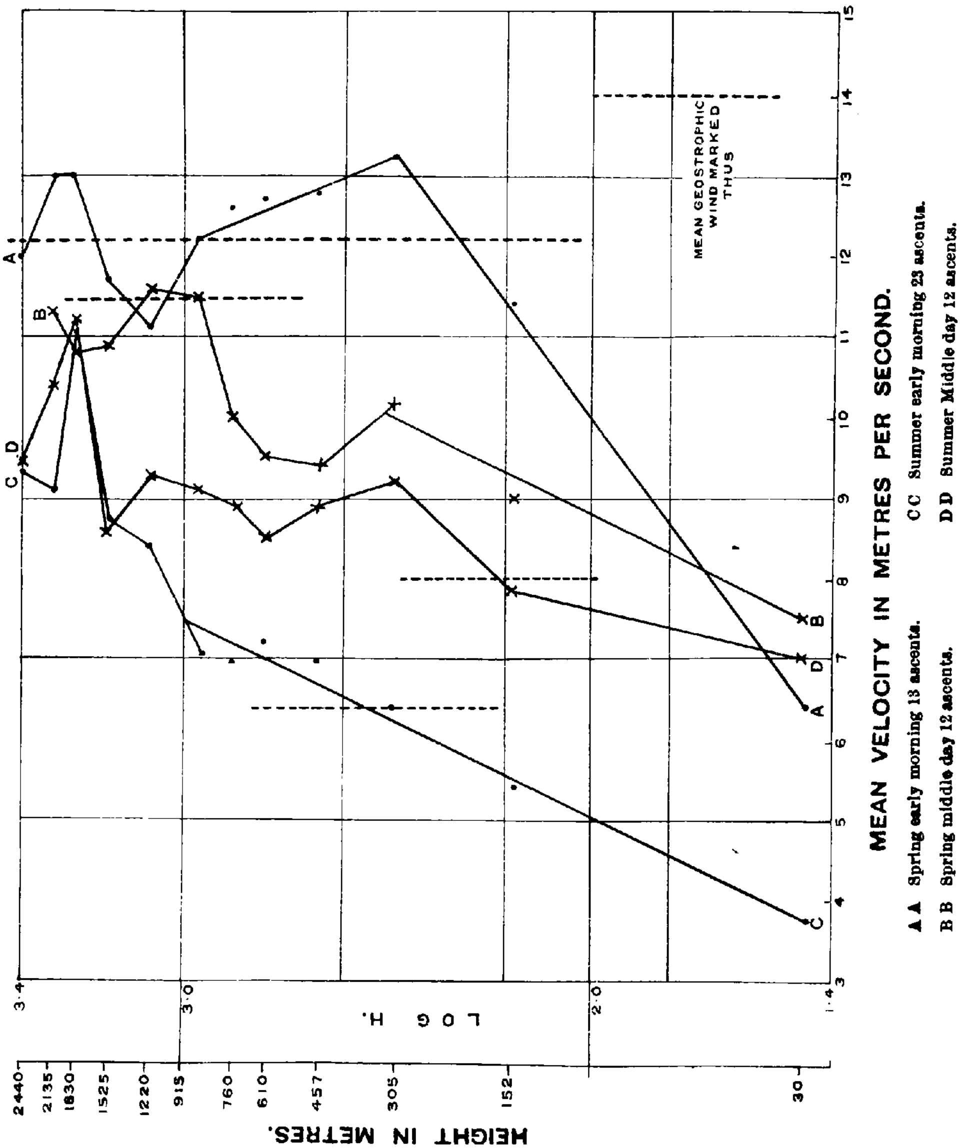
From Dobson's paper.



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log$H$ from 50 to 400 metres. The 1 p.m. set shows linearity from 50 to 300 metres. The 4 p.m. set shows linearity from 50 to 500 metres.

Figure XIX shows linearity between V and log$H$ as follows:—

(i) Ascents with strong vertical circulation, linearity erratic from 50–500 metres.

(ii) Ascents with lightly clouded sky, rough linearity from 50–500 metres.

(iii) Ascents with overcast sky, linearity good from 100 to 500 metres.

(iv) Ascents with weak vertical circulation, linearity good from 50–500 metres.

Figure XX shows the linearity of V with log$H$ for clouded and unclouded ascents made at 7 a.m., 1 p.m., and 4 p.m. The observations plotted against height are shown in Figure VI. In Figure XX it will be seen that for the three sets of "clouded" ascents V varies linearly with log$H$ very approximately. The "unclouded" ascents show linearity between V and log$H$ as follows:—7 a.m., 50–250 metres, 250–500 metres; 1 p.m., 50–500 metres, neglecting one mean velocity; 4 p.m., 50–400 metres.

Summing up from Dines' observations, which are valuable because of the varied manner of grouping the ascents, we see that mean wind velocity V is connected with height H by a formula of the type $V = a\log H + b$ for layers of the atmosphere at least from 100 to 300 metres above the surface, and frequently from 50 to 500 metres above the surface.

The grouping of the observations according to wind strength, time of day, state of sky, or vertical circulation does not make any noticeable difference in the linearity between V and log$H$.

Figures IX–XI from Dobson's observations were drawn chiefly because the author gave mean geostrophic wind velocities. They served to show that mean wind velocity increased up to 300 or 400 metres at which height the mean geostrophic wind velocity had been attained or exceeded. On looking through the actual figures given in Dobson's tables one cannot help being struck by the fact that the mean wind velocities at 305 metres are in most cases unexpectedly high. The mean wind velocity at 305 metres is particularly high for the NE. and SE. ascents. It is also high for the light winds (16 ascents). It is high in the spring ascents both early morning and mid-day. It is high in the summer mid-day ascents. This fact should be kept in mind when Figures XXI–XXIII are being considered. In these figures mean wind velocities are plotted for Dobson's ascents against log(Height). The values of mean wind velocities at 30 metres, 152 metres, 305 metres for NE., SE., SW., and NW. winds (Figure XXI) suggest linearity between mean wind velocity and log$H$. In Figure XXII mean wind velocities at the same heights for moderate winds are linear with log$H$. The corresponding mean wind velocities for strong winds are not linear, but the mean wind velocities at 152 metres, 305 metres, 457 metres for strong winds lie on a straight line. Two of the sets of observations in Figure XXIII, spring and summer early

$\log H$ from 50 to 400 metres. The 1 p.m. set shows linearity from 50 to 300 metres. The 4 p.m. set shows linearity from 50 to 500 metres.

Figure XIX shows linearity between V and $\log H$ as follows:—

(i) Ascents with strong vertical circulation, linearity erratic from 50–500 metres.
(ii) Ascents with lightly clouded sky, rough linearity from 50–500 metres.
(iii) Ascents with overcast sky, linearity good from 100 to 500 metres.
(iv) Ascents with weak vertical circulation, linearity good from 50–500 metres.

Figure XX shows the linearity of V with $\log H$ for clouded and unclouded ascents made at 7 a.m., 1 p.m., and 4 p.m. The observations plotted against height are shown in Figure VI. In Figure XX it will be seen that for the three sets of "clouded" ascents V varies linearly with $\log H$ very approximately. The "unclouded" ascents show linearity between V and $\log H$ as follows:—7 a.m., 50–250 metres, 250–500 metres; 1 p.m., 50–500 metres, neglecting one mean velocity; 4 p.m., 50–400 metres.

Summing up from Dines' observations, which are valuable because of the varied manner of grouping the ascents, we see that mean wind velocity V is connected with height H by a formula of the type $V = a\log H + b$ for layers of the atmosphere at least from 100 to 300 metres above the surface, and frequently from 50 to 500 metres above the surface.

The grouping of the observations according to wind strength, time of day, state of sky, or vertical circulation does not make any noticeable difference in the linearity between V and $\log H$.

Figures IX–XI from Dobson's observations were drawn chiefly because the author gave mean geostrophic wind velocities. They served to show that mean wind velocity increased up to 300 or 400 metres at which height the mean geostrophic wind velocity had been attained or exceeded. On looking through the actual figures given in Dobson's tables one cannot help being struck by the fact that the mean wind velocities at 305 metres are in most cases unexpectedly high. The mean wind velocity at 305 metres is particularly high for the NE. and SE. ascents. It is also high for the light winds (16 ascents). It is high in the spring ascents both early morning and mid-day. It is high in the summer mid-day ascents. This fact should be kept in mind when Figures XXI–XXIII are being considered. In these figures mean wind velocities are plotted for Dobson's ascents against log(Height). The values of mean wind velocities at 30 metres, 152 metres, 305 metres for NE., SE., SW., and NW. winds (Figure XXI) suggest linearity between mean wind velocity and $\log H$. In Figure XXII mean wind velocities at the same heights for moderate winds are linear with $\log H$. The corresponding mean wind velocities for strong winds are not linear, but the mean wind velocities at 152 metres, 305 metres, 457 metres for strong winds lie on a straight line. Two of the sets of observations in Figure XXIII, spring and summer early morning ascents, show a certain degree of linearity between mean wind velocity V and $\log H$.

The figures taken from Dobson's observations do not entirely agree with the suggested formula $V = a \log H + b$, but they serve to show that such a formula does not hold for heights above 300 or 400 metres, or, what is perhaps the same thing, after the mean gradient wind velocity has been attained.

For the sake of comparison the following equations, in which the velocities are all in metres per second, and the heights in metres, are given:—

Figure.	Observations.	Equation.	Linearity between V and log H.
XIII. XIV.	Set 1	$V = 5 \log H - 1$	From 12–900 metres
	" 2	$V = 4 \log H - 1$	" 75–600 "
	" 3	$V = 5 \log H - 2$	" 75–900 "
	" 4	$V = 3 \log H - 1$	" 75–1200 "
XVII.	Light or moderate winds	$V = 4 \log H - 3$	" 50–500 "
	Strong winds	$V = 9 \log H - 8$	" 100–300 "
XVIII.	7 a.m.	$V = 8 \log H - 9$	" 50–400 "
	1 p.m.	$V = 4 \log H - 2$	" 50–300 "
	4 p.m.	$V = 7 \log H - 7$	" 100–500 "
XIX.	Strong vertical circulation	$V = 6 \log H - 2$	" 50–500 "
	Weak vertical circulation.	$V = 4 \log H - 3$	" 50–500 "
	Lightly clouded	$V = 4 \log H - 1$	" 50–500 " roughly
	Overcast	$V = 6 \log H - 7$	" 100–500 "
XX.	7 a.m. clouded	$V = 8 \log H - 10$	" 50–400 "
	" unclouded	$V = 9 \log H - 12$	" 50–250 "
	1 p.m. clouded	$V = 5 \log H - 4$	" 50–500 "
	" unclouded	$V = 3 \log H + 3$	" 50–500 "
	4 p.m. clouded	$V = 7 \log H - 8$	" 50–500 "
	" unclouded	$V = 7 \log H - 7$	" 50–500 "
XXI.	N.E. winds	$V = 4 \log H + 0{\cdot}5$	" 30–305 "
	S.E. "	$V = 3 \log H + 3$	
	S.W. "	$V = 3 \log H + 1{\cdot}5$	
	N.W. "	$V = 3 \log H + 1{\cdot}5$	
XXII.	Light winds	$V = 0{\cdot}5 \log H + 2{\cdot}5$	" 30–305 "
	Moderate winds	$V = 3 \log H + 1{\cdot}5$	
	Strong winds	$V = 7 \log H - 3$	" 152–457 "

§ 8. THEORETICAL.

In his paper "Eddy Motion in the Atmosphere,"[14] G. I. Taylor gives the following equations for wind velocity at height z:—

$$v = A_2 e^{-Bz} \sin Bz + A_4 e^{-Bz} \cos Bz$$

$$u = A_2 e^{-Bz} \cos Bz - A_4 e^{-Bz} \sin Bz + G$$

where v, u are the components of wind velocity perpendicular to

[14] Phil. Trans. A, vol. 215.

and parallel to the direction of the geostrophic wind respectively. G is the geostrophic wind velocity.

$B = \sqrt{\dfrac{\omega \rho \sin \lambda}{\mu}}$ where ω = angular velocity of rotation of earth

ρ = density of air.

λ = latitude.

μ = eddy viscosity.

The values of A_2, A_4, are given in terms of a and G where a is the angle between the direction of the surface wind and the direction of the geostrophic wind.

$$A_2 = -\frac{\tan a\,(1 + \tan a)}{1 + \tan^2 a} G$$

$$A_4 = \frac{-\tan a\,(1 - \tan a)}{1 + \tan^2 a} G$$

We can write $\dfrac{v}{G} = k_2\, e^{-Bz} \sin Bz + k_4\, e^{-Bz} \cos Bz$

$$\frac{u}{G} = k_2\, e^{-Bz} \cos Bz - k_4\, e^{-Bz} \sin Bz + 1$$

where k_2, k_4 are constants depending on a.

Squaring and adding we get for the resultant velocity V.

$$\frac{V^2}{G^2} = k_2^2\, e^{-2Bz} + k_4^2\, e^{-2Bz} + 2e^{-Bz}\,(k_2 \cos Bz - k_4 \sin Bz) + 1$$

$$= k_2^2\, e^{-2Bz} + 2e^{-Bz}\, k_2 \cos Bz + \cos^2 Bz + k_4^2\, e^{-2Bz} - 2e^{-Bz}\, k_4 \sin Bz + \sin^2 Bz$$

$$= (k_2\, e^{-Bz} + \cos Bz)^2 + (k_4\, e^{-Bz} - \sin Bz)^2.$$

Giving a the value 20° as on p. 19 of Taylor's paper we get

$$k_2 = -\cdot 438$$
$$k_4 = -\cdot 204$$

Thus for $a = 20°$ we have

$$\frac{V^2}{G^2} = [-\cdot 438\, e^{-Bz} + \cos Bz]^2 + [-\cdot 204\, e^{-Bz} - \sin Bz]^2.$$

Values of $\dfrac{V}{G}$ according to the value of Bz can be calculated from this result. We get—

Bz	0	$\frac{\pi}{100}$	$\frac{\pi}{50}$	$\frac{\pi}{30}$	$\frac{\pi}{20}$	$\frac{\pi}{10}$	$\frac{2\pi}{10}$	$\frac{3\pi}{10}$	$\frac{4\pi}{10}$	$\frac{5\pi}{10}$	$\frac{6\pi}{10}$	$\frac{7\pi}{10}$	$\frac{8\pi}{10}$	$\frac{9\pi}{10}$	π
$\frac{V}{G}$	·60	·62	·64	·67	·70	·78	·90	·98	1·01	1·05	1·05	1·05	1·05	1·03	1·02

From these values of Bz and $\dfrac{V}{G}$ a curve can be drawn showing increase of wind velocity with height. This is done in Figure

XXIV, which is in reality Figure 4 of Taylor's paper on a larger scale. In Figure XXIV the velocities are given as fractions of the geostrophic velocity. Heights are given in terms of Bz. From Taylor's theoretical considerations the increase of wind velocity with height is not linear, but near the surface, from $Bz = 0$ to $Bz = \frac{\pi}{20}$ a straight line could be fitted to the four points shown in Figure XXIV. The equation to this straight line would be $\frac{V}{G} = \frac{2}{\pi} Bz + {\cdot}6$.

In a subsequent paper[15] Taylor gives a table of values of $\frac{1}{B\,G}$ for Salisbury Plain. The value for $\alpha = 20°$ is 21·9, i.e. for $\alpha = 20°$ $B = \frac{1}{21{\cdot}9\ G}$ in C.G.S. units.

Substituting for B in the equation just found we have :—

$$\frac{V}{G} = \frac{2}{\pi}\,\frac{1}{21{\cdot}9\ G}\ z + {\cdot}6$$

$$\text{or } V = {\cdot}029\ z + {\cdot}6\ G.$$

This linear equation holds from the surface up to a height of $Bz = \frac{\pi}{20}$. For Salisbury Plain this is equivalent to holding from the surface up to a height of $3{\cdot}44G$.

Thus according to Taylor's theoretical considerations when the angle between the direction of the surface wind and the direction of the geostrophic wind is 20° there is a linear law of increase of wind velocity with height over Salisbury Plain. This law holds from the surface to a height of $3{\cdot}44G$, where G is the geostrophic velocity in metres per second. Above this height wind velocity increases with height at a decreasing rate. For a geostrophic wind velocity of 10 metres per second this height would be 34 metres. It has been shown in Figures III and IV. that increase of *mean* wind velocity with height up to 30 metres is not linear. It must be remembered that Taylor's results apply to a single observation, and that in forming means for a set of observations the value of a does not remain constant, neither does the value of G. It would be interesting to obtain mean wind velocities for a set of observations for which a remained constant, say 20°

In Figure 5 of " Phenomena Connected with Turbulence in the Lower Atmosphere " Taylor gives a diagram of curves showing the variation of wind velocity with height for different values of a. Two height scales are provided on the right of the diagram : one for $G = 4{\cdot}6$ metres per second, the other for $G = 15{\cdot}6$ metres per second.

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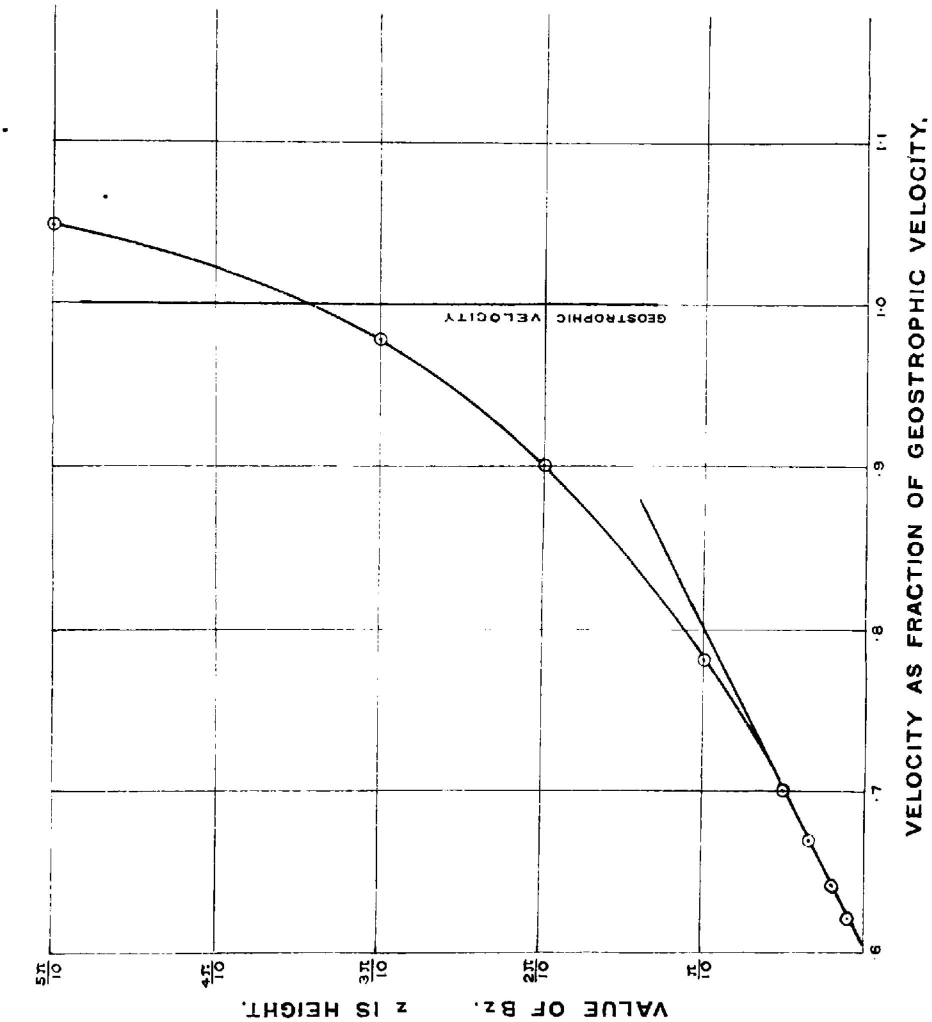
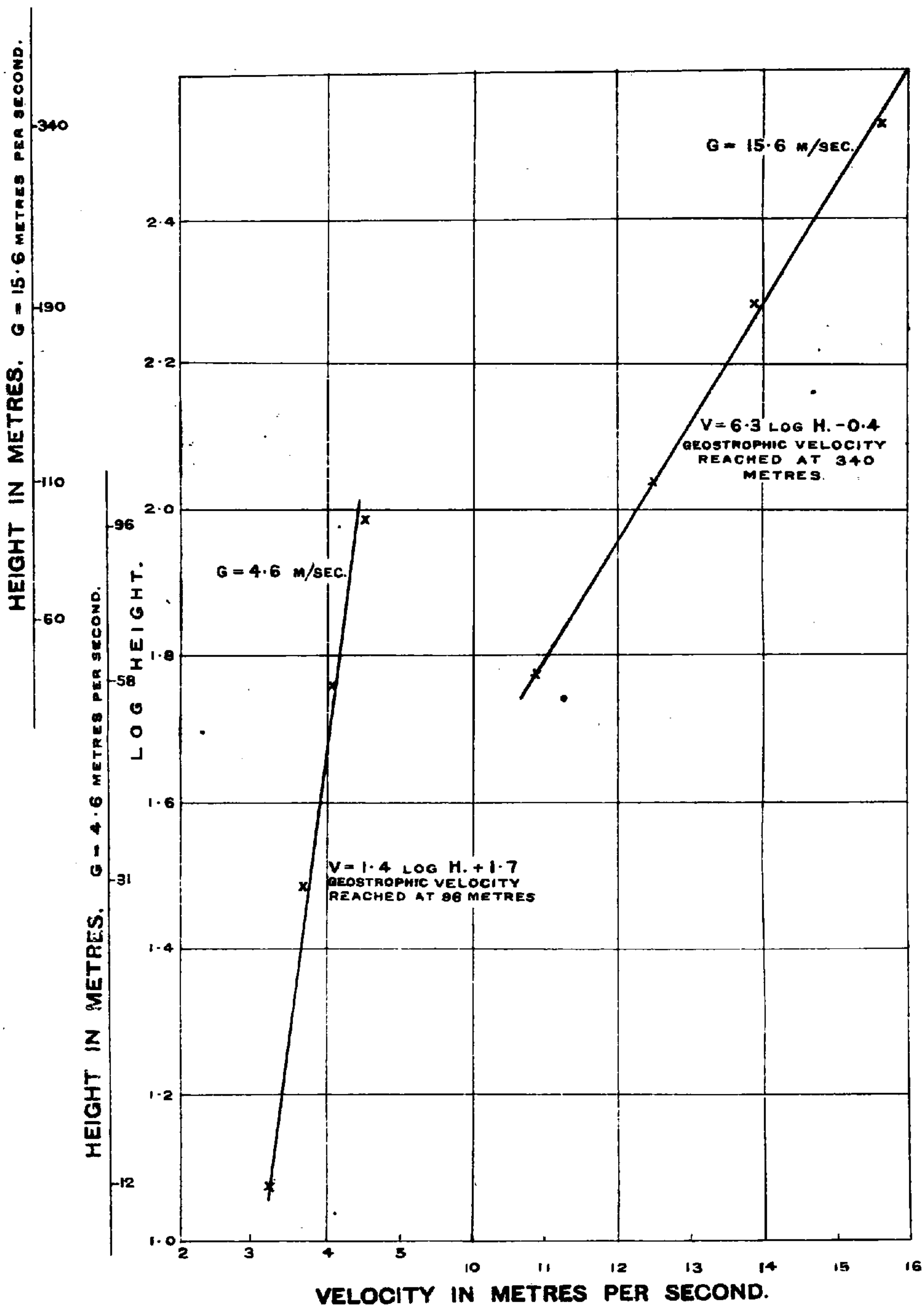
[15] Phenomena Connected with the Turbulence with Lower Atmosphere. Proc. Roy. Soc. A., vol. 94, 1917, p. 137.

From Taylor's paper.



Wyman & Sons, Ltd., M.O. Press, London, S.W. 7. Ps. 4421. 40660. S.O.P. 522. 600. 4/19.

From Taylor's paper.



Wyman & Sons, Ltd., M.O. Press, London, S.W. 7. Ps. 4421. 40660. S.O.P. 522. 600. 4 19.

For $a = 20^{\circ}$ we get the following:—

$G = 4\cdot6$ m/sec.			$G = 15\cdot6$ m/sec.		
Height.	Velocity.	Log$H$.	Height.	Velocity.	Log$H$.
12	3·2 m/sec.	1·08	60	10·9	1·78
31	3·7 „	1·49	110	12·5	2·04
58	4·1 „	1·76	190	13·9	2·28
96	4·6 „	1·98	340	15·6	2·53

These velocities are plotted against log$H$ in Figure XXV. The four points in each case are almost linear. Equations for the straight lines fitting the two sets of points are:—

$V = 6\cdot3 \log H - 0\cdot4$ for the stronger geostrophic velocity.
$V = 1\cdot4 \log H + 1\cdot7$ „ lighter „ „

The equation found in Figure XXII for strong winds (average geostrophic velocity 15·6 m./sec.) was $V = 7\log H - 3$. For light winds (average geostrophic 4·6 metres per second) the equation was $V = 0\cdot5 \log H + 2\cdot5$. The equations obtained from Figure XXII are at least comparable with those found from Taylor's diagram.

§ 9. ADDENDUM.

The present paper was commenced early in 1917 The possibility of a linear law between mean wind velocity and log(Height) was seen soon after the paper was commenced. When the paper was almost completed the author came across a very similar result obtained by Hellmann[16] from observations at Nauen. The formula for increase of mean wind velocity with height as given by Hellmann is

$$V = a \log(H + c) + b$$

where a, b, c are constants.

[16] Uber die Bewegung der Luft in der untersten Schichten der Atmosphäre—Preuss. Akad. der Wiss., Berlin, 10, pp. 174–197, 1917.