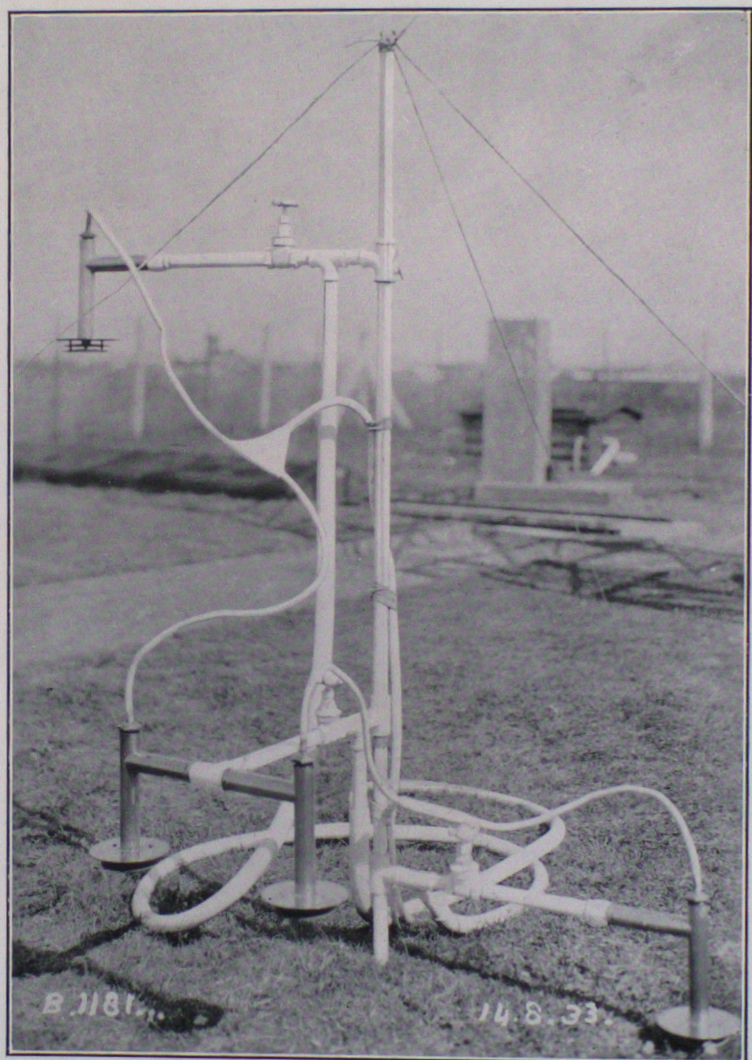

Fig. 1.—Temperature gradient apparatus.

M.O. 356h.

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TRANSFER OF HEAT AND MOMENTUM IN THE LOWEST LAYERS OF THE ATMOSPHERE

By A. C. BEST, B.Sc.

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TRANSFER OF HEAT AND MOMENTUM IN THE LOWEST LAYERS OF THE ATMOSPHERE

INTRODUCTION AND SUMMARY

During the period 1931 to 1933 the writer had occasion to investigate various aspects of turbulence in the layer of the atmosphere nearest the ground. Although these investigations were carried out separately the various phenomena are closely connected and accordingly the results have been collected and are given in the present paper.

From August 1931 to July 1933 temperature differences between the heights 2·5 cm. and 30 cm., and 30 cm. and 1·2 m., above close cropped grass were recorded continuously. Curves shewing the diurnal variation of these temperature differences for each month have been constructed and attention has also been given to the effect of the state of sky. The greatest values found for the temperature differences in each month have been extracted. The records from this apparatus have been combined with other records which give temperature differences between the heights 1·2 m. and 7·1 m., and 1·2 m. and 17·1 m., and hence some temperature height curves obtained for clear summer days. The times of maximum temperature and the diurnal ranges of temperature at heights from 2·5 cm. to 17·1 m. are also discussed. Finally the results so obtained are utilised to shew the variation of the coefficient of eddy conductivity with height. A brief account of this part of the work was given by the author to the British Association meeting at Leicester in 1933 and the results are described in some detail in the first part of the present paper.

The vertical gradient of wind velocity between 2·5 cm. and 5 m. above close cropped grass was examined in 1932 and 1933, the wind velocity being measured by means of a special hot wire anemometer and some small vane type air meters. The effects of vertical temperature gradient (measured by the temperature difference between 10 cm. and 110 cm. above the ground) and of wind velocity upon velocity gradient are considered. The applicability of a "power law" to the variation of wind velocity with height near the ground is discussed and the effects of various factors upon the index in the power law examined. This investigation is described in the second part of the paper.

The investigation of the vertical gradient of wind velocity provided a large number of measurements of the instantaneous wind velocity. For many of these measurements the ratio of the eddy velocity to the mean velocity has been calculated, and called the "fluctuation ratio." The variation of the mean fluctuation ratio with height, velocity and vertical temperature gradient and also the relative frequency of different sized fluctuations are discussed in the third part of the paper.

The last part of the paper deals with an investigation of gustiness in the lateral and vertical directions at heights up to 5 m. above close cropped grass by means of small bidirectional vanes. The effects upon gustiness of height above the ground, of vertical temperature gradient and of velocity are discussed. From the measurements of gustiness the magnitudes of the eddy velocities are deduced, also the influence upon these quantities of the factors already mentioned. Some figures are given shewing the accuracy to be expected in measuring the wind velocity by means of a single one minute run with a small vane type air meter.

No attempt has been made to discuss completely the results of this work in terms of any theory of atmospheric turbulence.

Finally, the author wishes to take this opportunity of acknowledging with gratitude the assistance given him in the course of this work by Mr. O. G. Sutton, B.Sc., and Mr. E. L. Davies, M.Sc., both of the Meteorological Office.

PART I. THE VERTICAL GRADIENT OF TEMPERATURE IN THE ATMOSPHERE NEAR THE GROUND

§ 1—GENERAL CONSIDERATIONS

In a memoir by N. K. Johnson (1)* the results were given of a systematic study of the temperature differences over the height intervals 1·2 m. to 7·1 m. and 1·2 m. to 17·1 m. during the years 1923–5. The present work is an endeavour to extend that investigation into the region below 1·2 m.

Johnson's experiments were carried out on the south-eastern edge of Salisbury Plain and the apparatus was " situated on the top of a horizontal ridge which runs west-north-west—east-south-east for about a kilometre on either side of the instrument. The top of the ridge is at a height of 111 m. (364 ft.) above mean sea level. The ground to the south slopes away for 700 m. at a mean slope of about one in thirty. On the north side of the ridge the ground falls away at about the same slope for some 250 m. and then becomes roughly horizontal." (Johnson, *loc. cit.* p. 3). The original apparatus was still in use during the period covered by the present work and the additional apparatus required to extend the results below 1·2 m. was erected three or four metres away. This additional apparatus was constructed to measure temperature differences over the height intervals 2·5 cm. to 30 cm. and 30 cm. to 1·2 m. By this means simultaneous records giving the temperature gradient between 2·5 cm. and 17·1 m. were obtained.

The present investigation took place between August 1, 1931, and July 31, 1933. No attempt is made to describe the temperature gradient above 1·2 m. (since this has already been fully dealt with by Johnson) except in so far as it relates to the conditions existing below 1·2 m. It will frequently be necessary to make reference to the different height intervals over which temperature differences were measured. In order to facilitate this the following reference letters will be used:

From	2·5 cm.	to 30 cm.	....	layer A
,,	30 ,,	,, 1·2 m.	....	,, B
,,	1·2 m.	,, 7·1 m.	....	,, C
,,	7·1 m.	,, 17·1 m.	....	,, D
,,	1·2 m.	,, 17·1 m.	....	,, CD

It should be noted that layer A is the lowest layer and layer D the highest.

§ 2—INSTRUMENTAL

Temperature gradient apparatus.—The apparatus used for measuring temperature differences over layers C and CD consisted of three platinum resistance thermometers operating differentially in two arms of a Wheatstone bridge, two of

* The numbers in brackets refer to the bibliography on p. 66.

the resistance thermometers being switched into one arm for alternate minutes in order to measure the temperature differences over two layers. Johnson gives a full description of this apparatus which, except in details, has not been altered. It was decided, however, to use thermo-electric couples for the lower intervals, although the resistance thermometers had been quite satisfactory. This dispensed with the need for batteries and bridge circuits and made frequent calibrations unnecessary. It may be pointed out here that the use of thermo-electric couples for recording temperature differences between two widely separated points introduces the difficulty that the circuit resistance becomes large and hence a more sensitive galvanometer is necessary.

Each thermo-electric couple was made by joining in series four lengths of S.W.G. No. 24 D.S.C. constantan wire and five lengths of S.W.G. No. 22 D.S.C. copper wire, each length being about six feet. The metals alternated and the junctions were made by welding. There were thus eight copper-constantan junctions in all and the two free ends of the composite wire were of copper. The wire was then arranged so that alternate junctions lay side by side and the two free ends of copper wire were joined to a length of two-core cable, each core consisting of six S.W.G. No. 22 copper wires. This cable led to the recording part of the apparatus. Thus each element consisted of four copper-constantan junctions. These junctions were insulated from each other with oiled silk and each set of four was inserted in a length of brass tubing of about 5/16 inch diameter and nine inches long. Lengths of composition tubing were threaded over the elements and joined up at the centre of the wires to the lead covering of the main cable, thus effectively weatherproofing the thermo-electric couples. The resistances of these pairs of thermo-electric couples were 14·5 ohms for one circuit and 16·8 ohms for the other. Each pair of thermo-electric couples was connected in series with a recording galvanometer, having a resistance of about 16 ohms, and an appropriate resistance inserted in order to obtain a convenient scale value on the galvanometer. This resistance had a value of about 30 ohms.

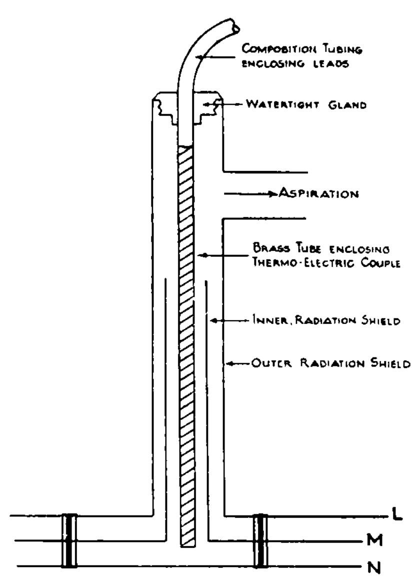


Fig. 2.—Diagram of housings for elements.

Each element was inserted in a special housing shewn diagramatically in Fig. 2. These housings consisted essentially of brass tubes with brass discs fixed to the ends. The discs M and N were slightly dished, being concave downwards, to allow rain to drain off. The tubes and discs were nickel plated and the outer tubes and discs were polished periodically. Artificial aspiration was provided by connecting up the side tubes to an electric fan which drew air past the elements. The photograph in Fig 1 shews the four housings and the aspiration pipes leading to the electric fan. Apart from the question of providing a radiation shield, the dimensions of the housings were chosen with a view to providing a reasonable rate of air flow past the elements and at the same time causing as little air disturbance as possible. Clearly the amount of air disturbance in the neighbourhood of the housing will be measured by the rate of flow of air over the edges of the discs M and N, and this must accordingly be kept as low as possible. With the dimensions chosen the rate of flow past the element is approximately nine times the rate of flow over the edges of the discs M and N. The actual rate of aspiration used gave a linear flow of about 2 m./sec. past the elements.

During some of the winter months the shadow of a nearby Stevenson screen fell on the housings about 1300 to 1400 G.M.T. On clear days this had a marked effect on the traces, the effect being much greater on the trace corresponding to layer A. Both housings corresponding to this trace were shaded on these occasions and undoubtedly the deflection of this trace is due to the cooling of the ground in the immediate vicinity of the 2·5 cm. element. In the case of layer B, only the 30 cm. housing was shaded and the deflection of this trace (usually about 0·5°F.) is probably due partly to the housing being cooled and partly to the cooling of the ground nearby. If, as is very improbable, it is assumed that the whole éffect here is due to the housing being cooled, it is clear that with both housings similarly exposed the differential effect due to sunshine must be very small. Thus, since temperature differences only are being measured, this slight inefficiency of the housings as radiation shields is unimportant. In a later section, when constructing diurnal variation curves for the winter months, the points which are clearly affected by this shadow have been neglected.

The four housings were mounted on a short mast and erected on a piece of level turf. The grass was kept cut as short as possible and occasionally the mast was rotated a few degrees in order that the grass immediately under the lower housing should not be perpetually in shadow. The heights of the housings were adjusted so that one circuit measured the temperature difference between 2·5 cm. and 30 cm. (i.e. 1 inch and 1 foot) and the other, the difference between 30 cm. and 1·2 m. (i.e. 1 foot and 4 feet). Naturally, owing to small irregularities in the ground surface, the height of the lowest housing can be taken as approximate only. All heights were measured from the surface to the centre of the space between discs M and N.

The recording part of the apparatus was a double galvanometer single thread recorder made by the Cambridge Instrument Company; the record consists of a series of dots, one dot being made every minute. This part of the apparatus was, of course, similar to the recorders used for layers C and CD.

Owing to the construction of the elements the lag was rather large. This was unavoidable since each of the four junctions in one element had to be insulated one from the other and the complete element rendered weather-proof. The lag of one pair was measured by immersing one element in water, thus maintaining it at a constant temperature, and drawing air past the other element at approximately 2 m./sec. The results shewed that 35 per cent., 65 per cent. and 80 per cent. of the final reading was reached after one, two and three minutes respectively. Since the investigation aimed primarily at giving mean values of the temperature gradient under different conditions this was considered satisfactory. For the purpose of comparison it may be mentioned here that Johnson gave the figures 81 per cent. after one minute and 96 per cent. after two minutes for the resistance thermometers used in layers C and CD.

In order the find the relation between temperature difference and deflection of the galvanometer, a calibration was carried out before the apparatus was set up by immersing the elements in different water baths. Further calibrations were carried out during the course of the work in a similar way. The results for one pair of elements are shewn in Fig. 3. It will be seen that there was no systematic change in the calibration over the period under review. Also as one would expect with the particular metals used, the calibration curve was linear over the range of temperature difference experienced.

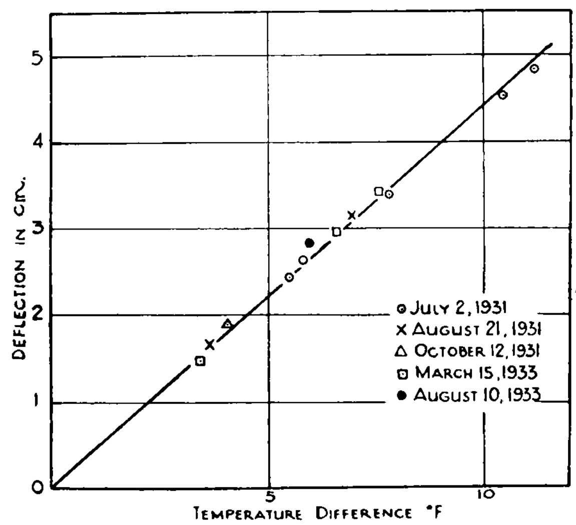


FIG. 3.—CALIBRATION CURVE FOR 2·5 CM. TO 30 CM. CIRCUIT.

Auxiliary apparatus.—In a subsequent section reference will be made to a black surface plate, the temperature of which was recorded from August 1932, until July 1933. The instrumental side of this part of the work was as follows :—A copper plate about 45 cm. square and 1·6 mm. thick was enamelled dull black on one side and one junction of a copper-constantan thermo-electric couple was soldered into a small hole in the centre of the plate. The latter was placed upon a bare earth surface, black side uppermost ; the other junction of the thermo-electric couple was enclosed in a thin brass tube and the whole immersed in a thermos flask containing oil. The flask was buried in the earth near the plate. The copper member of the thermo-electric couple was cut at approximately the centre and connected to one side of a double galvanometer thread recorder by means of a suitable cable and a small resistance, the latter being used to obtain a suitable scale. The other galvanometer of the recorder was used to record the temperature in the thermos flask as given by an ordinary resistance thermometer and Wheatstone bridge circuit. Thus, by combining the two traces from this recorder it was possible to obtain a daily temperature curve for the plate. The purpose of this apparatus was to measure the surface temperature of the plate. It is easily shewn that the mean temperature of the plate, which was the quantity actually measured, differs inappreciably either in magnitude or in phase from the temperature of the surface of the plate.

In addition to the instruments already mentioned for measuring temperature differences over different layers of air, and the surface plate described in the last paragraph, there were various other pieces of apparatus giving records of meteorological elements which were used in connexion with the present investigation. These consisted of

(a) A platinum resistance thermometer giving continuous records of the air temperature at a height of 1·2 m. This thermometer formed part of the apparatus used by Johnson (1) and was exposed in a louvred radiation shield three or four metres distant from the thermo-electric couples and was artificially aspirated.

(b) A Campbell-Stokes sunshine recorder.

TABLE I—MEAN HOURLY VALUES OF THE TEMPERATURE DIFFERENCES IN °F.

Over the height

Month \ Hour	0100	0200	0300	0400	0500	0600	0700	0800	0900	1000	1100
January ..	0·50	0·47	0·47	0·44	0·46	0·34	0·34	0·29	0·16	−0·23	−0·49
February ..	0·65	0·59	0·58	0·58	0·50	0·50	0·48	0·30	−0·13	−0·61	−0·92
March	1·02	0·93	0·87	0·83	0·73	0·69	0·28	−0·49	−1·18	−1·56	−1·87
April	0·74	0·67	0·66	0·56	0·52	0·24	−0·67	−1·29	−1·79	−2·11	−2·04
May	0·42	0·37	0·31	0·27	0·00	−0·57	−1·20	−1·77	−2·22	−2·28	−2·35
June	0·63	0·48	0·48	0·42	−0·21	−1·06	−1·77	−2·54	−2·87	−3·08	−3·24
July	0·36	0·27	0·27	0·15	−0·13	−0·35	−1·19	−1·79	−2·22	−2·26	−2·50
August ..	0·43	0·31	0·26	0·21	0·14	−0·23	−0·80	−1·25	−1·58	−2·14	−2·32
September ..	0·53	0·30	0·43	0·36	0·38	0·16	−0·37	−0·82	−1·21	−1·59	−1·71
October ..	0·69	0·70	0·66	0·53	0·52	0·48	0·23	−0·32	−0·83	−1·20	−1·27
November ..	0·58	0·48	0·52	0·51	0·56	0·57	0·53	0·38	−0·03	−0·48	−0·54
December ..	0·55	0·49	0·49	0·52	0·50	0·53	0·56	0·55	0·28	−0·05	−0·29

Over the height

January ..	0·42	0·40	0·46	0·54	0·46	0·34	0·31	0·28	0·14	−0·03	−0·17
February ..	0·37	0·40	0·34	0·38	0·33	0·34	0·30	0·16	−0·03	−0·24	−0·41
March	0·81	0·75	0·71	0·67	0·56	0·63	0·26	−0·16	−0·53	−0·73	−0·94
April	0·58	0·56	0·51	0·52	0·44	0·24	−0·19	−0·44	−0·70	−0·92	−0·93
May	0·52	0·49	0·49	0·44	0·20	−0·12	−0·39	−0·68	−0·82	−0·90	−0·98
June	0·61	0·47	0·45	0·40	0·19	−0·15	−0·40	−0·71	−0·87	−1·07	−1·10
July	0·41	0·38	0·36	0·29	0·12	−0·03	−0·20	−0·42	−0·52	−0·62	−0·70
August.. ..	0·33	0·33	0·31	0·26	0·22	0·00	−0·27	−0·44	−0·60	−0·79	−0·92
September ..	0·55	0·47	0·45	0·43	0·40	0·25	−0·06	−0·22	−0·40	−0·58	−0·63
October ..	0·62	0·57	0·66	0·54	0·52	0·46	0·35	−0·06	−0·33	−0·41	−0·53
November ..	0·42	0·34	0·35	0·31	0·37	0·34	0·31	0·21	−0·03	−0·22	−0·26
December ..	0·34	0·30	0·32	0·30	0·29	0·32	0·35	0·31	0·14	−0·03	−0·12

NOTES.—(1) A negative temperature difference denotes temperature decreasing upwards.
(2) The temperature differences corresponding to the dry adiabatic lapse rate are −0·0054°F. for the layer
* These values are in error, see

(c) A night sky camera. The shutter of this was opened shortly after sunset and closed shortly before sunrise, the camera being pointed at the pole star. The extent to which the trace given by the star was discontinuous was taken as a measure of the cloudiness of the sky during the night.

§ 3—MEAN HOURLY VALUES

Mean diurnal curves.—Hourly values have been extracted from all the temperature difference traces and from the temperature trace for 1·2 m., the value assigned to any particular hour being the mean value for a twenty-minute period centred at the hour. All temperatures are expressed in degrees Fahrenheit and G.M.T. is used throughout. Since temperature does not vary linearly with height near the surface, it has been deemed advisable to present the results in the form of temperature differences rather than as temperature gradients. The temperature differences

OVER TWO HEIGHT INTERVALS FOR EACH MONTH (AUGUST 1931 TO JULY 1933)

interval A, 2·5 cm. to 30 cm.

1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400
−0·51	−0·31	−0·18	0·16	0·52	0·79	0·71	0·61	0·60	0·53	0·57	0·59	0·54
−1·09	−0·85	−0·62	−0·19	0·38	0·78	1·03	0·94	0·95	0·88	0·86	0·82	0·77
−1·97	−1·82	−1·61	−1·08	−0·23	0·27	0·97	1·33	1·28	1·16	1·13	1·10	0·98
−2·19	−2·20	−1·78	−1·39	−0·75	−0·42	0·23	0·66	0·75	0·75	0·79	0·85	0·85
−2·38	−2·51	−2·02	−1·69	−1·05	−0·63	−0·06	0·25	0·50	0·45	0·50	0·49	0·44
−3·38	−2·92	−2·57	−2·07	−1·62	−0·94	−0·21	0·16	0·66	0·87	0·82	0·67	0·63
−2·49	−2·41	−2·08	−1·55	−1·21	−0·85	−0·14	0·22	0·52	0·56	0·54	0·45	0·41
−2·44	−2·27	−2·12	−1·51	−1·00	−0·64	0·21	0·65	0·63	0·64	0·55	0·50	0·50
−1·56	−1·45	−0·99	−0·83	−0·44	0·19	0·46	0·62	0·61	0·56	0·46	0·48	0·52
−1·30	−1·15	−0·98	−0·31	0·33	0·90	1·10	0·88	0·93	0·91	0·86	0·78	0·80
−0·60	+0·06 *	−0·18	0·17	0·63	0·71	0·73	0·69	0·60	0·63	0·69	0·63	0·55
−0·32	+0·21 *	0·00	0·40	0·73	0·75	0·71	0·72	0·66	0·69	0·77	0·66	0·67

interval B, 30 cm. to 1·2 m.

−0·12	−0·16	−0·10	0·08	0·35	0·53	0·56	0·52	0·50	0·50	0·54	0·54	0·42
−0·48	−0·40	−0·32	−0·12	0·11	0·31	0·51	0·57	0·52	0·50	0·50	0·43	0·48
−0·97	−0·93	−0·80	−0·59	−0·27	0·12	0·57	0·99	0·94	0·87	0·85	0·78	0·74
−1·01	−1·02	−0·81	−0·72	−0·47	−0·21	0·11	0·42	0·61	0·63	0·63	0·58	0·61
−1·03	−1·11	−0·91	−0·73	−0·55	−0·33	−0·08	0·15	0·44	0·52	0·56	0·56	0·58
−1·25	−1·09	−0·98	−0·87	−0·70	−0·41	−0·16	0·10	0·41	0·60	0·59	0·57	0·55
−0·78	−0·77	−0·74	−0·59	−0·46	−0·29	−0·09	0·09	0·34	0·42	0·42	0·38	0·41
−0·96	−0·98	−0·88	−0·68	−0·59	−0·31	0·01	0·36	0·53	0·54	0·55	0·51	0·50
−0·59	−0·59	−0·45	−0·35	−0·20	0·06	0·25	0·49	0·53	0·51	0·45	0·52	0·55
−0·52	−0·48	−0·40	−0·01	0·01	0·46	0·78	0·88	0·75	0·71	0·71	0·72	0·64
−0·25	−0·09	−0·11	0·17	0·36	0·46	0·48	0·46	0·42	0·42	0·42	0·40	0·38
+0·02 *	−0·07	−0·02	0·18	0·38	0·47	0·48	0·48	0·44	0·49	0·50	0·44	0·40

A, 2·5 cm. to 30 cm. and −0·016°F. for the layer B, 30 cm. to 1·2 m.
§2—*Temperature Gradient apparatus, p.* 6.

corresponding to the dry adiabatic lapse rate for the various layers considered are as follows :—

Layer A, 2·5 cm. to 30 cm. ....−0·0054°F.
,, B, 30 cm. ,, 1·2 m. ....−0·016°F.
,, C, 1·2 m. ,, 7·1 m. ....−0·11°F.
,, D, 7·1 m. ,, 17·1 m. ....−0·18°F.
,, CD, 1·2 m. ,, 17·1 m. ....−0·29°F.

It may be noted here that throughout the present paper a positive temperature difference denotes temperature increasing with height and *vice versa*. The word " lapse " is used only in cases where temperature decreases with height.

In Table I are given the mean hourly values of the temperature differences for the two lower layers for each month. The results have been plotted in Fig. 4, selected months only being reproduced. Corresponding tables and curves for the two upper layers were given in Johnson's paper, but for convenience the mean curves for the two layers C and CD for the months of January 1932 and 1933, and June 1932 and 1933, have been included in Fig. 4. These curves for the lower layers have much in common with the corresponding curves for the two upper layers given by Johnson. Thus they all show an inversion of temperature (i.e. temperature increasing upwards) during the night and a lapse during the daytime. The lapse period centres approximately at noon throughout the year but increases in duration as the season changes from winter to summer and *vice versa*. The magnitude of the lapse is usually greatest at noon, the value at this hour increasing progressively from January to June and then decreasing again. The parts of the curves corresponding to lapse conditions are approximately symmetrical about the noon line. The magnitude of the night inversion does not appear to undergo any systematic change from winter to summer, nor does it appear to depend upon the hour, apart from the periods immediately before or after the temperature difference changes sign.

An outstanding feature of the curves in Fig. 4 is the magnitude of the temperature lapses at midday. These lapses are shewn below in Table II in terms of the dry adiabatic temperature difference for January and June; figures for the same months for layers C and D are included for the purpose of comparison. From these figures it is clear that during both winter and summer there is a rapid decrease of temperature gradient with increasing height during the lapse period.

TABLE II.—RATIO OF MONTHLY MEAN MIDDAY LAPSE RATES TO THE DRY ADIABATIC LAPSE RATE

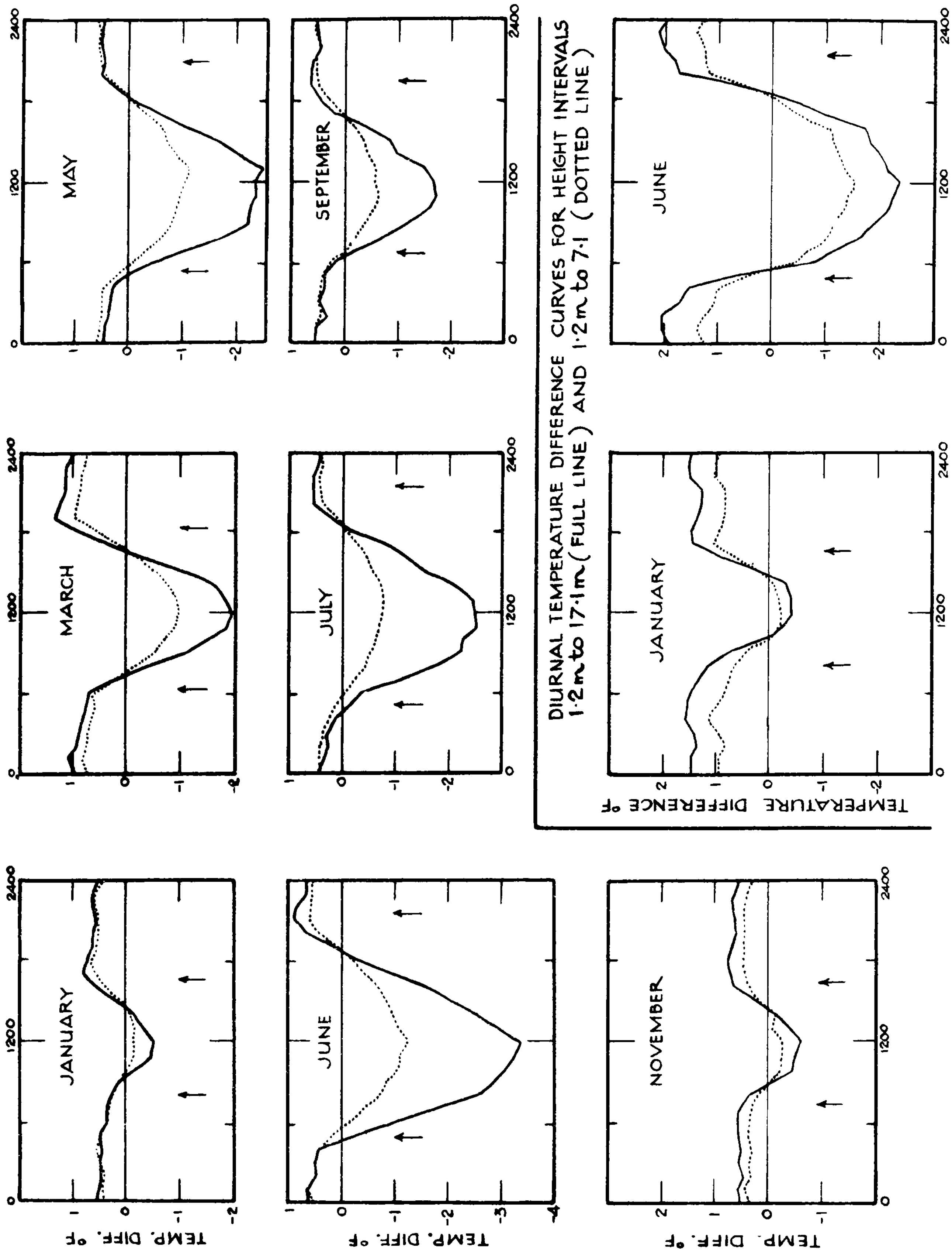
Layer	January 1932 and 1933	June 1932 and 1933
2·5 cm. to 30 cm. (A)	100	625
30 cm. to 1·2 m. (B)	11	78
1·2 m. to 7·1 m. (C)	2	14
7·1 m. to 17·1 m. (D)	1	5

The criterion as to the stability of the atmosphere under certain conditions is frequently expressed by giving the lapse rate as a multiple of the dry adiabatic lapse rate. For this reason the lapse rates in Table II have been expressed in terms of the dry adiabatic lapse rate as a unit.

A comparison between the curves of Fig. 4 and the corresponding curves from Johnson's memoir shews that although the midday lapse for layer A is usually greater than for layer CD, yet the nocturnal inversion for the lower layer does not reach the same magnitude as for the higher layer. This suggests that the rate of change of temperature gradient with height is much more rapid under lapse conditions than under inversion conditions.

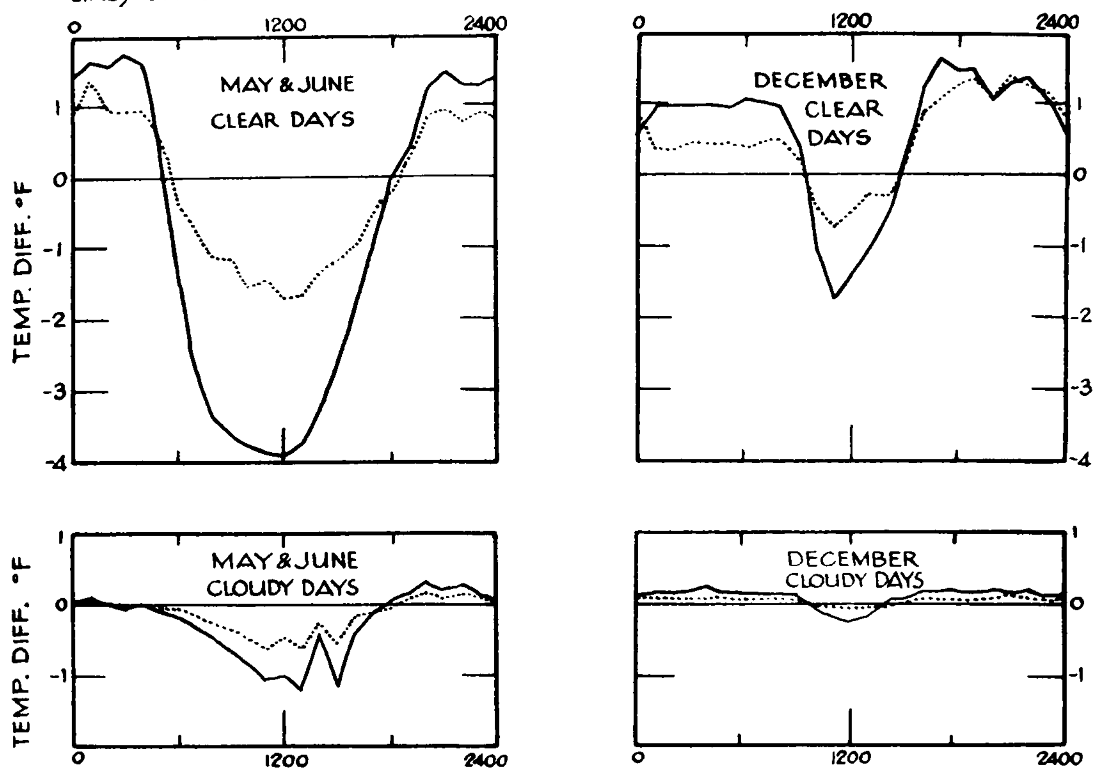
Time of crossover.—Twice during every twenty-four hours the temperature difference over each layer changes sign, shortly after sunrise and shortly before sunset. The curves given in Fig. 4 shew that there is a slight tendency for the crossover (i.e. the change of sign in the temperature difference) in layer B to take place after the crossover in layer A both in the morning and evening, though this tendency is much more marked in the morning than in the evening. The time of crossover must be governed by the time of sunrise (or sunset). The monthly mean curves in Fig. 4 have been examined with a view to finding how the time interval between sunrise or sunset and the crossover varies with the month. Since only two years' results are available, the scatter of points is rather large and only general conclusions can be given. It appears that for both layers A and B in both morning and evening this time interval is less in summer than in winter. This feature is most strongly marked in the variation of the time of morning crossover for layer A. Table III shews the time of crossover for each layer relative to sunrise or sunset for winter and summer. The winter figure is the mean time interval for the four months January, February, November and December, and the summer value was obtained from the months May, June, July and August. The time intervals for the individual months were formed by finding the interval between the time of crossover in Fig. 4 and the time of sunrise or sunset in the middle of the month.

FIG. 4—MEAN DIURNAL CURVES SHEWING TEMPERATURE DIFFERENCES OVER HEIGHT INTERVALS 2·5 cm to 30 cm (FULL LINE) 30 cm to 1·2 m (DOTTED ")



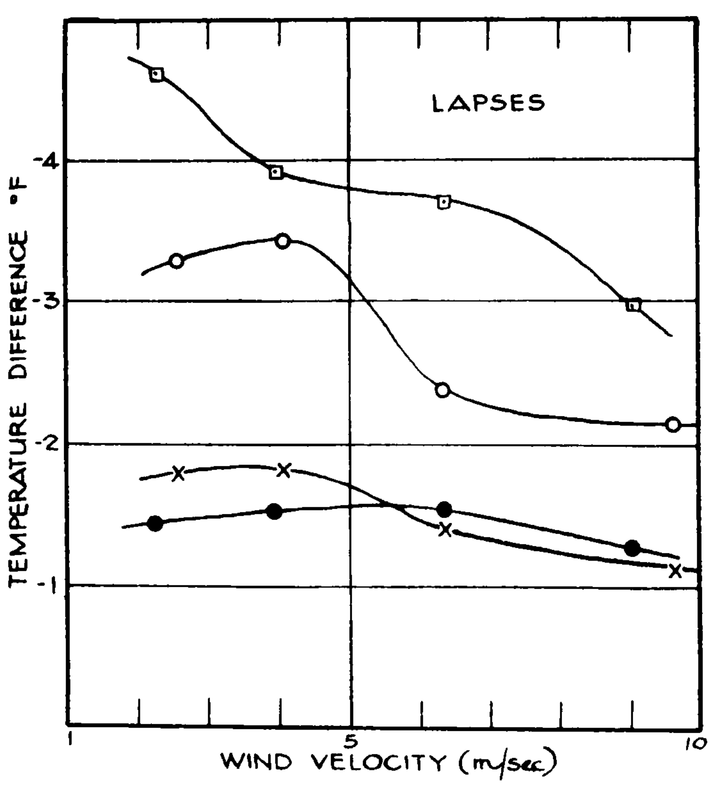
NOTE:—ARROW HEADS INDICATE TIMES OF SUNRISE AND SUNSET FOR MIDDLE DAY OF MONTH. TIMES ARE G.M.T.

FIG. 5—MEAN DIURNAL CURVES SHEWING TEMPERATURE DIFFERENCES OVER THE HEIGHT INTERVALS 2·5 cm. to 30 cm. (Full Line) & 30 cm. to 1·2 m. (Dotted Line) ON CLEAR & CLOUDY DAYS IN SUMMER & WINTER



Times are G.M.T.

FIG. 6.—TEMPERATURE DIFFERENCES NEAR MIDDAY & MIDNIGHT FOR CLEAR SKIES PLOTTED AGAINST WIND VELOCITY



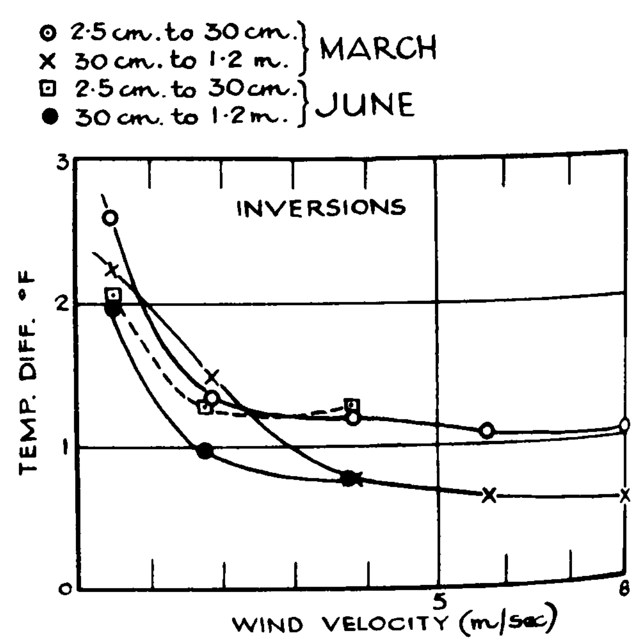


TABLE III.—TIME OF CROSSOVER

	Morning Crossover Minutes after Sunrise		Evening Crossover Minutes before Sunset	
Season.. ..	Layer A (2·5 cm. to 30 cm.)	Layer B (30 cm. to 1·2 m.)	Layer A (2·5 cm. to 30 cm.)	Layer B (30 cm. to 1·2 m.)
	min.	min.	min.	min.
Winter ..	91	96	115	111
Summer ..	35	78	107	98
Annual Mean ..	61	84	106	103

It will be noted that the " Annual Mean " figure for layer A for the evening crossover is less than the summer value. Since the variation from winter to summer is small this is a chance effect due to the small number of observations.

Mean monthly and yearly values.—The hourly values of temperature differences given in Table I have been meaned by months and the results are given in Table IV. It will be seen that in layer A the average gradient is negative for six months in the year and in layer B for five months in the year. From the figures given by Johnson the average gradient for layer C was negative for four months and for three months for layer D.

TABLE IV.—MEAN MONTHLY TEMPERATURE DIFFERENCES

Month	Height Interval	
	A (2·5 cm. to 30 cm.)	B (30 cm. to 1·2 m.)
	°F.	°F.
January	0·31	0·30
February	0·30	0·19
March	0·07	0·18
April	−0·35	−0·04
May	−0·70	−0·15
June	−0·94	−0·20
July	−0·73	−0·11
August	−0·55	−0·12
September	−0·20	0·08
October	0·16	0·28
November	0·36	0·25
December	0·46	0·29
Annual Mean ..	−0·15	0·08

NOTE.—The temperature differences corresponding to the dry adiabatic lapse rate are −0·0054°F. for the layer A, (2·5 cm. to 30 cm.) and −0·016°F. for the layer B, (30 cm. to 1·2 m.).

Thus the period during which the average monthly gradient is negative in the region considered becomes smaller as the height increases. The average values for the whole year are given at the foot of Table IV.

§ 4.—TEMPERATURE GRADIENT ON CLEAR AND CLOUDY DAYS

So far temperature gradient has been considered irrespective of other meteorological conditions. The state of sky, however, has a very great influence on temperature gradient and values of the latter on clear and cloudy days in winter and summer will now be discussed. Days on which there was a complete absence of sunshine and starshine trace were included in the cloudy category while days on which these traces were continuous or nearly continuous were counted as " clear " days. The month of December has been taken as representative of winter conditions for both clear and cloudy days, and the clear days in June are taken as typical of clear summer conditions. Unfortunately, during both the months of June considered, there was only one day satisfying the " cloudy " criterion. Accordingly the two days in May which were found to be cloudy have been included. The resulting curves are still irregular but suffice to give an idea of the temperature gradients probable under such conditions.

TABLE V—HOURLY VALUES OF TEMPERATURE DIFFERENCES

Period	Sky	Layer	0100	0200	0300	0400	0500	0600	0700	0800	0900	1000
Dec. 1931 1932	Cloudy	A, 2·5 cm. to 30 cm.	0·13	0·13	0·17	0·21	0·16	0·14	0·12	0·14	0·10	−0·07
		B, 30 cm. to 1·2 m.	0·07	0·07	0·06	0·07	0·08	0·05	0·05	0·04	0·01	−0·03
May and June 1932 1933	Cloudy	A, 2·5 cm. to 30 cm.	0·10	0·03	−0·03	0·00	−0·07	−0·17	−0·30	−0·47	−0·63	−0·83
		B, 30 cm. to 1·2 m.	0·03	0·03	−0·03	0·00	−0·03	−0·03	−0·13	−0·27	−0·37	−0·47
Dec. 1931 1932	Clear	A, 2·5 cm. to 30 cm.	0·97	0·93	0·97	0·97	0·93	1·03	1·00	0·93	0·43	−1·06
		B, 30 cm. to 1·2 m.	0·35	0·35	0·47	0·43	0·43	0·40	0·43	0·47	0·20	−0·48
June 1932 1933	Clear	A, 2·5 cm. to 30 cm.	1·65	1·56	1·77	1·57	0·24	−1·44	−2·66	−3·37	−3·61	−3·80
		B, 30 cm. to 1·2 m.	1·39	0·99	0·93	0·96	0·53	−0·36	−0·76	−1·11	−1·15	−1·54

NOTE.—The temperature differences corresponding to the dry adiabatic lapse rate are −0·0054°F. for the layer A

* These values are in error, see

The results of the above analysis are given in Table V and have been plotted in Fig. 5. The irregularity of the summer cloudy days curve is due to the small number of observations. In the construction of the winter clear day curves two points have been ignored for reasons already mentioned. It will be seen that on cloudy days the maximum lapse rate in layer A varies from 50 times the dry adiabatic lapse rate in winter to about 200 times in summer. In comparison with the mean monthly curves the cloudy day traces are flat and featureless, though for the winter month there is not a great difference in the midday lapses. On clear days in winter the midday lapse rate in layer A reaches a value somewhat greater than 300 times the dry adiabatic lapse rate, and in summer it goes up to the rather high value of 735 times the adiabatic. In connexion with this last value it is of interest to recall that Brunt (2) has suggested that the lapse rate near the ground cannot exceed a certain definite value. This upper limit is given by the formula $100p_w$ times the dry adiabatic lapse rate where p_w is the vapour pressure in millibars and the incoming radiation is 1 cal./cm.^{2} min. The mean vapour pressure at noon and at a height of 1·2 m. for the days used in constructing the clear day curve for June was 12 mb. Thus the appropriate upper limit for the lapse rate, as suggested by Brunt's formula, was 1200 times the dry adiabatic as compared with 735 times found for the mean value for clear summer days.

Effect of wind velocity.—The relation between wind velocity and temperature gradient was the next point examined. Since the presence of cloud introduced a complication, the effect of which it was difficult to assess, attention was confined to those hourly values of the temperature differences which were preceded by at least four hours of clear sky, as indicated by the sunshine or starshine traces.

ON CLEAR AND CLOUDY DAYS IN WINTER AND SUMMER

1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	No. of Days
−0·16	−0·24	−0·17	0·01	0·07	0·19	0·17	0·17	0·16	0·18	0·17	0·19	0·10	0·13	12
−0·06	−0·08	−0·07	−0·03	0·03	0·07	0·09	0·08	0·09	0·08	0·12	0·09	0·07	0·09	
−1·07	−1·00	−1·23	−0·40	−1·17	−0·40	−0·10	0·03	0·17	0·30	0·20	0·27	0·17	0·07	3
−0·60	−0·47	−0·60	−0·23	−0·53	−0·17	−0·10	−0·03	0·07	0·13	0·10	0·13	0·10	0·03	
−1·72	−1·44	+0·74*	−0·54	0·40	1·20	1·66	1·42	1·48	1·02	1·30	1·36	1·08	0·54	5
−0·74	+0·24*	−0·30	−0·30	0·16	0·88	1·08	1·26	1·36	1·06	1·40	1·28	1·18	0·88	
−3·90	−3·96	−3·76	−3·26	−2·63	−1·81	−0·99	−0·01	0·39	1·26	1·50	1·27	1·27	1·41	8
−1·45	−1·73	−1·65	−1·34	−1·19	−0·99	−0·57	−0·27	0·23	0·85	0·93	0·79	0·90	0·83	

(2·5 cm. to 30 cm.) and −0·016°F. for the layer B (30 cm. to 1·2 m.).
§ 2.—*Temperature gradient apparatus, p.* 6.

TABLE VI.—TEMPERATURE DIFFERENCES AND WIND VELOCITY ON CLEAR DAYS

	Lapses (1100, 1200, 1300 G.M.T.) Velocity Range, m./sec.					Inversions (2300, 2400, 0100 G.M.T.) Velocity Range, m./sec.				
	0 to 0·9	1 to 2·9	3 to 4·9	5 to 7·9	>7·9	0 to 0·9	1 to 2·9	3 to 4·9	5 to 7·9	>7·9
March										
No. of Obs.	1	6	13	25	16	11	13	21	18	2
Mean Velocity m./sec.	0·5	2·58	4·05	6·35	9·65	0·5	1·88	3·84	5·75	8·0
Temp. Diff. Layer A °F. ..	−2·70	−3·28	−3·42	−2·38	−2·14	2·59	1·33	1·20	1·09	1·05
Temp. Diff. Layer B. °F. ..	−1·00	−1·80	−1·82	−1·40	−1·12	2·24	1·49	0·77	0·62	0·55
June										
No. of Obs.	0	16	17	20	5	11	16	14	1	0
Mean Velocity m./sec.	—	2·28	3·93	6·37	9·04	0·5	1·78	3·80	5·0	—
Temp. Diff. Layer A °F. ..	—	−4·63	−3·92	−3·72	−2·96	2·06	1·28	1·29	0·60	—
Temp. Diff. Layer B °F. ..	—	−1·44	−1·52	−1·52	−1·26	1·97	0·96	0·76	0·20	—

The 1100, 1200 and 1300 G.M.T. observations which satisfied the above criterion were extracted and grouped according to wind velocity (as measured by a Dines pressure tube anemometer with the head 13·4 m. above the ground). They were then meaned in groups, the three hours being combined. Since the temperature difference varies only slowly in the middle of the day this method is justifiable. The 2300, 2400 and 0100 G.M.T. observations were treated similarly and the results are given in Table VI. Since the months of March and June appeared to have been particularly sunny only these months were examined. Naturally on most days on which one of the hourly observations at 1100, 1200 and 1300 G.M.T. was preceded by four hours of clear sky, the other two observations were also suitable for inclusion and similar considerations apply to the inversion group.

The figures in Table VI have been plotted in Fig. 6, the points corresponding to single observations being omitted. The curves shewing the effect of wind velocity on the inversion are as might be expected, i.e. the inversion drops fairly rapidly as the wind increases to 2 m./sec. and after that increasing wind has but little effect. In the case of the lapses there are no points corresponding to very light winds. The general tendency of the curves indicates that the lapse rate decreases as the wind increases for velocities above about 4 m./sec. Below this velocity three of the four curves shew a slight decrease in the lapse rate as the velocity decreases. The data available do not seem sufficient to be certain of this feature but, if the effect is genuine, it suggests the possibility that at low wind velocities on clear days there is some other factor operating which compensates for the small turbulence due to the low wind velocity. Possibly the convection currents due to the large lapse rates become more effective as mixing agents under these conditions. This should be compared with the variation of velocity gradient with velocity during lapses, treated later.

§ 5.—EXTREME VALUES OF THE TEMPERATURE GRADIENT

The analysis of hourly temperature differences has been examined and the maximum values of both lapse and inversion for each month have been extracted. These values are shewn in Table VII.

TABLE VII.—EXTREME HOURLY VALUES OF TEMPERATURE LAPSE AND INVERSION IN EACH MONTH

	Layer A 2·5 cm. to 30 cm.		Layer B 30 cm. to 1·2 m.		Layer A 2·5 cm. to 30 cm.		Layer B 30 cm. to 1·2 m.	
Month	Max. Lapse	G.M.T.	Max. Lapse	G.M.T.	Max. Inversion	G.M.T.	Max. Inversion	G.M.T.
January	−3·9	1300	−1·7	1100	4·1	1700	3·2	1800
February	−3·2	1100	−1·4	1200	5·2	2000	3·9	0600
March	−5·2	1200	−2·2	1200	7·8	1900	4·1	1900
April	−5·5	1200	−2·4	1100	6·0	2400	4·0	0400
May	−9·7	1400	−2·8	1300	3·1	2300	2·3	2400
June	−7·0	1200	−2·7	1200	3·6	2100	3·0	0100
July	−7·4	1000	−2·1	1100	2·2	0300	2·7	0200
August	−6·2	1300	−2·3	1100	2·9	0100	4·3	2400
September	−5·9	1100	−1·8	1200	2·6	0100	3·2	0200
October	−7·1	1200	−2·0	1200	7·8	1700	6·3	1900
November	−5·1	1200	−4·6	1200	5·7	1800	2·8	1700
December	−3·4	1100	−2·4	1100	4·3	1600	3·1	2100

NOTE.—The temperature differences corresponding to the dry adiabatic lapse rate are −0·0054°F. for the layer A (2·5 cm. to 30 cm.) and −0·016°F. for the layer B (30 cm. to 1·2 m.).

It will be seen that the times of occurrence of the maximum hourly lapses are closely grouped about noon. This, of course, would be expected from a consideration of the monthly mean curves in Fig. 4. The times of occurrence of the maximum hourly inversion do not appear to be closely grouped about any particular hour. The greatest value reached for the mean hourly lapse in layer A was −9·7°F., i.e.

about 1800 times the dry adiabatic lapse. The corresponding figure for layer B was −2·8°F., i.e. about 170 times the dry adiabatic lapse. This maximum hourly lapse of 1800 times the dry adiabatic for layer A can again be compared with the limit given by Brunt's formula. On this occasion the vapour pressure at a height of 1·2 m. was 11·7 mb. and thus Brunt's formula suggests 1170 times the dry adiabatic as the upper limit. Probably the discrepancy between 1800 and 1170 can be largely explained by the intensity of the incoming radiation. In order to get a figure of 1800 from Brunt's formula it would be necessary to assume a value 1·54 cal./cm.^{2} min. for the incoming radiation. Reference to several issues of the *Observatories' Year Book* shews that the noon radiation at Kew has approached this value several times in May (e.g. in May 1927, the maximum value measured was 1·39 cal./cm.^{2} min.). It seems then that Brunt's formula is in reasonable agreement with the facts.

Looking at Table VII as a whole it appears that the maximum inversion in any one month is much greater in spring and autumn than at other seasons of the year and that it is also slightly greater in winter than in summer. The figures for the greatest lapses shew certain irregularities, but the general trend indicates that the greatest hourly lapse in any one month increases from winter to summer.

Details as to wind and sky corresponding to these maximum lapses and inversions have not been included in the table, but an inspection of the available data supports the conclusion arrived at by Johnson, viz. that the maximum inversions occur with a clear sky and little or no wind and that the conditions necessary for a large lapse are similar except that the wind may reach a moderate velocity.

It has already been explained that the hourly values discussed above are mean values over twenty minute periods centred at the hour. Each trace consists of a sequence of dots made at intervals of one minute and, following Johnson, it has been considered advisable to include Table VIII, indicating the greatest excursion made by an individual dot on both the lapse and the inversion side. These values have been termed the " Absolute Extremes " by Johnson and this term will be retained here.

TABLE VIII.—ABSOLUTE EXTREME VALUES OF INSTANTANEOUS TEMPERATURE LAPSE AND INVERSION IN EACH MONTH

	Layer A 2·5 cm. to 30 cm.		Layer B 30 cm. to 1·2 m.		Layer A 2·5 cm. to 30 cm.		Layer B 30 cm. to 1·2 m.	
Month	Max. Lapse	G.M.T.	Max. Lapse	G.M.T.	Max. Inversion	G.M.T.	Max. Inversion	G.M.T.
January	− 4·9	1120	−2·0	1045	5·2	1705	5·1	2335
February	− 5·8	1150	−2·1	1220	6·0	1745	4·5	2115
March	− 6·5	1050	−3·0	1035	11·0	2125	5·4	2005
April	− 7·1	1045	−3·7	1200	6·8	0340	5·2	0550
May	−10·8	0955	−3·5	1350	4·8	0145	4·2	0140
June	− 8·0	1205	−3·5	1140	5·7	2210	4·7	0045
July	− 8·4	1010	−2·7	1100	3·0	0045	4·2	0040
August	− 8·6	1230	−3·1	1230	4·6	0140	5·8	0200
September	− 7·8	1110	−2·5	1135	5·2	0115	6·0	0125
October	− 8·3	1230	−3·4	1125	8·6	1740	8·6	1850
November	− 5·6	1205	−1·7	1100	7·5	1750	4·7	2120
December	− 5·1	1210	−2·3	1405	5·5	1600	4·8	2140

NOTE.—The temperature differences corresponding to the dry adiabatic lapse rate are −0·0054°F. for the layer A (2·5 cm. to 30 cm.) and −0·016°F. for the layer B (30 cm. to 1·2 m.).

With the exception of the magnitudes of the temperature gradients, Table VIII giving absolute extremes is very similar in its main characteristics to the table of extreme hourly values. The values given in the former table are, of course, somewhat greater than those in the latter. The greatest inversion recorded in the table of absolute extremes is 11·0°F. for the lower interval. This dot was on the extreme edge of the chart and it is just possible that on that occasion the inversion may have exceeded this value temporarily. The greatest lapses recorded for layers A and B are about 2000 times and 230 times the dry adiabatic lapse rate respectively. These values cannot be compared with Brunt's formula since they usually represent the top of a peak on the trace and so correspond to unstable conditions.

§ 6—SOME TEMPERATURE HEIGHT CURVES

As has already been pointed out the variation of temperature with height is much more marked on clear days than when the sky is clouded over. Also during the lapse period the variation is more marked in summer than in winter. Accordingly the eight clear days utilised to give the temperature differences for clear days in June in Table V have been selected to provide some temperature height curves. For this purpose the temperature at 1·2 m. and the temperature differences over the layers C and CD have been included, thus giving five points on each curve. The curves are shewn in Fig. 7 and are divided into four groups.

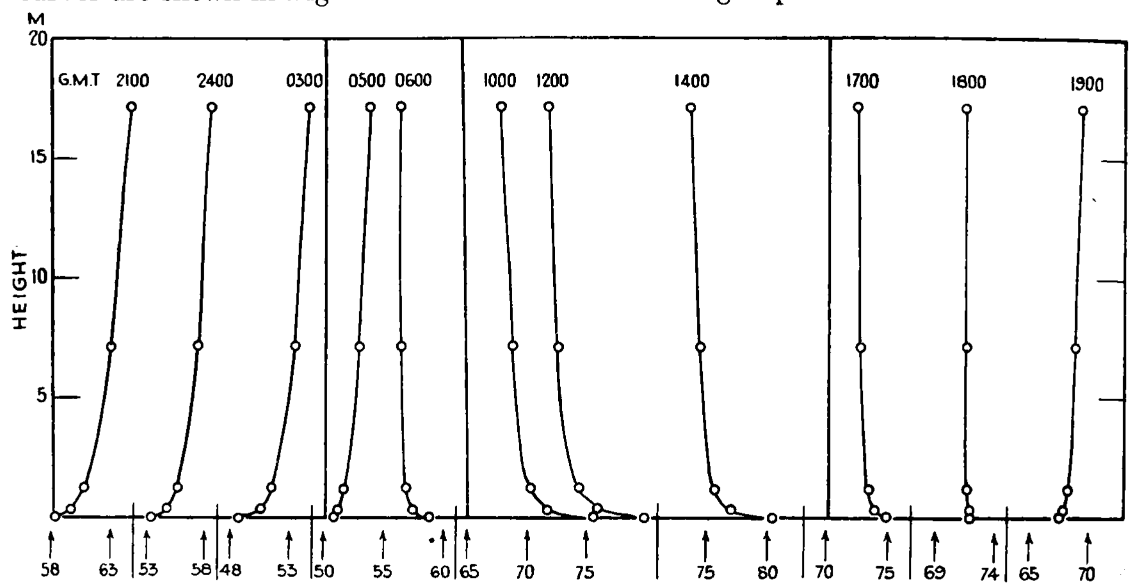


FIG. 7.—HOURLY TEMPERATURE—HEIGHT CURVES. CLEAR DAYS IN JUNE.

The first group shews the temperature variation at three different hours during the period of inversion. These three are very similar except for the progressive fall of temperature from 2100 to 0300 G.M.T. Each one shews the marked decrease of temperature gradient as the height increases.

The next group shews the conditions obtaining just before and just after the temperature gradient changes from inversion to lapse.

In the third group three curves are given shewing the temperature distribution under lapse conditions. These three again exhibit similar characteristics. The very rapid change of temperature gradient with height near the surface is well marked and is clearly greater than in the first group.

The last group is similar to the second group with the order reversed except that the "crossover" from lapse to inversion occurred very near 1800 G.M.T. and accordingly three curves have been included. It can be seen that the 1800 curve shews zero gradient over the lowest interval while a lapse still exists above.

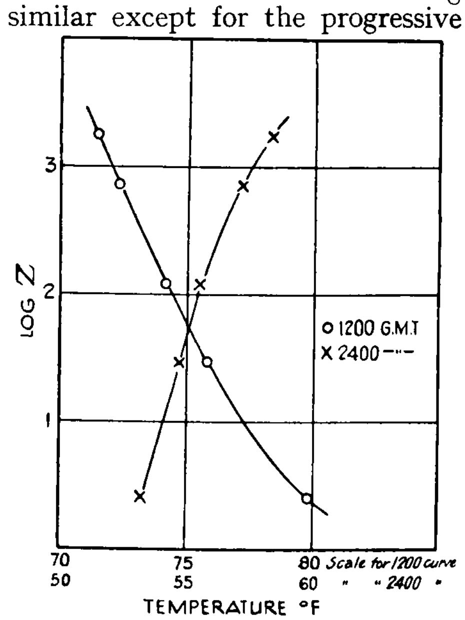


FIG. 8.—TEMPERATURE AS A FUNCTION OF LOG z. CLEAR DAYS IN JUNE.

The observations upon which the first and third groups in Fig. 7 are based have been examined to see whether there is any simple law which will express the relation between temperature and height. There does not appear to be any simple expression which will do this adequately over the whole range of height considered. However, if temperature is plotted against log z, where z represents height, for the first and third groups it may be seen that during the period 1000–1400 G.M.T. temperature is approximately a linear function of log z from 30 cm. to 17·1 m., and that during the inversion period temperature is approximately a linear function of log z from 2·5 cm. to 1·2 m. but above this height the temperature increases more rapidly. This is illustrated in Fig. 8, where temperature is plotted against log z for 1200 G.M.T. and 2400 G.M.T. These two curves are typical of the curves in their respective groups.

§ 7—TIME OF MAXIMUM TEMPERATURE AT VARIOUS HEIGHTS

Considerable interest attaches to the variation of the time of maximum temperature with height, and since the actual temperature at 1·2 m. was being recorded as well as the temperature differences over various intervals it was decided to attempt to investigate this question.

In connexion with this it would be useful to know the time of maximum temperature at the surface. Unfortunately, the recording of the surface temperature of the earth is a matter of considerable difficulty and no suitable apparatus was available. Inasmuch as the actual surface temperature was not required, but only the time at which it reached a maximum, it was decided to record the temperature of a blackened copper plate resting on the surface of the earth. The time at which the surface temperature reaches a maximum will be the time at which the heat received by the surface is equal to the heat lost and since the rates of loss of heat for a close cropped grass surface and a blackened copper plate may be different it is clear that the plate and the grass may not reach a maximum temperature at the same time. Although no figures were available to establish a connexion between the plate and the grass surface it was considered that the times of maximum temperature for these two surfaces would not be widely different. Nevertheless the records from the surface plate must be utilised only with great caution.

It is quite useless attempting to find the variation in the time of maximum temperature at the low heights under consideration on occasions when the sky is partially clouded since the masking and unmasking of the sun produces an irregular temperature time curve with no well defined maximum. Inspection of the monthly mean curves in Fig. 4 shews that almost all of them have obvious irregularities due, probably, to cloud. Thus taking mean curves for a period of two months is insufficient to provide the smooth curve necessary for finding a time of maximum temperature. Accordingly clear day records have been selected by examining the charts individually in conjunction with the sunshine records from a Campbell Stokes sunshine recorder. The criterion for a clear day adopted for this work was a continuous, or almost continuous, sunshine trace from 0700 to 1600 G.M.T. Unfortunately such days are rare. The most fruitful period was March 1933, during which there were nine suitable days. During March 1932 also, there were four days with the requisite weather. On several other occasions the day appeared to have been clear when judged upon the sunshine record only, but the temperature trace for 1·2 m. shewed various irregularities. On other occasions there would be but one suitable day in a particular month. Isolated days such as these were of little use.

Having selected the charts suitable for this part of the work the following analysis was carried out. The mean value over twenty minutes, centred on successive half hours, was found for each trace (1·2 m. temperature, various temperature difference traces, and the plate trace) for each day between 1100 and 1700 G.M.T. These values were then averaged for the nine clear days in March 1933 and for the four clear days in March 1932. From these figures temperature time curves were constructed for the plate and for the heights 2·5 cm., 30 cm., 1·2 m., 7·1 m. and 17·1 m.

The temperature time curves for the clear days in March 1933 are given in Fig. 9. It will be seen that the curves for those clear days are all fairly satisfactory except that for 7·1 m. At this height there are three points at the same temperature. There was no obvious explanation of this, so one of the points had to be neglected in order to get a reasonable curve. Since the time of maximum temperature at 7·1 m. must lie between the corresponding times for 1·2 m. and 17·1 m. the point for 1530 G.M.T. was neglected. This makes the time of maximum temperature at 7·1 m. somewhat doubtful. The curves for the clear days in March 1932 were similar in general form to those for March 1933, though not so regular.

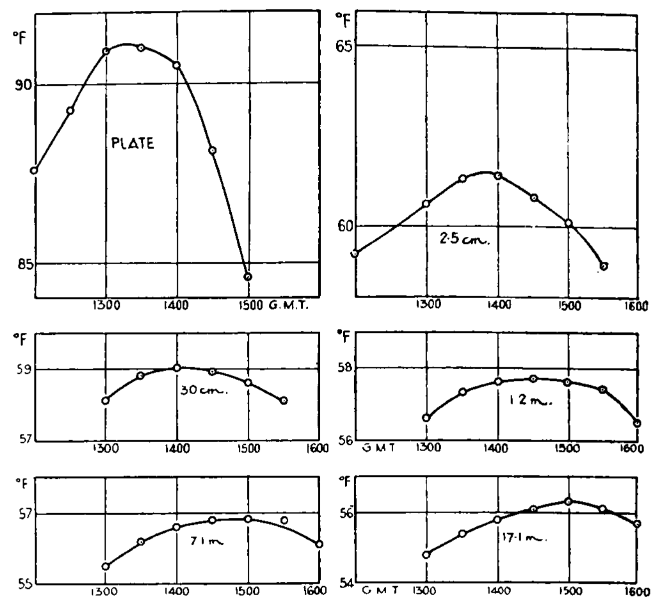


FIG. 9.—TEMPERATURE—TIME CURVES AT VARIOUS HEIGHTS NEAR TIME OF MAXIMUM TEMPERATURE. CLEAR DAYS IN MARCH, 1933.

From these temperature time curves the time of maximum temperature has been read off and the results are given in Table IX. Since the surface plate was not installed until August 1932, it was possible to get a result from this only in March 1933.

TABLE IX. TIMES OF MAXIMUM TEMPERATURE AT VARIOUS HEIGHTS

Height	Four Clear Days March, 1932	Nine Clear Days March, 1933
	G.M.T.	G.M.T.
Plate	—	1317
2·5 cm.	1403	1348
30 cm.	1424	1405
1·2 m.	1445	1430
7·1 m.	1518	(1445)*
17·1 m.	1533	1500

* This time is somewhat doubtful, see above.

Although no theoretical discussion of turbulence is included here, it is desirable to bear in mind the difficulties associated with this branch of meteorology. Some of these difficulties are connected with the solution of the differential equations. These equations are frequently most tractable when the expressions involved are powers of the variables, and a considerable amount of work has been done in which it has been assumed that various entities in the atmosphere vary as a power of the height. Thus it is of interest to see how closely any set of observations can be represented by a power law.

As an example of the use of power laws some results due to Köhler (3) will be quoted. Köhler starts from the assumption that the " Austausch " coefficient A can be represented by $A=A_1 z^{p/(p+1)}$ where A_1 and p are constants. If the variation of the density of the air with height is neglected this is equivalent to assuming that K, the coefficient of eddy conductivity (see § 9 p. 27) varies as $z^{p/(p+1)}$. From this starting point Köhler deduces an expression for the temperature distribution which can be written

$$T=M.\, z^{\frac{2-q}{4(q-1)}} \exp\left(-a.\, z^{\frac{q}{2(q-1)}}\right).\ \cos\left(nt-az^{\frac{q}{2(q-1)}}+b\right) \ldots\ldots\ldots (\text{i})$$

where t is time, $q=p+2$, and M, a, b and n are various constants.

It is clear that according to equation (i) above the time of maximum temperature at height z is given by

$$t = \frac{a}{n} z^{\frac{q}{2(q-1)}} - \frac{b}{n} \ldots\ldots\ldots\text{(ii)}$$

The figures given in Table IX have been tested to see how closely they can be represented by a relation such as equation (ii). Unfortunately, without knowing the value of $-b/n$, which is the time of maximum temperature at the surface, there appears to be no way of plotting the figures in Table IX which will permit a direct measurement of p. For reasons already given it was undesirable to assume $-b/n$ to be the same as the time of maximum temperature of the plate. Accordingly the following expedient was adopted. If t_1, t_2 and t_3, are the times of maximum temperature at heights z_1, z_2 and z_3, then from equation (ii) above

$$\frac{t_3-t_2}{t_2-t_1} = \frac{z_3^{\alpha}-z_2^{\alpha}}{z_2^{\alpha}-z_1^{\alpha}} \text{ where } \alpha = \frac{q}{2(q-1)}$$

The function on the right hand side of the above equation has been calculated for various values of α and for each combination of three heights. These values were then compared with the appropriate value of $\frac{t_3-t_2}{t_2-t_1}$ for the clear days in March 1933. As a result of testing different values of α in this way it appeared that the value 0·19 might be suitable. In Fig. 10 the times in Table IX have been plotted against $z^{0\cdot19}$. It will be seen that each set of points gives a reasonably good straight line. The 1·2 m. point for the clear days in March 1933 is somewhat off the line, but even here the discrepancy is only about 10 minutes.

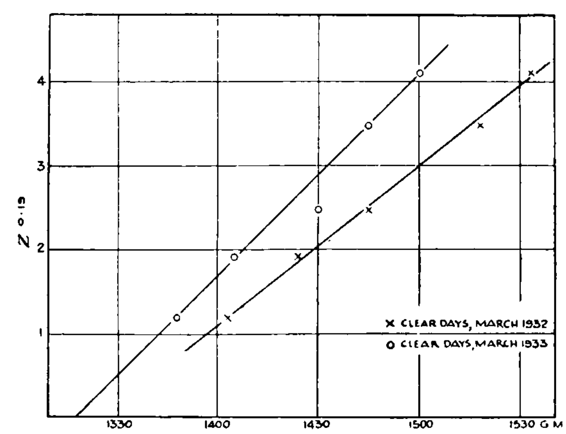


FIG. 10.—VARIATION OF TIME OF MAXIMUM TEMPERATURE WITH HEIGHT.

It is worthy of note that the line corresponding to the clear days in March 1933 cuts the time axis at 1318 G.M.T.; the time of maximum temperature of the plate was found to be 1317 G.M.T. It must be emphasised that the times of maximum temperature given in Table IX are liable to errors of a few minutes solely due to the difficulty in fixing the exact peaks of the curves in Fig. 9. Since the differences between these times are themselves small a few minutes' error might be important. Accordingly, the result given above, that the time of maximum temperature is a linear function of $z^{0\cdot19}$ must be regarded as approximate only. Thus from equation (i) the variation of time of maximum temperature with height may be deduced if $\frac{q}{2(q-1)} = 0\cdot19$. This gives a value of $-0\cdot61$ for q and hence $p = -2\cdot61$. Köhler assumed K to vary as $z^{p/(p+1)}$ as his starting point. Putting in the value $-2\cdot61$ for p, it appears that K must vary as $z^{1\cdot62}$. This point will be examined further in § 9, p. 27.

It may be noted here that the observations used above to test Köhler's theory are all confined to a small part of the day during which the temperature gradient does not vary much. It has been pointed out by several writers, e.g. Sutton (4), that the influence of temperature gradient must be taken into account when the effects of turbulence are considered and it appears that the index p must vary considerably with temperature gradient. Köhler's treatment makes no allowance for this and so it would be useless attempting to test the theory by means of observations which extend over a period of the day during which the temperature gradient varies appreciably.

§ 8—RANGE OF TEMPERATURE AT VARIOUS HEIGHTS

At low heights, such as are considered in the present paper, the temperature usually reaches a minimum near the time of sunrise and a maximum in the afternoon. The range of temperature is, of course, greatest near the surface. The manner in which this range of temperature varies with height will now be examined.

Cloud tends to decrease the range of temperature at any fixed height and variations in cloud during the day may make comparisons of temperature range at any fixed heights difficult, if not unjustifiable. There are two ways of overcoming this difficulty. Either clear days only can be considered, or the mean range of temperature over a long period can be examined, thus smoothing out irregularities due to cloud. Both these methods have been adopted here.

For this part of the work a clear day has been defined as a day on which the night-sky camera record and the sunshine trace were practically continuous from 0000 to 1700 G.M.T. and since observations based upon single days are of little value all clear days during one calendar month have been combined. As has been mentioned before, such clear days are very rare. Examination of the records shewed the nine clear days in March 1933 used in § 7 satisfied the criteria mentioned above, and by combining June 1932 with June 1933 another group of eight clear days was found. The four clear days in March 1932 used in § 7 did not all satisfy the present criteria. Since small irregularities in the 1·2 m. temperature trace were unimportant for this part of the investigation, some other days in March 1932 were considered suitable and a group of six days was eventually found for this month. For reasons to be given later, the two groups of clear days in March 1932 and 1933 have been kept separate.

In order to obtain figures relating to " average " conditions as distinct from clear days the mean ranges of temperature for each height were obtained for the months of December, March and June. Since temperature differences were recorded over two years each of these figures is, therefore, the mean figure for two months. Diurnal temperature curves usually shew that the temperature is changing only slowly for several hours before sunrise and also for about two hours near the time of maximum temperature. Accordingly it was considered sufficiently accurate to obtain the mean range of temperature at any particular height by forming the mean hourly values of the temperature, over the period considered, and taking the difference between the greatest and least of these hourly values.

The maximum temperature and the range of temperature for each height and period considered are given in Table X.

TABLE X—MEAN MAXIMUM TEMPERATURES AND DIURNAL RANGES

Height	6 Clear Days March, 1932		9 Clear Days March, 1933		8 Clear Days June, 1932 and 1933	
	Max.	Range	Max.	Range	Max.	Range.
	°F.	°F.	°F.	°F.	°F.	°F.
2·5 cm.	49·6	22·5	61·4	30·5	80·3	32·3
30 cm.	47·1	19·2	59·0	26·6	77·0	27·5
1·2 m.	45·8	17·4	57·6	23·9	75·7	25·2
7·1 m.	44·6	14·9	56·8	21·5	74·2	21·5
17·1 m.	43·8	13·4	56·3	20·0	73·5	19·6
s	−0·07		−0·06		−0·06	

	Two Complete Months					
	Dec., 1931 and 1932		Mar., 1932 and 1933		June, 1932 and 1933	
	°F.	°F.	°F.	°F.	°F.	°F.
2·5 cm.	45·6	6·6	52·8	17·4	70·8	21·3
30 cm.	45·4	5·9	51·2	15·1	68·2	18·3
1·2 m.	45·4	5·5	50·5	13·7	67·2	17·0
7·1 m.	45·4	4·9	49·8	12·1	65·9	14·9
17·1 m.	45·1	4·3	49·3	11·0	65·2	13·9
s	−0·04		−0·06		−0·06	

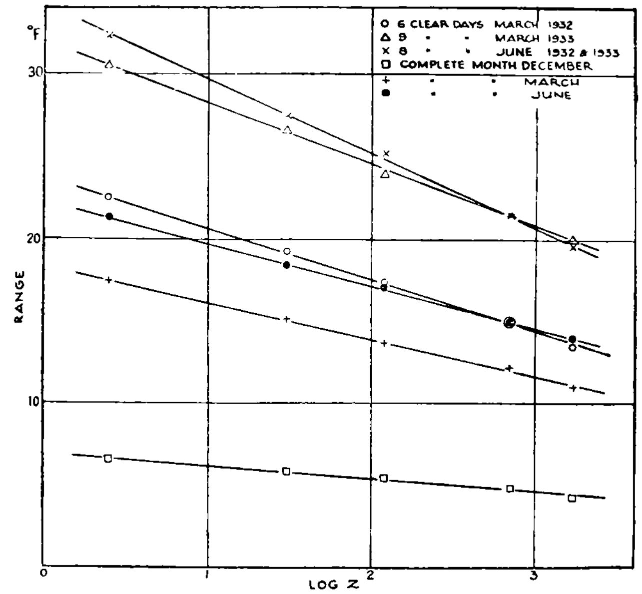


FIG. 11.—DIURNAL RANGE OF TEMPERATURE AS A FUNCTION OF LOG z.

In Fig. 11 diurnal range of temperature has been plotted against log z. Each set of points gives a fairly good straight line shewing that the range of temperature decreases as a linear function of log z. If log (Range) is plotted against log z it will be seen that although the resulting graphs are slightly curved yet a straight line can be drawn in every case to be a reasonably good fit to the points from 2·5 cm. to 1·2 m. This is of interest as it indicates that over this height interval the range of temperature R can be represented as a function of the height z by an equation of the form

$$R = Bz^s$$

where B and s are constants. The values of R in Table X have been plotted in this way and the resulting values of s are given at the foot of Table X. It must be remembered that these values of s are valid only over the height interval 2·5 cm. to 1·2 m. and that in any case R can be more accurately represented by a linear function of log z.

§ 9—THE COEFFICIENT OF EDDY CONDUCTIVITY

The equation for the conduction of heat in the atmosphere is given by Brunt (5) as

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} K \left(\frac{\partial T}{\partial z} + \beta\right) \ldots\ldots\ldots \text{(iii)}$$

where T = absolute temperature, t = time, z = height, β = the dry adiabatic lapse rate and K = the coefficient of eddy conductivity. Although it seems probable that K is not independent of z in the lower layers of the atmosphere it is useful to assume that K is constant over certain shallow layers and hence obtain values of K for these various layers. If K is assumed to be constant, equation (iii) become

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} \ldots\ldots\ldots \text{(iv)}$$

A solution of equation (iv), which is periodic in time and becomes zero at an infinite height, is given by

$$T = A_1 \exp(-b_1 z) \sin\left(\frac{2\pi t}{P_1} - b_1 z + \alpha_1\right) + A_2 \exp(-b_2 z) \sin\left(\frac{2\pi t}{P_2} - b_2 z + \alpha_2\right) \ldots\ldots\ldots \text{(v)}$$

subject to the conditions $b_1^2 = \pi/P_1K$, and $b_2^2 = \pi/P_2K$..........(vi)

The quantities A_1, A_2, b_1, b_2, α_1, α_2, P_1 and P_2 are all constants and from the physical nature of the problem it is clear that when the diurnal variation of temperature is under consideration the period P_1 must be one day, and P_2 must be a half-day. The right hand side of equation (v) can obviously be extended to include as many similar terms as are desirable.

From the experimental data giving the diurnal variation of temperature at one particular height, the variable part of the temperature can be expressed as a Fourier series of the form

$$T = c_1 \sin(t + \phi_1) + c_2 \sin(2t + \phi_2) + \ldots\ldots\ldots \text{(vii)}$$

A different series is, of course, required for each height. In practice the amplitude of the terms of the series (vii) decreases fairly rapidly and T can be adequately represented by a trigonometrical series containing about three or four terms. If then the solution (v) to equation (iv) is also limited to the same number of terms

T is given by two trigonometrical polynomials having the same number of terms. Accordingly the two polynomials can be identified term by term,

$$\text{i.e.} \qquad A_r \exp(-b_r z) \sin\left(\frac{2\pi t}{P_r} - b_r z + \alpha_r\right) = c_r \sin(rt + \phi_r)$$

$$\therefore A_r \exp(-b_r z) = c_r$$

If the two heights z_1 and z_2 are considered

$$\exp(-b_r(z_1 - z_2)) = (c_r)_{z_1} / (c_r)_{z_2}$$

$$\text{and } b_r = \frac{\log_e\left[(c_r)_{z_1} / (c_r)_{z_2}\right]}{(z_2 - z_1)} \quad \ldots\ldots\ldots\text{(viii)}$$

but since from equation (vi) $b_r^2 = \pi / P_r K$, equation (viii) provides a means of determining K.

A value for K can also be obtained by comparing the phases of corresponding terms in the two Fourier series for heights z_1, and z_2. Thus, the time of maximum value of the r^{th} term of (v) is given by

$$\frac{2\pi t}{P_r} - b_r z + \alpha_r = \frac{\pi}{2}$$

Hence if t_2 and t_1 are the times for heights z_2 and z_1

$$\frac{2\pi}{P_r}(t_2 - t_1) = b_r(z_2 - z_1)$$

$$\text{and } b_r = \frac{2\pi}{P_r}(t_2 - t_1) / (z_2 - z_1) \quad \ldots\ldots\ldots\text{(ix)}$$

Numerical values for t_2 and t_1 can be obtained from the appropriate Fourier series. Then combining (ix) with (vi), values for K can be obtained.

The above analysis makes no assumption as to the form of the diurnal temperature curve at any height. It should be noticed that if all the terms of the Fourier series except the first can be neglected then the temperature curve at any height becomes a pure sine wave, $2c_1$ becomes the diurnal range of temperature, and the time of maximum value of the first term becomes the time of maximum temperature.

Diurnal variation of temperature expressed as a Fourier series.—The constants in the first two terms of Fourier series to fit the data giving the diurnal variation of temperature at each height for the months of December, March and June have been found. These are given in Table XI. Thus the temperature is represented by the equation

$$T = c_1 \sin(x + \phi_1) + c_2 \sin(2x + \phi_2) \quad \ldots\ldots\ldots\text{(x)}$$

where x = time after midnight (x is measured in degrees and 15° = 1hr.),
ϕ_1 and ϕ_2 are given in degrees (angular) and c_1 and c_2 are given in degrees Fahrenheit.

TABLE XI—COEFFICIENTS IN FOURIER SERIES FOR DIURNAL TEMPERATURE CURVES

Month	Height	c_1	ϕ_1	c_2	ϕ_2
		°F.	°F.	°F.	°F.
December ..	2·5 cm.	2·46	243·9	1·52	59·2
	30 cm.	2·22	237·5	1·29	54·5
	1·2 m.	2·09	232·6	1·16	51·7
	7·1 m.	1·86	224·8	0·97	44·6
	17·1 m.	1·68	218·2	0·83	39·6
March ..	2·5 cm.	8·60	243·7	2·91	62·5
	30 cm.	7·37	237·4	2·33	56·9
	1·2 m.	6·63	232·9	1·92	58·0
	7·1 m.	5·70	225·1	1·40	40·4
	17·1 m.	5·11	219·5	1·43	44·2
June ..	2·5 cm.	10·40	245·5	0·87	108·0
	30 cm.	9·26	238·1	0·52	106·7
	1·2 m.	8·50	234·7	0·44	110·1
	7·1 m.	7·39	227·7	0·50	107·3
	17·1 m.	6·77	222·8	0·55	102·5

The relative importance of the two terms in equation (x) is measured by the relative magnitudes of c_1 and c_2. Thus it can be seen from Table XI that in December the second term is comparable with the first and hence the diurnal temperature curve cannot be represented as a pure sine wave with much accuracy. In March the relative importance of the second term is much less and in June the second term is almost negligibly small compared with the first term. Thus in June the diurnal temperature curve approximates very closely to a pure sine wave, in March the approximation is only fair, and in December it is very poor. These remarks apply, of course, to the question of representing the temperature time curve for the whole twenty-four hours by a sine wave. It should be noted also that although the data used in this harmonic analysis are based upon measurements over two complete months it might be that another month with very different cloud amounts and distributions would yield appreciably different values for the constants.

It is of interest to compare the figures in Table XI with equation (v) of § 9, p. 23 If equation (v) can truly represent the temperature distribution in the atmosphere then the four constants tabulated in Table XI should decrease with increasing height. This does occur in the case of c_1 and ϕ_1 for each month. In the case of the second harmonic term however the agreement is not so good. For the month of December c_2 and ϕ_2 do decrease with increasing height. In March c_2 decreases with increasing height up to 7·1 m. and the variation of ϕ_2 appears to be irregular. In June c_2 decreases with increasing height for the three lowest heights and the variation of ϕ_2 is again irregular.

Thus in December the second harmonic term is comparable in importance with the first and the four constants all decrease with increasing height. It is possible that values of K may be obtained from both the harmonic terms by each method. In March the importance of the second harmonic term is rather small. It appears possible that values for K may be obtained from the first harmonic term by both methods. It is clear that any attempt to evaluate K from the second harmonic term by the phase method would be useless and that the method depending upon the values of c_2 (to be called the " range " method) can be used only below 7·1 m. In June values of K may be obtained from the first harmonic. Again the second harmonic term cannot be used for this purpose except by the range method for heights up to 1·2 m.

To summarise the above : only the more important terms of a Fourier series can be used with any hope of obtaining consistent values for K. The explanation of this probably lies in the fact that equation (iv) represents the temperature variation in the atmosphere only approximately.

It has already been appointed out that if the temperature variation at any one height can be represented approximately by a single sine wave, then the range of temperature is equal to $2c_1$ and the time of maximum temperature is the time of maximum value of the first term of the Fourier series. The extent to which this is justifiable can be estimated from Table XII which gives the ratio of $2c_1$ (from Table XI) to R (from Table X) for the months of March and June.

TABLE XII—RATIO OF $2c_1$ TO R

Height	2·5 cm.	30 cm.	1·2 m.	7·1 m.	17·1 m.
March	0·99	0·98	0·97	0·94	0·93
June	0·98	1·01	1·00	0·99	0·97

It appears then that little error is likely to arise if the ratio of the diurnal ranges at two heights is taken as the ratio of the values of c_1 for these two heights under the conditions postulated above. Accordingly, in the next section values of K have been calculated using the times of maximum temperature on certain clear days from § 7 and the diurnal ranges of temperature on clear days from § 8.

Variation of K with height.—It has been shewn by many other workers that K is not independent of height (6). Nevertheless it is of interest to assume that K is constant over certain shallow layers of the atmosphere and to calculate mean values of K for these layers. This has been done for the layers A, B, C and D. The methods of doing this have been described in the preceding sections and the values obtained are given in Table XIII. The method by which each series of values was obtained is also indicated. In view of the approximations involved it is considered that the values in Table XIII shew reasonably good internal agreement. The following points concerning some of the individual values may be noted.

The rather high value for layer C in series 2 is probably due to the late time of maximum temperature at 1·2 m. for this period (cf. § 7 p. 20). This, of course, implies that the value for layer B is rather low, as in fact it is.

The first five series were all obtained by assuming the temperature curves to be pure sine waves.

The values obtained from the second harmonic term of the Fourier series shew fairly good agreement with those obtained from the first harmonic in the case of December. The discrepancy is greatest in the case of the values depending upon the phase of the second harmonic. For reasons already given it was useless getting more values from the second harmonic term for the other months than those shewn.

TABLE XIII—VALUES OF K IN cm.^{2}/sec.

	Period Examined	Data in Section	Method	Layer			
				A 2·5 cm. to 30 cm.	B 30 cm. to 1·2 m.	C 1·2 m. to 7·1 m.	D 7·1 m. to 17·1 m.
1	Four clear days, March, 1932 ..	7, p. 20	a	3·3	35	610	8500
2	Nine ,, ,, 1933 ..	,,	a	5·0	25	3000	8500
3	Six ,, ,, 1932 ..	8, p. 22	b	1·1	30	520	3200
4	Nine ,, ,, 1933 ..	,,	b	1·4	26	1100	6900
5	Eight ,, June, 1932 and 1933	,,	b	1·0	39	510	4100
6	Two months, Dec., 1931 and 1932 ..	9, p. 24	c	2·6	84	930	3600
7	,, ,, ,, ,, ..	,,	d	2·2	40	680	2700
8	,, ,, ,, ,, ..	,,	e	2·0	52	800	3000
9	,, ,, ,, ,, ..	,,	f	8·2	250	1650	9500
10	,, ,, March, 1932, and 1933..	,,	c	1·2	27	560	3100
11	,, ,, ,, ,, ..	,,	d	2·3	48	680	3800
12	,, ,, ,, ,, ..	,,	e	1·1	16	250	—
13	,, ,, June ,, ..	,,	c	2·1	40	650	4900
14	,, ,, ,, ,, ..	,,	d	1·7	83	850	5000
15	,, ,, ,, ,, ..	,,	e	0·21	21	—	—
16	Mean of Series 1, 2, 3, 4, 5, 6, 7, 10, 11, 13, 14	—	—	2·17	43·4	917	4940

Method a consists of using times of maximum temperature.
,, b ,, ,, ,, diurnal range of temperature.
,, c ,, ,, ,, the amplitude of the first harmonic term in the Fourier series.
,, d ,, ,, ,, the time of maximum value of the first harmonic term in the Fourier series.
,, e ,, ,, ,, the amplitude of the second harmonic term in the Fourier series.
,, f ,, ,, ,, the time of maximum value of the second harmonic term in the Fourier series.

It should be noted that the values obtained by methods a, d and f depend upon the difference between two times both of which occur in the afternoon and accordingly these values may be considered as being appropriate to afternoon conditions. The other series appear to give values appropriate to the average conditions between the time of minimum temperature and the time of maximum temperature. It is probable that during the inversion period K may have quite different values.

Regarding the Table as a whole the outstanding feature is the enormous variation in K with height. The values found for the upper layer are comparable with those usually associated with the lower regions of the atmosphere, but the values found in the lowest layer are remarkably low. Rossi (7) quotes some two hourly observations of temperature, due to Homens, at four heights over a stretch of moorland. These observations covered four days. Some two hourly observations of temperature at eight different heights made by Rossi himself are also given. Values of K for various layers have been calculated by the range method from these observations and are given in Table XIV.

TABLE XIV—SOME VALUES OF K FOR VARIOUS LAYERS

From Homens' Observations		From Rossi's Observations	
Layer	K	Layer	K
0 to 1·0 m.	13	1 cm. to 25 cm.	1·7
1·0 to 2·0 m.	95	25 cm. to 1·0 m.	10
2·0 to 5·0 m.	440	1·0 m. to 2·0 m.	27
		2·0 m. to 4·0 m.	126

The eleven series in Table XIII which do not depend on the second harmonic term of a Fourier series have been meaned and the mean values are given at the foot of the table. In Fig. 12 these values have been used to plot log K against log z. The height of the mid point of the layer has been taken as the value of z corresponding to the appropriate value of K. It can be seen that the resulting graph approximates very closely to a straight line and the best straight line to fit these points has been drawn in. Finally the points based on the observations of Homens and of Rossi have been plotted in the same figure. It can be seen that the figures based on Homens' observations are in remarkable agreement with those obtained from the present work. The figures based on Rossi's observations do not shew such good agreement, though even in this case the agreement is probably as good as could be expected.

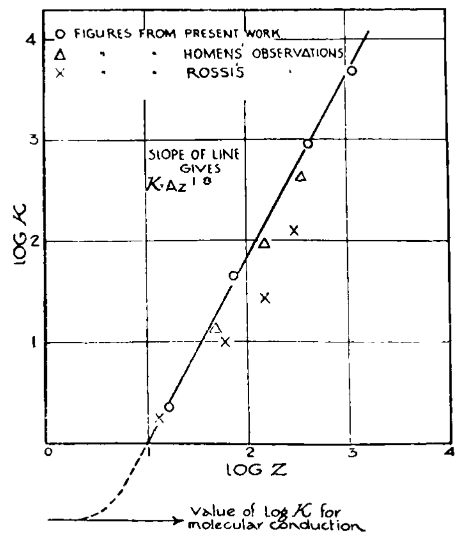


FIG. 12.—VARIATION OF THE COEFFICIENT OF EDDY CONDUCTIVITY WITH HEIGHT.

Measurement of the slope of the line in Fig. 12 suggests that K is connected with z by an equation of the form $K = Az^{p/(p+1)}$ where A is a constant and $p/(p+1)$ has a value of about 1·8. This relation clearly cannot hold good right down to the surface and in any case must be regarded as approximate only. The value 1·8 obtained for $p/(p+1)$ here is in good agreement with the value 1·62 obtained in § 7 p. 21 using Köhler's treatment.

If it is assumed that the coefficient of eddy conductivity is the same as the coefficient of eddy viscosity then Ertel (8) has shewn that the variation of wind velocity V with height is given by $V = V(1)z^{1/(p+1)}$, where V (1) is the velocity at unit height provided $K=Az^{p/(p+1)}$. Putting $p/(p+1) = 1{\cdot}8$ gives $p = -2{\cdot}25$ and hence $V=V(1)z^{-0{\cdot}8}$; an impossible result. Although the value of p used here is approximate only it is improbable that there is sufficient error to account for the absurd index found for the power law for the variation of wind with height, and it seems that the assumption that the mode of variation with height of the coefficient of eddy conductivity is the same as that of the coefficient of eddy viscosity is probably false. In connexion with this it should be noted that Taylor (9) has suggested that eddy motion is characterised by transfer of vorticity rather than transfer of momentum and has shewn that as a consequence temperature distribution in a turbulent fluid is not necessarily the same as the velocity distribution.

If the line in Fig. 12 is extrapolated downwards it indicates that K falls to the value 0·18 at a height of about 4 cm. This value of K, viz. 0·18 is the value corresponding to conduction by molecular processes. Such extrapolation is, perhaps, unjustifiable, but it does suggest the possibility that when the surface of the earth is covered by close cropped grass there is a layer of air having a depth of the order of a few centimetres in which the conduction of heat is by molecular processes.

PART II—THE VERTICAL GRADIENT OF WIND VELOCITY NEAR THE GROUND

§ 10—GENERAL CONSIDERATIONS

During the summer and autumn of 1932 a number of experiments were carried out to determine the vertical gradient of wind velocity near the ground and the extent to which this velocity gradient is influenced by wind velocity and the vertical gradient of temperature. The method adopted consisted of measuring simultaneously the wind velocity at a standard height and at some other height which varied from experiment to experiment. While these velocities were being measured the temperature gradient near the ground was also measured. The standard height was fixed at one metre and the auxiliary heights at which the simultaneous velocity measurements were made were 2·5, 5, 10, 25, 50 and 200 cm., all heights being measured from the actual ground surface.

Since velocity measurements at a height of only 2·5 cm. were contemplated it was necessary to pay particular attention to the ground over which the experiments were carried out. The site selected was a cricket field, all the work being performed on the leeward side of the actual pitch. Thus for a distance of at least 10 m. upwind the height of the grass was only about 0·5 to 1 cm. while beyond this distance the grass was still short. The field was level and the nearest obstruction was a small hut about 100 m. away.

It was thought that to use an ordinary air meter as near to the ground as 2·5 cm. might be open to criticism. Furthermore the velocities experienced at this height are frequently too low to be measured accurately with an ordinary air meter. Accordingly a special hot wire anemometer (to be referred to hereafter as the H.W.A.) was constructed.

The object of the work was to find how the wind velocity varies from 2·5 cm. to 2·0 m., and how this variation is affected by different degrees of thermal stability (as measured by temperature gradient) and by the scale of wind velocity. It was clearly desirable that the temperature gradient should be measured over approximately the same layer of air as the velocity gradient and it would have been possible to arrange the temperature gradient apparatus so that temperature differences were measured between the same two heights as the velocity gradient in each experiment. This, however, would have made it impossible to combine the results to give a picture of the velocity variation over the whole range 2·5 cm. to 2·0 m. for a definite thermal structure. Accordingly it was decided that the temperature difference between the heights 10 cm. and 110 cm. should be measured during every experiment. It must be emphasized that where the temperature gradient results are expressed in °F./m. they give the mean temperature gradient only over the height interval 10 cm. to 110 cm.

During the summer and autumn of 1933 a number of experiments (described in Part IV) on gustiness were carried out. In the course of this work simultaneous measurements were made of the wind velocity at 1 m. and 2 m. and of the vertical temperature gradient between 10 cm. and 110 cm., all measured over grass about 1 cm. to 2 cm. long. In a number of these experiments the velocity at either 25 cm., 50 cm. or 5 m. was also obtained together with the other readings. These results have been used to supplement and extend the results obtained in 1932.

§ 11—INSTRUMENTAL

The temperature gradient apparatus was based on the usual Wheatstone bridge circuit employing two platinum resistance thermometers working differentially as described by Johnson (**1**). The housings for these elements were similar to the housings used for the thermo-electric couples described in the first part of this paper and were mounted on a hollow mast which also served as an aspiration pipe. Aspiration was provided by a small six volt electric motor mounted half way up the mast which caused a flow of air past the elements of about 2·5 m./sec. The housings were so constructed as to be easily interchangeable, complete with element. With portable apparatus of this description it was of course impracticable to use a recording galvanometer, and instead a Cambridge Instrument Co. " Unipivot " galvanometer was used as an indicator of the instantaneous temperature gradient. Calibration of this apparatus for the purpose of determining any zero error was carried out by enclosing the mast, housings and elements in a small chamber 1 m. cube. During these calibrations the air was mixed thoroughly by means of an electric fan in order to ensure an even temperature throughout the chamber and the aspiration motor was also kept running. As a further precaution the housings were interchanged at the beginning of each day's work in the field.

This apparatus was also used during the investigation of gustiness in 1933.

Three air meters were used in the course of the work carried out in 1932. These were of the conventional vane type, the diameter of the vane circle being about 7 cm. Each air meter was fixed on a small direction vane of the R.A.E. type, these direction vanes being mounted on ball bearings on light tubular masts. Thus each air meter was always facing into the wind. Two of these air meters, used at the heights of 1 m. and 2 m., were fixed above the direction vanes. The third air meter was used at various heights down to 5 cm. above the ground and in order to make this possible it was necessary to fix the air meter below the direction vane. The height of an air meter was, of course, measured to the centre of the vane circle.

These air meters had to be calibrated rather indirectly. One of the three was adopted as a standard and the other two were compared with it by taking a number of simultaneous runs, each of one minute duration, with the air meters at the same height and about one to three metres apart. These comparisons were carried out over the whole range of velocity experienced during the investigation. Finally the air meter which had been adopted as a standard was compared with a fourth air meter which had been recently calibrated at the National Physical Laboratory. Thus calibration curves were obtained for each air meter. It was considered that these air meters might not give reliable readings in the fluctuating winds experienced in the open if the mean velocity fell below 1·5 m./sec. and so no reading below this figure obtained from these instruments has been used.

In 1933 the apparatus was augmented by two special low speed air meters (designed by E. Ower and made by Messrs. Short and Mason, Type No. 3140) which were calibrated at the National Physical Laboratory for velocities as low as about 0·3 m./sec. These instruments had a vane circle diameter of about 10 cm. and were fixed above small direction vanes in a similar manner to the others. All readings obtained from these air meters, which were used at 1 m. and 2 m. only, were utilised unless a lull in the wind caused the air meter vanes to stop turning momentarily during the experiment.

The hot wire anemometer.—The hot wire anemometer (H.W.A.) was designed and made for this particular investigation, and since the measurements made with it were afterwards re-analysed in order to examine the gustiness of the wind (see Part III) it is described in some detail here.

The requirements of this instrument were that it should be portable, cover the range of velocity likely to be experienced, and the sensitive part of the anemometer had to be of such a size as to permit velocity measurements within 2·5 cm. of the ground. A further restriction was imposed by the fact that since the velocities to be measured were not steady no null reading method could be used. The circuit finally adopted consisted of a Wheatstone bridge of which one arm was the sensitive element. This element is heated to a high temperature in still air. The wind however cools the element and so alters the resistance. This in turn causes an out-of-balance current in the bridge, the magnitude of which indicates the wind velocity.

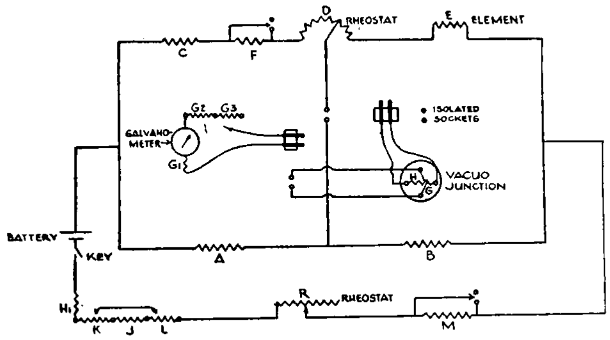


FIG. 13.—HOT WIRE ANEMOMETER CIRCUIT.

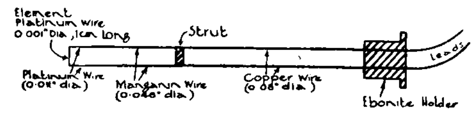


FIG. 14.—DIAGRAM (APPROXIMATELY TO SCALE) OF ELEMENT AND SUPPORTS OF HOT WIRE ANEMOMETER.

The circuit of the H.W.A. is shewn diagramatically in Fig. 13 and the resistances of the various coils, which were made of a suitable resistance wire, are given in Table XV. The sensitive element consists of about 1 cm. of platinum wire of 0·025 mm. diameter welded on to suitable supports and is represented by E in Fig. 13. This element and the supports are shewn in Fig. 14. The galvanometer is a "Unipivot" microammeter and is connected to the bridge part of the circuit by means of a double plug and socket. By using two plugs and three sockets alternative circuits were arranged. Either the galvanometer was connected directly across the bridge or, alternatively, the heater coil of a Cambridge Instrument Co. vacuo-junction replaced the galvanometer and the galvanometer itself was connected to the thermo-electric couple which is in thermal contact with the heater coil. The reason for this is given later. One resistance, G_1, was always in series with the galvanometer, the value of this coil being chosen to give a convenient range of wind velocity over the galvanometer scale. The resistances G_2 and G_3 were used to decrease the galvanometer sensitivity during the initial adjustments. Arrangements were also made whereby resistances F and M could be short circuited as required. In order to get fine adjustments the rheostat R was made of two single coils of S.W.G. No. 26 manganin wire wound on an ebonite disc. This gave a very small total resistance and so the fixed resistances K, J and L, which could be short circuited, were necessary.

The initial adjustments to the instrument were carried out as follows. Resistance M was included in the circuit, thus limiting the current to a very low value. This caused no appreciable heating in E and with F short circuited the bridge was balanced by adjusting the slider on D. During this adjustment the galvanometer, with maximum sensitivity, was connected directly across the bridge, the vacuo-junction being out of use. This adjustment makes allowance for the effect of air temperature and is fully described in Ower's treatise on air flow measurements (10). Having obtained a first balance in this way the resistance F was then included in the circuit. This put the bridge out of balance. On short circuiting M, however, a heavy current flowed through the bridge thus raising the temperature of E. Balance was restored by adjusting the value of this current, the adjustment being carried out by short circuiting K, J and L, or such of them as was necessary, and by positioning the slider on R. During this process the galvanometer was connected directly across the bridge, in its most insensitive condition at first, and finally with G_3 cut out. The purpose of this adjustment was to ensure that the same current flows through E irrespective of the state of the battery and was carried out with E enclosed in a draught-proof case. Finally the heater coil of the vacuo-junction was connected across the bridge and the galvanometer, with G_1 only in series, was connected to the thermo-electric junction. The instrument was then ready for use.

Some approximate figures relating to the H.W.A. are given in Tables XV and XVI. The temperatures were calculated from the resistance of the element.

TABLE XV—RESISTANCES OF COILS IN THE H.W.A.

Coil	Resistance	Coil	Resistance	Coil	Resistance	Coil	Resistance
	ω		ω		ω		ω
A	20	F	5·2	Galvo	8·9	L	1·4
B	20	G(2)	7·8	H(3)	33·9	M	490
C	2·6	G_1	30	H_1	12	R	1·5
D	0·8	G_2	214	J	4·0		
E(1)	2·9	G_3	3070	K	2·7		

(1) Sensitive element of H.W.A.
(2) Thermo-electric couple in Vacuo-Junction.
(3) Heater coil in Vacuo-Junction.

TABLE XVI—SOME CHARACTERISTICS OF THE H.W.A.

Air Speed	Resistance of Element	Temperature of Element	Current through heater of Vacuo-Junction
m./sec.	ω	°C.	milliamps
0·0	8·1	470	0·0
1·5	5·5	260	7·0
3·0	4·9	200	9·2
6·1	4·33	150	11·3
9·1	4·07	120	12·0

Battery Voltage..........12 volts.
Battery Current..........approximately 0·40 amps.

The sensitive part of the element was annealed before use by heating to a dull red colour for a few hours. It was found necessary to wind those coils which carry the heavy current in an open form between two small pillars in order to avoid excessive heating.

Calibration of the H.W.A. was carried out by inserting the element E into a tube and drawing air past it. The sensitive part of the element was positioned in the centre of the tube and was perpendicular to the line of mean flow. The mean rate of flow through the tube was obtained by connecting in series a shaped nozzle having a 15 mm. throat diameter. From this value of the mean rate of flow the axial velocity was calculated by means of the curve shewing the relation between these two quantities in the treatise by Ower referred to above (1927 edition, p. 43, Fig. 18). Since the velocity does not change rapidly near the axis it was assumed that the element E was experiencing a wind with a velocity equal to the axial velocity. Owing to the limited facilities available for getting a high air speed it was necessary to use a small tube (3·6 cm. diameter) for calibration at the higher velocities (3·4 to 15 m./sec.) and a larger tube (8·0 cm. diameter) for calibration at the lower velocities (1·3 to 3·0 m./sec.). It was impracticable to use the small tube for low velocities since at low values of Reynold's number the relation between mean velocity and axial velocity could not be determined accurately from the curve given by Ower referred to previously. The two calibration curves obtained from these two tubes joined up with no apparent discontinuity either in position or slope.

Most of the field work on velocity gradient was done during two periods of about one month separated by an intervening period of about two months. During each working period the H.W.A. was calibrated five or six times. No systematic change in calibration was observed during either set of calibrations, but there was an appreciable difference between the mean calibrations for the two periods. During each period the scatter of calibration points was reasonably small, there being very few points differing from the mean curve by as much as 6 per cent. Accordingly a mean calibration curve for each of the two periods was constructed and used for converting to velocities all the galvanometer readings obtained during that period.

A feature of many hot wire anemometers is their great sensitivity at low velocities. In the present instrument if the galvanometer were connected directly to the bridge the instrument would have a very open scale at the low wind velocities and a very close scale at the higher velocities. This defect has been partially overcome by the use of the vacuo-junction. Since the heating effect of a current is proportional to the square of the current it is clear that the scale will be magnified at the higher velocities. Unfortunately, even with the use of the vacuo-junction, it was not possible to attain the ideal of a straight line calibration curve.

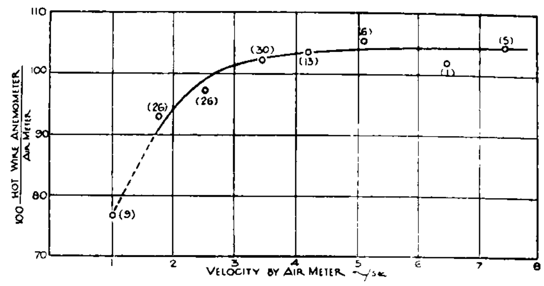


FIG. 15.—COMPARISON OF HOT WIRE ANEMOMETER AND AIR METER.

During the field work the H.W.A. was compared with an ordinary type air meter in a manner to be described later. A large number of these comparisons were made, covering a fairly wide velocity range, and the results have been grouped and meaned over small velocity bands and are shewn in Fig. 15, the figures in brackets denoting the number of observations for each point. It will be seen that for speeds greater than about 4·5 m./sec. the percentage difference between the two instruments is constant with a value of about 4·5 per cent. Below this speed the curve falls off and at very low speeds there is a serious discrepancy. With any vane type air meter used for measuring a natural wind velocity there is a lower limit of velocity below which the readings are unreliable. It is considered that with the instruments used in these comparisons this lower limit is about 1·5 m./sec. At this speed the discrepancy between the air meter and the H.W.A. is about 10 per cent.

The method of calibrating the H.W.A. involved the values of various coefficients which have been determined empirically and it is possible that on account of this fact the calibration of the H.W.A. may be in error by a small constant amount (about 4·5 per cent.) over the whole range of velocity ; there seemed no reason why this percentage error should vary at the lower speeds. It is considered that the departure of the curve in Fig. 15 from a horizontal straight line at the lower velocities is due to the air meters overestimating a gusty wind, this percentage overestimation increasing with increasing wind velocity. This characteristic of vane type air meters is discussed by Ower (loc. cit.). Accordingly it has been assumed that the air meters were in error and all air meter readings have been corrected by means of the curve in Fig. 15. This procedure may mean that all velocities have been overestimated by a constant amount of about 4·5 per cent. Such an error is, however, quite unimportant since it applies equally to all velocities. If it had been assumed that the air meters were correct and all H.W.A. readings had been corrected by means of the curve in Fig. 15, the final results would have been affected only for those cases where two simultaneous velocities, differing widely and both low, were compared, and even in these cases the effect would be small. Accordingly, when examining the results obtained in 1933 (for heights of from 25 cm. to 5 m.) by means of air meters only, this effect of gustiness upon the air meter readings has been ignored, since simultaneous readings at different heights did not differ sufficiently to make the effect appreciable.

§ 12—GENERAL PROCEDURE

The aim of every experiment was to measure the wind velocity during a three minute period at a height of 1 m. and simultaneously at one other of the auxiliary heights z. During the three minute period the temperature gradient was also measured. The velocity at 1 m. height was measured by means of an air meter and it was found possible to operate a second air meter at 2 m. height simultaneously. In general the following method was adopted. Having chosen a site on the leeward edge of the cricket pitch two small tubular masts were erected. On one was mounted an air meter at a height of 1 m. ; on the second were two air meters, one at a height of 2 m. and the other at the auxiliary height z. All the air meters were fixed on direction vanes. The element of the H.W.A. was mounted on a retort stand at height z with the sensitive part vertical and the support pointing into the wind. The temperature gradient apparatus was also erected nearby. The three air meters were run concurrently for three minute periods and during these periods the galvanometers of the H.W.A. and the temperature gradient apparatus were read every ten seconds, thus giving nineteen readings on each galvanometer. From these two sets of readings the mean velocity at height z and the mean temperature gradient were obtained. Thus each experiment provided the following information :—

V (100) = Mean velocity at 1 m. by air meter.
V (200) = ,, ,, ,, 2 m. ,, ,, ,,
V (z) = ,, ,, ,, z ,, ,, ,,
V'(z) = ,, ,, ,, z ,, H.W.A.
Mean temperature gradient between 10 cm. and 110 cm.

During different experiments z took the values 2·5, 5, 10, 25 and 50 cm. Obviously it was not possible to get the centre of an air meter as low as 2·5 cm. and all observations at this height were provided by the H.W.A. only. At the other auxiliary heights both the air meter and the H.W.A. were utilised, as some doubt was entertained as to how near the ground air meter readings were valid. It may be stated here, however, that the difference between the air meter and the H.W.A., after the corrections discussed above had been applied, was small enough to be neglected even down at the lowest height (viz. 5 cm.) at which it was possible to mount an air meter. Nevertheless the H.W.A. readings were necessary on account of the very large number of air meter readings at the lower heights which had to be discarded owing to the velocities being below the lower limit at which the air meters were reliable.

Usually a series of experiments was preceded by some preliminary experiments in which the H.W.A. was mounted at a height of two metres for the purpose of comparing it with an air meter. These preliminary runs were similar in every other respect to the main experiments, except that only one air meter was used, at 2 m. The temperature gradient was measured with a view to obtaining information on gustiness. The main experiments in which the auxiliary height was 50 cm. were combined with these preliminary experiments to provide the curve in Fig. 15.

In 1933, when the H.W.A. was not in use, the procedure was simpler. Three minute runs with one air meter at 1 m., one at 2 m., and sometimes one at either 25, 50, or 506 cm. were carried out. During these experiments the temperature gradient apparatus was in use, the galvanometer being read every 30 seconds as experience had shewn that more frequent readings of temperature gradient were unnecessary.

§ 13—REDUCTION OF RESULTS

The nineteen instantaneous velocity readings obtained during each run with the H.W.A. were meaned to give the average velocity during that three minute period. Having done this the curve in Fig. 15 was constructed in the manner described above. From this curve correction factors were applied to correct all air meter readings to equivalent H.W.A. readings. All air meter readings (after correction) giving a velocity of less than 1·5 m./sec. were discarded. This, of course, eliminated most of the air meter readings at 5 cm. and many at 10 cm. Usually the values of $V(z)$ and $V'(z)$ agreed within two or three per cent. This can be seen by the fact that for the twenty-one occasions on which simultaneous velocity readings were obtained by H.W.A. and air meter at 5 cm. the mean velocity by H.W.A. differed from the mean velocity by air meter by only 1·06 per cent. Finally all velocities were expressed as percentages of the simultaneous velocity at 1 m. During 1933 the H.W.A. was not in use and only air meter readings were available. Of these any readings below 1·5 m./sec. obtained on the ordinary air meters were discarded. The low readings obtained with the special low speed air meters were, of course, retained. All these velocities were also expressed as percentages of the simultaneous velocity at 1 m.

Finally these percentage velocities, denoted by $U(z)$ where z may take any of the values 2·5, 5, 10, 25, 50, 200 and 506 cm., were grouped and meaned for each height according to temperature gradient and the velocity at 1 m. Since a value for U (200) was obtained from every experiment there are very many more results available for this value of z than for any other value.

§ 14—EFFECT OF TEMPERATURE GRADIENT AND VELOCITY UPON VELOCITY GRADIENT

The values obtained for $U(z)$ for each height are shewn arranged according to temperature gradient and velocity at 1 m. in Table XVII. It has already been

TABLE XVII—VELOCITY AT HEIGHT z AS PERCENTAGE OF VELOCITY AT 1 M.

		Temperature Gradient °F./m.									
z cm.	Velocity at 1 m. m./sec.	< −3·6°	−3·6° to −2·7°	−2·7° to −1·8°	−1·8° to −0·9°	−0·9° to 0·0°	0·0° to 0·9°	0·9° to 1·8°	1·8° to 2·7°	2·7° to 3·6°	> +3·6°
506	1·5 to 4·0					122·9 (3)	122·1 (7)	132·1 (4)			
	4·0 to 8·0				122·8 (7)	126·5 (13)	123·8 (27)	123·4 (8)			
200	0·5 to 1·5		106·4 (2)	107·5 (2)	104·4 (1)	127·0 (1)	127·7 (8)	138·5 (1)	134·7 (4)	117·2 (1)	137·6 (1)
	1·5 to 2·5	107·0 (3)	104·1 (9)	106·6 (15)	107·4 (14)	111·3 (10)	112·4 (36)	114·1 (21)	119·2 (2)		
	2·5 to 4·0	108·6 (18)	108·5 (50)	110·4 (30)	110·9 (37)	112·7 (36)	111·9 (52)	116·7 (13)			
	4·0 to 6·0	107·5 (7)	108·3 (40)	109·6 (20)	109·8 (36)	109·5 (32)	110·7 (43)	110·8 (13)			
	6·0 to 8·0			109·8 (2)	112·8 (12)	111·9 (24)	111·0 (31)	113·8 (1)			
50	1·5 to 4·0	91·2 (9)	90·5 (14)	89·6 (24)	90·4 (19)	90·8 (7)	90·6 (23)	89·3 (9)	82·7 (1)		
	4·0 to 8·0	88·2 (4)	90·1 (9)	90·3 (4)	90·3 (9)	91·1 (18)	91·0 (9)				
25	1·5 to 4·0		80·7 (5)	81·9 (12)	79·3 (5)	81·0 (5)	78·1 (14)	75·0 (3)	81·5 (1)		
	4·0 to 8·0		78·6 (7)	81·5 (9)	78·7 (2)	81·6 (5)	80·9 (7)				
10	1·5 to 4·0		65·8 (10)	67·6 (11)	66·4 (13)	62·6 (15)	62·8 (12)	59·6 (8)			
	4·0 to 8·0				64·4 (2)	68·2 (4)	67·1 (9)				
5	1·5 to 4·0		51·7 (4)	49·4 (7)	53·5 (10)	48·4 (10)	47·7 (9)	50·5 (6)			
	4·0 to 8·0		57·4 (4)	56·3 (4)	54·6 (2)		54·0 (7)	44·9 (3)			
2·5	1·5 to 4·0		47·2 (1)	41·0 (6)	39·7 (13)	34·0 (8)	36·3 (4)				
	4·0 to 8·0					30·1 (1)	38·1 (9)				

NOTE.—Numbers in brackets denote number of experiments.

pointed out that very many more values of $U(z)$ were obtained for 2 m. than for any other height. Accordingly it was possible to use narrower velocity bands over which to mean the results for this height than for the other heights. In order that the relative reliability of the figures in Table XVII may be assessed the number of three minute runs upon which each figure is based has been put in brackets underneath the figure. It will be seen that, despite the more detailed analysis, the 2 m. figures are based upon more experiments than the figures for the other heights. Thus the 2 m. results are less liable to be affected by casual variations than the others, and it appears that the effect of temperature gradient and velocity can best be determined by examining the 2 m. figures first, and then scrutinising the figures for the other heights to see how far the conclusions based on the 2 m. figures are confirmed.

Effect of temperature gradient.—It is impossible completely to separate the effects of temperature gradient and velocity since they are, to a certain extent interconnected. As far as possible, however, this will be done by examining the effect of temperature gradient alone for each velocity band. For this purpose some of the figures in Table XVII have been plotted in Figs. 16a and 16b.

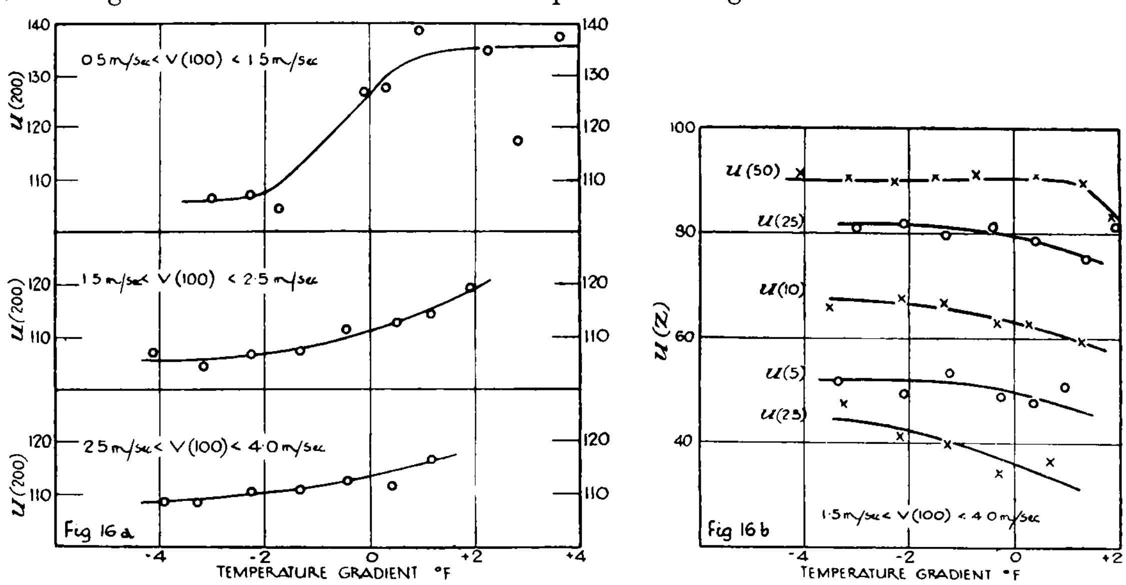


FIG. 16.—EFFECT OF TEMPERATURE GRADIENT UPON VELOCITY GRADIENT.

The top curve in Fig. 16a shews that at the very low velocities U (200) increases considerably as the temperature gradient changes from lapse to inversion. The number of observations in this velocity band is small and so the points are rather scattered. For the next two velocity bands more observations are available and the scatter of points is considerably reduced. For these medium velocities also it is clear that the velocity gradient increases as temperature gradient changes from lapse to inversion. The effect, however, is not so marked as in the case of the low velocities. No curves have been drawn for the higher velocity bands as it is clear from the figures in Table XVII that the variation of velocity gradient presents similar features when the velocity lies between 4·0 m./sec. and 6·0 m./sec., though on a diminished scale, while for the highest velocity band the variation is probably very small though here the number of observations during large temperature gradients is too small to form definite conclusions. This restricted range of temperature gradient at high velocities is, of course, inevitable since high velocities and very large temperature gradients, either negative or positive, are mutually exclusive.

In Fig. 16b curves are given shewing the variation of $U(z)$ with temperature gradient for each of the other heights (except 506 cm. for which there are too few observations) for velocities at 1 m. between 1·5 m./sec. and 4·0 m./sec. Here the points are rather scattered again owing to the small number of observations but, except for the 50 cm. curve, they shew an increasing velocity gradient as the temperature gradient changes from lapse to inversion. (It should be noted that a fall in a curve for a height below 1 m. corresponds to a rise in a curve for a height above 1 m.). There is no obvious reason why the 50 cm. curve should shew this feature to such a small degree only, and in view of the evidence of the curves for the greater and lesser heights it must be ascribed either to casual variation or to experimental error. The numbers of observations at higher velocities at these other heights do not warrant the construction of curves, especially since the evidence obtained at 2 m. indicates that the effect of temperature gradient at these higher velocities is small.

Thus, in general, velocity gradient increases as temperature gradient changes from lapse to inversion, the magnitude of the effect decreasing with increasing wind velocity. This variation will be discussed quantitatively in a later paragraph when the applicability of a " power law " is considered.

Effect of wind velocity.—The effect of velocity upon velocity gradient can be examined in the 2 m. results only. In Fig. 17 two curves are given, one shewing the variation of velocity gradient with velocity for lapses between $-0{\cdot}9°$F./m. and $-1{\cdot}8°$F./m. and the other for inversions of a similar magnitude. The rather curious fact emerges that during all lapses greater than $-0{\cdot}9°$F./m. the velocity gradient increases with increasing velocity. The chief factor which tends to minimise velocity gradients is the mixing due to turbulence, and it would appear from the above that the mixing accompanying a high lapse rate is greatest when there is little or no wind. It is of interest to recall a phenomenon described by S. Mal (11). He states that a thin layer of liquid in which a super-adiabatic lapse rate exists shews a cellular formation when there is little or no general flow. These cells are sometimes known as Bénard cells and when there is no general movement of the fluid they take the form of polygonal cylinders with vertical upward currents in the centres of the polygons and downward currents along the sides of the polygons. As the general flow increases the cells become elongated into strips and the shear increases. Thus there appears to be an analogy between the convection currents in the atmosphere (during lapses) and the vertical currents in the centres of the cells. In the presence of very light winds these convection currents are almost vertical and produce almost complete mixing, thus reducing the velocity gradient. As the wind increases the convection currents become bent over at an angle and become less efficient as mixing agents. The increased turbulence due to the increase in wind velocity is not sufficient to counteract this effect. During inversions of course there are no complications due to convection currents and the velocity gradient decreases as velocity increases.

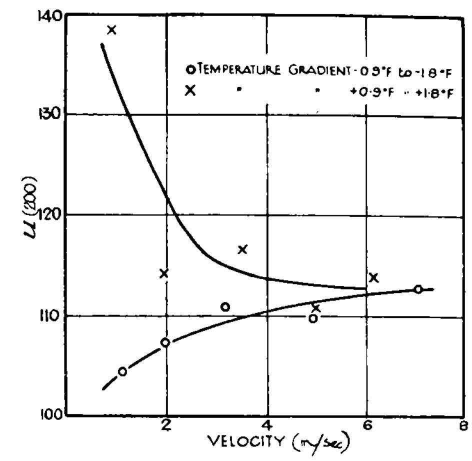


FIG. 17.—EFFECT OF VELOCITY UPON VELOCITY GRADIENT.

The gradient of wind velocity has also received consideration by M. A. Giblett and others (12) who give details of observations of wind velocity at 150 feet and 50 feet. The ratio of the wind at these two heights is tabulated according to the temperature difference between 4 feet and 143 feet and the wind at 150 feet. The figures in this table vary in a very similar manner to the 2 m. figures in Table XVII. They shew an increase as the temperature gradient changes from lapse to inversion, the variation being more marked at low velocities, and an increase as the velocity increases during lapses with the reverse effect in inversions.

§ 15—VARIATION OF VELOCITY WITH HEIGHT

In order to illustrate the variation of velocity with height the values of $U(z)$ for three definite temperature gradients corresponding to velocities between $1{\cdot}5$ m./sec. and $4{\cdot}0$ m./sec. have been obtained from the curves in Figs. 16a and 16b. The figures for 2 m. are mean values obtained from the two appropriate curves in Fig. 16a and the figure for 5 m. was estimated from Table XVII. These values are given in Table XVIII.

TABLE XVIII—VALUES OF $U(z)$ FOR VELOCITIES (AT 1 m.) BETWEEN 1·5 AND 4·0 m./sec.

z	2·5 cm.	5 cm.	10 cm.	25 cm.	50 cm.	100 cm.	200 cm.	506 cm.
Temperature Gradient								
−3°F./m.	42·9	52·0	66·7	81·0	90·3	100	107·1	—
Zero	36·4	49·0	63·0	79·2	90·3	100	111·7	122·5
1°F./m.	33·9	47·8	60·0	77·1	89·5	100	114·3	—

If the values of $U(z)$ for zero temperature gradient are plotted against log z the resulting curve departs but little from a straight line and over the range of height considered it seems that a good approximation is obtained by writing $U(z)$ as a linear function of log z. It is notable that Prandtl (13) has deduced theoretically the relation

$$\text{Velocity} = \text{const. log } z/z_0$$

where z_0 is a " roughness " constant. In the case of a sand covered surface Prandtl gives 1/30 of the diameter of a grain of sand for z_0.

Although the curvature of the graph connecting $U(z)$ and log z was small it seemed worth while, bearing in mind the fact that the velocities were measured over grass approximately 1 cm. high, plotting $U(z)$ against log $(z-1)$ where z is measured in centimetres. The result of this is shewn in Fig. 18. The difference between this graph and the one discussed above is apparent mainly in the position of the points corresponding to the lower heights. It will be seen that the points in Fig. 18 lie very close to a straight line the equation of which is

$$U(z) = 34{\cdot}5 \log (z-1) + 30$$

Obviously this equation must not be considered as applying to heights either appreciably less than 2·5 cm. or appreciably greater than 5 m.

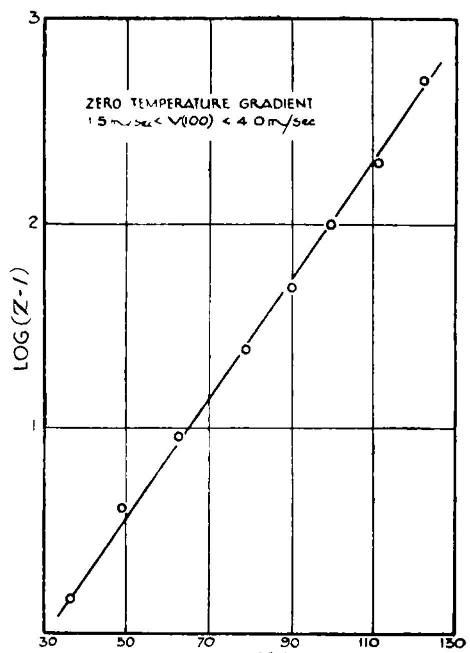


FIG. 18.—VELOCITY AS A FUNCTION OF LOG $(z-1)$.

The variation of $U(z)$ with temperature gradient is small compared with the variation with height and accordingly curves, corresponding to Fig. 18 for the other temperature gradients in Table XVIII, have not been drawn though the figures in Table XVIII shew that such curves would approximate fairly closely to straight lines having slightly different slopes from that of the line in Fig. 18.

There does not appear to have been a great deal of work done on the variation of wind with height at the low heights considered in the present paper and, as far as the writer has been able to ascertain, the effect of temperature gradient upon velocity gradient has never before been considered explicitly for these surface layers. Since, as has been shewn, temperature gradient near the surface can vary between fairly wide limits and since the effect of temperature gradient upon velocity gradient can be large under suitable conditions, this makes it rather difficult to compare the results given above with those of other workers.

It is of interest however to note that Chapman (14), when considering some results due to Stevenson, found that the velocity increased as a linear function of log z from the surface up to 51 ft. The constants in the equation connecting velocity and height varied from one occasion to another but probably this could be explained by the different surfaces over which the measurements were made and by possible variations in temperature gradient. Chapman also shews that the ratio of the mean wind at heights from 50 cm. to 30 m. to the mean wind at 10 m., as given in a Meteorological Office publication (15), can be expressed as a linear function of log z. These results indicate a much more rapid increase of velocity between 50 cm. and 5 m. than is given by Table XVIII for zero temperature gradient. Although a note accompanying these results suggests that they apply to conditions under which the temperature gradient would probably be small, the type of surface, which is of course important in velocity measurements at low heights, is not defined.

Velocity gradient expressed by a " Power Law."—As has been pointed out in the first part of this paper, it is very convenient to be able to express the relationship between two meteorological elements by a " power law." In this particular case it is desirable to test the possibility of expressing $U(z)$ by an equation of the form

$$U(z) = Az^{\alpha}$$

where A and α are constants. In order to do this log $U(z)$ has been plotted against log z in Fig. 19, the values of $U(z)$ being those appropriate to zero temperature gradient in Table XVIII. It is at once apparent that α, which is measured by the slope of the line in Fig. 19 is not a constant but varies with height. However, for any three consecutive points a straight line can be drawn which is a reasonably good fit. Approximately, alternate points in Fig. 19 represent heights which are in a ratio of about four to one. Thus it appears that, within the height interval 2·5 cm. to 5 m., the velocity can be represented by a power law over any shallow layer with a fair degree of accuracy, provided the greatest height in the shallow layer is not more than about four times the smallest height. The index will of course depend upon the particular layer considered. Thus examining the slope of the appropriate parts of the curve in Fig. 19, it can be seen that α will have the value 0·39 for the layer 2·5 cm. to 10 cm. and 0·13 for the layer 1 m. to 5m.

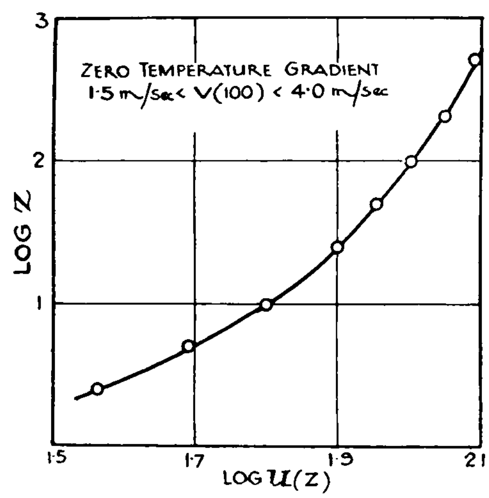


FIG. 19.—VELOCITY AS A FUNCTION OF A " POWER " OF THE HEIGHT.

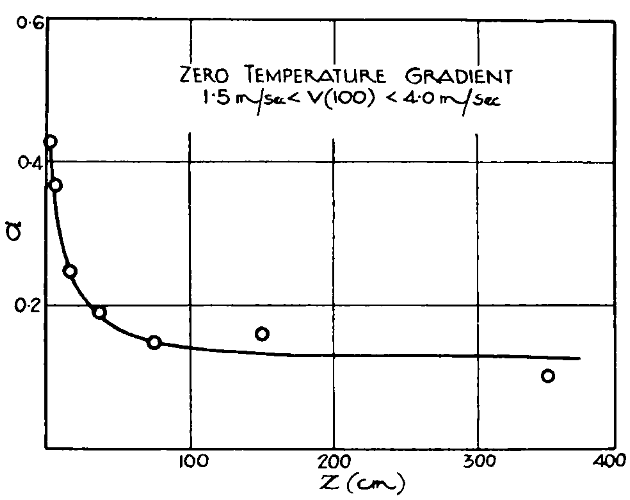


FIG. 20.—VARIATION WITH HEIGHT OF α IN THE RELATION $U(z) = Az^{\alpha}$.

So much interest attaches to this question of representing the velocity by a power of the height that the value of α corresponding to each pair of consecutive points in Fig. 19 has been calculated. These values are shewn plotted against the mid-height of the layer in Fig. 20. It can be seen that in the first metre α decreases very rapidly as the height increases. Above this height however the variation is small.

It may be noted here that the curvature in Fig. 19 cannot be eliminated by plotting log $(z-a)$ instead of log z (where a is a positive constant) as was done in the last section.

The application of a power law provides a convenient means of discussing quantitatively the effect of temperature gradient. If the four points corresponding to 25, 50, 100 and 200 cm. in Fig. 19 are examined it will be seen that they are nearly collinear. These points have been plotted for each of the temperature gradients in Table XVIII, the best straight line drawn in each case and the value of α determined from the slope. These values are given in Table XIX. During a heavy lapse, the velocity gradient between 25 cm. and 2 m. is given approximately by a power law having an index 0·14 and during a moderate inversion the index is 0·19.

TABLE XIX—VARIATION OF INDEX IN POWER LAW WITH TEMPERATURE GRADIENT

Temperature Gradient	Index	Simultaneous Conditions
−3·0°F./m.	0·14	Layer 25 cm. to 2 m. Velocity at 1 m. between 1·5 m./sec. and 4·0 m./sec.
Zero	0·17	
+1·0°F./m.	0·19	

During lower wind velocities the effect of temperature gradient is more marked as may be seen in Fig. 16a. The top curve of this figure shews that for velocities between 0·5 and 1·5 m./sec. *U* (200) varies from about 106·0 in heavy lapses to 134·5 in large inversions. These values correspond to indices of 0·084 and 0·43 for the layer between 1 m. and 2 m. Even 0·43 does not represent the greatest value for the index which can be attained under suitable conditions. On one occasion the two air meters, one at 2 m. and one at 1 m., were moving at such markedly different speeds as to attract attention. Close watch was maintained throughout the three minute run to ensure that the instrument at one metre did not stop temporarily. During the whole period both air meters appeared to turn fairly uniformly. The resulting velocity readings were 0·59 m./sec. at 1 m. and 1·06 m./sec. at 2 m., the temperature difference between 10 cm. and 110 cm. indicating an inversion of 2·17°F./m. These figures correspond to a value of 180 for *U* (200) and 0·85 for the index. This, of course, was one isolated case but it seems probable that streamline flow in which the velocity varies linearly as the height may be approached very closely in the lowest layers under suitable conditions. Anticipating results described later in this paper, it might be expected that under such conditions gustiness would be reduced to a very small quantity.

It was mentioned in the first part of this paper that Ertel (8) has shewn that if the law of variation of wind with height is

$$V(z) = V(1)z^{1/(p+1)}$$

then the law of variation of the coefficient of eddy conductivity $K(z)$ is

$$K(z) = K(1)z^{p/(p+1)}$$

$V(1)$ and $K(1)$ being the values of $V(z)$ and $K(z)$ at unit height. The curve in Fig 20 shews that above 1 m. in the air near the ground $1/(p+1)$ is probably not very different from 0·13. This would indicate that $K(z)$ varies as $z^{0·87}$ for this region. Below a height of about one metre the variation of velocity with height is no longer satisfied by a power law and so strictly the variation of K cannot be deduced. It seems probable however that since $1/(p+1)$ becomes larger in the lowest layer the variation of K (z) with height will be slower. It may be repeated here that in order that $K(z)$ should vary more rapidly than z (as was found from temperature considerations in the first part of this paper) $1/(p+1)$ would have to be negative.

§ 16—CONCLUSIONS

Thus, summarising these results on velocity gradient, in the first five metres the velocity over close cropped grass can best be represented as a linear function of log $(z-c)$ where c is a constant. A power law can be used provided only a shallow layer is considered, the accuracy of such a law increasing with increasing height. If a power law is used the index for zero temperature gradient may vary from about 0·43 in the lowest layers to about 0·13 at greater heights. This index can increase considerably during light winds and big inversions.

Finally it must be remembered that all these results apply to average conditions. Owing to the mechanism of turbulence individual measurements of velocity gradient, especially if partaking of the nature of " snap " readings, might give results differing appreciably from those described above.

PART III—WIND FLUCTUATIONS IN THE MEAN WIND DIRECTION AS MEASURED BY THE HOT WIRE ANEMOMETER

§ 17—GENERAL CONSIDERATIONS

In the course of the investigation of velocity gradient described in the second part of this paper a very large number of measurements of velocity were made with the hot wire anemometer. These were measurements of the instantaneous velocity and on that account they are suitable for examining the fluctuations of wind velocity. The original purpose of the experiments however was the study of velocity gradient; the examination of velocity fluctuations is thus somewhat handicapped by the fact that the data used are obtained from experiments which were not designed for such work. The instrumental details and the method of carrying out the experiments have already been described.

Let the components of the instantaneous wind velocity, referred to a system of rectangular axes, be u, v and w. The mean values (with respect to time t) of these components during the period of the experiment T are defined by

$$\bar{u} = \frac{1}{T}\int_{t-\frac{1}{2}T}^{t+\frac{1}{2}T} u\,dt, \text{ etc.}$$

It is assumed that conditions are such that the orientation of the axes can be fixed so that

$$\bar{u} > 0 \text{ if } z > 0, \bar{v} = \bar{w} = 0 \text{ for all values of } z.$$

The " eddy velocities " are then defined by the equations

$$u' = u-\bar{u}\ ;\ v' = v-\bar{v} = v\ ;\ w' = w-\bar{w} = w$$

The absolute value of the eddy velocity $|u'|$, expressed as a percentage of the mean velocity $\bar{u}$, will be termed the " fluctuation ratio " and will be denoted by g.

$$\text{Thus } g = 100\,|u'|/\bar{u}$$

The mean value of g during an experiment will be termed the mean fluctuation ratio of that experiment and will be denoted by $\bar{g}$.

$$\text{Thus } \bar{g} = 100\,\overline{|u'|}/\bar{u}$$

Laboratory experiments had shewn that when the element of the anemometer was turned parallel to the mean wind direction the indicated velocity fell to about 15 to 20 per cent. of the true velocity. There is little doubt that this residual reading of 15 to 20 per cent. was caused by eddy components perpendicular to the element and that an air flow which is truly parallel to the element has but a neglible effect on the anemometer reading. When used in the open the element of the anemometer was always vertical. Accordingly the effect of w' may be neglected.

The effective instantaneous wind velocity is thus

$$(\bar{u}^2+2\bar{u}u'+u'^2+v'^2)^{\frac{1}{2}} = \bar{u}[1+2u'/\bar{u}+(u'^2+v'^2)/\bar{u}^2]^{\frac{1}{2}}$$

Expanding and neglecting terms of the order of $(u'/\bar{u})^3$ and $(v'/\bar{u})^3$ and of higher orders

$$\text{Effective instantaneous velocity} = \bar{u}(1+u'/\bar{u}+v'^2/2\bar{u}^2).$$

This is the velocity actually measured by the anemometer. The mean value of this for one experiment is $\bar{u}[1+\overline{(v'^2/2\bar{u}^2)}]$. Hence

$$\begin{aligned}\text{Instantaneous velocity—mean velocity} &= \bar{u}(1+u'/\bar{u}+v'^2/2\bar{u}^2)-\bar{u}[1+\overline{(v'^2/2\bar{u}^2)}]\\ &= u'+\bar{u}[v'^2/2\bar{u}^2-\overline{(v'^2/2\bar{u}^2)}]\end{aligned}$$

In the last part of this paper it will be seen that $|v'/\bar{u}|$ has an extreme value of the order of 0·6 and a mean value of the order of 0·4. Thus the part in brackets in the expression for the difference between instantaneous and mean velocity has an extreme value of about 0·1 and is usually appreciably less than this. Scrase (17) has shewn that at a height of 1·5 m. above the ground the components of eddy velocity u' and v' are not widely different. Thus $|u'/\bar{u}|$ will have a mean value of the order of 0·4. Accordingly, to a fairly close approximation,

$$\text{Instantaneous velocity} - \text{mean velocity} = u'$$

Moreover in the expression for the mean velocity, viz.: $\bar{u}[1+\overline{(v'^2/2\bar{u}^2)}]$, the term $\overline{(v'^2/2\bar{u}^2)}$ can be neglected compared with unity and the mean velocity becomes $\bar{u}$. Thus, approximately, the velocity readings can be treated as though they represented velocities along the x axis.

The anemometer was used in experiments of three minutes duration, during which nineteen instantaneous velocity measurements were made at ten second intervals. The velocity gradient experiments referred to previously included readings with the hot wire anemometer at 2·5, 5, 10, 25 and 50 cm. and the experiments carried out to compare the anemometer with an air meter gave anemometer readings at 2m. The data derived from these experiments have been utilised for an examination of the mean fluctuation ratio and also to give frequency curves for fluctuations of various sizes.

§ 18—METHOD OF ANALYSIS

The nineteen values of g for each experiment were calculated and hence the value of $\bar{g}$. Cases where the wind velocity appeared to be either steadily increasing or steadily decreasing were discarded. The values of $\bar{g}$ were then grouped and meaned for each height according to the wind velocity at 2 m. (as measured by an air meter) and temperature gradient.

The frequency of fluctuation ratios of different magnitudes was examined for various ranges of velocity (measured at 2 m.) and temperature gradient (between 10 and 110 cm.), at the three heights 10 cm., 50 cm. and 200 cm. in the following manner. The number of fluctuation ratios lying between 0 and 5 per cent., 5 and 15 per cent., 15 and 25 per cent., etc., and the mean value of g for each group were found. In order to allow for the unequal ranges of g and to facilitate comparison between different frequency curves based upon different total numbers of fluctuations, the number of fluctuations for each range of g was expressed as a percentage of the total number of fluctuations and divided by the range of g. The resultant number is denoted by f and gives the percentage of the total number of fluctuation ratios which fall within a range of g of unit width. Some preliminary work had indicated that the frequency curves were not obviously asymmetrical if positive and negative values of u' were differentiated. Altogether over 3,300 instantaneous velocity readings were used to provide the frequency curves given below.

§ 19—EFFECT OF TEMPERATURE GRADIENT AND VELOCITY UPON THE MEAN FLUCTUATION RATIO

The values of $\bar{g}$ for various velocities (measured at 2 m.), temperature gradients, and heights are set out in Table XX, the figure in brackets below each entry indicating the number of experiments contributing to the mean values. Here, as in the case of velocity gradient, the effects of temperature gradient and of velocity are interdependent.

At the three lowest heights, viz., 2·5, 5·0 and 10·0 cm. the mean fluctuation ratio appears to be very little affected by temperature gradient over the range of the latter factor covered by the observations. At 25 cm. and 50 cm. the mean fluctuation ratio clearly decreases as the temperature gradient changes from lapse to inversion for velocities up to 4 m./sec. For the highest velocities however, temperature gradient apparently has little or no effect. The same variation is evident in the figures for 200 cm., though here the limiting velocity is 2·5 m./sec. It should be noted that the 200 cm. results do not extend beyond small inversions. Had results for larger inversions been available for this height there seems little doubt that the effect of temperature gradient would have been shewn at higher velocities. The comparative failure of thermal stratification (between 10 cm. and 110 cm.)

TABLE XX—MEAN FLUCTUATION RATIOS

z cm.	Velocity at 2 m. m./sec.	Temperature Gradient °F/m. < −2·7	−2·7 to −1·8	−1·8 to −0·9	−0·9 to 0·0	0·0 to 0·9	0·9 to 1·8	> 1·8
200	0·5 to 1·5		21·9 (1)	15·3 (1)				
	1·5 to 2·5	20·4 (1)	15·4 (5)	14·3 (2)	13·8 (1)	12·1 (12)		
	2·5 to 4·0	13·2 (2)	17·1 (5)	12·7 (8)	14·6 (9)	14·6 (3)		
	4·0 to 6·0	14·2 (1)	12·7 (2)	12·7 (2)	16·6 (3)	12·3 (2)		
	> 6·0	10·1 (1)		16·7 (3)	12·8 (3)	10·8 (4)		
50	0·5 to 1·5					14·8 (2)		
	1·5 to 4·0	19·1 (1)	17·0 (9)	18·5 (5)		14·0 (5)	10·8 (3)	8·1 (1)
	4·0 to 8·0		10·4 (3)	12·6 (3)	13·3 (2)	12·8 (9)	13·6 (3)	
25	0·5 to 1·5			12·2 (1)				
	1·5 to 4·0	21·2 (2)	20·8 (6)	15·2 (6)	16·1 (2)	15·9 (8)	13·1 (3)	12·9 (3)
	4·0 to 8·0		12·0 (2)	14·5 (1)	14·8 (2)	12·7 (7)		
10	0·5 to 1·5							28·3 (1)
	1·5 to 4·0	17·6 (6)	19·6 (2)	15·5 (8)	14·6 (8)	17·4 (9)	15·6 (7)	
	4·0 to 8·0		18·0 (3)		21·5 (3)	14·1 (7)	18·7 (3)	
5	0·5 to 1·5		17·1 (1)			19·0 (3)	18·1 (3)	
	1·5 to 4·0	15·5 (3)	19·5 (4)	17·2 (11)	17·2 (5)	17·1 (9)	15·6 (6)	
	4·0 to 8·0	13·8 (2)	15·8 (3)	9·7 (1)	17·5 (2)	13·4 (7)		
2·5	0·5 to 1·5					23·0 (4)		
	1·5 to 4·0		18·3 (6)	13·6 (10)	21·2 (7)	18·3 (6)		
	4·0 to 8·0			18·5 (3)	14·5 (4)	13·7 (10)	16·0 (3)	

NOTE.—Numbers in brackets denote number of experiments.

to damp out fluctuations at the lowest levels is probably due to the fluctuations at these heights being largely governed by the roughness of the ground.

Considering Table XX as a whole, it appears that for lapses greater than $-0\cdot9°$F., there is a tendency for $\bar{g}$ to decrease with increasing velocity. During small lapses the evidence is indecisive. During small inversions $\bar{g}$ appears to decrease with increasing velocity. There are too few observations available for the larger inversions but it is clear that if $\bar{g}$ decreases as the temperature gradient changes from lapse to inversion, and the effect is more marked at low velocities than at high velocities, then when the inversion becomes big enough $\bar{g}$ must be smaller at the low velocities than at the high velocities. This is supported by the values of $\bar{g}$ for inversions between $0\cdot9$ and $1\cdot8°$F. at 50 cm. and at 10 cm. The corresponding values at 5 cm. do not support this view but it is considered that this may be due to errors in the value of $\bar{g}$ for the very low velocities.

§ 20—VARIATION OF MEAN FLUCTUATION RATIO WITH HEIGHT

Since only one hot wire anemometer was used there were no experiments giving the mean fluctuation ratio measured at two heights simultaneously. Accordingly the following procedure was adopted. The mean value of $\bar{g}$, irrespective of velocity and temperature gradient, was obtained for each height. The mean values of the temperature gradient, the velocity at 2 m., and the velocity at the height of the anemometer were also obtained for each height. These results are set out in Table XXI. It can be seen that the mean temperature gradient and the mean velocity at 2 m. are nearly the same for all heights and thus the mean values of $\bar{g}$ will indicate the variation of $\bar{g}$ with height when the temperature gradient is about $-0\cdot50°$F. and the velocity at 2 m. is about $3\cdot7$ m./sec.

The mean eddy velocity $\overline{|u'|}$ for each height may be obtained from the equation

$$\overline{|u'|} = \bar{u}\,\bar{g}/100$$

where $\bar{u}$ and $\bar{g}$ refer to the particular height under consideration.

TABLE XXI—VARIATION OF $\bar{g}$ WITH HEIGHT

Height z (cm.)	2·5	5	10	25	50	200
Mean Value of $\bar{g}$	16·9	16·5	16·1	15·5	14·3	14·0
Mean Temperature Gradient °F.	−0·40	−0·54	−0·47	−0·40	−0·58	−0·86
Mean Velocity at 2m. (m/sec.)	3·64	3·55	3·46	3·45	3·62	3·75
No of Experiments	53	60	57	43	46	71
$\frac{(\text{Mean Velocity at } z) \times 3\cdot7}{\text{Mean Velocity at 2m.}}$	1·25	1·72	2·16	2·66	3·05	3·70
Mean Eddy Velocity $\overline{\vert u'\vert}$ for velocity of 3·7 m./sec. at 2m.	0·21	0·28	0·35	0·41	0·44	0·52
$\frac{\text{Mean Velocity at } z}{\text{Mean Velocity at 2m.}}$	0·34	0·46	0·58	0·72	0·82	1·00
$V(z)/V(\text{2m.})$ for lapse of −0·50°F. (Figs. 16a, 16b.)	0·34	0·45	0·57	0·71	0·80	1·00

Since $\bar{g}$ varies so little with height $\overline{|u'|}$ will depend very largely upon $\bar{u}$. Table XXI shews that there were slight variations in the mean velocity at 2m., and hence the mean values obtained for $\bar{u}(z)$ for the various heights are not strictly comparable. In order to allow for this, each mean value of $\bar{u}(z)$ has been multiplied by $3\cdot7/\bar{u}$ (2m.) where $\bar{u}$(2m.) is the corresponding mean velocity at 2m. Thus the velocities as set out in the fifth line of Table XXI all correspond to a velocity of $3\cdot7$ m./sec. at 2m. Using these values of $\bar{u}$ the mean eddy velocity $\overline{|u'|}$ has been calculated for each height.

As a matter of interest, the ratio of the mean velocity at height z to the simultaneous mean velocity at 2m. has been calculated for each height and the same ratio has been obtained from Figs. 16a and 16b of the second part of this paper for a temperature gradient of $-0{\cdot}5°$F. These ratios are given in the last two lines of Table XXI. As might be anticipated, the agreement is good.

Table XXI shews that $\bar{g}$ decreases very slowly with height. If $\bar{g}$ is plotted against log z it will be found that the points lie approximately on a straight line indicating that $\bar{g}$ may be expressed by an equation of the form

$$\bar{g} = m \log z + c$$

where m has the value $-1{\cdot}52$, and z is measured in cm. Since $\bar{g}$ varies so slowly with height the variation of the mean eddy velocity $\overline{|u'|}$ will be governed very largely by the variation of $\bar{u}$. In the second part of this paper it was shewn that $\bar{u}$ is a linear function of log $(z-1)$. Hence $\overline{|u'|}$ must follow a similar law fairly closely. If $\overline{|u'|}$ is plotted against log $(z-1)$ it will be found that there is a slight curvature in the resulting graph but it appears that $\overline{|u'|}$ can be represented with fair accuracy by the equation

$$\overline{|u'|} = m \log (z-1) + c$$

where m has the value $0{\cdot}15$, the units being the centimetre and the second.

§ 21—RELATIVE FREQUENCY OF DIFFERENT SIZED FLUCTUATIONS

The relative frequency of fluctuation ratios of different magnitudes has been found for the three heights 10, 50 and 200 cm. as explained in § 18. The frequency is denoted by f which gives the number of fluctuations (as a percentage of the total number examined) per unit range of g. The experiments used for Table XX were also used for this part of the work and the values obtained for f are set out in Table XXII, which also indicates the conditions of velocity and temperature gradient applying to the various values of f. The mean values of g corresponding to each value of f are not given since for most of the lower ranges of g the mean value is not very different from the mid point of the range.

The last column of Table XXII shews the relative frequency of the fluctuation ratios meaned over all temperature gradients and velocities and the three heights. The shape of the curve given by these figures is a matter of some interest. Hesselberg and Björdkal (16) have shewn that, under certain assumptions on the nature of the flow, the distribution of eddy velocities follows a law analagous to that deduced by Maxwell for the distribution of molecular velocities. The Maxwell relation

$$f = f_0 \exp(-kg^2)$$

where f_0 and k are constants, will therefore be examined to see if it is satisfied in the present case. In order to do this $\log_{10} f$ has been plotted against g^2 in Fig. 21. It can be seen that except for the three lowest points (which correspond to the larger fluctuations), the points fit a straight line very closely. These three points, however, together represent less than 1 per cent. of the observations, and thus more than 99 per cent. of the total number of fluctuation ratios measured conform to the "normal error" law. Measurement of the slope of the line in Fig. 21 gives a value $1{\cdot}46 .\ 10^{-3}$ for k.

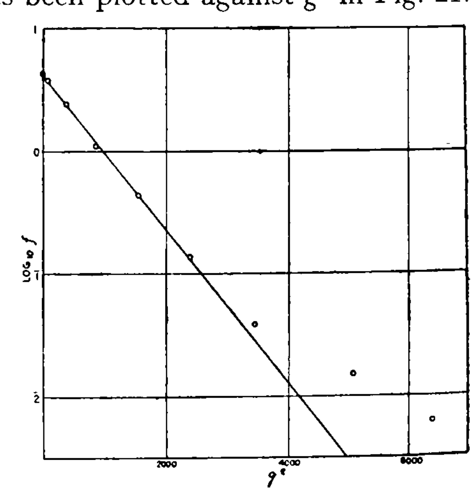


FIG. 21.—DISTRIBUTION OF THE FLUCTUATIONS OF WIND VELOCITY EXPRESSED BY A "NORMAL ERROR" LAW.

It might be anticipated that the shape of the frequency curves would be affected by temperature gradient and velocity only in those cases in which the mean fluctuation ratio was affected. That this is so can be seen by a comparison of Tables XX and XXII.

At 200 cm. the mean fluctuation ratio was very little affected by velocity during lapses, and Table XXII shews that the shape of the frequency curves for the three velocity ranges during lapses differed but slightly.

TABLE XXII—FREQUENCY OF FLUCTUATION RATIOS OF VARIOUS MAGNITUDES

z (cm.)	200					50			
Temperature Gradient.	Lapses.			Inversions		Lapses.		Inversions.	
V (2m.) m/sec.	< 2·5	2·5 to 4·0	> 4·0	< 4·0	> 4·0	< 4·0	> 4·0	< 4·0	> 4·0
g	Frequency f								
0%— 5%	4·40	4·42	4·14	4·98	5·61	3·08	5·39	4·90	4·56
5%—15%	3·30	3·84	3·79	3·79	4·21	3·44	4·21	4·31	4·04
15%—25%	2·44	2·26	2·35	3·01	2·28	2·70	2·16	2·06	2·67
25%—35%	1·34	1·23	1·26	0·49	0·53	1·05	0·66	0·77	0·79
35%—45%	0·38	0·33	0·46	0·11	0·18	0·81	0·20	0·34	0·18
45%—55%	0·14	0·07	0·04	0·07	..	0·35	0·07	0·10	0·04
55%—65%	0·10	..	0·04	0·04	..	0·07	..	..	..
65%—75%	0·10	0·02	..	..	..	0·04	..	..	..
75%—85%	..	0·04	..	..	..	..	..	..	..
No. of Experiments	11	24	15	15	6	15	8	11	12
No. of Fluctuations	209	456	285	285	114	285	152	209	228

z (cm.)	10				200	50	10	All	
Temperature Gradient.	Lapses.		Inversions.		All	All	All	All	
V (2m.) m./sec.	< 4·0	> 4·0	< 4·0	> 4·0	All	All	All	All	
g	Frequency f								
0%— 5%	3·81	2·81	3·22	4·00	4·58	4·30	3·56	4·18	
5%—15%	3·59	3·34	3·35	3·90	3·75	3·95	3·60	3·76	
15%—25%	2·54	2·36	2·45	2·20	2·45	2·45	2·36	2·43	
25%—35%	1·25	1·76	1·55	1·21	1·04	0·85	1·38	1·10	
35%—45%	0·59	0·88	0·59	0·42	0·30	0·42	0·59	0·43	
45%—55%	0·18	0·26	0·19	0·26	0·07	0·16	0·20	0·14	
55%—65%	0·11	..	0·06	..	0·03	0·02	0·06	0·04	
65%—75%	..	..	0·03	..	0·02	0·01	0·01	0·02	
75%—85%	..	..	..	..	0·01	..	..	0·01	
No. of Experiments	24	6	17	10	71	46	57	174	
No. of Fluctuations	456	114	323	190	1349	874	1083	3306	

At 50 cm. however the mean fluctuation ratio shewed a marked decrease as the velocity increased (during lapses) and also as the temperature gradient changed from lapse to inversion (for velocities less than 4·0 m./sec.). Table XXII shews that the smaller mean fluctuation ratio is accompanied by a suppression of the larger fluctuations and a corresponding increase in the number of small fluctuations. Three of the four curves for 50 cm. have been plotted as in Fig. 21 and the values of k measured with the following results

During lapses, velocity at 2 m. less than 4·0 m./sec. .. $k = 0{\cdot}98.10^{-3}$
During lapses, velocity at 2 m. greater than 4·0 m./sec. .. $k = 2{\cdot}54.10^{-3}$
During inversions, velocity at 2 m. less than 4·0 m./sec... $k = 1{\cdot}82.10^{-3}$

The mean frequency curve for all velocities and temperature gradients has also been plotted for each of the heights 10 cm., 50 cm. and 200 cm. and the values of k measured. The values were as follows

At 200 cm. $k = 1{\cdot}75.10^{-3}$
,, 50 ,, $k = 1{\cdot}59.10^{-3}$
,, 10 ,, $k = 1{\cdot}20.10^{-3}$

Thus it appears that near the ground the number of fluctuation ratios of different magnitudes is given by a law of the normal error form, the constant in the index having a value which may vary between about $1{\cdot}0.10^{-3}$ and $2{\cdot}5.10^{-3}$ according to the conditions existing at the time.

PART IV—THE LATERAL AND VERTICAL COMPONENTS OF WIND GUSTINESS IN THE LOWEST FIVE METRES OF THE ATMOSPHERE

§ 22—SUMMARY

The examination of the lateral and vertical components of wind gustiness ("gustiness" being interpreted as the ratio of eddy velocity to mean velocity) at heights up to 5 m. above close cropped grass has been carried out by using two small universally pivoted vanes. The records obtained permitted the measurement of both "extreme" and "mean" gustiness, the interpretation of the two terms "extreme" and "mean" being given in § 25, p. 49. The effects of wind velocity, temperature gradient (between 10 cm. and 110 cm. above the ground), and height are considered.

It is found that extreme gustiness does not depend upon velocity during moderate or large lapses, but during small lapses and inversions gustiness increases with increasing velocity. As the temperature gradient changes from lapse to inversion, with constant velocity, the extreme gustiness decreases, the effect being most marked at small velocities and almost negligible at large velocities.

As the height above the ground increases from 25 cm. to 5 m. the lateral component of extreme gustiness decreases rather slowly and the vertical component increases more rapidly. At 5 m. however the vertical component is still less than the lateral component.

The ratio of mean gustiness to extreme gustiness is found to depend upon velocity only to a close approximation, the ratio increasing with velocity. The variations of mean gustiness with velocity, temperature gradient and height are thus deduced from the corresponding variations of extreme gustiness.

The effects of velocity, temperature gradient and height upon eddy velocity are then deduced, the variation of mean velocity with height necessary for a discussion of the first factor being obtained from the second part of this paper.

Finally the accuracy with which mean velocity can be measured with a small air meter is discussed.

§ 23—GENERAL CONSIDERATIONS

Due to the eddy structure of the atmosphere the wind at a fixed point in space is subject to rapid and more or less irregular changes in direction and velocity. Let a system of rectangular axes and the components of wind velocity be specified as in § 17, p. 40 in the third part of this paper. If a small direction vane, which is free to turn about a vertical axis, is held in the wind the instantaneous angular deflection of the vane from the mean position will be arc tan $(v'/(\bar{u}+u'))$. Approximately this can be taken as arc tan $(v'/\bar{u})$. If the vane is also free to move about a horizontal axis the deflection in the vertical plane will similarly be arc tan $(w'/\bar{u})$. In cases where the angular deflection is small the angle may be represented by its tangent and the angular deflections will be $v'/\bar{u}$ and $w'/\bar{u}$. These quantities, $v'/\bar{u}$ and $w'/\bar{u}$, will be referred to as the y and z components of gustiness.

Scrase (17) gives the results of some experiments with a small bi-directional vane of the type described above. In 1933 it was decided to extend the results obtained by Scrase to cover more varied conditions. Only the layer of air near the ground was under consideration and it appeared that the factors which might affect gustiness in this layer would be

(a) The nature of the surface of the ground.
(b) The height above the ground.
(c) The wind velocity.
(d) The thermal structure of the atmosphere near the ground.
(e) The length of the period over which gustiness is measured.

In the present work an endeavour has been made to determine the influence of (b), (c) and (d) upon gustiness in the layer of air below 5 m. and a small number of measurements upon the effect of (e) were also made. As far as possible, however, all the experiments were performed over a similar surface.

§ 24—INSTRUMENTAL

The apparatus used in this work comprised two types of air meter mounted on direction vanes, a temperature gradient apparatus which measured the temperature difference between 10 cm. and 110 cm. above the surface, and two bi-directional vanes. The air meters and temperature gradient apparatus have already been described in this paper and no further reference appears necessary.

The bi-directional vanes were constructed for this particular investigation and in their construction special attention was paid to the following points :—

(a) The vanes should be as sensitive as possible to light winds.
(b) The sensitivities in horizontal and vertical directions should be as nearly equal as possible.
(c) The supports and chart holders should be arranged to cause the least possible interference with the vane.
(d) The two vanes should possess similar characteristics.
(e) The vanes should be fairly robust in order to prevent accidental damage.

The first two points were satisfied by making the vanes light, reducing the friction at the bearings to a minimum, making the moment of the pressure due to the wind about the axis as large as possible and making the vanes symmetrical about their axes. Allowance was made for the third point by ensuring that the centre of the vane should be at least five times the height of the chart holder above the base of the chart holder.

A photograph of one of the vanes is shewn in Fig. 22 and a diagrammatic sketch giving various dimensions in Fig. 23. As may be seen the actual vane was constructed of stiff wire and cardboard with a brass balance weight at the nose. The bearings were all point bearings and stops were provided to prevent the vanes being deflected too far in either direction. Suspended from the framework of the vane was a wire pen arm to the end of which was attached a small glass pen. From the diagram it will be seen that every movement of the point of suspension of the pen on the vane arm was reproduced, to a close approximation on the same scale, on a chart which formed part of a cylinder having the same vertical axis as the vane. The glass pen was made by drawing out a piece of glass tubing and bending the fine end suitably. The pen was filled with a suitable ink and

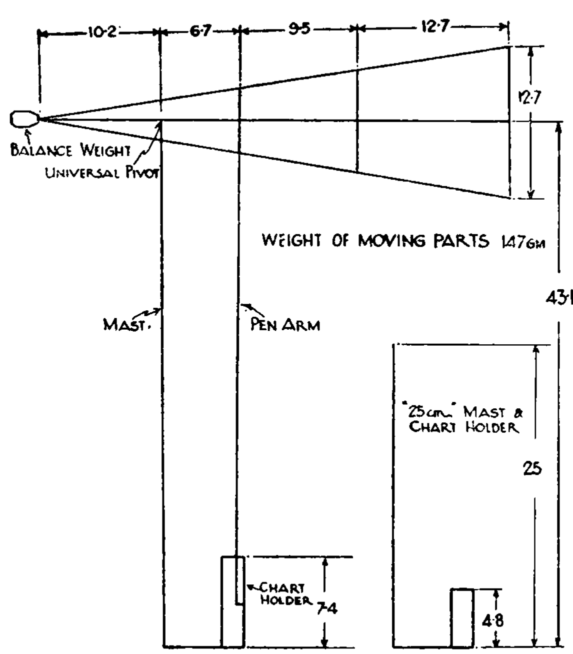


FIG. 23.—DIAGRAM SHOWING DIMENSIONS OF BI-DIRECTIONAL VANE IN CENTIMETRES.

normally would make about ten records without further attention. Sufficient pressure between the pen and the chart was maintained partly by the pen arm being inclined slightly to the vertical and partly by the wind pressure on the wire of the pen arm. The pen could be kept clear of the chart holder for changing charts, etc., by moving a lever attached to the chart holder shewn in Fig. 22. The friction between the pen and the chart was very small since the ink acted as a lubricant, the pressure necessary to produce satisfactory traces was small and the point of the pen was always ground fairly smooth.

It was thought desirable to get records with the vane as low as 25 cm. above the ground. It can be seen from Fig. 23 that this would be impossible with the taller stand shewn there, so a special short stand was made for this purpose. Since this brings the vane nearer to the chart holder the height of the latter was reduced proportionately in order to minimise any interference.

§ 25—GENERAL DESCRIPTION OF TRACES

Most records were obtained by allowing the bi-directional vane to move freely in the wind for a period of three minutes. Since no provision was made for a time scale the resulting traces consisted of masses of continuous lines covering areas roughly oval. In most cases these traces had a central portion where the line density was very great. Outside this central portion the line density becomes lower and the edge of the trace was generally irregular owing to stray loops projecting beyond the general confines. Some typical examples are shown in Fig. 25. These examples consist of two pairs of traces, each pair being made simultaneously. The conditions under which they were made are indicated, the traces being chosen to illustrate the effects of height, velocity and temperature gradient. These effects will be discussed fully later on.

The dimensions of these traces are clearly an indication of the gustiness of the wind, but some difficulty was experienced in deciding how to measure them. It is clear that a circumscribing oval can be drawn to include most of the trace and the diameters of this oval will furnish a measure of certain aspects of the gustiness. Without a more precise definition however it would be possible to draw several ovals of different sizes for one particular trace. It was considered undesirable for the oval to be large enough to include all the trace on each chart as this might place too much importance on one or two exceptionally large gusts. Ultimately the following definition was framed:

> " The circumscribing oval shall have the two main diameters approximately vertical and horizontal and shall be the smallest oval which will include most of the trace, not more than three loops of the trace per minute duration of the record being allowed outside the oval."

Thus on a three minute trace not more than nine loops were allowed outside the oval. The choice of the figure of three loops per minute duration was quite arbitrary and was made after inspecting a large number of records. On some traces which had a fairly regular outline the number of excluded loops would be appreciably less than nine. As a further safeguard, in order to minimise any progressive change in the method of drawing the ovals, the writer drew the ovals for the first 850 traces (there were 876 records altogether) in as short a period as was consistent with care.

TABLE XXIII—MEAN ERRORS IN DRAWING OVALS

		First Fifty	Second Fifty
		mm.	mm.
Mean lateral (y) diameter	.. ..	75·5	69·1
,, vertical (z) ,,	.. ..	37·4	35·8
Mean error by Observer A,	—lateral ..	+1·26	+0·38
first estimate.	—vertical ..	+0·32	+1·32
Mean error by Observer B	—lateral ..	−0·24	—
	—vertical ..	−2·32	—

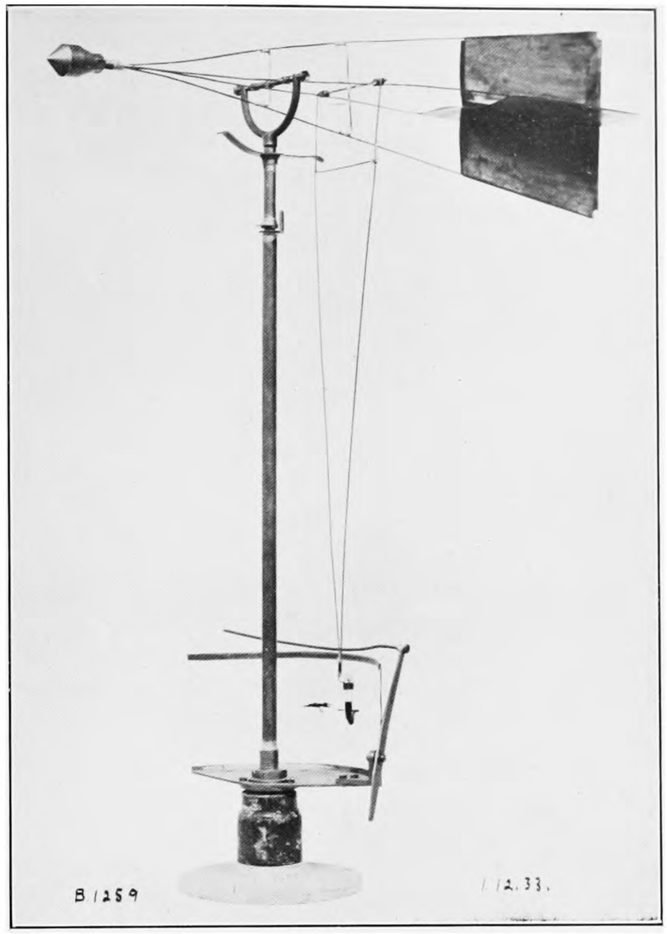


FIG. 22.—BIDIRECTIONAL VANE.

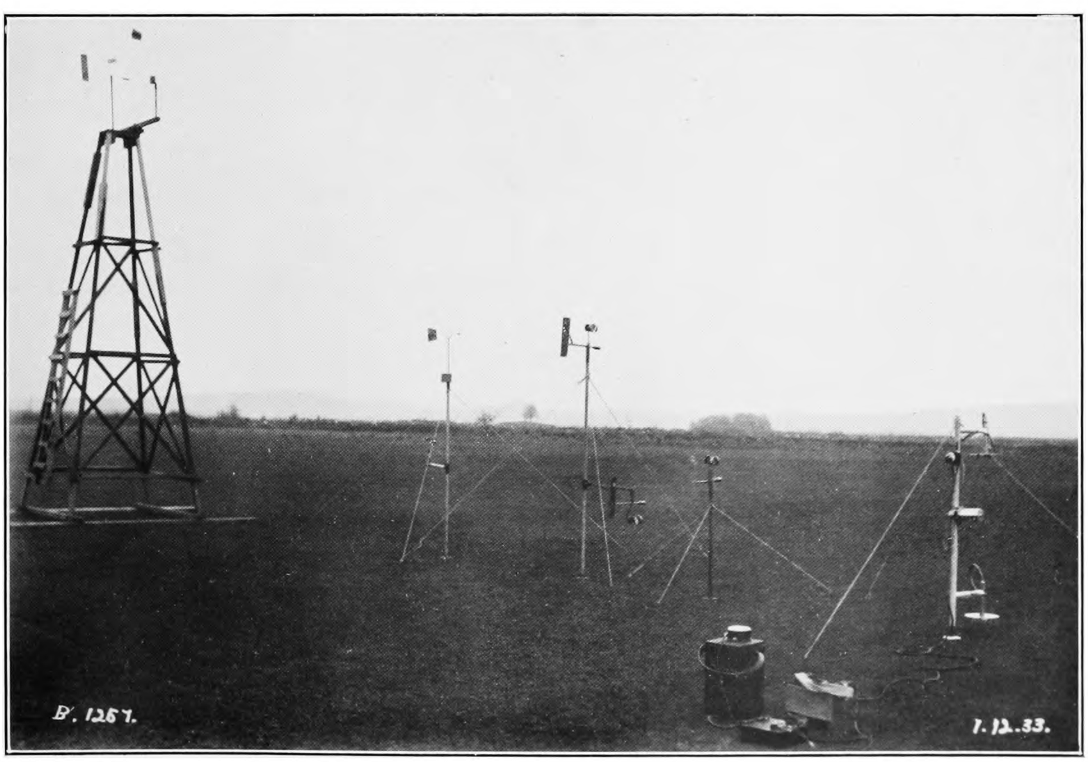


FIG. 24.—APPARATUS FOR MEASURING GUSTINESS.

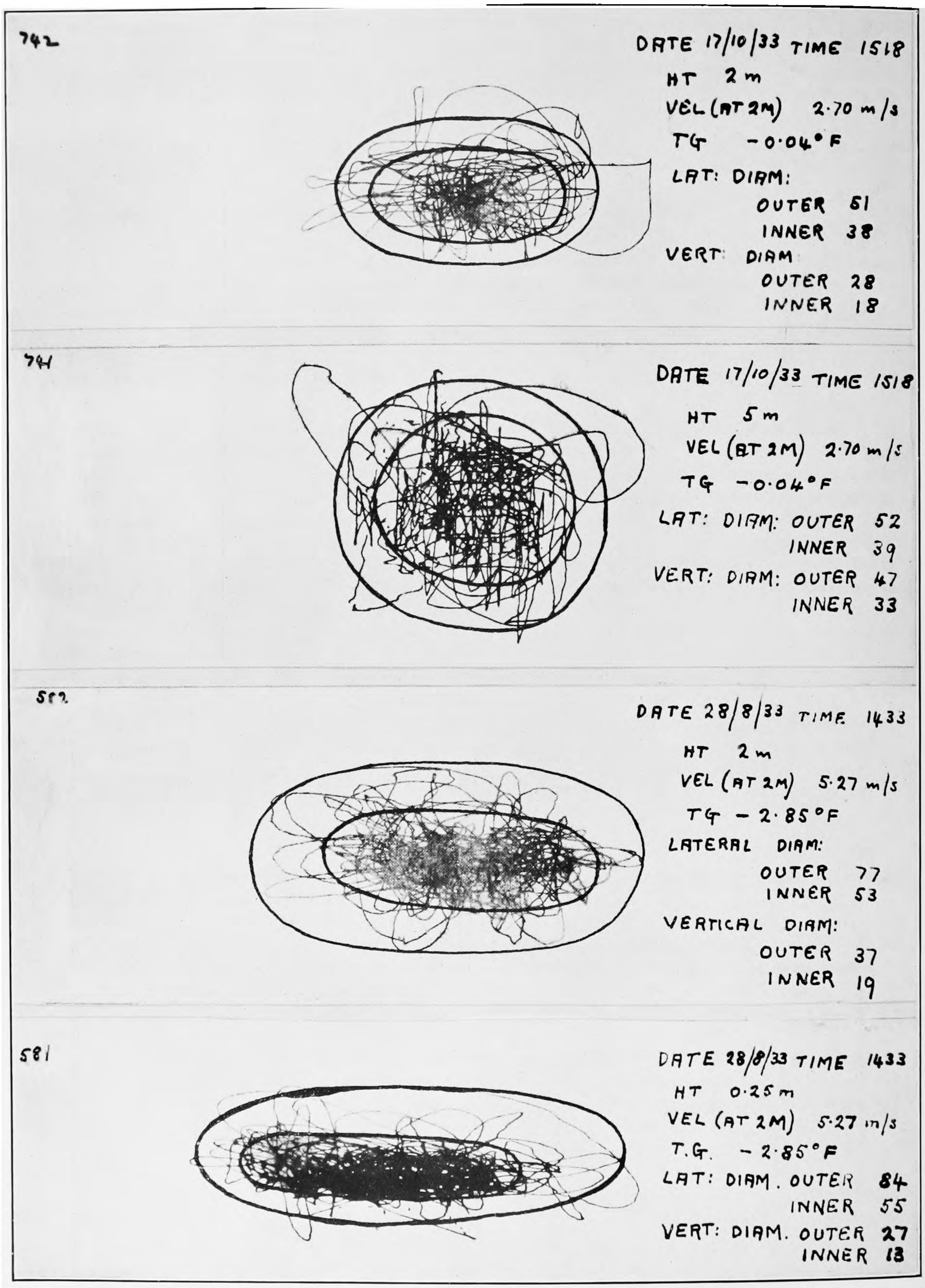


FIG. 25.—SOME TYPICAL TRACES OBTAINED FROM BIDIRECTIONAL VANES.

It may appear that despite these precautions the drawing of the ovals was still subject to considerable uncertainty. In order to test this the following procedure was adopted. After drawing the first 850 ovals the writer (Observer A) erased the first hundred and redrew them. The first fifty were then erased again and Observer B drew them. Assuming that the second estimate by A gave the true values the mean errors of the other estimates were calculated. They are given in Table XXIII. It is clear that the error in drawing the ovals is fairly small.

An attempt was also made to find the dimensions of the inner part of the trace where the line density is fairly high. It is difficult to describe this central portion clearly ; its nature can best be appreciated by examining the typical traces shewn in Fig. 25. It will be seen that inside the inner ovals on these traces the line density is much higher than in the area between the two ovals. It appears that the dimensions of the inner oval correspond approximately to a mean gustiness and the dimensions of the outer oval to the extreme gustiness. It was found impossible to formulate a definition of this inner oval, but since a distinction undoubtedly did exist between the inner and outer parts of many of the traces these inner ovals were drawn wherever the trace appeared suitable. Owing to the uncertainty of the exact position of the inner oval, however, the results of such measurements are not as reliable as those based upon the outer oval.

Anticipating results somewhat it may be mentioned here that under certain conditions of very low gustiness (e.g. low wind velocity and a fairly large temperature inversion) some of the traces were of a radically different nature. On a few occasions the vane did not move at all and the resulting trace was just a dot. On other occasions the vane apparently moved just once and the trace consisted of a short line (about one to two centimetres) more or less straight. When dealing with such traces geometrical definitions were disregarded and the oval circumscribing a dot was assumed to have two main diameters of one millimetre each, while an oval circumscribing either a horizontal or a vertical line was assumed to have the appropriate diameter one millimetre in length. The occasions on which such traces occurred will be indicated later.

§ 26—DESCRIPTION OF EXPERIMENTS

The site of the experiments was a level open field containing a hockey pitch and a cricket pitch (the same, in fact, as the site of the experiments upon velocity gradient), the nearest obstruction being a small hut. When, owing to the direction of the wind, the apparatus had to be set up downwind from this hut, the site chosen was never nearer than 100 m. from the hut. The average height of the grass on this ground was about two to three centimetres.

The general procedure followed during these experiments was very similar to that of the velocity gradient experiments. One bi-directional vane was mounted on a light tubular mast at a height of 2 m. during every experiment while a second bi-directional vane was mounted at one of the auxiliary heights 25 cm., 49 cm., 1 m., or 5 m., the intention being to use the 2 m. results to examine the effect of velocity and temperature gradient upon gustiness while the relation between the traces obtained at the auxiliary heights and those at 2 m. should indicate the effect of height above the ground. It may appear inconsistent that 2 m. should be chosen as a " standard " height in this investigation while 1 m. was adopted as the standard height in other parts of this paper. It was originally intended to extend these results on gustiness to a greater height than 5 m. and with the facilities available 2 m. would have been a more convenient standard height for this purpose. Unfortunately circumstances prevented the original programme being completed. In additon to the bi-directional vanes, air meters were mounted on direction vanes at 1 m. and 2 m. and also at the auxiliary height. The portable temperature gradient apparatus was erected nearby, the galvanometer being read every 30 seconds during an experiment.

All the apparatus, except that used at 5m., was mounted on light tubular masts. The bidirectional vane and air meter used at 5m. were mounted on a light wooden tower. This tower was constructed so as to offer as little obstruction to the general air flow as possible. The general disposition of the instruments with one bidirectional vane and one air meter on the wooden tower is shewn in Fig. 24. It should, perhaps, be mentioned that the apparatus was erected especially for this photograph and that under working conditions it would not be erected with the rough meadowland, seen in the background, so near upwind of the instruments.

Most experiments lasted three minutes, though a few were made having a different period. It will be assumed in the succeeding sections that the results under consideration were obtained from three minute runs unless it is specifically stated otherwise.

No attempt was made always to use the same bidirectional vane at 2m. so that any individual peculiarities of either vane probably affect the traces at all heights equally when the results are meaned. Nevertheless, since the method of examining the effect of height upon gustiness depends essentially on comparing simultaneous traces made at two different heights by two different vanes, it was thought desirable to compare the two vanes at the same height. For this purpose it was usual to precede every series of experiments by three or four experiments in which the two vanes were mounted both at 2m. and separated horizontally by about 1 m. across wind.

It has been mentioned that in order to get a vane within 25 cm. of the ground a special short stand with a shallow chart holder was necessary. It seemed possible that the vane might be differently affected by eddies caused by the comparatively shallow chart holder near the vane and the normal width chart holder at a greater distance. Accordingly a number of experiments were carried out with one vane mounted on the special 25 cm. stand, but raised until it was at a height of 49 cm., and the other vane mounted at 49 cm. on a normal stand. These experiments could not be carried out at any other height since 49 cm. was the lowest height the vane could be erected on a normal stand, and above this height the vertical diameters of the traces were too big to be recorded on the narrow charts used in the 25 cm. chart holder.

§ 27—METHOD OF ANALYSIS

As has already been indicated the gustiness was assessed by drawing circumscribing ovals round the traces, subject to certain restrictions, and measuring the two main diameters. In cases where the trace appeared suitable an inner oval containing the dense central portion was also drawn and the two main diameters measured. All diameters are expressed in millimetres and it is assumed, unless stated otherwise, that the horizontal and vertical diameters are proportional to $v'/\bar{u}$ and $w'/\bar{u}$ respectively. This, of course, is an approximation, but where the variation of gustiness with respect to some other factor is being considered it is probably sufficiently accurate.

It is clear that the outer ovals correspond to " average extreme " values of $v'/\bar{u}$ and $w'/\bar{u}$, the word " average " being used to indicate that a few isolated gusts of exceptional magnitude have been neglected. It is not quite so certain what the inner ovals represent, since their definition is too vague, but after drawing a large number of them it appears to the writer that they probably correspond fairly closely to a mean gustiness. The evidence for this lies in the fact that, despite the lack of a formal specification of the inner oval, very little difficulty was experienced in most cases in deciding where it should be drawn. This appears to be due to the fact that near the inner oval the line density of the trace frequently changes very rapidly. Owing to the uncertainty about the inner ovals the results based upon the measurement of the outer ovals are regarded as being much more capable of a definite interpretation and evaluation.

In what follows the term " gustiness " will be used with its customary meaning, i.e., the ratio of eddy velocity to mean velocity. The words " extreme gustiness " will be used to denote the gustiness measured by the outer ovals and the words " mean gustiness " to denote the gustiness measured by the inner ovals. It is obvious that the inner oval can represent the true mean gustiness only approximately.

There were many more records obtained at 2m. than at any other height. Accordingly the lateral and vertical diameters of the outer ovals, denoted by D_y and D_z respectively, obtained at 2m. have been grouped and meaned according to velocity at 2 m. and temperature gradient to shew the effect of these two factors upon gustiness. In order to shew the effect of height the diameters of records at the auxiliary heights have been expressed as fractions of the corresponding diameters of the records obtained simultaneously at 2m. These fractions will be denoted by r_y and r_z. Thus

$$r_y(z) = \frac{D_y \text{ for record at auxiliary height } z}{D_y \text{ for simultaneous record at 2 m.}} \text{ etc.}$$

The diameters of the inner ovals will be denoted by d_y and d_z. These have all been expressed as fractions R_y and R_z of the corresponding diameters of the outer oval of the same trace. Thus

$$R_y = d_y/D_y \quad \text{etc.}$$

§ 28—COMPARISON OF TWO BIDIRECTIONAL VANES

It has been mentioned that each series of experiments was preceded by three or four experiments in which both bidirectional vanes were mounted at 2m. The diameters were meaned for each set of three or four. These means however shewed no systematic variation with time, indicating that the characteristics of the vanes did not change during the experiments, so finally the mean values of D_y and D_z for all the traces so obtained were calculated for each vane and are given in Table XXIV. The mean diameters from the comparison runs using a normal stand and the special 25 cm. stand are also shewn.

TABLE XXIV—COMPARISON BETWEEN TWO BIDIRECTIONAL VANES

	Vane	No. of Runs.	Lateral Diameter D_y	Vertical Diameter D_z
			mm.	mm.
Both vanes on normal stands	No. 1 No. 2	83	68·5 68·1	39·0 37·4
One vane on normal stand and one on special 25 cm. stand	No. 1 No. 2	11	74·9 73·7	28·1 27·6

It is clear from Table XXIV that there was no substantial difference between the two vanes irrespective of the stands on which they were mounted.

§ 29—EFFECT OF WIND VELOCITY AND TEMPERATURE GRADIENT ON EXTREME GUSTINESS AT 2 M. ABOVE THE GROUND

The diameters D_y and D_z of the outer ovals of the traces obtained at 2 m. have been grouped and meaned according to the wind velocity at 2 m. and the temperature gradient in a similar way to that in which the velocity gradients were grouped and meaned in the second part of this paper (except that the height at which velocity was measured was 1 m. in the case of velocity gradient). The results are given in Table XXV. Each group in the table consists of three figures, viz., the mean values of D_y and D_z and the number of observations N upon which the means are based. A key is provided in the bottom right hand corner of the table. Certain groups in the top right hand corner of the table are separated from the rest of the table by a thick line. It is thought that the actual numerical values of the diameters in these groups are liable to considerable error both on account of the few observations on which they are based and also because the traces were found to shew large departures from the normal oval shape. This latter fact is obvious from an examination of the values given for the diameters. Nevertheless these groups are of value as they do indicate that under such conditions the gustiness, particularly in the vertical direction, is very small.

Here, again, the effects of velocity and temperature gradient are not independent, and the effect of varying the one factor while restricting the other to a narrow range of values will be considered.

Effect of temperature gradient.—For the lowest velocity band (0·5 to 1·0 m./sec.) the number of observations is quite insufficient to do more than indicate roughly the effect of temperature gradient upon gustiness, though it seems fairly safe to conclude that gustiness is much less during inversions than during heavy lapses.

In the next velocity group (1·0 to 1·5 m./sec.) the figures definitely shew that gustiness, both lateral and vertical, decreases as the temperature gradient changes from lapse to inversion though the values shew casual variations.

TABLE XXV—MEAN VALUES OF D_y AND D_z OBTAINED AT 2 M.

Temperature Gradient / Velocity (2m). m/sec.	< −3·6°F.	−3·6°F. to −2·7°F.	−2·7°F. to −1·8°F.	−1·8°F. to −0·9°F.	−0·9°F. to 0·0°F.	0·0°F. to 0·9°F.	0·9°F. to 1·8°F.	1·8°F. to 2·7°F.	2·7°F. to 3·6°F.	> 3·6°F.
0·5 to 1·0			71 41 (2)				36 1 (1)	26 3 (1)	15 4 (2)	1 1 (1)
1·0 to 1·5		85 37 (4)	73 43 (2)	91 39 (2)	19 11 (1)	22 12 (5)	17 6 (1)	39 15 (2)		7 2 (1)
1·5 to 2·5	73 43 (4)	88 43 (6)	65 33 (2)		51 29 (2)	47 27 (9)	42 24 (15)	19 8 (1)		
2·5 to 4·0	72 37 (16)	82 41 (38)	76 38 (7)	70 35 (5)	70 36 (17)	52 33 (56)	55 27 (4)			
4·0 to 6·0	91 42 (15)	73 38 (55)	76 41 (14)	71 40 (42)	66 39 (25)	68 40 (31)	67 42 (6)			
6·0 to 8·0		77 42 (2)	65 40 (5)	71 41 (9)	68 39 (24)	71 43 (18)	63 39 (2)			
> 8·0				67 41 (7)	74 42 (5)	67 41 (4)				D_y D_z (N)

This result is confirmed in the next two velocity groups (1·5 to 2·5 m./sec. and 2·5 to 4·0 m./sec.). The effect is smaller however, and is evidently decreasing in magnitude as the velocity increases. The figures for these two groups are plotted in Figs. 26a and 26b.

For velocities between 4·0 and 6·0 m./sec. the lateral gustiness shews a slight decrease as the temperature gradient changes from lapse to inversion but there is no apparent change in the vertical gustiness. For the higher velocities there appears to be no variation in gustiness with changing temperature gradient.

Before leaving the discussion of the effect of temperature gradient upon gustiness there are two points which should be mentioned. Very few experiments were carried out during heavy lapses and very light winds since, under such conditions, the wind direction changes through very wide angles and makes it impossible to set up the apparatus across the mean wind direction. Probably this long period wind swinging, which gives rise to abnormally large values of D_y, should not be considered as gustiness in the same sense as the more normal oscillations of shorter period and less amplitude which are the main subject of this section. The second point refers to the fact that owing to the impossibility of working in the darkness most of the inversion records were obtained shortly after the evening " crossover " of temperature gradient. If there is any appreciable lag in the damping out of gustiness by the inversion the figures in Table XXV will thus overestimate the gustiness during inversions.

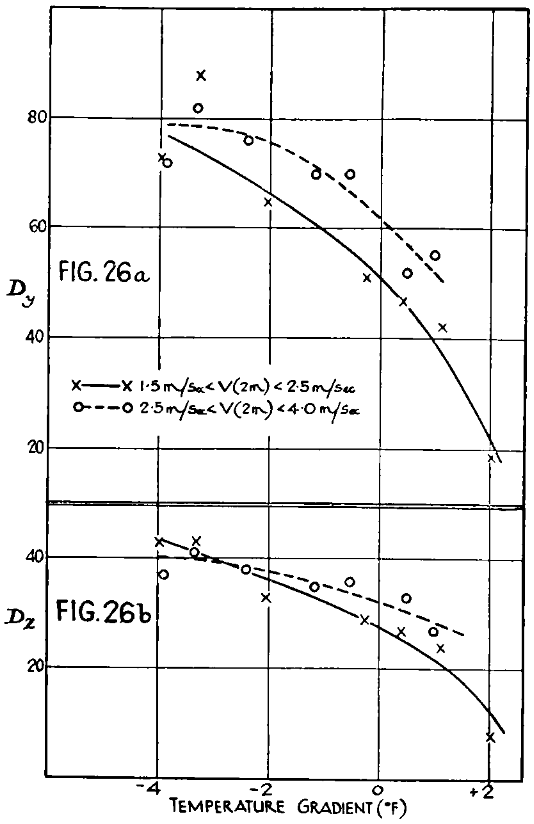


FIG. 26.—VARIATION OF GUSTINESS WITH TEMPERATURE GRADIENT.

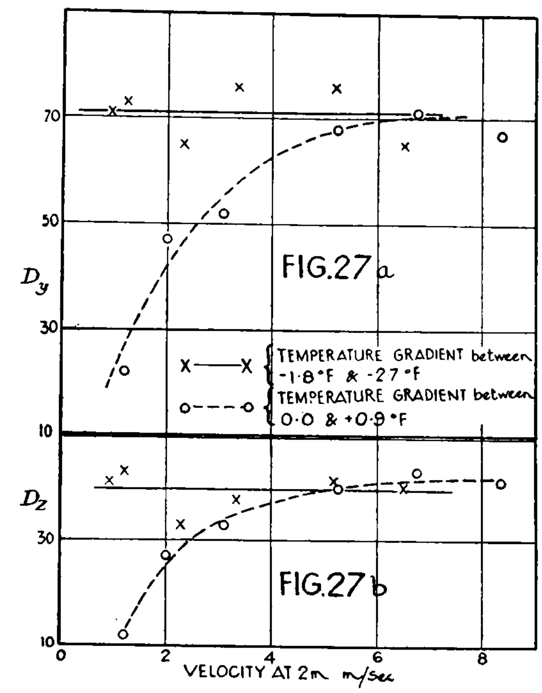


FIG. 27.—VARIATION OF GUSTINESS WITH VELOCITY.

Effect of wind velocity upon gustiness.—For lapses greater than —0·9°F. the results are somewhat affected by casual variations. The heaviest lapses shew an apparent increase in lateral gustiness with increasing velocity. Lapses between —2·7°F. and —3·6°F. however shew a slight decrease in lateral gustiness with increasing velocity. The other two columns (lapses —1·8°F. to —2·7°F. and —0·9°F. to —1·8°F.), shew no progressive variation of lateral gustiness with velocity. On the whole it seems probable that lateral gustiness is independent of velocity for lapses greater than —0·9°F. This conclusion applies with greater certainty to the vertical gustiness as there is very little variation in D_z, provided the lapse is greater than —0·9°F. For lapses less than —0·9°F. and for inversions gustiness, in both lateral and vertical directions, quite obviously increases with increasing velocity. The small gustiness is maintained for higher velocities in the greater inversions. The variation of gustiness with velocity for lapses of —1·8°F. to —2·7°F. and for inversions of 0·0°F. to 0·9°F. is shewn in Figs. 27a and 27b, these two temperature gradient ranges being selected as representative of the two types of variation. These curves emphasize the fact that at high velocities there is no difference between gustiness in large lapses and gustiness in inversions.

Owing to the effects of friction the measurement of gustiness at low velocities is rather difficult when the measuring instrument contains moving parts which are moved by the wind either directly, as with the present bidirectional vanes, or indirectly as with a Dines float anemometer. If the gustiness appears to decrease with decreasing velocity there is frequently a temptation to blame the instrument and it seems desirable to examine this point in connexion with the present results. Every endeavour was made to reduce friction in the construction of the bidirectional vanes by employing point bearings but it could not, of course, be eliminated entirely.

Referring to Table XXV it will be seen that gustiness decreases as the temperature gradient changes from lapse to inversion with constant velocity. This effect is well marked for all velocities up to 4·0 m./sec. though it is greater at the lower velocities. Since this decrease in gustiness occurs with constant velocity it seems impossible that it should be due to friction. The effect may be accentuated slightly by the residual friction in the instrument becoming more important as the true gustiness decreases, but the prime cause of the apparent reduction in gustiness must be a real reduction in gustiness. Thus the small values obtained for the gustiness during inversions and low velocities are real and not due to friction. The large values obtained during inversions and high velocities cannot be caused by friction and hence, during inversions, gustiness must increase with increasing velocity and this effect, as shewn in Table XXV. is not due to friction.

There remains the possibility that the effect attributed to velocity may really be due to temperature gradient. This might arise if, e.g. in the column of Table XXV shewing the effect of velocity upon D_y and D_z for a temperature gradient between 0·0°F. and 0·9°F., the mean temperature gradient corresponding to the lower velocity groups is appreciably larger (algebraically) than that corresponding to the higher velocity groups. That this is not so is shewn by Table XXVa.

TABLE XXVa—MEAN TEMPERATURE GRADIENTS FOR COLUMN OF TABLE XXV CORRESPONDING TO A TEMPERATURE GRADIENT 0·0°F. TO 0·9°F.

Velocity range m./sec. ..	1·0 to 1·5	1·5 to 2·5	2·5 to 4·0	4·0 to 6·0	6·0 to 8·0	> 8·0
Mean temperature gradient °F/m.	+0·25	+0·41	+0·50	+0·35	+0·43	+0·17

The effect of velocity and temperature gradient upon gustiness may be summed up by saying that the figures on the left hand side of Table XXV are fairly large and that they decrease towards the right hand side, the decrease being large at the top of the table and small at the bottom. The figures for light winds and big inversions shew that under such conditions the gustiness is reduced to a very small amount. It will be remembered that it was under such conditions that a very large velocity gradient was measured as described in the second part of this paper. These gustiness figures confirm the suggestion made there that the flow of air may, at times, approach very closely to a streamline flow.

Ratio of lateral gustiness to vertical gustiness.—It is of interest to consider the ratio D_y/D_z. This ratio was calculated for every pair of values in Table XXV except the pairs in the top right hand corner separated from the rest of the table by a thick line. These pairs were not included since, as has been explained, the traces were not of a suitable type for drawing circumscribing ovals. On examining the values of the ratio D_y/D_z it was seen that there was no progressive variation with temperature gradient but there did appear to be a variation with velocity. Thus the shape of the trace does not depend upon temperature gradient (over the range of temperature gradient for which it is practicable to draw circumscribing ovals) but it does upon velocity. Accordingly the values of D_y/D_z were meaned irrespective of temperature gradient for each velocity band, due regard being paid to the number of observations contributing to each value of D_y/D_z. These results are given in Table XXVI.

TABLE XXVI—RATIO OF LATERAL GUSTINESS TO VERTICAL GUSTINESS AT 2 M.

Mean Velocity m./sec.	0·92	1·22	2·01	3·24	5·04	6·76	8·55	All
No. of Records ..	2	14	38	143	188	60	16	461
Ratio D_y/D_z	1·73	2·01	1·80	1·83	1·83	1·70	1·67	1·81

It can be seen from this table that D_y/D_z decreases slightly for the higher velocities. The amount is small however and a fairly good approximation is obtained by assuming the ratio to have the value 1·81 irrespective of velocity. This value 1·81, obtained by meaning all the results, is somewhat greater than the value (1·61) obtained by Scrase(17).

Effect of velocity gradient upon gustiness.—It had seemed possible that although there is a connexion between temperature gradient and velocity gradient, when a number of observations are averaged, gustiness might be affected more directly by velocity gradient. Accordingly the values of D_y and D_z at 2 m. were grouped and meaned according to velocity (at 2 m.) and velocity gradient [(Velocity at 2 m.)/(Velocity at 1 m.)]. It was found however that the observations were not so well distributed over the velocity gradient range as over the range of temperature gradient. The resulting table shewed the same general variation as Table XXV (with large velocity gradients corresponding to inversions), but it was clear that for the present observations no advantage was to be gained by using velocity gradient as an independent variable rather than temperature gradient.

§ 30—VARIATION OF EXTREME GUSTINESS WITH HEIGHT

As has been already explained the method adopted for determining the effect of height upon gustiness was to find the ratios r_y and r_z of the diameters of the circumscribing oval at height z to the corresponding diameters of the circumscribing oval of a simultaneous trace at height 2 m.

More traces were obtained at a height of 1 m. than at any other auxiliary height and accordingly the quantities r_y and r_z for 1 m. have been grouped and meaned according to velocity and temperature gradient and are given in Table XXVII. The numbers of observations contributing to each group are mostly rather small with the result that the values of r_y and r_z shew some irregular variations. An examination of the table however shews no progressive variation of either r_y or r_z with either velocity or temperature gradient. Putting this fact another way—the effect of velocity and temperature gradient upon gustiness appears to be the same at 1 m. as at 2 m. Owing to the comparatively small number of records obtained at 1 m. this statement cannot be regarded as having been proved conclusively. It

TABLE XXVII—RATIO OF GUSTINESS AT 1 M. TO GUSTINESS AT 2 M.

Temperature Gradient / Velocity (2m). m/sec.	< −3·6°F.	−3·6°F. to −2·7°F.	−2·7°F. to −1·8°F.	−1·8°F. to −0·9°F.	−0·9°F. to 0·0°F.	0·0°F. to 0·9°F.	0·9°F. to 1·8°F.	1·8°F. to 2·7°F.	2·7°F. to 3·6°F.	> 3·6°F.
0·5 to 1·0							0·03 1·00 (1)	0·08 1·67 (1)	0·15 0·37 (2)	1·00 1·00 (1)
1·0 to 1·5		0·99 0·68 (2)	0·96 0·69 (2)		0·79 0·45 (1)	0·90 0·44 (5)	0·94 0·83 (1)	0·38 1·55 (2)		2·57 3·00 (1)
1·5 to 2·5	1·12 0·94 (3)	1·07 0·81 (1)	1·08 0·79 (2)		1·05 0·91 (1)	0·93 0·71 (7)	0·96 1·05 (9)	2·00 0·75 (1)		
2·5 to 4·0	1·05 0·82 (5)	1·02 0·85 (15)	1·03 1·00 (4)	1·04 0·96 (2)	0·98 0·76 (5)	1·07 0·78 (12)	1·02 0·90 (4)			
4·0 to 6·0	1·01 0·82 (2)	1·00 0·91 (17)	0·98 0·89 (7)	1·05 0·92 (13)	1·04 0·95 (6)	1·04 0·81 (1)				
6·0 to 8·0		0·96 0·84 (1)	1·05 0·94 (3)	0·97 0·98 (1)	0·98 1·03 (3)	1·01 0·92 (1)				
> 8·0				1·13 0·93 (2)	0·96 0·93 (1)					r_y r_z (N)

seems safe to assume however that r_y and r_z do not vary much with velocity and temperature gradient. In this paper they will be assumed to be constant and the values will be obtained by meaning all the figures in Table XXVII (excluding certain values in the top right hand corner of the table for reasons already given).

At the other auxiliary heights only a few records were obtained. Accordingly only the mean values for these other heights are given, no attempt being made to tabulate them according to velocity and temperature gradient. The mean values for all heights and the number of observations upon which they are based are given in Table XXVIII. Instead of tabulating r_y however the values of $1{\cdot}81r_y$ are given. The columns for $1{\cdot}81r_y$ and r_z thus give the relative values of lateral and vertical gustiness on the same scale, at various heights. The last column gives the ratio $1{\cdot}81r_y/r_z$ for each height. It is clear that lateral gustiness decreases with height, but the variation is quite small. Vertical gustiness increases with height, the variation being roughly twice as much as in the case of lateral gustiness. The last column of Table XXVIII shews how the shape of the trace, i.e. the ratio of lateral gustiness to vertical gustiness, changes with height. It is clear that the two components of gustiness approach equality as the height is increased, but that near the ground the lateral component of gustiness is much greater than the vertical component.

TABLE XXVIII—RELATIVE VALUES OF LATERAL AND VERTICAL GUSTINESS AT VARIOUS HEIGHTS

Height z	No. of Experiments.	Mean Value $1{\cdot}81r_y$	Mean Value r_z	$1{\cdot}81r_y/r_z$
cm.				
25	19	1·99	0·68	2·93
49	59	1·91	0·74	2·58
100	138	1·83	0·86	2·13
200	—	1·81	1·00	1·81
506	71	1·61	1·15	1·40

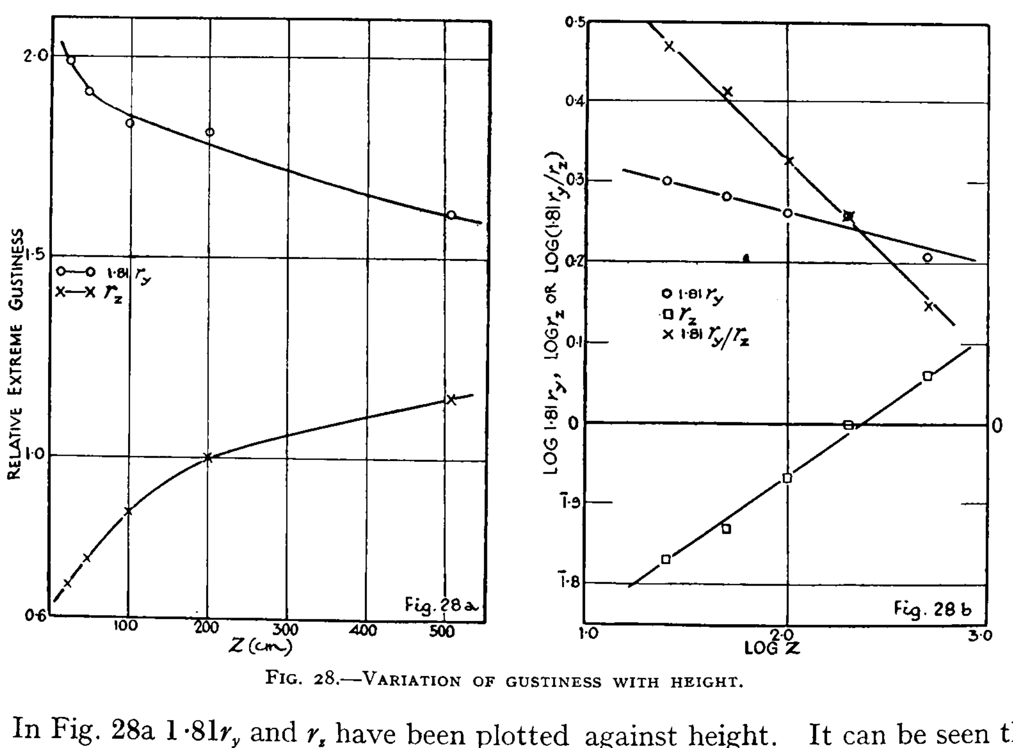


FIG. 28.—VARIATION OF GUSTINESS WITH HEIGHT.

In Fig. 28a $1{\cdot}81r_y$ and r_z have been plotted against height. It can be seen that r_z increases with height throughout the whole range though the rate of increase falls off. It seems very unlikely that r_z could continue to increase indefinitely with height and it is probable that, above a certain limiting value of the latter, vertical gustiness may even decrease. The lateral component of gustiness increases as the surface is approached, the rate of increase becoming larger near the surface. This does not imply that the actual movements of the air due to turbulence increase near the surface (as will be seen later) since the mean velocity decreases near the surface.

There is a rather remarkable resemblance between the curves in Fig. 28a and two curves given by Fage and Townend(18), who examined the quantities corresponding to what are here called components of extreme gustiness for the flow of water through a square section pipe having sides of 0·89 inch, and give curves shewing the variation of lateral and vertical gustiness with distance from the side of the pipe. Thus the vertical component corresponds to the lateral component of this paper and *vice versa*. Fage and Townend shew that the vertical component (in the pipe) decreases, rapidly at first and then more slowly, as the distance from the side of the pipe increases. This is exactly as shewn by the curve for $1{\cdot}81\ r_y$ in Fig. 28a. The lateral component of gustiness in the pipe increases rapidly at first with distance from the side, reaches a maximum value which is less than the vertical component at the same place, and then falls off slowly. Eventually the lateral and vertical components reach approximate equality. Probably the curve for r_z in Fig. 28a corresponds to the first part of Fage and Townend's curve for lateral gustiness before it reaches a maximum.

In Fig. 28b log z has been plotted against log $1{\cdot}81r_y$, log r_z and log $(1{\cdot}81r_y/r_z)$. As can be seen all the resulting graphs approximate fairly closely to straight lines. Measurement of the slopes of these lines indicates that the lateral component $(1{\cdot}81r_y)$ and the vertical component (r_z) of gustiness are linear functions of $z^{-0{\cdot}062}$ and $z^{0{\cdot}175}$ respectively. The variation of the shape of the trace is also obtained as a linear function of $z^{-0{\cdot}24}$. These relations cannot hold good very far above 5 m. since it is fairly clear that as the height increases and the influence of the ground becomes less the lateral and vertical components of gustiness must become equal. The data available here unfortunately does not extend high enough to shew at what height this equality is reached, but the slopes of the lines in Fig. 28b suggest that it should take place at about 25 m. This however is somewhat speculative.

§ 31—EFFECT OF DURATION OF TRACE

Scrase(17) has shewn that the diameters of the records increase rapidly during the first minute but afterwards only slowly and steadily. As a matter of interest a few records of different durations were obtained and the diameters afterwards expressed as fractions of the corresponding diameters of a three minute record for similar conditions, the three minute diameters being estimated from Table XXV. The mean result for each period is shewn in Table XXIX.

TABLE XXIX—EFFECT OF DURATION OF RECORD ON DIAMETERS

Duration in Minutes.	1	2	3	5	10
Relative y diameter ..	0·83	0·85	1·00	0·90	1·04
,, z ,, ..	0·87	1·01	1·00	0·87	1·02
No. of Experiments ..	11	5	—	4	3

Despite the scatter due to the small number of observations it is clear that the duration has very little effect provided it exceeds one minute.

§ 32—MEAN GUSTINESS

Ratio of mean gustiness to extreme gustiness.—So far only the gustiness described as " extreme gustiness " has been considered. As has been explained however many of the records shewed a central part which appeared to correspond to a " mean gustiness" and these will now be considered. Since the inner oval could be drawn only on some of the traces and since the definition of it is rather vague it was thought desirable to express the diameters of the inner oval as fractions of the diameters of the outer oval on the same record rather than consider the actual measurement of the inner oval. It had been thought that the ratio of mean gustiness to extreme gustiness might be more or less independent of other factors and that this procedure would shew this independence, or otherwise, more clearly than by examining the mean gustiness directly.

TABLE XXX—RATIO OF MEAN GUSTINESS TO EXTREME GUSTINESS

Temperature Gradient / Velocity (2m). m/sec.	< −3·6°F.	−3·6°F. to −2·7°F.	−2·7°F. to −1·8°F.	−1·8°F. to −0·9°F.	−0·9°F. to 0·0°F.	0·0°F. to 0·9°F.	0·9°F. to 1·8°F.	1·8°F. to 2·7°F.
				Height 2 m.				
1·0 to 1·5					0·53 0·64 (1)	0·59 0·47 (3)		0·56 0·46 (1)
1·5 to 2·5		0·45 0·41 (1)	0·48 0·47 (1)		0·58 0·70 (1)	0·63 0·59 (6)	0·57 0·46 (15)	0·63 0·75 (1)
2·5 to 4·0	0·67 0·45 (10)	0·57 0·47 (22)	0·65 0·51 (3)	0·64 0·56 (2)	0·63 0·58 (10)	0·66 0·59 (39)	0·67 0·52 (3)	
4·0 to 6·0	0·68 0·54 (12)	0·68 0·61 (43)	0·75 0·59 (13)	0·67 0·60 (35)	0·65 0·59 (22)	0·69 0·63 (23)	0·67 0·62 (6)	
6·0 to 8·0		0·71 0·67 (2)	0·72 0·67 (5)	0·69 0·66 (5)	0·71 0·67 (19)	0·72 0·67 (15)	0·75 0·67 (2)	
> 8·0				0·75 0·63 (7)	0·69 0·55 (3)	0·69 0·66 (2)		R_y R_z (*N*)
				Height 1 m.				
1·5 to 2·5		0·40 0·40 (1)			0·57 0·52 (1)	0·66 0·52 (6)	0·56 0·54 (4)	
2·5 to 4·0	0·69 0·46 (4)	0·59 0·49 (10)	0·53 0·39 (1)	0·57 0·45 (1)	0·71 0·54 (5)	0·66 0·56 (12)	0·64 0·45 (3)	
4·0 to 6·0	0·70 0·60 (2)	0·68 0·62 (17)	0·71 0·56 (7)	0·66 0·57 (12)	0·62 0·51 (6)	0·62 0·59 (1)		
6·0 to 8·0		0·63 0·50 (1)	0·56 0·54 (3)	0·78 0·45 (1)	0·66 0·58 (4)	0·59 0·46 (3)		
> 8·0				0·71 0·69 (2)	0·76 0·61 (1)			R_y R_z (*N*)

The records obtained at 1 m. and 2 m. on which an inner oval could be drawn covered a fairly wide range of velocity and temperature gradient but at the other heights comparatively few records were available. Accordingly Table XXX gives values of R_y and R_z and the number of records contributing to them for various velocities and temperature gradients for 1 m. and 2 m. only.

The effect of temperature gradient will be considered first. The scatter of the points in Table XXX makes it rather difficult to say whether R_y and R_z are affected by temperature gradient or not. At the higher velocities it appears fairly certain that they are not, but at the lower velocities there is a suggestion that R_y and R_z increase as the temperature gradient changes from lapse to inversion. Bearing in mind the small number of observations contributing to some of the figures in Table XXX and the uncertainty associated with the measurement of the inner ovals it is not possible to be definite on this point. The position can be summed up by saying that at low velocities R_y and R_z are probably slightly greater in inversions than in lapses and at higher velocities R_y and R_z are independent of temperature gradient. The physical significance of the first part of this statement is that at low velocities the damping effect of the stable density gradient associated with inversions acts first on the larger gusts. This may be compared with a remark in the memoir by M. A. Giblett and Others(12) referred to previously. There (p. 52) it is stated " the falling off of the eddy energy after the formation of the inversion is pronounced for the larger eddies, but not for the smaller eddies." Apart from this, temperature gradient appears to affect extreme and mean gustiness equally. Since the effect of temperature gradient on R_y and R_z is so uncertain and is in any case small it will now be neglected.

The effect of velocity upon R_y and R_z is obvious from an inspection of Table XXX. If temperature gradient is to be neglected however the effect of velocity can be studied more easily by finding mean values of R_y and R_z for the various velocity bands irrespective of temperature gradient. This has been done and the results are given in Table XXXI, the corresponding figures for the other heights also being given. The number of records contributing to each mean value of R_y and R_z is given and also the mean value of the temperature gradient.

TABLE XXXI—RATIO OF MEAN GUSTINESS TO EXTREME GUSTINESS

N	Mean Temperature Gradient °F./m.	Velocity at 2 m. m./sec.	R_y	R_z	N	Mean Temperature Gradient °F./m.	Velocity at 2 m. m./sec.	R_y	R_z	N	Mean Temperature Gradient °F./m.	Velocity at 2 m. m./sec.	R_y	R_z
25 cm.					49 cm.					100 cm.				
										12	+0·21	2·07	0·60	0·52
					19	−2·18	3·46	0·65	0·51	36	−1·28	3·15	0·65	0·51
11	−2·33	5·38	0·69	0·60	39	−2·32	4·70	0·68	0·52	45	−2·13	5·02	0·67	0·58
8	−1·14	7·04	0·74	0·61	8	−1·48	6·57	0·71	0·65	12	−1·09	6·63	0·63	0·52
										3	−1·30	8·59	0·73	0·66
200 cm.					506 cm.					Mean for all heights				
4	+0·23	1·21	0·58	0·51						4	+0·23	1·21	0·58	0·51
24	+0·56	2·02	0·58	0·50	6	+1·29	1·96	0·62	0·58	42	+0·56	2·03	0·59	0·52
89	−1·21	3·27	0·63	0·54	10	+0·39	3·42	0·71	0·62	154	−1·24	3·28	0·64	0·53
154	−1·63	4·99	0·68	0·60	36	+0·06	5·30	0·72	0·65	285	−1·62	5·01	0·68	0·59
48	−0·52	6·76	0·71	0·67	17	+0·23	6·62	0·73	0·68	93	−0·59	6·74	0·71	0·65
12	−0·81	8·65	0·72	0·62	1	−0·08	8·27	0·69	0·59	16	−0·86	8·60	0·72	0·63

Before considering the effect of velocity as shewn by Table XXXI it will be profitable to compare the results for different heights. It will be seen that for approximately the same velocity at 2 m. there is no progressive change in R_y and R_z with changing height except that the values for R_y and R_z for low velocities at 5 m. appear to be rather high. If now, the mean temperature gradients corresponding to these observations are examined, it will be seen that the average temperature gradients corresponding to the 5 m. results are inversions (except for the highest velocity point) whereas most of the values for the other heights correspond to mean lapses. Thus the small increase in R_y and R_z at 5 m. for low velocities, as compared with other heights, is probably attributable to the slight effect of temperature gradient (as described in § 32 p. 59) rather than to the effect of height. If this be so then the ratio of mean gustiness to extreme gustiness is independent of height within the region examined.

Since R_y and R_z are independent of height, and nearly independent of temperature gradient it is advantageous to average the results for all heights and all temperature gradients for each velocity band, the small and possibly doubtful effect of temperature gradient being neglected. This has been done and the results are given in the last part of Table XXXI. It is apparent that the ratio of mean gustiness to extreme gustiness increases with increasing velocity. These values are plotted in Fig. 29.

Variation of mean gustiness.—If the variation of mean gustiness is considered separately, as distinct from the ratios R_y and R_z, then the results of the above paragraphs must be combined with those of § 29 and § 30. If this is done it will be seen that the variations of the components of mean gustiness with temperature gradient are similar to the variation of D_y and D_z (Figs. 26a and 26b and Table XXV) though possibly slightly smaller for the low velocities. During medium and large lapses, D_y and D_z were found to be substantially independent of velocity. The variation of mean gustiness under these conditions will therefore be indicated by

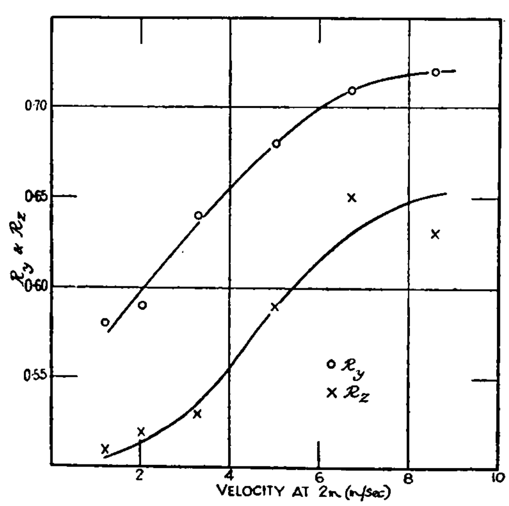


FIG. 29.—EFFECT OF VELOCITY UPON THE RATIO OF MEAN GUSTINESS TO EXTREME GUSTINESS.

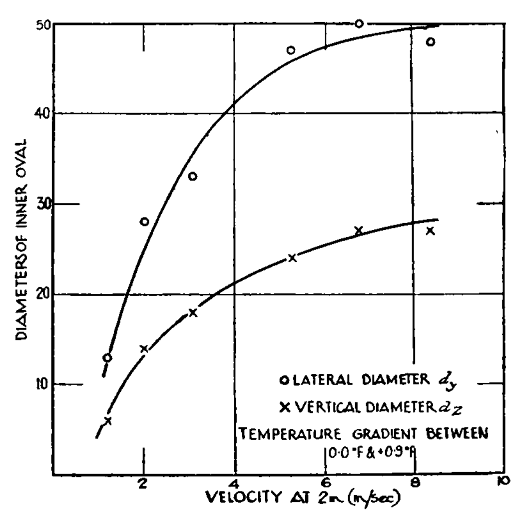


FIG. 30.—VARIATION OF MEAN GUSTINESS WITH VELOCITY DURING INVERSIONS.

Fig. 29. During inversions and small lapses, however, it was found that the components of extreme gustiness increased with increasing velocity. Since R_y and R_z also increase with velocity, the variation in mean gustiness under these conditions will be even more marked. Numerical values to illustrate this can be obtained by multiplying the appropriate figures in Table XXV by values of R_y and R_z obtained from Fig. 29. In order to make this procedure clearer the variation with velocity of mean gustiness for a temperature gradient of 0·0 to 0·9°F./m will be considered.

TABLE XXXII—VARIATION OF MEAN GUSTINESS AT 2M. WITH VELOCITY. TEMPERATURE GRADIENT 0·0 TO 0·9°F./M.

Mean Velocity	D_y (Table XXV)	R_y (Fig. 29)	d_y	D_z (Table XXV)	R_z (Fig. 29)	d_z
*m./sec.						
1·22	22	0·58	13	12	0·51	6
2·01	47	0·60	28	27	0·52	14
3·09	52	0·63	33	33	0·53	18
5·25	68	0·69	47	40	0·59	24
6·77	71	0·71	50	43	0·63	27
8·37	67	0·72	48	41	0·65	27

* These are the mean velocities corresponding to the values of D_y and D_z and have not been given before.

The values obtained for d_y and d_z (the components of mean gustiness) at 2m. for a temperature gradient between 0·0 and 0·9°F./m. are set out in Table XXXII and are shewn plotted in Fig. 30. Since R_y and R_z are independent of height and since the variation of extreme gustiness with velocity and temperature gradient is similar at all heights, it follows that similar curves to those in Fig. 30 would be obtained for any height, though the actual numerical values would have to be multiplied by the appropriate factor from Table XXVIII.

It must be emphasized that the results based on the measurements of the inner ovals are not as reliable as those based on the outer ovals owing to the lack of an adequate definition of the inner oval. Yet the fact that the writer seldom found any difficulty in deciding where to draw the inner oval shews that the line density on the traces usually changed rather suddenly at the inner oval. Thus, though the inner oval does not correspond exactly to a theoretical mean gustiness, it is thought that the variations of the inner oval probably represent fairly closely variations in the mean gustiness.

§ 33—EDDY VELOCITIES

Movements of the vane in a lateral direction caused the pen to move in a circle in a horizontal plane, the radius of the circle being the radius of the chart holder. If θ_y be the angular deflection of the vane in a horizontal direction, then, as explained in § 23

$$\frac{v'}{\bar{u}} = \tan\theta_y \text{ and } \theta_y = \left(\frac{1}{c}\right) \times \text{deflection of pen}$$

where c = radius of chart holder.

Thus the values of the extreme eddy velocities in a lateral direction are given by

$$v' = \bar{u}\tan(D_y/2c)$$

The movement of the pen in a vertical plane is slightly more complicated since the pen moved on a vertical line whereas the axis of suspension of the pen described a circle. It will probably be sufficiently accurate to assume

$$w'/\bar{u} = \tan\theta_z = D_z/2c$$

$$w' = \bar{u}\, D_z/2c$$

For a constant velocity and a fixed height the variation of eddy velocity with temperature gradient is the same as the variation of gustiness and so need not be discussed any further. The cases which need discussion are the variation of both mean and extreme eddy velocity with mean velocity and with height.

Effect of wind velocity upon eddy velocity.—It was shewn in § 29 *Effect of wind velocity upon gustiness, p.* 53, that extreme gustiness does not vary appreciably with velocity during medium and large lapses. Accordingly, under these conditions, the variation of extreme eddy velocity will be identical with the variation of velocity. During small lapses and inversions however this is not the case : extreme gustiness increases with velocity and so extreme eddy velocity increases even more rapidly. The variation is the same for all heights considered in this paper and in order to illustrate it the variation of both extreme and mean eddy velocities at 2 m., for inversions between 0·0 and 0·9°F., will be examined. In Table XXXII the values of D_y and D_z and of d_y and d_z at 2 m. for an inversion between 0·0 and 0·9°F. and for various velocities are given. From these values the extreme and mean eddy velocities have been calculated and are given in Table XXXIII. The formulæ used are

$$\text{Extreme } v' = \bar{u} \tan (D_y/2c) \qquad \text{Extreme } w' = \bar{u} \ (D_z/2c)$$
$$\text{Mean } v' = \bar{u} \tan (d_y/2c) \qquad \text{Mean } w' = \bar{u} \ (d_z/2c)$$
$$c = \text{radius of chart holder} = 67 \text{ mm.}$$

It must be emphasized that Table XXXIII applies only to inversions between 0·0 and 0·9°F. During medium and large lapses extreme eddy velocity is proportional to mean velocity and during large inversions it increases more rapidly than is indicated in Table XXXIII.

TABLE XXXIII—VARIATION OF EDDY VELOCITIES AT 2 M. WITH MEAN VELOCITY. TEMPERATURE GRADIENT BETWEEN 0·0 AND 0·9°F./M.

Mean Velocity $\bar{u}$	Extreme Eddy Velocities.		Mean Eddy Velocities.	
	Lateral	Vertical	Lateral	Vertical
m./sec.	m./sec.	m./sec.	m./sec.	m./sec.
1·22	0·21	0·11	0·12	0·05
2·01	0·74	0·40	0·42	0·21
3·09	1·27	0·77	0·80	0·40
5·25	2·94	1·57	1·94	0·94
6·77	4·00	2·17	2·71	1·35
8·37	4·60	2·59	3·18	1·67

Since the ratio of mean to extreme gustiness increases with velocity it follows that even during large lapses the mean eddy velocity must increase more rapidly than the mean velocity. It is clear from Table XXV that the rate of increase of extreme gustiness with velocity falls off at the higher velocities and probably approaches a limiting value. Also although Table XXXI shews the ratio of mean to extreme gustiness increasing with velocity yet this ratio can never exceed unity. Thus ultimately, as the mean velocity continues to increase, the rate of increase of both extreme and mean eddy velocities must become proportional to the rate of increase of mean velocity.

Effect of height upon eddy velocity.—The variation of eddy velocity with height will be considered next, the method adopted being as follows. Table XXV gives the components of extreme gustiness at 2 m. for various velocities and temperature gradients and Table XXVIII the ratio of the corresponding components at other heights, these being independent of velocity and temperature gradient. From these two tables, together with the rate of increase of mean velocity with height as given in the second part of this paper, the variation of extreme eddy velocity with height can be calculated for any particular temperature gradient and velocity at 2 m. Since velocity gradient depends upon temperature gradient and upon velocity the variation of extreme eddy velocity so found will hold only for the particular values chosen of these two factors.

By using the appropriate values of R_y and R_z from Table XXXI (or Fig. 29) the variation of the components of mean eddy velocity can be obtained. It should be noted however that since R_y and R_z are independent of height the variation of the components of mean eddy velocity will be the same as the variations of the components of extreme eddy velocity.

In order to illustrate these relationships the effect of height upon extreme and mean eddy velocity for zero temperature gradient and a mean velocity of 3·7 m./sec. at 2 m. will be examined. From Figs. 26a and 26b the values of D_y and D_z at 2 m. for these conditions are found to be 61 and 33 respectively. The values of D_y and D_z at other heights are then calculated from Table XXVIII. The ratio of mean to extreme gustiness depends only upon velocity, and the appropriate ratios for the present case can be obtained from Fig. 29. They are 0·65 and 0·55 for the lateral and vertical components respectively. Thus the values of d_y and d_z are obtained. Finally the actual eddy velocities are obtained from the formulæ used in § 33 *Effect of wind velocity upon eddy velocity*, p. 61. The mean velocity $\bar{u}$ in these formulæ varies with height and the values are given in the second column of Table XXXIV. They were obtained by using Table XVIII in the second part of this paper. The last column of Table XXXIV shews some values of u'. These are the mean eddy velocities obtained in the third part of this paper and are included for comparison.

TABLE XXXIV—VARIATION OF EDDY VELOCITIES WITH HEIGHT. ZERO TEMPERATURE GRADIENT AND VELOCITY AT 2 M. = 3·7 M./SEC.

z	$\bar{u}$	D_y	D_z	d_y	d_z	Extreme		Mean		
						v'	w'	v'	w'	u'
cm.	m./sec	mm.	mm.	mm.	mm.	m./sec.	m./sec.	m./sec.	m./sec.	m./sec.
25	2·62	67	22	43	12	1·44	0·43	0·86	0·24	0·41
49	2·99	64	24	41	13	1·55	0·54	0·96	0·29	0·44
100	3·31	62	28	40	15	1·66	0·70	1·03	0·37	..
200	3·70	61	33	39	18	1·81	0·91	1·11·10	0·50	0·52 14
506	4·05	54	38	35	21	1·58	1·15	1·09	0·64	..

The values of v' and w' in Table XXXIV have been plotted against height in Fig. 31a. As can be seen the mean and extreme values of v' shew a maximum value between 2 m. and 5 m. though in the case of the mean lateral eddy velocity there is scarcely sufficient evidence to decide that the curve has a maximum rather than a constant value above 2 m. height. The curves for the vertical component of eddy velocity shew that they continue to increase with height between 25 cm. and 5 m. These curves are somewhat different from the corresponding curves given by Scrase(17) which shewed both lateral and vertical components of eddy velocity having a well defined maximum value at 1·4 m.

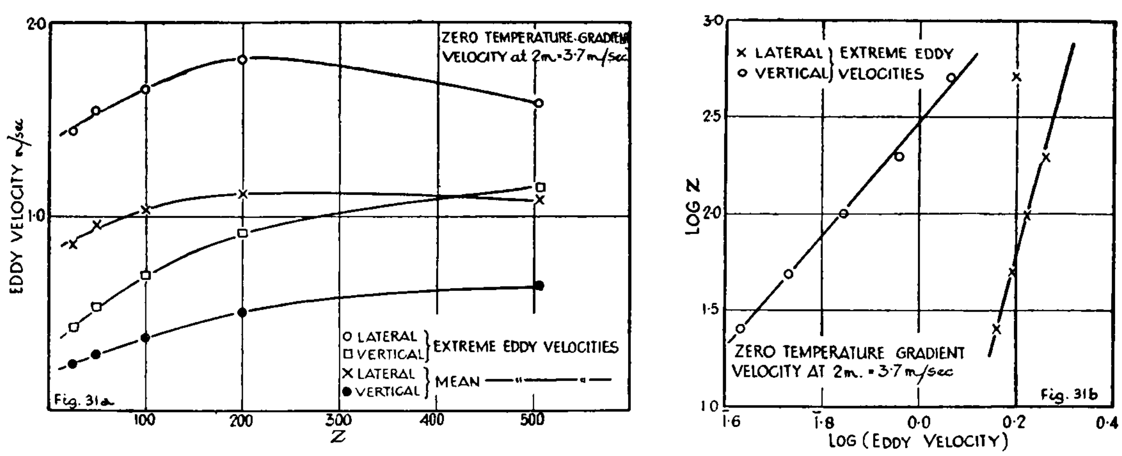


FIG. 31.—VARIATION OF EDDY VELOCITIES WITH HEIGHT.

In the paper by Fage and Townend(18), previously mentioned, curves were given shewing the variation of u', v' and w' with distance from the side of the square sectioned pipe. The length of the side of the square was $2s$ and the eddy velocities probably corresponded to what are here called extreme eddy velocities, the v' and w' components being interchanged owing to the distance being measured from a vertical wall. Describing these curves the authors say:

" Over the middle of the pipe these components are roughly equal to each other and they have a value of about 20 per cent. of the mean rate of flow U_0, or 14 per cent. of the velocity at the centre of the pipe U_c. As the wall is approached the component v_1 at first rises to a maximum value of $0{\cdot}25U_0$ at a distance of $0{\cdot}25s$ from the wall and then falls steadily to zero at the wall.

"The other two components, u_1 and w_1 also rise, but at a greater rate than v_1 and eventually reach maximum values of $0{\cdot}34U_0$ and $0{\cdot}39U_0$ respectively at a distance of about $0{\cdot}15s$ from the wall."

It seems clear that the curves for the vertical component of extreme eddy velocity in Fig. 31a must reach a maximum value at no great height above 5 m. If this is accepted the resemblance between the two curves for the extreme components in Fig. 31a and the description of Fage and Townend's curves is rather remarkable both in respect of the probable relative magnitudes and the relative positions of the maximum values.

In Fig. 31b log (eddy velocity) has been plotted against log z for the two components of extreme eddy velocity. It can be seen that the vertical component can be adequately represented by a power law up to 5 m. and the lateral component up to 2 m. By measuring the slopes of the lines in Fig. 31b the relations between z and v' and w' are found to be

$$v' = k_1 z^{0{\cdot}24} \text{ and } w' = k_2 z^{0{\cdot}11}$$

where k_1 and k_2 are constants for the height intervals stated. The indices in these equations could have been obtained by finding a power law to represent the wind velocity between 25 cm. and 506 cm. (or 200 cm. for the lateral component) from Fig. 19 of the second part of this paper and combining it with the laws shewing the variation of gustiness with height in § 30, p. 55.

§ 34—MEASUREMENT OF WIND VELOCITY

Many of the experiments described in this paper have involved the measurement of wind velocity and in some of them the success of the experiment depended very largely upon the accuracy of this measurement. Owing to the turbulent nature of the flow of air in the open the instantaneous velocity is not necessarily the same at two points even when the latter are at the same height above the ground and separated by only a short distance horizontally. If the mean velocity over a definite period of time is measured at each of these two points one might expect the results to shew a fairly close agreement, and it is clear that the greater the period the closer will be the agreement. It seems that this paper might appropriately close with a few figures which indicate the magnitude of this incidental effect of turbulence.

Most of the velocities mentioned above were measured by means of a vane type air meter. Since it was not possible to employ the same instrument for all measurements it was necessary to compare one with another the various air meters used. During the course of the work one air meter, which was adopted as a standard, was compared with each of three others. The comparisons were carried out by mounting this standard air meter and one other at the same height (in most cases 2 m. though on a few occasions the height was 1 m.) separated by a horizontal distance of about 1 to 3 m. Both air meters were then run simultaneously for one minute. Since there were slight unavoidable differences in the characteristics of the instruments it was assumed that the standard air meter was correct and mean calibration curves were constructed for each of the other three from these results. The readings of these three were then corrected by means of these calibration curves. After these corrections had been made the simultaneous readings still shewed small differences and these differences were expressed as percentages of the "standard" reading.

 65

The total number of comparison runs of one minute duration was 482. The number of occasions on which the difference between the simultaneous readings (after correction) lay between 0 and 1 per cent., 1 and 2 per cent., etc., of the mean velocity are given in Table XXXV.

TABLE XXXV.—DISTRIBUTION OF PERCENTAGE DIFFERENCES BETWEEN SIMULTANEOUS VELOCITY MEASUREMENTS

Percentage Difference	0 to 1%	1% to 2%	2% to 3%	3% to 4%	4% to 5%	5% to 6%	6% to 7%	7% to 8%	8% to 9%	9% to 10%	10% to 11%	11% to 12%	All
Number ..	172	126	85	66	14	10	3	2	2	1	0	1	482

Undoubtedly the distribution of the figures in Table XXXV is affected to a certain extent by the characteristics of the instrument used for measuring the velocity as well as by the degree of turbulence. But, to whatever ultimate cause the effect is attributed, these figures do indicate the extent to which one can regard a single velocity measurement made with this type of air meter and lasting one minute as representative of the velocity at any point at the same height in the immediate neighbourhood.

Originally these results were divided into two groups corresponding to velocities above and below about 5 m./sec. and there was some indication that the figures for the higher velocities gave a more rapid fall off than the figures for the lower velocities. It was decided, however, that the number of observations available were insufficient to justify any attempt to separate various conditions, especially since no temperature gradient measurements were made during the comparison runs.

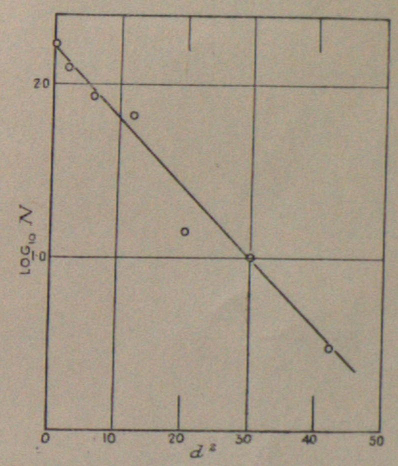


FIG. 32.—DISTRIBUTION OF DIFFERENCES BETWEEN TWO SIMULTANEOUS MEASUREMENTS OF WIND VELOCITY.

Assuming that the figures in Table XXXV can be represented approximately by the usual law

$$N = N_0 \exp(-kd^2)$$

where N is the number of occasions on which the percentage difference between the two air meters lay between $(d+\frac{1}{2})$ and $(d-\frac{1}{2})$, $\log_{10}N$ has been plotted against d^2 in Fig. 32, the points corresponding to differences greater than 7 per cent. being omitted since there are too few observations for these to be reliable. The points are a reasonably good fit to a straight line. The slope of this line is $-k \log_{10}e$, and from a measurement of the slope in Fig. 32 the value of k is obtained as 0·094.

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