M.O. 524f

AIR MINISTRY

METEOROLOGICAL OFFICE

PROFESSIONAL NOTES NO. 106

(Sixth Number of Volume VII)

OCCURRENCE OF HIGH RATES OF ICE ACCRETION ON AIRCRAFT

By A. C. BEST, M.Sc.

LONDON
HER MAJESTY'S STATIONERY OFFICE

1952

Decimal Index
551·574·7

NINEPENCE NET

Prof. Notes met. Off., London, 7, No. 106, 1952

CONTENTS

OCCURRENCE OF HIGH RATES OF ICE ACCRETION ON AIRCRAFT

By A. C. Best, M.Sc.

Summary.—Measurements of liquid-water content in strongly convective cloud at great heights in America are compared with theoretical values, and good agreement is found. The same method of computing the amount of liquid water in clouds with much higher temperatures at the base is thus justified. After taking account of the variation with drop diameter of the temperature of spontaneous freezing of small drops the probability of an aircraft encountering various concentrations of supercooled liquid water in strongly convective cloud in low latitudes is assessed.

Introduction.—The present-day tendency for airline operators to plan in terms of jet- or gas-turbine-engined aircraft flying at greater heights than has hitherto been common practice has concentrated attention on the maximum rates of ice accretion which are of probable occurrence. The problem of ice accretion is not new, of course, but hitherto it has been encountered more frequently in temperate regions than in low latitudes and more frequently in winter than in summer. This has been because in low latitudes, or in temperate latitudes in summer, many types of aircraft can conveniently operate at heights below the freezing level. The newer types of aircraft are designed to operate at greater heights, and it appears that many aircraft of the future will operate at heights where the temperature is frequently below 0°C. even in tropical regions.

It is generally accepted that the highest rates of ice accretion occur in strongly convective cloud, and it is clear that the free-water content of a convective cloud will increase as the cloud-base temperature increases, other things being equal. One would therefore expect the highest rates of ice accretion to occur in convective cloud in the tropics. During recent years there have been many observations made of liquid-water content and mean effective drop diameter in cloud. Most of these measurements have been made in the United States of America during winter or spring. The results so obtained have direct application to the operation of present-day aircraft in temperate latitudes, but it is not immediately obvious how these results should be modified for application to future aircraft or to other latitudes.

The purpose of the present note is to summarize the information available from certain American sources on the occurrence of supercooled water in the atmosphere, and to compare this information with what would be expected from theoretical arguments. If the agreement is good the theoretical arguments can then be used with confidence to predict the concentration of supercooled water in lower latitudes, where the air temperature is higher and the liquid-water content of the clouds is also likely to be higher. A similar task has already been performed by Jones and Lewis[1]* for present-day aircraft operating in America. Many of the data and much of the theoretical treatment in the present paper derive from the publications of the second of these two authors.

Experimental data.—The experimental data used in the present note comprise measurements of liquid-water content and of mean effective drop diameter

* The index numbers refer to the bibliography on p. 21.

published in three *Technical Notes of the National Advisory Committee for Aeronautics* (Nos. 1424, 1793 and 1904)[2, 3, 4]. The areas and seasons in which the measurements were made are shown in Table I.

TABLE I—DETAILS OF EXPERIMENTAL DATA USED

Technical Notes from which data were extracted	Place	Time	Cloud types
No. 1424	Mainly northern States of the United States	Winter, 1946–47	Mainly stratocumulus
No. 1793	Great Lakes and surrounding States	Winter, 1947–48	Stratus and stratocumulus
No. 1904	Northern United States	January to May, 1948	Mainly convective

It can be seen that the results of these measurements may faithfully represent conditions during winter in temperate regions, but will need modification before they are applied to summer flying or to flying in low latitudes.

The authors of these three papers have of course discussed their measurements in their respective papers. However, the purpose of the present note—to get some guidance regarding icing rates in low latitudes—is different from that of the original authors. It has therefore seemed desirable to use only the tabulated data in these three *Technical Notes*, to combine them and to examine certain aspects of them, bearing in mind the present requirement.

The frequency distribution of values of the liquid-water content of the air and of drop size were accordingly examined. Then, on the basis of the method of assessing the icing rate as described on p. 6, the frequency distribution of various rates of icing was also studied.

The experimental data were obtained on experimental flights carried out for research purposes during which icing conditions were sought. The frequency of encounter with icing conditions would thus bear no relation to the frequency with which an airline aircraft would encounter icing conditions. It is likely, however, that this dissimilarity would not be so marked with regard to the relative frequency rates of icing.

Observed frequency distribution of liquid-water content in temperate latitudes. —*Convective cloud.*—The data were taken from *Technical Notes* Nos. 1904 and 1424.* Preliminary examination indicated that the value of the liquid-water content of the air, W, was not related to the temperature at the same height except that there was a suggestion that, in cumulonimbus cloud, large values of W were more frequent at the lower temperatures. The data in *Technical Notes* No. 1904, however, showed clearly that large values of W are more frequent in cumulus than in cumulonimbus cloud. For example, the percentage number of observations of W exceeding 0·6 gm./m.^{3} was 27·3 for cumulus cloud (99 observations) but only 8·5 for cumulonimbus cloud (94 observations). The data are too few to justify dividing into groups according to all the parameters which might be significant. Accordingly results from all measurements in cumulus and cumulonimbus clouds given in both reports have been grouped together to give the cumulative percentage frequency figures in Table II.

* Except where stated otherwise the value denoted by w_1 in *Technical Notes* No. 1904 has been used since this corresponds to W in the other papers.

The convective cloud in Table II refers to well developed convective cloud since it is clear that an aircraft seeking icing conditions would not examine small cumulus clouds.

TABLE II—CUMULATIVE PERCENTAGE FREQUENCY DISTRIBUTION OF *W*

Cloud type	W (gm./m.^{3}) 0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·2	1·4	1·63	No. of obs.
	Percentage number of measurements < W													
Convective	17·2	34·0	51·1	61·4	72·5	82·8	88·3	90·6	94·5	97·2	99·0	99·5	100·0	215
Layer	19·2	53·8	76·9	91·0	98·1	100·0	..	..	..	..	..	..	..	156
Medium	59·3	91·5	98·6	100·0	..	..	..	..	..	..	..	..	..	140

The greatest value of *W* measured in these flights is discussed on p. 14. The value 1·63 gm./m.^{3}, quoted in Table II, was the greatest value averaged over the period of exposure, generally a few minutes, of the rotating cylinders (the instrument used for measuring *W*). A somewhat higher value, 1·72 gm./m.^{3}, was obtained as the highest average over any single period during which the aircraft was in cloud. The authors of *Technical Notes* No. 1904 also quote the highest value of *W* averaged over 10 sec. found on each flight. The two greatest values in this category were 1·88 and 2·0 gm./m.^{3}.

The temperature and height at which these large values of *W* were obtained will be considered at a later stage. The values 1·63 and 1·72 gm./m.^{3} were obtained on the same occasion (flight 179) when the aircraft was at a height of 14,500 ft. and the temperature was 6°F. in cumulonimbus cloud. The form in which the data are presented does not permit one to determine with certainty the temperature and height at which the 10-sec. values of 1·88 and 2·0 gm./m.^{3} were measured, but it seems probable that the value of 2·0 gm./m.^{3} for *W* was obtained at the same time as the values 1·63 and 1·72 already quoted, and that the value 1·88 gm./m.^{3} was obtained at about 15,000 ft. and a temperature of about −10°F. (flight 201).

One further comment on the figures in Table II is desirable. The rotating cylinder method of measuring *W* has been criticized on the ground that at temperatures only slightly below freezing point the cylinders do not collect all the supercooled water. Of the 215 measurements summarized in Table II only 13 were obtained at a temperature above 19°F. so that the frequency distribution is not likely to be seriously in error on this account.

Layer cloud.—Data relating to stratus and stratocumulus cloud are given in all three *Technical Notes*. Grouping the data together there was definite evidence that the frequency of large values of *W* decreased as the temperature decreased. For example, at temperatures from 29° to 10°F., 26·1 per cent. of the observations indicated a value of *W* exceeding 0·3 gm./m.^{3} (134 measurements) but at temperatures below 10°F. (22 measurements) only 4·5 per cent. gave *W* greater than 0·3 gm./m.^{3}. If the effect of temperature is ignored the figures in Table II for cumulative percentage frequency are obtained.

Medium-type cloud.—Data relating to altocumulus, altocumulus castellatus, altostratus and altocumulus embedded in altostratus cloud (from *Technical Notes* Nos. 1904 and 1424) were grouped under this heading. The range of *W* was small, and with the unit of *W* employed here (0·1 gm./m.^{3}) no effect of temperature on the distribution of *W* was apparent. Cumulative percentage frequencies as given in Table II were obtained.

Frequency distribution of mean effective drop size.—*Convective cloud.*—There was no evidence that temperature has any effect on the frequency distribution of the mean effective drop diameter, *d*, i.e. the median volume diameter, either in cumulus or cumulonimbus cloud, but *Technical Notes* No. 1904 shows that large values of *d* occur more frequently in cumulonimbus than in cumulus cloud. For example, the mean effective drop diameter exceeded 24 microns in 14·4 per cent. of the measurements in cumulus cloud (total number 97) but in 39·4 per cent. in cumulonimbus cloud (total number 89).

Grouping all convective cloud together the cumulative percentage frequencies in Table III were obtained.

TABLE III—CUMULATIVE PERCENTAGE FREQUENCY DISTRIBUTION OF MEAN EFFECTIVE DROP DIAMETER*

Cloud type	*d* (microns) 6	9	12	15	18	21	24	27	30	35	40	45	50	No. of obs.
	Percentage number of measurements $< d$													
Convective	..	2·4	11·0	25·0	40·4	57·6	76·4	89·9	93·2	95·6	..	97·5	98·1	208
Layer	1·9	20·1	51·0	74·3	85·5	93·1	96·2	97·5	98·8	99·4	100·0	..	..	159
Medium	0·7	5·2	17·0	37·0	54·9	69·6	84·5	88·9	90·4	91·1	..	..	97·1	135

* The totals in this table differ from those in Table II because *W* and *d* were not always both quoted.

Layer cloud.—The small number of observations at low temperature makes it difficult to be certain of the effect of temperature on the frequency distribution of *d* for layer-type cloud, but there was a suggestion from the data that large values of *d* do not occur at very low temperature. Ignoring any possible temperature effects and grouping all the observations together, the results in Table III were obtained.

Medium-type cloud.—Again the data suggested that very large values of *d* occur more frequently at high temperature than at low temperature but the evidence was not conclusive. Table III gives the frequency distribution of *d* for medium-type cloud.

Quantitative assessment of icing rate.—"*Standard*" *ice collector.*—The rate of ice accretion experienced by an aircraft flying through a cloud of supercooled water drops may vary over a very wide range, and there is an obvious need for some means of forming a numerical assessment of the icing rate in specified conditions. A method of doing this has apparently been adopted by the Weather Bureau of the United States, but an outline of the procedure is given below since the literature describing the basis of the method is not very accessible and since also it is desired to examine the extent to which the method fulfils the requirements.

The rate of ice accretion depends upon the liquid-water concentration in the air, the drop size, the air speed past the aircraft, the size of the affected part of the aircraft, and the efficiency of catch of the affected part. By efficiency of catch is meant the ratio of the amount of water caught to the amount of water in the swept path.

The problem of the efficiency of catch of a solid object moving through a drop-laden atmosphere was first considered by Glauert[5] and later extended by Langmuir and Blodgett[6]. Glauert considered the efficiency of catch of aerofoils and of circular cylinders ; Langmuir and Blodgett dealt with circular cylinders, spheres and ribbons. The parts of an aircraft have various shapes and for simplicity attention will be confined here to circular cylinders. Langmuir and Blodgett have shown that the efficiency of catch, E, of a circular cylinder is related to a quantity k by curves of the type shown in Fig. 1, where

$$k = \frac{2\varrho}{9\mu}\frac{a^2U}{R} \qquad \text{........(1)}$$

a = drop radius (in centimetres)
U = air speed (in centimetres per second)
R = radius of cylinder (in centimetres)
ρ = density of drops (1 for water)
μ = coefficient of viscosity of air.

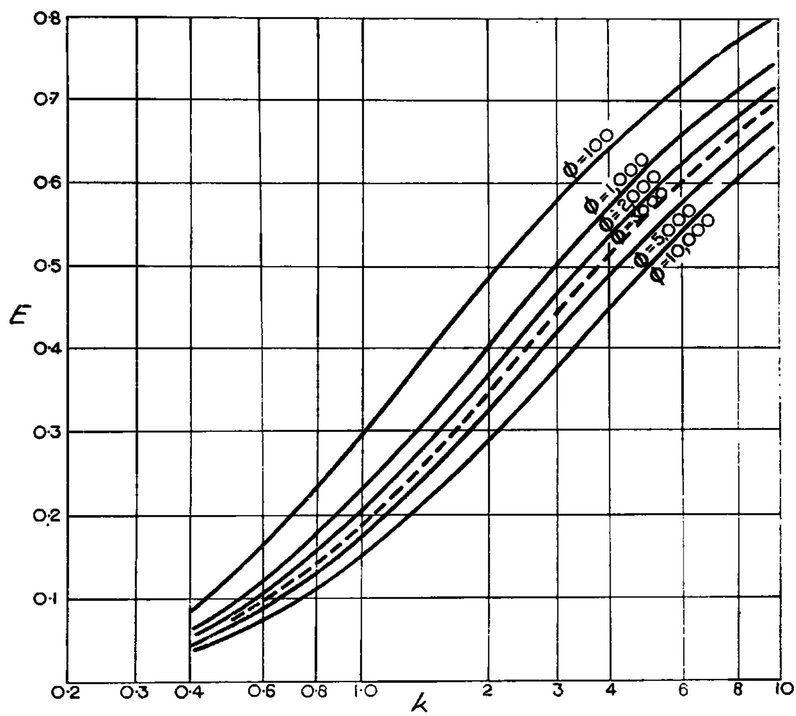

FIG. 1—COLLECTION EFFICIENCY, E, OF A CYLINDER AS A FUNCTION OF k

There are several curves in Fig. 1 and the precise curve to use depends upon another parameter ϕ defined by the relation

$$\phi = \frac{18\,\sigma^2}{\mu\rho}RU, \qquad \text{........(2)}$$

where σ is the density of the air.

The variables fall naturally into three groups : those characteristic of the air (σ and μ), those related to the ice collector (R and U) and those related to the cloud (ρ and a). The atmospheric characteristics can be standardized by assuming that the aircraft is flying at a height of 3 Km. in the international standard atmosphere. Then putting $\sigma = 0 \cdot 9095 \times 10^{-3}$ gm./cm.³ and $\mu = 1 \cdot 692 \times 10^{-4}$ poise, and, at the same time, changing the units for drop radius to microns and for air speed to miles per hour, equation (1) reduces to

$$k = 5 \cdot 87 \times 10^{-4} \frac{a^2 U}{R} . \qquad \ldots\ldots\ldots(3)$$

In passing it may be noticed that Langmuir and Blodgett adopted average conditions at the top of Mount Washington and put $\mu = 1 \cdot 658 \times 10^{-4}$ poise and $\sigma = 1 \cdot 040 \times 10^{-3}$, thus getting $5 \cdot 99$ for the numerical coefficient on the right-hand side of equation (3).

The ice collector can now be standardized by defining it as a cylinder of diameter 3 in. moving at 200 m.p.h. through the air and equation (3) becomes

$$k = 0 \cdot 0308\, a^2 \qquad \ldots\ldots\ldots(4)$$

with a measured in microns. In the same conditions

$$\phi = 2{,}994 \simeq 3{,}000. \qquad \ldots\ldots\ldots(5)$$

Equation (5) determines the curve in Fig. 1 which will be used (the curve for $\phi = 3{,}000$ has been interpolated from Langmuir and Blodgett's original curves) and equation (4) defines k in terms of drop size. Having evaluated k the value of E, the efficiency of catch, can be read from the curve for $\phi = 3{,}000$ in Fig. 1.

If the liquid-water concentration in the air is W gm./cm.³ it is easily seen that the rate of ice accretion R_g is given by

$$R_g = 3{,}600\ UWE \text{ gm./cm.}^2\text{/hr.} \qquad \ldots\ldots\ldots(6)$$

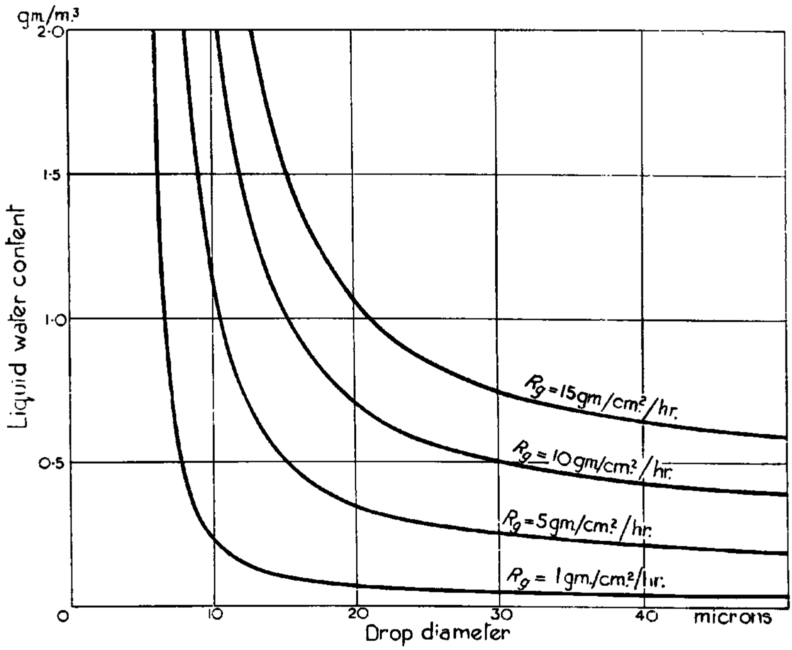

FIG. 2—ICING RATE, R_g, AS A FUNCTION OF DROP SIZE AND LIQUID WATER CONTENT

where U is measured in centimetres per second. Putting $U = 8{,}940$ cm./sec. (200 m.p.h.) then

$$R_g = 3{\cdot}218 \times 10^7\, WE \text{ gm./hr./unit area of cross-section.} \quad \ldots . (7)$$

Using equation (6) isopleths of R_g have been plotted in Fig. 2 on a drop-size–liquid-water-content diagram. This diagram emphasizes the sensitivity of R_g to variation in drop size when the drops are small.

The theory indicates that no deposition occurs on the collector when k falls below a critical value which, for a cylinder, is 1/8. This implies that, for the standard ice collector defined above, no ice accretion will occur if $a < 2{\cdot}03$ microns.

Effect of variation from standard ice-collecting conditions.—The standard collector has been defined as a 3-in. diameter cylinder moving at 200 m.p.h. through the international standard atmosphere at a height of 3 Km. The variations to be expected when the conditions departs from standard must now be assessed.

Variations in ϕ have a small effect on E ; for example, if $a = 10$ microns then $k = 3{\cdot}08$ and the values of E are 0·50, 0·44 and 0·38 for $\phi = 1{,}000$, 3,000 and 10,000 respectively.

Variations in R and U correspond to variations in the structure of the aircraft and in its speed. It is rather outside the province of the meteorologist to take account of such factors, but it may be noted here that equation (1) indicates the reason for the greater tendency of small objects, such as aerial masts, to collect ice, and also for the greater tendency for high-speed aircraft to suffer from icing. The latter tendency is additional to the effect of the greater volume of the swept path per unit of time. In fact the two aspects are represented by the occurrence of the two factors U and E on the right-hand side of equation (6).

Variations in k owing to variations in the characteristics of the atmosphere arise from the effect of the latter on $1/\mu$ (for k) and σ^2/μ (for ϕ). The ratios are tabulated below of these quantities at various heights in the international standard atmosphere and summer tropical atmosphere to the corresponding values (denoted by subscript zero) at 3 Km. in the international standard atmosphere.

	International standard atmosphere			Summer tropical atmosphere	
	1Km.	3Km.	10Km.	5Km.	10Km.
$(1/\mu)/(1/\mu)_0$ ..	0·97	1·00	1·17	0·97	1·06
$(\sigma^2/\mu)/(\sigma^2/\mu)_0$ ..	1·44	1·00	0·24	0·58	0·23

Thus the variation in ϕ may be by a factor of 4 at great heights, but it has already been seen that if $a = 10$ microns a factor of 3 in ϕ increases E by only about 14 per cent. The variation in k arising from the variations in $1/\mu$ will obviously have only a small effect on E.

Bearing in mind the other sources of uncertainty in this kind of work (such as estimation of W and a, the assumption that the drops are of uniform size, etc.) it appears that little would be gained by trying to make allowance for the temperature and density of the air at the height of the aircraft. Accordingly Fig. 4 (p. 17) will be regarded as providing a reasonable numerical assessment of the icing rate in all circumstances.

Frequency distribution of computed rate of icing in temperate latitudes.—For each occasion on which both liquid-water content and mean effective drop diameter are quoted in the three *Technical Notes* the rate of icing R_g has been assessed by the method described on p. 6.

The results for cumulus and cumulonimbus from the data of *Technical Notes* No. 1904, were examined, but there appeared to be no significant difference between the two cloud types in the frequency distribution of R_g. It appears that the greater frequency of large values of W in cumulus cloud was balanced by the greater frequency, already noted, of large values of d in cumulonimbus cloud.

The effect of temperature was also examined. The data for convective cloud covered a reasonably large number of observations at temperatures down to −10°F., but there was no evidence of a significant correlation between temperature and frequency distribution of R_g. In layer-type cloud the number of observations was small at very low temperatures, but there was a definite indication of a decrease in frequency of large values of R_g at lower temperatures. In medium-type cloud it was not possible to be confident of the importance of temperature. Grouping together the results for all temperatures, Table IV was obtained to show the cumulative percentage frequency of various values of R_g.

TABLE IV—CUMULATIVE PERCENTAGE DISTRIBUTION OF RATE OF ICING

Cloud type	Value of R_g: 0·5	1·0	1·5	2·0	3	4	5	6	7	8	10	12	14	17	21	33	No. of obs.
	Percentage number of measurements $< R_g$																
Convective	..	17·7	..	29·3	..	49·5	..	69·1	..	76·7	86·4	93·5	96·5	98·0	99·0	100	198
Layer	18·7	38·1	56·7	65·7	87·1	91·5	96·7	98·6	99·4	100	..	..	..	..	..	..	155
Medium	35·0	51·1	67·9	74·5	88·3	94·9	98·5	99·2	100	..	..	..	..	..	..	..	137

Theoretical computation of liquid-water content in cloud.—*Three models of a convective cloud.*—The simplest model of a convective cloud consists of a vertical current of air in which water condenses as a result of cooling by ascent, the temperature falls with height at the saturated adiabatic lapse rate and the cloud is isolated from its surroundings so that no air is entrained* from outside the cloud. There is little doubt that such a model is much oversimplified except in a very few cases which are probably of transitory existence. A more elaborate model is obtained if one assumes that air is entrained into the cloud from its surroundings, and this seems likely to approximate more closely to reality provided a suitable rate of entrainment is postulated. With this second model any calculations involve some assumption concerning the lapse rate of temperature in the cloud. Here again the simplest assumption is that the temperature follows the wet adiabatic lapse rate. This, however, is inconsistent with the assumption of unsaturated air being entrained from outside. In the third model air is entrained into the cloud and the lapse rate of temperature in the cloud is adjusted to some value intermediate between the dry and saturated adiabatic lapse rates.

Although the three models described seem unlikely to be equally probable there are advantages in computing W according to each model. The first model

* Entraining is the process whereby a moving stream pulls in or drags along part of the surrounding air.

(no entrainment) will give an upper limit which W cannot exceed ; it may correspond to the core of a strongly convective cloud for a short period of time. The second model may correspond to some inner part of the cloud (but not the core). It is, of course, assumed that all condensed water is carried up in the rising air current.

Symbols.—At any height z above cloud base let

T = temperature (in degrees Absolute)
P = barometric pressure (in millibars)
p = saturation vapour pressure (in millibars) of water at T°A.
ρ = saturation vapour density (in grammes per cubic metre) of water at T°A.
σ = air density at P mb. and T°A.
W = liquid-water concentration (in grammes per cubic metre),

and let the suffix zero denote the conditions at the base of the cloud.

Convection with no entrainment.—Let unit volume of air at the cloud base rise to height z above the cloud base. The volume then increases to P_0T/PT_0.

The mass of water vapour required to saturate the new volume is $\rho P_0T/PT_0$. The mass of water vapour in the original unit volume was ρ_0. Thus the condensed water in the volume P_0T/PT_0 is $(\rho_0 - \rho P_0T/PT_0)$ and

$$W = \frac{PT_0}{P_0T}\left(\rho_0 - \rho\frac{P_0T}{PT_0}\right). \qquad \ldots\ldots\ldots(8)$$

Now if p is measured in millibars and ρ in grammes per cubic metre,

$$\rho = 216{\cdot}7\frac{p}{T},$$

and substituting in equation (8)

$$W = \frac{216{\cdot}7}{T}\left(\frac{P}{P_0}p_0 - p\right). \qquad \ldots\ldots\ldots(9)$$

If the values of T_0 and P_0 are known the value of T at any pressure level P can be found from the tephigram by following the ascent along the wet adiabat, and values of W can thus be computed. Corresponding values of z can also be found from the tephigram.

Convection with entrainment.—The rate of entrainment of the outside air must now be defined. Consider an element of cloud having mass M (i.e. M is the mass of the air in the element of cloud) and suppose it increases by an amount dM as it rises from pressure level P to level $P + dP$. The rate of entrainment is defined by

$$\frac{dM}{M} = -c\,dP \qquad \ldots\ldots\ldots(10)$$

where c is a constant whose value is still to be determined. Integrating equation (10) between the two levels P_1 and P_2

$$\log\frac{M_2}{M_1} = -c(P_2 - P_1) \qquad \ldots\ldots\ldots(11)$$

Let the rate of entrainment be such that the mass of the cloud is doubled as it ascends between two pressure levels 400 mb. apart (that this rate of entrainment is reasonable is shown in "The thunderstorm".[7] Then to determine c

$$\log 2 = 400\, c. \qquad \ldots\ldots\ldots(12)$$

If the relative humidity of the air outside the cloud is $100f$ and if the temperature of the ambient air is the same as the cloud temperature at the same level, then the mass of water vapour added to the cloud element as it passes from P to $P + dP$ is

$$f\rho \frac{dM}{\sigma} = - cf \frac{\rho M}{\sigma}\, dP.$$

The original vapour content of the cloud element was ϱ_0/σ_0 and the water vapour required for saturation at the level P is $M\varrho/\sigma$, the volume of the element now being M/σ. Hence

$$W = \frac{\sigma}{M}\left(\frac{\rho_0}{\sigma_0} - \frac{M\rho}{\sigma} + c\int_P^{P_0} \frac{f\rho M}{\sigma}\, dP\right) \qquad \ldots\ldots\ldots(13)$$

$$= \frac{\rho_0 T_0}{P_0}\frac{P}{MT} - \rho + \frac{P}{MT} fc \int_P^{P_0} \rho \frac{MT}{P}\, dP \qquad \ldots\ldots\ldots(14)$$

where M is defined by

$$\log M = c(P_0 - P) \qquad \ldots\ldots\ldots(15)$$

from equation (11) and c is given by equation (12). The integral on the right-hand side of equation (14) must be evaluated graphically. The value of T at any level is obtained by following a wet adiabat (with respect to water) on the tephigram from the point corresponding to the base of the cloud.

Convection with entrainment and adjusted lapse rate in the cloud.—As pointed out above the rate of decrease of temperature with height must exceed the saturated adiabatic lapse rate in a convective cloud in which entrainment occurs. There is, however, very little reliable information available about the lapse rate in convective cloud or about the difference between the temperature in a convective cloud and the ambient temperature at the same level. Accordingly the following artificial expedient was adopted in order to define the lapse rate of temperature in the cloud. It was assumed that entrainment took place only at multiples of 50 mb. and that the temperature of the entrained air was given by dry adiabatic ascent from the preceding entrainment level. Now from equation (11) it can be shown that if $P_1 - P_2 = 50$ mb. then $M_2/M_1 = 1{\cdot}09$. Thus if T_w is the temperature reached at P_2 along the saturated adiabat from P_1 and T_d the temperature reached along the dry adiabat, then the cloud temperature at P_2 will be given by $T_d + (T_w - T_d)/1{\cdot}09$.

It must be emphasized that this artificial expedient was adopted only for the purpose of defining the temperature lapse rate in the cloud. Obviously it gives a rate of decrease of temperature slightly greater than the saturated adiabatic lapse rate and to that extent is likely to correspond more closely to reality. For the computation of water vapour brought into the cloud by the entrained air it is still assumed that the ambient air temperature is equal to the cloud temperature at the same height.

Some numerical results for convective cloud.—The methods indicated above have been used to compute W at various heights for three different cloud-base temperatures (38°, 70° and 82°F.) all at 950 mb. The results are given in Table V. All heights are above cloud base. Model 1 is the case of no entrainment, model 2 allows entrainment as indicated above but with the saturated adiabatic lapse rate in the cloud. At a given pressure level the height (above cloud base) and temperature will be the same in model 2 as in model 1. Model 3 takes account of the adjustment of lapse rate in the cloud. The entries in the various parts of the table terminate at a level where the temperature is substantially below — 40°F.

TABLE V—VALUES OF LIQUID-WATER CONTENT IN VARIOUS MODELS OF CONVECTIVE CLOUD

Pressure	Cloud model 1			2		3			
				Ambient relative humidity 70%	Ambient relative humidity 100%			Ambient relative humidity 70%	Ambient relative humidity 100%
	Height above 950 mb.	Temperature	W	W	W	Height above 950 mb.	Temperature	W	W
mb.	ft.	°F.	gm./m.^{3}	gm./m.^{3}	gm./m.^{3}	ft.	°F.	gm./m.^{3}	gm./m.^{3}
	Cloud-base temperature 38°F. at 950 mb.								
400	21,068	−50·0	3·01	1·55	1·74	20,996	−52·8	1·55	1·73
500	16,036	−25·2	3·27	1·89	2·15	15,988	−27·3	1·91	2·16
600	11,704	− 5·3	3·18	2·00	2·31	11,676	− 7·1	2·07	2·37
700	7,896	10·4	2·71	1·82	2·15	7,884	8·8	1·94	2·27
800	4,501	22·8	1·89	1·35	1·63	4,496	21·9	1·46	1·74
900	1,433	33·3	0·72	0·54	0·67	1,433	32·9	0·61	0·74
950	0	38·0	0·00	0·00	0·00	0	38·0	0·00	0·00
	Cloud-base temperature 70°F. at 950 mb.								
250	34,212	−42·0	5·99	2·84	3·35	33,909	−50·1	2·88	3·36
300	30,042	−21·8	6·58	3·27	3·89	29,819	−29·0	3·37	3·96
400	23,082	5·8	7·18	3·77	4·63	22,949	0·7	4·05	4·88
500	17,406	24·9	6·91	3·70	4·78	17,339	20·5	4·19	5·20
600	12,618	38·6	6·14	3·33	4·54	12,583	35·4	3·96	5·12
700	8,473	49·5	4·91	2·72	3·92	8,460	47·3	3·29	4·45
800	4,811	58·7	3·19	1·78	2·74	4,810	57·6	2·22	3·15
900	1,526	66·5	1·08	0·59	1·00	1,526	66·0	0·89	1·30
950	0	70·0	0·00	0·00	0·00	0	70·0	0·00	0·00
	Cloud-base temperature 82°F. at 950 mb.								
200	40,861	−30·8	6·87	3·26	3·94	40,153	−49·0	3·31	3·90
250	35,611	− 9·0	7·66	3·72	4·59	35,105	−24·8	3·99	4·74
300	31,141	6·5	8·17	4·09	5·06	30,777	− 6·6	4·50	5·43
400	23,811	27·6	8·50	4·14	5·59	23,607	18·6	5·09	6·36
500	17,911	43·2	7·91	3·72	5·48	17,799	36·5	5·05	6·64
600	12,951	54·9	6·76	3·05	4·94	12,899	50·1	4·51	6·27
700	8,683	63·8	5·43	2·44	4·26	8,659	60·7	3·87	5·59
800	4,923	71·6	3·70	1·56	3·16	4,917	69·9	2·67	4·05
900	1,561	78·6	1·44	0·75	1·37	1,560	78·1	1·13	1·73
950	0	82·0	0·00	0·00	0·00	0	82·0	0·00	0·00

Comparison of the various columns in the table shows that cloud model 1 leads to a much higher value of *W* than models 2 and 3. The two latter models do not differ very greatly, but if one is concerned to know the greatest probable value of *W* it seems preferable to use model 3, both because this leads to somewhat higher values of *W* and also because the model seems likely to approximate more closely to reality.

If the maximum value of W for any given cloud model is plotted against cloud-base temperature it will be seen that the resulting points approximate reasonably closely to a straight line so that the maximum value of W for other cloud-base temperatures can be obtained with fair accuracy by linear interpolation.

Liquid-water content of layer-type clouds.—Clouds, such as stratocumulus which are formed by turbulence, not by bodily lifting of the whole air mass, are considered here. An upper bound to the liquid-water content is given by assuming that the air is lifted—by eddies—along the wet adiabatic ascent curve from the base of the cloud to the top. It is clear, however, that this value will seldom be attained since the value is appropriate only to the top of the cloud, and, by definition, the cloud is being continuously mixed by turbulence. It has been suggested by Lewis, Kline and Steinmetz[2] that a practical upper limit to the concentration of free water is given by half the value computed on the basis of adiabatic lifting from base to top.

Clouds of this type are seldom more than about 3,000 ft. deep though they may extend horizontally for many miles. Table VI shows the liquid-water content of air which has risen along the saturated adiabat through a height of 3,000 ft. above the cloud base to reach the temperatures shown in the table at a pressure level 900 mb. (computed from equation (9)).

TABLE VI—LIQUID-WATER CONTENT IN LAYER CLOUD

Cloud depth 3,000 ft.

	Temperature at top of cloud (°F.) 32	14	−4	−22	−40
	grammes per cubic metre				
W	1·59	1·05	0·65	0·39	0·18

At lower pressures the liquid-water content is less. For example an ascent of 3,000 ft. above the cloud base to a temperature of 32°F. at 700 mb. led to a free-water content of 1·47 gm./m³.

Comparison of observed and theoretical values of liquid-water content.—The results of the computations show that there is a temperature and height at which the liquid-water content has a maximum value, and that for a considerable depth of cloud centred at this height of maximum W the value of W varies but little. For ease of comparison it is convenient to summarize these results as follows :—

Cloud-base temperature	Maximum value of W	Temperature at maximum value of W	Height at maximum value of W
°F.	gm./m.³	°F.	ft.
38	2·07	− 7·1	11,500
70	4·2	20·5	17,500
82	5·1	27·5	20,700

Turning now to the observations made in America, consideration of layer-type cloud may be dismissed very rapidly. The greatest values quoted for W in layer-type cloud are 0·53 gm./m.³ (at 19°F.) and 0·57 gm./m.³ (at 20°F.) in

Technical Notes No. 1424. These values confirm the suggestion on p. 14 that the value of *W* is seldom more than half the value obtained by adiabatic lifting through 3,000 ft.

In *Technical Notes* No. 1904 Lewis and Hoecker quote measurements of liquid-water content measured during a number of flights in convective cloud. Corresponding to each measurement of *W* the temperature at the same height in the cloud is also given. For obvious reasons the tabulated data do not give the temperature and height of base of cloud. For many flights *W* and the corresponding temperature are given for several different heights in, apparently, the same cloud. Some of these flights were made near the western seaboard of the United States in weather conditions in which one would expect a cloud base at about 950 mb. and a near-wet adiabatic lapse rate in the cloud. Eight such flights were selected (Nos. 154, 155, 171, 175, 176, 179, 200 and 201) and the temperature at the cloud base estimated. The estimate was made by using the observed temperature at a stated height and tracing a wet adiabat from the corresponding point on the tephigram to the 950-mb. level. In practice two estimates were made for each selected flight, using the highest and lowest temperatures (at different heights of course) measured in the particular cloud. The difference between the two estimates exceeded 3°F. for one flight only (No. 154—with a difference of 5°F.) The mean of the two estimates varied from 31° to 47·5°F. with an average value of 38°F. For this reason 38°F. was chosen for numerical computation. On the two flights on which the greatest values of *W* were observed, Nos. 179 and 201, the estimated cloud-base temperatures (as described above) were 47·5° and 39°F. respectively. Interpolation in the theoretical values quoted above for convective cloud suggests that with these cloud-base temperatures the maximum values of *W* should be about 2·6 and 2·1 gm./m.^{3} respectively; the comparable measured 10-sec. values were 2·0 and 1·88 gm./m.^{3} respectively. It is considered that this agreement is reasonable in view of all the approximations involved, and that the agreement justifies the calculations and the use of cloud model 3 with an ambient relative humidity of 70 per cent.

The height and temperature at which the maximum values of *W* were measured are also in reasonable accord with expectations based upon the theoretical values when it is borne in mind that the theoretical heights are above cloud base at 950 mb., that the value of *W* shows very little variation with height in the neighbourhood of the height at which the maximum value occurs, and that on flight 179 the cloud-base temperature was about 10°F. higher than that adopted in the calculations.

Frequency distribution of liquid-water content—generalization.—Now, as already explained, the main difficulty lies in the fact that there are no data giving the frequency distribution of *W* for high cloud-base temperatures such as may occur in low latitudes. The close agreement between computed and measured values for maximum *W* with low cloud-base temperatures justifies the same computational method for assessing the maximum value of *W* with high cloud-base temperatures. Of the 215 measurements of *W* in convective cloud summarized in Table II, only 17 were made at a temperature outside the range 19° to − 20°F. Now reference to Table V shows that with cloud model 3, 70-per-cent. ambient relative humidity and cloud-base temperature 38°F. *W* varies only between about 1·65 and 2·07 gm./m.^{3} in this temperature range. To a reasonable approximation, therefore, the large majority of measurements were made at a temperature at which *W* might be expected to have a value close to the maximum value. In fact, of course, the measurements gave a frequency distribution of *W* as indicated by Table II. With the assumption that the frequency distribution of *W*, expressed as a fraction of the maximum value corresponding to the cloud-base temperature, is independent of cloud-base

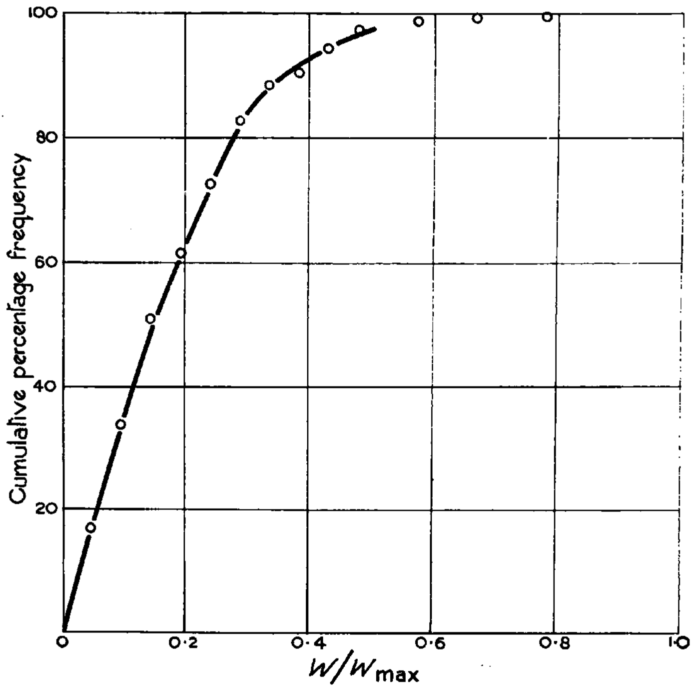

FIG. 3—CUMULATIVE PERCENTAGE FREQUENCY OF GIVEN FRACTION OF MAXIMUM LIQUID-WATER CONTENT IN CONVECTIVE CLOUD

temperature, the frequency distribution in Table II for convective cloud (layer and medium cloud are clearly unimportant in this connexion) has been plotted in Fig. 3, W being expressed as a fraction of 2·07 gm./m.^{3} the maximum value of W for a cloud-base temperature of 38°F., cloud model 3 and 70-per-cent. ambient relative humidity from Table V. The shape of the curve in the upper parts must obviously be unreliable owing to the small number of observations involved but it seems likely to be reliable up to about $W/W_{max} = 0 \cdot 4$.

Frequency distribution of icing rate—generalization.—Means have been devised above of assessing the frequency distribution of W in convective cloud, using cloud-base temperature as the independent variable. As explained, however, drop size is also of importance in determining icing rates. It might be possible to devise means of assessing the frequency distribution of drop size on lines similar to those employed for W, but this would be of little help unless the relation between drop-size distribution and W were known. Fortunately it appears that a simpler procedure is suitable.

In Fig. 4, two curves have been plotted. One is the frequency distribution of R_g for convective cloud, based upon Table IV. The other is the frequency distribution of R_g' where R_g' is the icing rate which would have existed for all convective cloud occasions used in Table II if the mean effective drop diameter had been equal to the median drop diameter for convective cloud obtained from Table III (19·7 microns). It can be seen that the two curves are practically identical. The utility of this lies in the fact that the frequency distribution of icing rate can thus be assessed from the frequency distribution of W (discussed on p. 15) and an assessment of the median mean effective drop diameter. Since this paper is concerned with the higher icing rates layer-

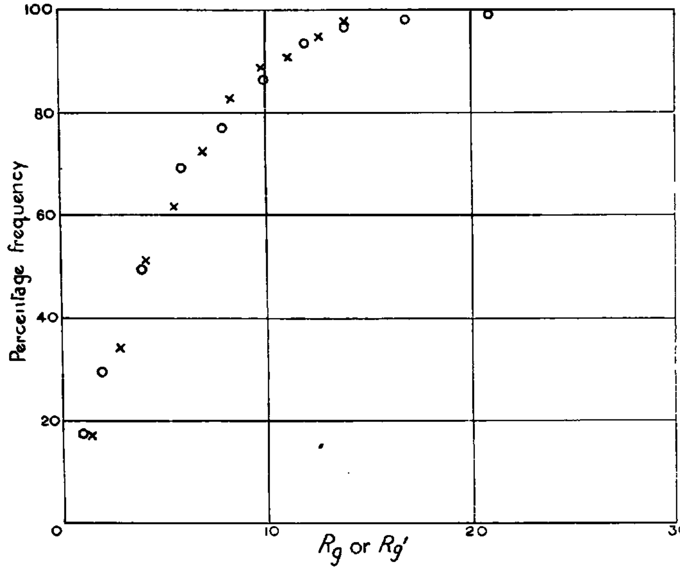

FIG. 4—CLOUD-BASE TEMPERATURE ABOUT 38° F.

o Frequency distribution of computed icing rate R_g.

× Frequency distribution of icing rate, R_g' based on median mean effective drop diameter and frequency distribution of W.

or medium-type cloud need scarcely be considered any more for, as seen above, both W and the median value of the mean effective drop diameter are appreciably greater in convective cloud than in cloud of any other type.

Conditions in low latitudes.—It is not unusual for the mean vapour pressure at the surface, to have values between 30 and 33 mb. at places such as Calcutta, Lagos and Singapore. When convective cloud occurs in such circumstances the cloud-base temperature is likely to be not far removed from the surface dew point, i.e. 75° to 78°F.

The effect of temperature upon mean drop size is somewhat difficult to assess. Howell[8] has dealt with drop size in convective cloud but only for a cloud-base temperature of 32°F. The present writer[9] has considered the factors, including temperature, which determine the size of the drops in layer-type cloud. Both treatments probably oversimplify the problem, but it is reasonably certain that higher temperatures lead to larger drops though it is difficult to say how much larger. The icing rate will not be over-estimated if a mean effective drop diameter of 20 microns is assumed.

Assuming now a maximum cloud-base temperature of 76°F.—a conservative figure in view of the dew points quoted above—and a drop diameter of 20 microns, the following inferences using Figs. 2 and 3 can be made about the liquid-water content and icing rates likely to be encountered in low latitudes in well developed convective cloud.

Maximum liquid-water content about 4·6 gm./m.^{3}

No. of occasions %	W gm./m.^{3}	R_g gm./cm.^{2}/hr.
75	>4·6 × 0·07 = 0·32	> 4·5
50	>4·6 × 0·15 = 0·69	> 9·9
25	>4·6 × 0·25 = 1·15	>16·5
10	>4·6 × 0·36 = 1·65	..
5	>4·6 × 0·44 = 2·02	..

The slow variation of *W* with temperature in Table V also suggests that these values of *W* would not be reduced below about three quarters of the quoted value even at temperatures as low as – 40°F. In deriving these figures certain assumptions have been made about the temperature structure of the cloud. For this reason it would probably be unwise to apply the reduction factor of 3/4 at low temperatures. However, at temperatures as low as – 40°F. the physical processes implied on p. 10 must be reconsidered.

Freezing of small droplets.—It now seems to be generally accepted that clouds at a temperature near – 10°C. (14°F.) probably contain a few ice nuclei. At still lower temperatures the number of ice nuclei increases and at – 32°C. (– 25·6°F.) there is a marked increase in the number of ice nuclei (in air which has not been carefully cleaned). At – 41°C. (– 41·8°F.) there is a further large increase in the number of ice nuclei. The experiments upon which these results are based were laboratory experiments, but nevertheless they do indicate a probability that in the atmosphere the number of ice nuclei may increase markedly as the temperature falls to – 40°F. If there are sufficient ice nuclei present the amount of liquid water is likely to decrease by sublimation, and so the icing risk to aircraft diminishes. Unfortunately the laboratory work mentioned above gives no guide to the amount of water which is likely to remain liquid at various temperatures. Accordingly, after noting the probability that serious icing is unlikely at temperatures below – 40°F., the laboratory work of Dorsch and Hacker [10] will now be considered. Heverley [11] has shown that the temperature at which a drop of water freezes spontaneously is lower for the smaller drops. Dorsch and Hacker extended this work and showed that the temperature of spontaneous freezing of a droplet is not uniquely determined by the size of the drop. There is, in fact, a range of temperature at which freezing occurs for each drop size. If the number of drops (all of the same size) which freeze at a specified temperature is plotted against the temperature, a frequency curve approximating to the conventional "normal" shape is obtained, the peak occurring at a temperature which depends upon the drop size. The standard deviation of temperature of spontaneous freezing for a given drop size is of the order of 5°F. Dorsch and Hacker give the average temperature of spontaneous freezing for a number of drop sizes and Table VII is based upon their results.

TABLE VII—AVERAGE TEMPERATURE OF SPONTANEOUS FREEZING AS A FUNCTION OF DROP DIAMETER

	Drop diameter (microns)							
	90	35	18·5	13	10·5	6·5	4·5	4·0
	degrees Fahrenheit							
Temperature of spontaneous freezing	–12·5	–15·0	–20	–22	–25	–30	(–35)	(–40)

The two bracketed values for very small drops were obtained by extrapolation and are probably very unreliable.

These results will now be applied to clouds in the atmosphere. There are, of course, obvious objections to doing this. The conditions of the laboratory experiments are not those of the free atmosphere, and moreover Table VII ignores the fact that drops of the same size do not all freeze at the same temperature. Nevertheless there seems no other way of forming a numerical estimate of the decrease of liquid water at very low temperatures.

It has been shown elsewhere [12] that the distribution of drop size in cloud can be represented by

$$1 - F = \exp\left[-(d/a)^n\right] \qquad \ldots\ldots\ldots(16)$$

where F is the fraction of free water in the cloud comprised of drops with diameter less than d, n and a are constants, the former having a value about 3·3. This distribution law is assumed valid at temperatures above − 12·5°F. and that thereafter, as the drops are carried up in the rising air current, the vapour pressure is so small that the major factor affecting the liquid-drop-size distribution is the successive freezing of the larger liquid drops as the temperature falls. To evaluate " a " it is easily shown from equation (1) that 50 per cent. of the water is comprised of drops with diameter less than $0{\cdot}892a$. A reasonable value for this diameter would be the median value from Table III, i.e. 20 microns ; from this $a = 22{\cdot}4\mu$. However, since there is some uncertainty about the appropriate value of " a " the result should also be considered of putting $0{\cdot}892a$ equal to 15 and 25 microns, leading to $a = 16{\cdot}8\mu$ and $28{\cdot}0\mu$ respectively. From this and using equation (16) and Table VII the figures in Table VIII are deduced.

TABLE VIII—FRACTION OF FREE WATER WHICH REMAINS LIQUID

		Temperature (°F.)							
	a	−12·5	−15	−20	−22	−25	−30	−35	−40
	μ								
Liquid fraction	28·0	1·0	0·88	0·22	0·076	0·038	0·008	(0·002)	(0·002)
	22·4	1·0	0·99	0·41	0·15	0·079	0·017	(0·005)	(0·004)
	16·8	1·0	1·0	0·75	0·35	0·19	0·043	(0·013)	(0·009)

The figures quoted for temperatures of − 35° and − 40°F. are unreliable since they were obtained by extrapolating the data of Dorsch and Hacker.

In each case the fraction of water remaining liquid decreases markedly at temperatures below − 15°F., becoming almost negligible at − 30°F. Within this temperature range the rate of decrease with decreasing temperature is rather sensitive to the value of a. Now it was noted above that with higher cloud-base temperatures the drop size (and hence a) is likely to be greater, and larger drops are likely to lead to greater rates of icing. From Table VIII, however, it can be seen that larger drops lead to a smaller fraction of water remaining liquid. Since this section is concerned with maximum rates of icing and safety considerations are involved, it therefore seems reasonable to adopt the values corresponding to $a = 22{\cdot}4\mu$ in Table VIII.

Significance of the numerical icing rate.—The question of whether a particular combination of circumstances leads to " severe " icing can be answered only by the pilot of the aircraft, and so the answer must, to some extent, be subjective. On the other hand a numerical scale of icing is of little use unless it can be interpreted in terms used by the pilot. Lewis [13] has suggested that the ranges 0 – 1 to 6 – 12 gm./cm.^{2}/hr. for R_g should be classified as light, moderate and heavy, but for our present purpose it seems better to take advantage of some comments in a report by British European Airways. This report [14] lists the liquid-water content and drop size measured on a number of flights into icing clouds. The text of the report contains remarks which, on a few occasions, classify the icing as severe or approaching severe. The value of R_g has been determined from Fig. 2 for these occasions. The results are as follows :—

W gm./m.³	d microns	R_g gm./cm.²/hr.	Remarks in Report
0·35	11	2·0	Severe
0·35	10	1·6	Approaching severe
0·32	18	4·0	Severe
0·33	10	1·5	Severe
0·28	13	2·2	Approaching severe
0·82	12	5·5	Severe
0·74	15	7·3	Severe

From these comments by an experienced aircrew it would seem reasonable to assume that on any occasion when R_g exceeds 4·0 the rate of icing would be regarded as severe.

Protection of aircraft against icing.—Probably it is impracticable to protect an aircraft against ice accretion in the worst possible combination of circumstances which can be envisaged. If that is so, the duty of the meteorologist is to advise the aircraft designer of the frequency of the various degrees of severity. It is then incumbent upon the designer and/or operator to decide what risk shall be taken.

The present paper is concerned primarily with the greatest rate of ice accretion which is probable in strongly convective cloud. It is now proposed to construct a table showing the severity of icing which is likely to be exceeded only on a specified percentage number of occasions of icing in strongly convective cloud in low latitudes. To do this the percentage frequencies on p. 17 are combined with the correction factors for $a = 22{\cdot}4\mu$ in Table VIII. Since safety considerations are involved it has seemed desirable to increase these correction factors to 0·5, 0·2, 0·1 and 0·05 for − 20°F., − 22°F., − 25°F. and − 30°F. respectively. For example, from the table on p. 17, on 10 per cent. of occasions W exceeds 1·65 gm./m.³, but at − 25°F. only 0·1 of this amount, i.e. 0·17 gm./m.³, remains liquid. As the range of sizes of drops remaining liquid diminishes the mean size approaches more and more closely to the maximum size still liquid. Bearing in mind again that safety considerations are involved it is therefore desirable to couple the maximum size of drop still in the liquid state with the values of W. Table IX is thus obtained.

TABLE IX—VALUE OF LIQUID-WATER CONTENT EXCEEDED ON A SPECIFIED PERCENTAGE NUMBER OF OCCASIONS OF ICING IN STRONGLY CONVECTIVE CLOUD

Temperature	Drop diameter	Percentage number of icing occasions in convective cloud 75	50	25	10	5	0
°F.	μ	*grammes per cubic metre*					
32 to − 15	20	0·32	0·69	1·15	1·65	2·02	4·60
− 20	19	0·16	0·35	0·57	0·82	1·01	2·30
− 22	13	0·06	0·14	0·23	0·33	0·40	0·92
− 25	11	0·03	0·07	0·11	0·17	0·20	0·46
− 30	7	0·02	0·03	0·06	0·08	0·10	0·25
− 40	7	0·02	0·03	0·06	0·08	0·10	0·25

It is of course clear that these figures apply to low latitudes since a cloud-base temperature of 76°F. is assumed.

Jones and Lewis[1] have proposed specifications of icing severity in various circumstances. They considered not only the severity in strongly convective cloud but also conditions in areas in which there is widespread convection and in layer-type cloud. Their icing condition designated as "instantaneous maximum" corresponds approximately to the last column of Table IX, and indeed the results are very similar. It seems desirable therefore to compare and contrast their treatment with that adopted above. Jones and Lewis computed the theoretical value for W appropriate to a cloud-base temperature of 85°F. at 950 mb. using cloud model 3 with a 70-per cent. relative humidity (the conception of cloud model 3 derives of course from the work of Jones and Lewis). They do not describe the method of computation in detail, but the numerical results obtained are very similar to those for cloud-base temperature of 82°F. Having obtained these theoretical values Jones and Lewis then scale them down at very low temperatures but it appears that the reason for so doing is based upon the increase in the number of ice crystals at these low temperatures. There does not appear to be any quantitative argument in support of the values they give.

If the values in Table IX are examined in conjunction with Fig. 2 it is seen that R_g will be less than 4 on 95 per cent. of occasions at -22°F. and on 100 per cent. of occasions at -25°F. In fact the spontaneous freezing of droplets should lead to a marked absence of high rates of icing on the airframe at temperatures below a critical value which is in the neighbourhood of -32°C. ($-25 \cdot 6$°F.). It is not clear whether the similarity between this temperature and the temperature at which the number of ice nuclei increases markedly is more than a coincidence.

Relative frequency of severe icing in various parts of the world.— The preceding sections have been concerned only with the percentage frequency distribution of various rates of icing; the actual frequency of high icing rates has not been considered. However it has been shown that the very severe rates of icing (e.g. R_g greater than about 8) occur only in strongly convective cloud, and that even occasions of $R_g = 4$ are very much more frequent in strongly convective cloud than in layer- or medium-type cloud. From this it is a reasonable inference that the frequency of very severe icing over various routes is proportional to the frequency of thunderstorms over those routes. Now a considerable amount of information exists on the frequency of thunderstorms at various places, and it thus appears that this information might be used as a guide to the relative frequency of high rates of icing. This method of approach does not, of course, give any information about the absolute frequency of high icing rates, about the height at which icing occurs or about the lower rates of icing, but it should lead to a reliable estimate—for planning purposes—of the relative frequency of very severe icing conditions.

BIBLIOGRAPHY

1. JONES, A. R. and LEWIS, W.; Recommended values of meteorological factors to be considered in the design of aircraft ice-prevention equipment. *Tech. Note nat. adv. Comm. Aero., Washington D.C.*, No. 1855, 1949.

2. LEWIS, W., KLINE, D. B. and STEINMETZ, C. P.; A further investigation of the meteorological conditions conducive to aircraft icing. *Tech. Note nat. adv. Comm. Aero., Washington D.C.*, No. 1424, 1949.

3. KLINE, D. B.; Investigation of meteorological conditions associated with aircraft icing in layer-type clouds. *Tech. Note nat. adv. Comm. Aero., Washington D.C.*, No. 1793, 1949.

4. LEWIS, W. and HOECKER JR., W. H.; Observations of icing conditions encountered in flight during 1948. *Tech. Note nat. adv. Comm. Aero., Washington D.C.*, No. 1904, 1949.

5. GLAUERT, M.; A method of constructing the path of raindrops of different diameters moving in the neighbourhood of (1) a circular cylinder, (2) an aerofoil, placed in a uniform stream of air; and a determination of the rate of deposit of the drops on the surface and the percentage of drops caught. *Rep. Memor. aero. Res. Comm., London*, No. 2025, 1940.

6. LANGMUIR, I. and BLODGETT, K. B.; A mathematical investigation of water droplet trajectories. *Tech. Rep. U.S.A.A.F., Air Material Command, Dayton Ohio*, No. 5418, 1946.

7. Washington D.C., U.S. Department of Commerce, Weather Bureau. The thunderstorm. Washington D.C., 1949, p. 37.

8. HOWELL, W. E.; The growth of cloud drops in uniformly cooled air. *J. Met., Lancaster Pa.*, **6**, 1949, p. 134.

9. BEST, A. C.; The size of cloud droplets in layer-type cloud. *Quart. J. R. met. Soc., London*, **77**, 1951, p. 241.

10. DORSCH, R. G. and HACKER, P. T.; Photomicrographic investigation of spontaneous freezing temperatures of supercooled water droplets. *Tech. Note nat. adv. Comm. Aero., Washington D.C.*, No. 2142, 1950.

11. HEVERLY, J. R.; Supercooling and crystallization. *Trans. Amer. Geophys. Un., Washington D.C.*, **30**, 1949, p. 205.

12. BEST, A. C.; Drop size distribution in cloud and fog. *Quart. J.R. met. Soc., London*, **77**, 1951, p. 418.

13. LEWIS, W.; A flight investigation of the meteorological conditions conducive to the formation of ice on airplanes. *Tech. Note nat. adv. Comm. Aero., Washington D.C.*, No. 1393, 1947.

14. British European Airways, Project Development Department. Vickers Viking fluid de-icing trials, 1950. London, 1950.

PRINTED AND PUBLISHED BY HER MAJESTY'S STATIONERY OFFICE

To be purchased from

York House, Kingsway, LONDON, W.C.2 423 Oxford Street, LONDON, W.1

P.O. Box 569, LONDON, S.E.1

13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF

39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1

2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or from any Bookseller

1952

Price 9*d* net

PRINTED IN GREAT BRITAIN

(52724) Wt. 4527/M.3291 K.6 8/52 Hw. S.O. Code No. 40-57-10-6