

Meteorological Office

Boundary Layer Branch (Met O 14)

Turbulence and Diffusion Note No. 2.

PRELIMINARY OBSERVATIONS AT CARDINGTON CONCERNING
THE TURBULENT ENERGY BALANCE IN THE PLANETARY
BOUNDARY LAYER.

BY

F. Pasquill and J.B. Tyldesley

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1. Introduction

One of the basic problems of turbulence calling for much further investigation is the question of the generation and dissipation of the turbulent kinetic energy. For the evaluation of the various terms concerned in the balance of this energy measurements are required of the fluctuations of wind and temperatures and of the vertical gradients of the mean values of these quantities. Most of the data obtained so far refer to the relatively low heights (< 100 m) accessible from masts and towers. With the object of obtaining data at greater heights in the planetary boundary layers the use of suitable instruments mounted on the captive balloon cable has been under development at Cardington for some time. The present note is concerned with the preliminary observations carried out with these instruments on three days in May 1965 (12th, 13th and 14th). As will be seen, these observations are lacking in both quality and comprehensiveness, but it was hoped that they would be adequate for some useful lessons to be learnt, especially as regards the planning and conduct of further observations.

A full account of the instruments and technique will be given separately. Briefly, they provide measurements of the following quantities:

Fluctuations of wind inclination (hot wire yawmeter), wind speed (sensitive cup anemometer), air temperature (platinum resistance thermometer), all at one level.

Mean wind speeds (cup anemometers) at two levels, one above and one below the level at which the fluctuations are measured.

Mean difference in temperature between the same two levels (thermistors). Continuous electrical signals from the various instruments were transmitted to ground by cables and were there recorded, the fluctuation signals being recorded both digitally (paper tape) and as pen records of the mean deviation from band-pass filters. The supporting data on mean wind speed were read off counters at suitable intervals and those on mean temperature difference were recorded on a pen trace.

The Eddy Correlation Data

(a) Sensors The response times of the instruments used were approximately as follows:

Wind speed	0.3 sec
Wind inclination	0.005 sec
Temperature	0.015 sec

(b) Data Logging Equipment Every second, the three outputs above were sampled once. The samples were instantaneous and simultaneous to within 0.001 sec. These samples were digitised sequentially and the results punched on five-hole paper tape. (Three digits per channel). It is argued that because of the simultaneous sampling, the high-frequency response of the system was limited by the sensors and not by the data-logging equipment itself.

(c) Computer Analysis The Ferranti Mercury computer at CDEE Porton was used, and the programming work was done there by Mr. A.C. Thomas and Miss S. A. Matthews.

The data-logging tapes were not in a format that was acceptable to Mercury's standard input routine. A special routine was therefore written to read in the tape character by character, check the format, and assemble the digits into numbers. When format errors occurred, the reader was stopped to enable the tape to be marked, and an indication of the type of fault was printed. Such errors were then corrected by hand. Not many were found, as the tapes had already been manually edited. The numbers were stored as concisely as possible as 10-bit integers. About 38,400 can be stored, corresponding to a maximum observation period of about $3\frac{1}{2}$ hours.

The following steps were carried out in analysing the data:

(1) Inclination I, total wind in vertical plane V and temperature T were brought to their proper units by applying zero and calibration factors.

(2) The horizontal component of wind u and vertical component w were found as

$$u = V \cos I$$

$$w = V \sin I$$

(3) The data were divided into sections of various lengths, and best-fittings straight lines were found by the least-squares method for u, w and T and for each section. This was done for the following combinations of sampling and averaging times:

Sampling time	Averaging time
1000 sec	} 1 sec
2000 sec	
.	
.	
.	
max round no. of thousands of secs.	
whole period of record	1,2,5,10, 20,50,100,500, 1000 secs

(4) The differences of u , w and T from the straight lines found in (3) were formed (called u' , w' , T' respectively).

(5) The mean products $\overline{u'w'}$ and $\overline{T'w'}$ were formed for each sampling and averaging time.

(6) The following results were printed out:

(a) $\overline{u'w'}$ and $\overline{T'w'}$ for each sampling and averaging time

(b) the constants of the best-fitting straight lines found in (3)

(c) certain intermediate results, useful for checking.

(d) Hand Check Parts of the computation are being repeated by hand, to check that the computer program was working properly, the period 13 October 1965, cycles 2900-3899, i.e. 1000 seconds of record, with 20 sec averaging. The 20 sec averages of temperature, horizontal wind and vertical wind were checked against the computer output before going on to calculate the best-fitting straight lines and fluxes. Satisfactory agreement has been demonstrated for the w and T data.

(e) Results

(1) The effect of averaging time on the mean products $\overline{u'w'}$ and $\overline{T'w'}$ for the whole period of each record was examined. In figs. 1 and 2 it can be seen that the computed products vary little with averaging time up to 5 or 10 secs, and it is concluded that the system records the high frequency components of the fluxes adequately. On the other hand the curves do not in general drop to zero for large averaging times, indicating that important low-frequency components are present. It has to be borne in mind that the recorded low-frequency components may be spurious to some extent, due to balloon movement.

This applies particularly to the horizontal wind component (and therefore to the momentum flux) but also to the vertical component, because of the way it is obtained from the total wind speed. No measurements of balloon movement were made on the occasions of the flux measurements, but a little information is available for five days in September and October 1965, when instruments were flown at comparable heights. Measurements were made using either an inclinometer on the balloon cable, or a camera obscura. The general picture is that horizontal speeds of up to 1 m/sec, measured over intervals of 30 or 60 seconds, were recorded with mean wind speeds of 5-10 m/sec. Movements of this sort are clearly sufficient to influence the low frequency measurements.

The values of $\overline{u'w'}$ and $\overline{T'w'}$ which are used in the comparison with the band pass measurements, are for the whole period of each record and 1 second averaging.

(2) The effect of sampling time was similarly examined. The periods considered were the first 4,000 secs of record on the 12th and 13th, and the first 10,000 secs of record on the 14th. The results are tabulated below:

Date	Sampling time (secs)	$\overline{u'w'}$	$\overline{T'w'}$
12.5.65	1000	0.0492	0.0384
	2000	0.0817	0.0452
	4000	0.0960	0.0493
13.5.65	1000	-0.0331	0.0162
	2000	-0.0598	0.0126
	4000	-0.0747	0.0108
14.5.65	1000	0.0406	0.0120
	2000	0.0387	0.0119
	5000	0.0254	0.0011
	10,000	0.0292	0.0016

It is clear that $\overline{u'w'}$ and $\overline{T'w'}$ do not in general rise towards constant values for large sampling time. The conclusion in (1), that low frequencies have an important effect, is confirmed, but here it is also apparent that the effect is complex.

(3) The $\overline{u'w'}$ values for 1000 sec periods were examined in relation to the mean wind gradients for the same periods. This was done using fluctuations referred to the best-fitting straight line for each period. Fig 3 shows the results. The wind gradients and the $\overline{u'w'}$ values using 1000 sec straight lines

tend to move together on the 12th and 14th. This is particularly so for the 12th, the only day on which the table in (2) above shows the expected increase of $\overline{u'w'}$ with sampling time. Otherwise no obvious relation emerges.

(4) The $\overline{T'w'}$ values were similarly examined in relation to the temperature gradient, see fig 4. As well as the temperature difference between the two levels bracketing the fluctuation measurement, the dry adiabatic lapse over the same interval is also shown. This shows that although there was a lapse of temperature throughout each occasion, there was an inversion of potential temperature. The computed $\overline{T'w'}$ values indicate a heat flux against the potential temperature gradient for nearly all the 1000-second periods. No similarity of trend can be seen.

(5) Initially the best-fitting straight lines were fitted to the 1-second data, but later the effect of fitting to averaged data was tried. Generally the trend constants are insensitive to change of averaging time, changes of up to 1% for averaging time up to 10 sec being normal. The exception is the slope constant for w , which at times changed by up to 10% in the same range of averaging time. The effect on the computed fluxes was always small, less than 1% to 50 sec averaging time. This difference of procedure probably has no theoretical significance, but it could be of practical importance if longer runs, or runs with more channels recorded, are to be undertaken in future. A run of three hours, recording three channels, practically fills the store of Mercury, so that greater quantities of data would have to go into the store and be processed in sections. If the straight lines for the whole period could be fitted to averaged data, then the procedure would be considerably simplified as all the averaged data could be accommodated at once.

The band-pass data

Consecutive 5-minute averages were read off the pen traces representing the mean deviations in the various spectral bands, and these were meant to give overall averages for the total period of observation on each occasion. In all cases the first section of the trace, amounting to 15-30 minutes, was omitted to avoid any initial spurious effects from the smoothing operation which was imposed in order to facilitate direct reading of the mean deviations. On the same account the variability of the 5 minute values is an underestimate of the true intermittency. However, to give a rough indication of the sequential

nature of the intermittency, and also of the stationarity the 5-minute values (of inclination) are listed in Table 1 as a fraction of the overall (average) magnitude of the mean deviation.

The overall average mean deviations of both inclination and temperature, together with the supporting data on mean wind speed at two levels and mean temperature differences between the same two levels, are summarized in Table 2. Considering first the implied shape of spectrum, note that if inertial sub-range conditions hold, and energy density is proportional to $n^{-5/3}$, then the ratios of the mean deviations in consecutive bands should be $6^{1/3}$ or 1.82. In the case of the inclination data it is seen that the ratios are not widely different from this, so providing some support for the use of the inertial sub-range relations. The data for temperature are less well-defined in this respect, no doubt partly because of the generally small values of temperature fluctuation, but the data could be said to be not grossly inconsistent with the inertial sub-range relations.

For velocity fluctuation the explicit relation between energy density $S_w(\kappa)$ and wavenumber κ is

$$S_w(\kappa) = C_w \epsilon^{2/3} \kappa^{-5/3}$$

where $\int_0^\infty S_w(\kappa) d\kappa = \overline{w^2}$ (w = vertical component)

ϵ is the rate of dissipation of turbulent kinetic energy (per unit mass) C_w is a universal constant.

For the constant C Panofsky and Pasquill (1963) give the best estimate for the 'longitudinal' form of spectrum as 0.47, with κ in radians per unit length. The above C_w refers to the 'transverse' form of spectrum and is $0.47 \times \frac{4}{3}$ or $0.47 \times \frac{4}{3} \div 2\pi^{2/3}$ if κ is in cycles per unit length, i.e. 0.184.

In terms of the mean deviation of wind inclination $|\overline{\phi}|$ for a given band the above relation becomes (Pasquill, 1963, Eq. 15)

$$|\overline{\phi}|^2 = \frac{3.453 a C_w \epsilon^{2/3}}{f^2 u^{4/3} n_2^{2/3}}$$

where ϕ is in radians

n_2 is the higher value of the frequency for 50% power transmission, in cycles/sec.

- a is a factor correcting for the fact that the band-pass filter is not 'square-shouldered', and equals 1.05 for a $n^{-5/3}$ spectrum
- f is the form factor relating standard deviation and mean deviation, which we take to be 1.25, appropriate to a Gaussian distribution.
- \bar{u} is the mean wind speed at the height at which θ is measured.

Rearranging the above expression, taking θ in degrees, and putting in the value for C_w given above

$$\epsilon = 1.89 \times 10^{-5} \bar{u}^2 |\bar{\theta}|^3 n_2 \text{ cm}^2 \text{ sec}^{-3}$$

The analogous expression for the high-frequency spectrum of temperature fluctuation is

$$S_{\theta}(\kappa) = C_{\theta} \epsilon^{-1/3} \chi \kappa^{-5/3}$$

where $\int_0^{\infty} S_{\theta}(\kappa) d\kappa = \overline{\theta'^2}$, θ' being the eddy fluctuation of temperature and χ is the rate of reduction of $\overline{\theta'^2}$ by molecular action. Comparing this with the expression for velocity fluctuation

$$\frac{\chi}{\epsilon} = \frac{C_w}{C_{\theta}} \frac{S_{\theta}(\kappa)}{S_w(\kappa)}$$

and assuming the θ' distribution to be similar to the w distribution this means

$$\frac{\chi}{\epsilon} = \frac{C_w}{C_{\theta}} \frac{|\bar{\theta}|^2}{|\bar{\theta}|^2} \frac{57.3^2}{\bar{u}^2}$$

where $|\bar{\theta}|$ is the corresponding band-pass output in $^{\circ}\text{C}$ and $|\bar{\theta}|$ is in degrees. An unpublished estimate of C_{θ} (by Ellison) is 0.37 (for κ in radians)

so

$$\frac{C_w}{C_{\theta}} = \frac{0.47 \times 4}{0.37 \times 3} \quad \text{and}$$

$$\frac{\chi}{\epsilon} = 5.56 \times 10^3 \times \frac{|\bar{\theta}|^2}{|\bar{\theta}|^2} \bar{u}^2$$

Balance in turbulent kinetic energy
and mean square temperature
fluctuation

$$\frac{dE}{dt} = \overline{-u'w'} \frac{du}{dz} + \frac{g}{T} \overline{\theta'w'} - \epsilon - \frac{d}{dz} (F_E)$$

(neglecting pressure fluctuation terms) where E is total turbulent kinetic energy per unit mass, i.e. $\frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$. The first term on the R.H.S. is the energy transferred from the mean flow by the action of the Reynolds stress, and is usually referred to as the mechanical production term. The second term is the energy produced or extracted by buoyancy forces ($\overline{\theta'w'}$ ρC_p is the vertical flux of heat, positive when upwards). The fourth term represents the contribution from the vertical diffusion of kinetic energy, F_E being the vertical flux of total turbulent kinetic energy. Of these terms we can evaluate the rate of dissipation from the band-pass measurements, and the mechanical and buoyancy production terms from the shearing stress and heat flux estimates and the vertical gradient of wind speeds.

The corresponding expression for temperature fluctuation is

$$\frac{d\overline{\theta'^2}}{dt} = \overline{-\theta'w'} \frac{d\overline{\theta}}{dz} - \chi - \frac{d}{dz} (F_{\theta'})$$

where $\frac{d\overline{\theta}}{dz}$ is the vertical gradient of potential temperature and $F_{\theta'}$ is the vertical flux of θ'^2 . Here also estimates can be attempted for the production term and the dissipation term.

The various estimates are now listed in Table 3. Without attaching too much significance to the precise numerical values the following features stand out:

- (a) The mechanical energy production term is an order of magnitude less than either the buoyancy production term or the dissipation. The mechanical production has so far been considered entirely in terms of u' , the fluctuation in the direction of the mean wind. Actually the measurements refer to U' , the fluctuation in the total horizontal component, but in Appendix 1 it is argued that to an acceptable approximation $\overline{u'w'} \approx U'w'$. However, there should also be a production

term $-\overline{u'v'} \frac{d\bar{v}}{dz}$, for which no direct estimate is available. An indirect estimate, assuming that $\overline{u'w'}$ and $\overline{v'w'}$ are similarly related to the vertical gradients of \bar{u} and \bar{v} , is simply

$$-\frac{\overline{u'w'} \left(\frac{d\bar{v}}{dz} \right)^2}{\frac{d\bar{u}}{dz}}$$

This means that the production term involving the v-component is $\left(\frac{d\bar{v}/dz}{d\bar{u}/dz} \right)^2$ times that involving the u-component. If $\frac{d\bar{v}}{dz}$ is not substantially greater than $\frac{d\bar{u}}{dz}$ the conclusion regarding the unimportance of mechanical production and the importance of diffusion of energy is unaffected. Although similarity of $\frac{d\bar{v}}{dz}$ and $\frac{d\bar{u}}{dz}$ might be expected from classical analyses there is unfortunately some doubt in the present cases where the Bulthun data indicate appreciable variation of wind direction with height.

(b) The dissipation substantially exceeds buoyancy production. The difference is in the region of $5 \text{ cm}^2 \text{ sec}^{-3}$ which over a period of one hour amounts to a loss of $2 \text{ m}^2 \text{ sec}^{-2}$. It is obvious that there is no reduction in turbulent energy comparable with this (σ_w^2 averaged over the whole period on 12/5/65 is only about $0.5 \text{ m}^2 \text{ sec}^{-2}$) and the most reasonable interpretation is that energy is appearing at the 600 m level as a result of diffusion from another (presumably lower) level.

(c) For temperature fluctuation the production term is negative, a consequence of the apparently counter-gradient heat flux (i.e. upward flux through a stable environment). There is nothing obviously unreasonable in this situation, which is predicted by the Priestley-Swinbank treatment of heat transfer in a stratified medium. It is also apparent in Telford and Warner's measurement from an aircraft at similar heights. Furthermore, Deardorff (1966) has analysed Telford and Warner's results to yield values of $-\overline{w'\theta'} \frac{d\bar{\theta}}{dz}$ similar to those listed here, and also values of the diffusion term which to some extent are in balance with the negative values of production. From the present data all that can be said is that the dissipation and production are of opposite sign, implying that

$$\frac{d\overline{\theta'^2}}{dt} + \frac{d}{dz}(\overline{F_{\theta'}}) = -10^{-4} \text{ } \theta_c^2 \text{ sec}^{-1}$$

From the band-pass and low-pass traces of temperature fluctuation $\overline{\theta'^2}$ probably averages about $10^{-2} \text{ }^\circ\text{C}^2$, with no substantial change throughout the period of observation, implying that the first of the above two terms is negligible in relation to the second. So in the temperature fluctuation, as well as the velocity fluctuation, there is a fairly strong indication that the diffusion term is important.

APPENDIX I

INTERPRETATION OF THE EDDY CORRELATION MEASUREMENT

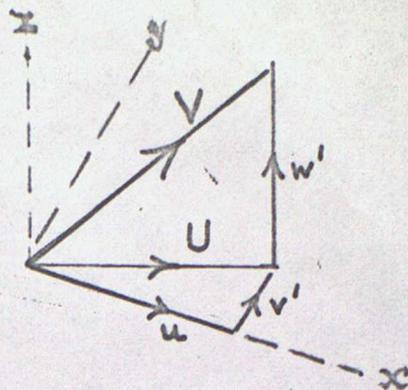
with axes as opposite we
measure V and ϕ (x along
mean wind direction)

we evaluate $U = V \cos\phi$

$$U' = U - \bar{U} \text{ (in effect)}$$

we can write $U = (u^2 + v'^2)^{\frac{1}{2}} \approx u + \frac{v'^2}{2u}$ if $\frac{v'^2}{u^2} \ll 1$

as expected.



$$\text{So } -\overline{w'U'} \approx -\overline{w' \left[u - \bar{u} + \frac{v'^2}{2u} - \frac{1}{2} \left(\frac{v'^2}{u} \right) \right]}$$

$$\approx -\overline{w'u'} - \overline{w' \left[\frac{v'^2}{2u} - \frac{1}{2} \left(\frac{v'^2}{u} \right) \right]}$$

An overestimate of the second term may be attempted by taking $\frac{1}{2} \left(\frac{v'^2}{u} \right)$ zero and writing

$$\frac{\overline{w'v'^2}}{2u} \approx \overline{w'u} \frac{\overline{v'^2}}{2u^2} \approx \overline{w'u'} \frac{\overline{v'^2}}{2u^2}$$

which suggests that to a reasonable approximation

$$-\overline{w'U'} \approx -\overline{w'u'}$$

Table 1. $\bar{\phi}$ -min averages of the band-pass traces expressed as a fraction of the value of $|\bar{\phi}|$ for the whole period.

<u>12/5/65</u>	4	1.05	1.07	0.10	0.75	0.78	> 1.46	> 1.46	1.21	1.27	1.03	1.24	1.11	> 1.46	> 1.46	0.89		
	3	0.96	1.14	0.93	0.58	0.92	1.28	1.23	0.87	0.92	0.84	1.09	1.12	1.20	1.29	0.75		
Band	2	0.90	1.29	0.95	0.71	0.99	> 1.56	1.33	0.88	0.87	0.77	1.01	1.16	1.25	1.18	0.72		
	1	0.87	1.39	0.77	0.67	1.01	1.77	1.28	0.77	0.88	0.84	1.07	1.05	1.19	0.91	0.54		
	0	0.93	1.48	0.59	0.68	1.13	1.86	1.13	0.71	0.87	0.90	1.13	1.13	1.14	0.79	0.54		
<u>13/5/65</u>	4	0.76	0.98	> 1.50	> 1.50	> 1.50	0.75	0.73	1.09	1.11	1.30	1.03	0.91	1.19	1.15			
	3	0.65	0.88	1.46	1.31	1.12	0.78	0.74	0.92	1.03	1.02	0.82	1.04	> 1.71	1.35			
Band	2	0.77	0.90	1.18	1.20	1.06	0.82	0.94	0.91	0.98	1.01	0.79	1.02	> 1.67	1.43			
	1	0.72	0.99	1.24	1.10	0.95	0.79	0.86	0.88	0.97	0.83	0.76	1.10	1.73	1.07			
	0	0.71	1.01	1.30	1.12	0.85	0.73	0.81	0.84	0.98	0.71	0.81	1.26	1.82	1.02			
<u>14/5/65</u>	4	1.17	1.45	1.18	0.60	0.66	0.91	0.72	0.53	0.94	1.07	0.70	0.74	1.36	1.24	1.29	1.21	1.33
	3	1.13	1.31	1.11	0.66	0.62	0.71	0.79	0.75	0.91	0.99	0.76	1.03	1.22	1.01	0.97	1.11	1.17
Band	2	1.22	1.35	1.03	0.63	0.70	0.84	0.90	0.86	0.93	0.95	0.75	0.89	1.16	1.06	1.07	1.19	1.16
	1	1.34	1.49	1.03	0.60	0.75	0.93	0.96	0.93	0.92	1.03	0.75	0.93	1.21	1.03	1.08	1.21	1.15
	0	1.36	1.53	0.94	0.57	0.75	0.92	0.94	0.94	0.87	0.94	0.75	0.98	1.28	0.94	1.11	1.23	1.13
<u>14/5/65 cont</u>	4	1.45	0.97	0.60	0.93	1.15	0.68	0.58	0.72	0.65	0.75	0.97	> 1.85	> 2.01	1.26	1.58	1.61	
	3	1.26	1.01	0.79	1.28	1.18	0.76	0.63	0.61	0.63	0.65	0.75	1.41	1.41	1.26	1.54	1.47	
Band	2	1.13	1.04	0.99	1.24	1.15	0.81	0.69	0.61	0.56	0.56	0.77	1.36	1.40	1.22	1.46	1.31	
	1	1.08	1.04	1.03	1.21	1.07	0.70	0.65	0.58	0.56	0.59	0.72	1.37	1.40	1.14	1.33	1.19	
	0	1.11	1.08	0.98	1.28	0.94	0.66	0.66	0.57	0.55	0.66	0.75	1.53	1.32	1.17	1.36	1.15	

Table 2 Summary of band-pass data
(inclination and temperature fluctuations
measured at 600 m)

Band	4		3		2		1		0		$u_{\text{msec}^{-1}}$	$\Delta\theta_{\text{OC}}$ (z_1-z_2)
	n_m c/sec	T , s. sec	n_m c/sec	T , s. sec	n_m c/sec	T , s. sec	n_m c/sec	T , s. sec	n_m c/sec	T , s. sec		
12/5/65 M.D. 1148-1303	0.0060 180,30	>5.72 0.043	0.036 30, 5	4.88 0.022	2.48 0.009	5, 5/6	1.29 5/6, 5/36	0.67 -	7.77 5/36, 5/216	3.6 3.5	-4.0	
13/5/65 M.D. 1125-1235	>5.62 0.040		4.70 0.027		>2.32 0.014		1.34 0.008	0.67 -		4.0 2.6	-2.9	
14/5/65 M.D. 1149-1434	4.60 0.041		3.70 0.018		1.85 0.005		1.05 0.005	0.51 -		4.6 3.9	-3.7*	

* -2.1 (1149-1233), -4.5 (1248-1434)

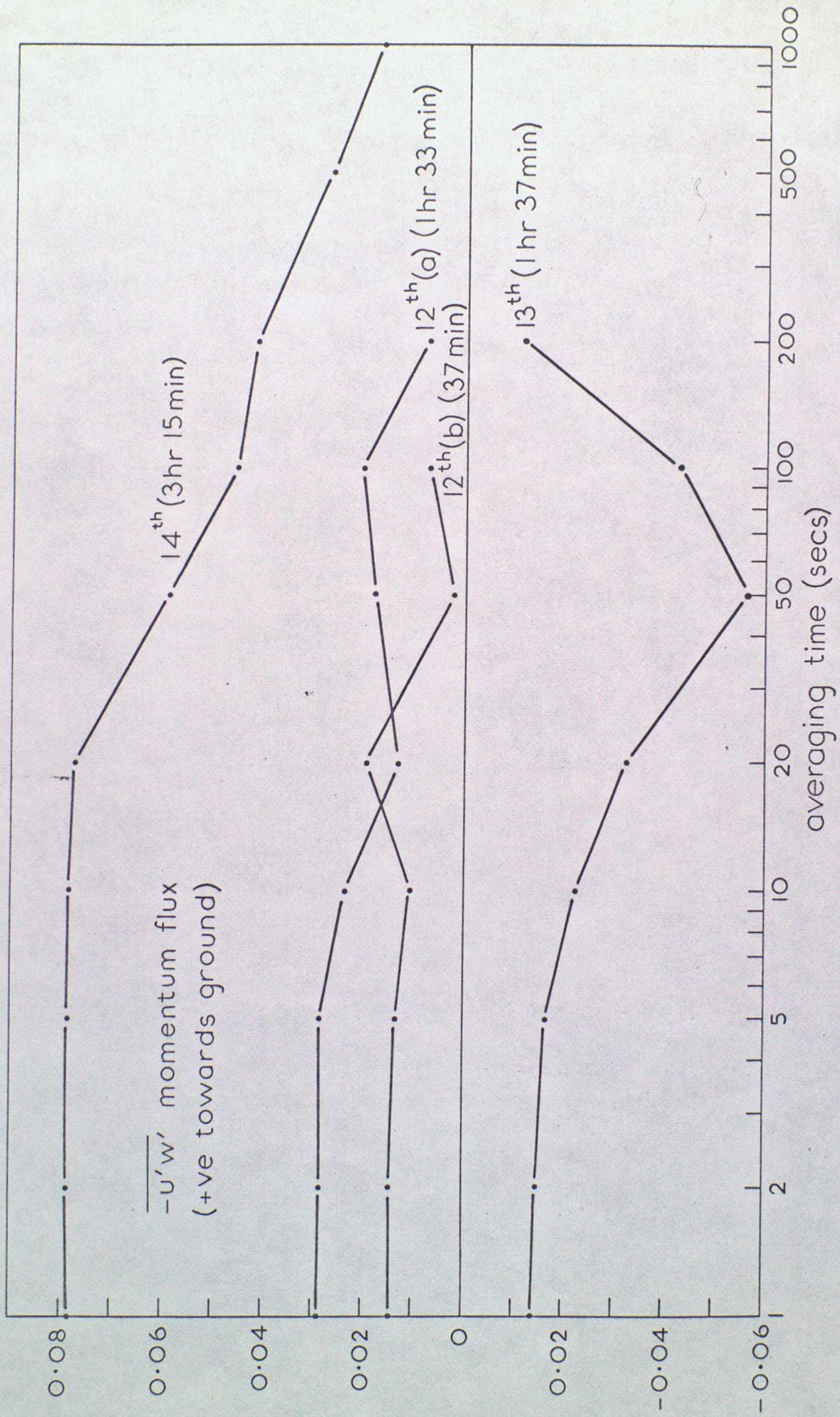


Fig 1

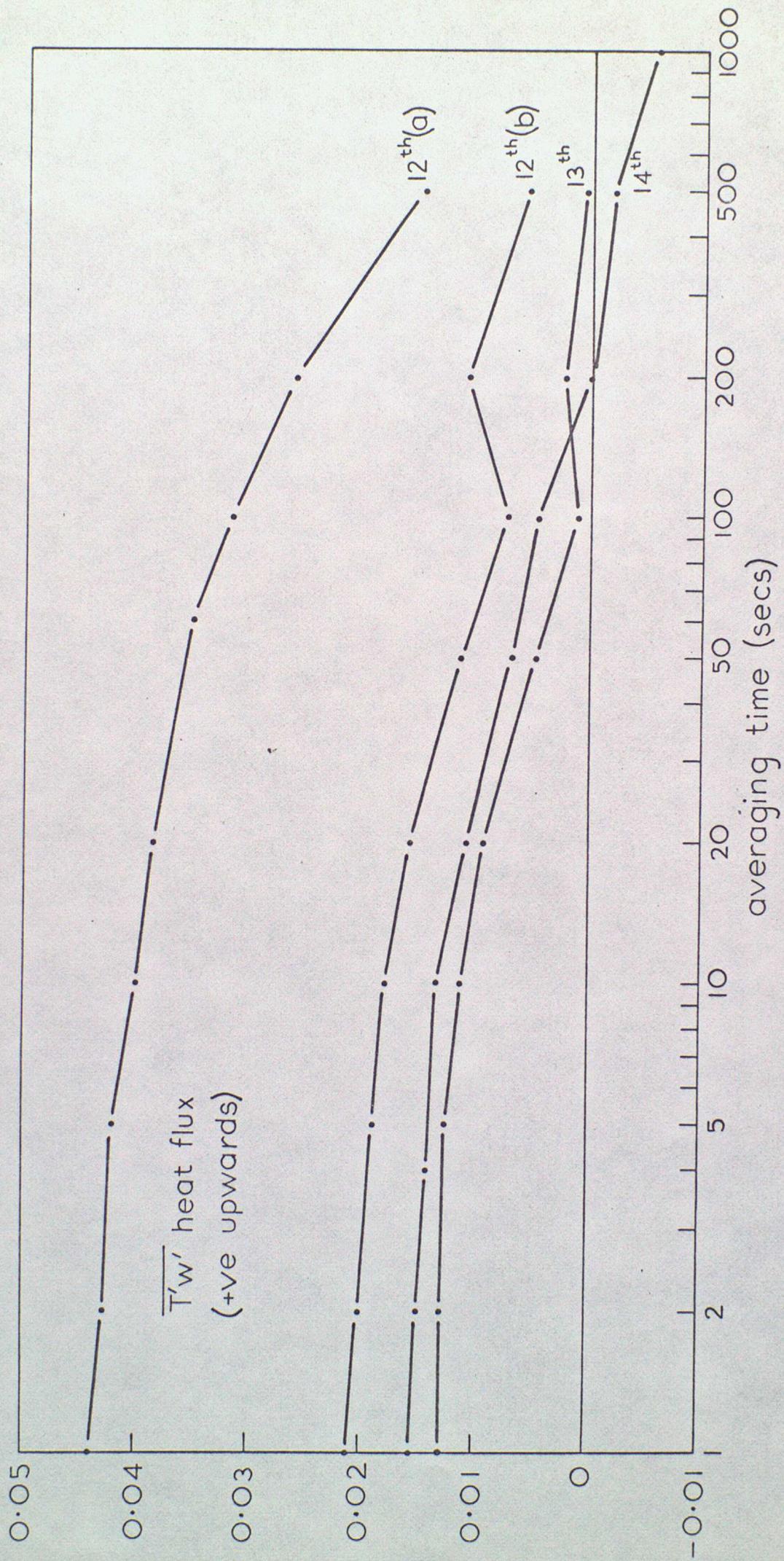


Fig 2

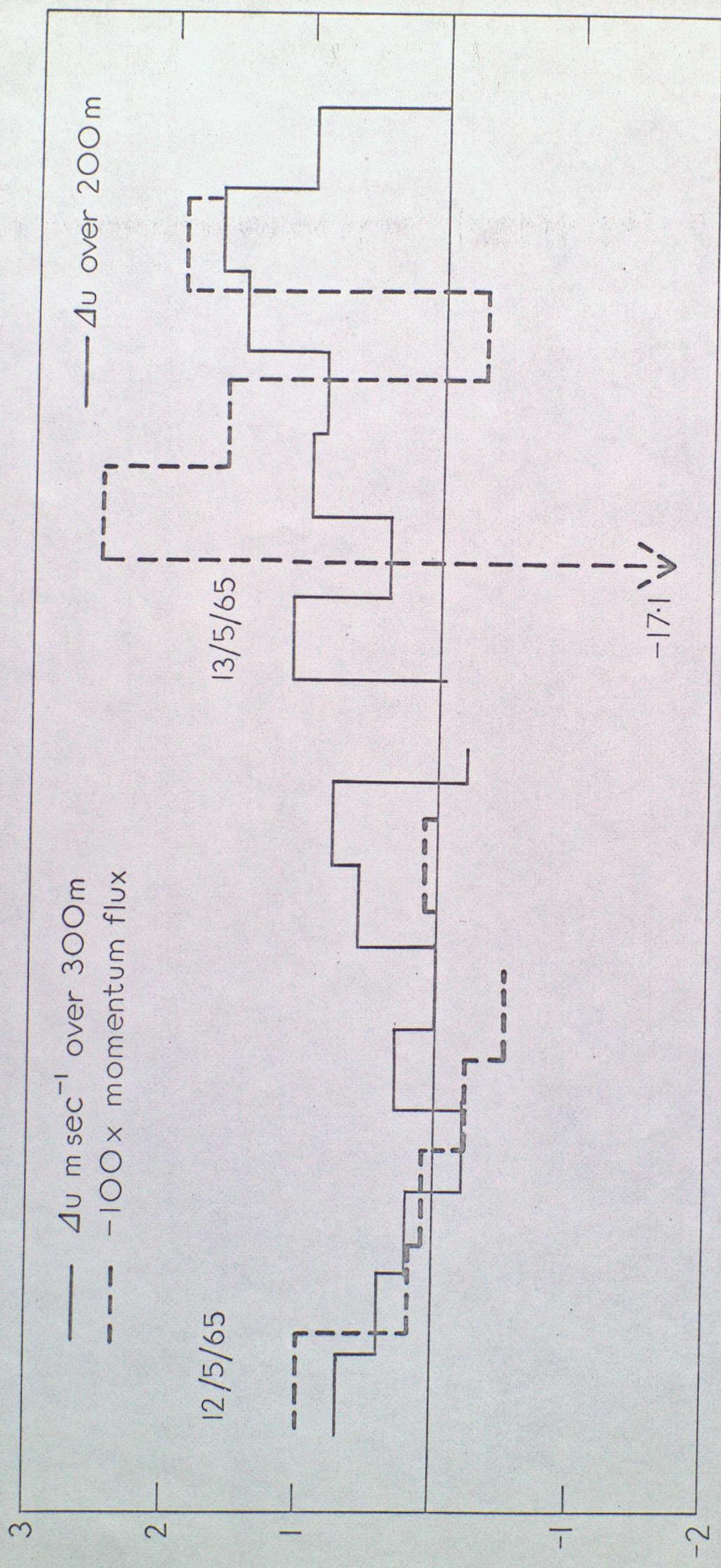


Fig 3 (a) Momentum flux and wind gradient over 10000 sec periods

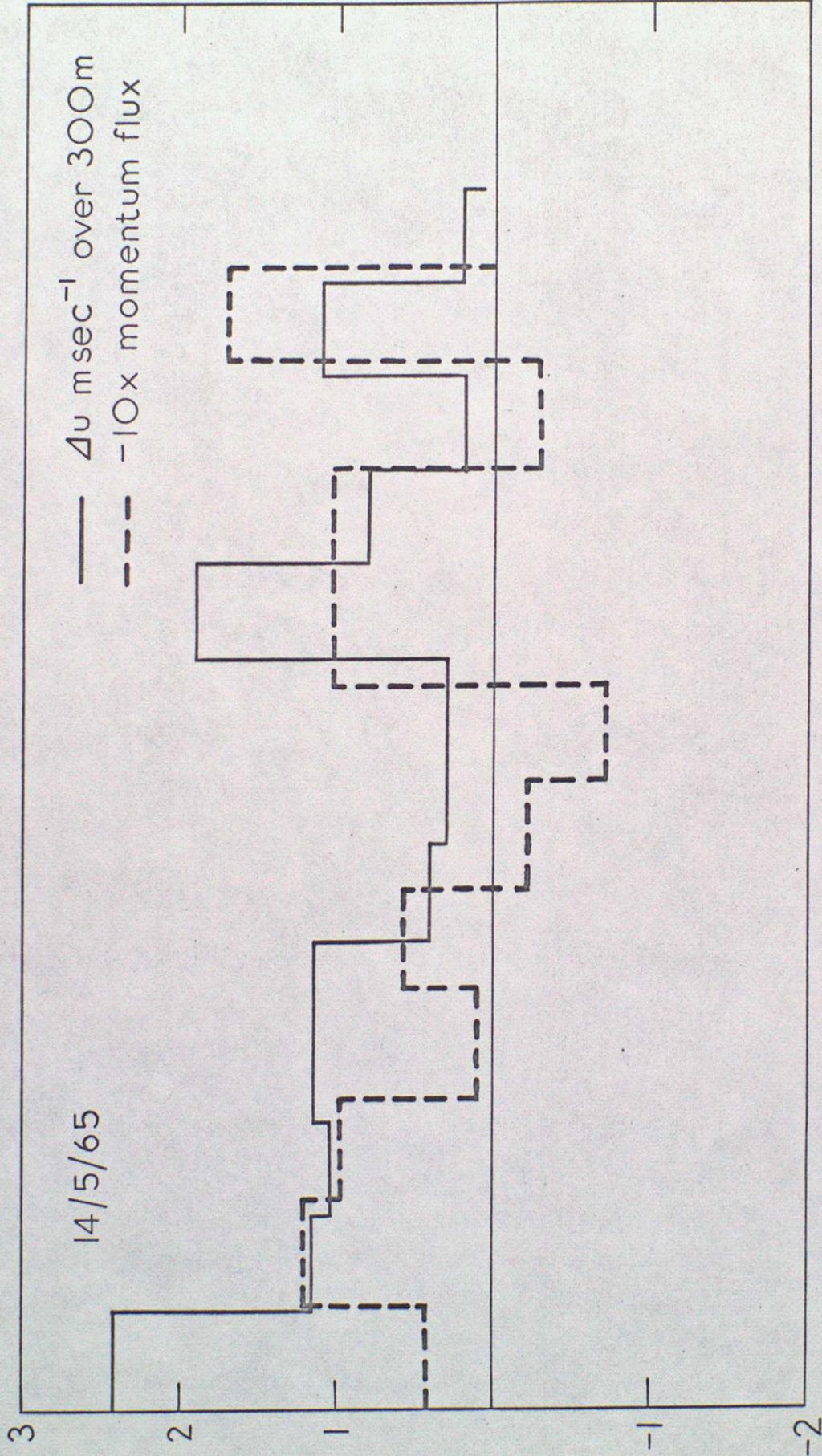


Fig 3 (b) Momentum flux and wind gradient over 1000 sec periods

— $\frac{1}{10}$ temperature difference ($^{\circ}\text{C}$) over intervals stated
 (dry adiabatic difference indicated by - - -)

- - - $10 \times$ heat flux

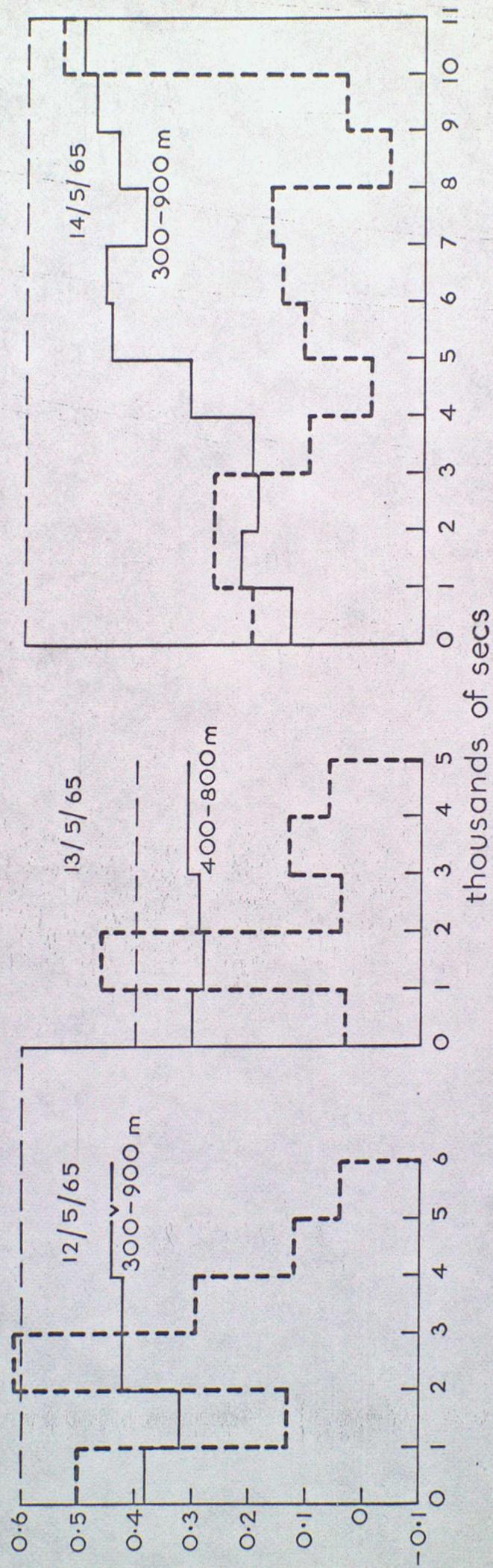


Fig 4 Heat flux and temperature gradient

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