METEOROLOGICAL OFFICE

INVESTIGATIONS DIVISION TECHNICAL NOTE NO 8

The use of Cornford's wind shear tables to estimate the frequency of occurrence of a given wind shear in level flight.

W R Sparks.

1. The variation of the correlation of vertical wind shears with horizontal distance

1.1 Definition of variables.

V_1 is the deviation of the wind from its mean value at level 1 at some arbitrary origin

V_2 is the deviation of the wind from the mean value at level 2 at the origin

V_1^1 is the deviation of the wind from its mean value at level 1 distance d from the origin

V_2^1 is the deviation of the wind from its mean value at level 2 distance d from the origin

$S = V_2 - V_1$ ie the deviation of the shear from its mean at the origin

$S^1 = V_2^1 - V_1^1$ ie the deviation of the shear from its mean distance d from the origin

$\sigma^2_{V_1} = \sigma^2_{V_1^1}$ is the variance of the wind at level 1

$\sigma^2_{V_2} = \sigma^2_{V_2^1}$ is the variance of the wind at level 2

r_{xy} is the correlation coefficient between two variables x and y.

1.2 The variance of the shear between two levels is given by

$$\sigma_S^2 = \sigma^2_{V_2 - V_1} = \frac{1}{N}\left\{\Sigma V_2^2 + \Sigma V_1^2 - 2\Sigma V_1 V_2\right\}$$
$$= \sigma^2_{V_2} + \sigma^2_{V_1} - 2\sigma_{V_1}\sigma_{V_2} r_{V_1 V_2} \qquad (1)$$

If $\sigma_{V_1} = \sigma_{V_2} = \sigma_V$

$$\sigma_S^2 = 2\sigma_V^2(1 - r_{V_1 V_2}) \qquad (2)$$

1.3 The correlation of shears distance 'd' apart.

$$r_{ss'} = \frac{\frac{1}{N}\sum ss'}{\sigma_s \sigma_{s'}} = \frac{\frac{1}{N}\sum (V_2-V_1)(V_2'-V_1')}{\sigma_s \sigma_{s'}}$$

If $\sigma_s = \sigma_{s'}$

$$r_{ss'} = \frac{1}{N\sigma_s^2}\left[\sum V_2V_2' - \sum V_2V_1' - \sum V_1V_2' + \sum V_1V_1'\right]$$

$$= \frac{1}{\sigma_s^2}\left[\sigma_V^2 r_{V_2V_2'} - \sigma_V^2 r_{V_2V_1'} - \sigma_V^2 r_{V_1V_2'} + \sigma_V^2 r_{V_1V_1'}\right] \quad (3)$$

If it is assumed that the only link between V_1 and V_2' is through the horizontal and vertical correlations $r_{V_1V_1'}$ and $r_{V_1'V_2'}$ then it is easily shown that

$$r_{V_1V_2'} = r_{V_1V_1'} \cdot r_{V_1'V_2'} \quad (4)$$

If it is also assumed that $r_{V_1V_1'} = r_{V_2V_2'} = r_d$

and $r_{V_1V_2} = r_{V_1'V_2'} = r_{12}$

then (4) becomes $r_{V_1V_2'} = r_d r_{12}$

and (3) may then be written

$$r_{ss'} = \frac{2\sigma_V^2}{\sigma_s^2}\left[r_d - r_d r_{12}\right] = \frac{2\sigma_V^2 r_d}{\sigma_s^2}(1 - r_{12}) \quad (5)$$

Substituting from (2)

$$\underline{r_{ss'} = r_d} \quad (6)$$

2. The distance over which a single observation of shear may be regarded as representative

It can be shown (Brooks and Carruthers 1953) that if we define a persistence factor p by $p = \frac{1}{1-r_d} = \frac{2\sigma^2}{\sigma_d^2}$ (7)
where σ^2 is the true variance of the variable, r_d is the correlation coefficient of observations of the variable at distance 'd' apart and σ_d^2 is the variance of the differences of those observations, then the distance over which a single observation is representative is $(2p - 1)d$. (8)

Hawson (1970) gives statistics of upper winds near the Brotish Isles from which p and d can be calculated at standard pressure levels from 850 mb to 200 mb. The maximum value of (2p-1)d occurs at the 200 mb level and is given in the table below.

Table 1

The product (2p-1)d at 200 mb near the British Isles

distance (d) km	10	50	300	500
p	242	60.5	8.00	4.94
(2p-1)d	4830	6000	4500	4440

The value of (2p-1)d at 50 km does not agree well with the values for the other distances. Hawson's tables show that the root-mean-square (RMS) variation of wind over the various distances is the same at 200 mb and 500 mb except at the 50 km distance.

If we substitute the 500 mb value of the RMS difference at 50 km (4.5 m/s) in place of the 200 mb value (4.0 m/s) we find p = 47.8 and (2p-1)d = 4730 km which is in good agreement with the values for other distances. At 200 mb then, the mean distance between independent shears is about 4600 km and this value is larger than at any other level up to 200 mb.

Lenhard (1973) measured the variability of wind over a distance of 16.25 km over New England during January-March 1969. This was a period of high cyclonic activity and his minimum value of r_d (0.92) found at about 250 mb is probably well below the representative annual value for the whole of the northern hemisphere. However, that value of r_d gives p = 12.5 (2p-1)d = 390 km.

There is, therefore, an uncertainty of more than one order of magnitude in the mean distance between independent shears.

3. The distance that must be flown by an aircraft in level flight to sample the same number of independent shears that are sampled in a single batch in the tables.

The number of independent shears in the northern hemisphere at a given time is $N = 4A/\pi d^2$ where A is the area of the hemisphere and d is the average diameter of the area representative of a single observation of shear.

$$A = 8 \times 10^8/\pi \quad km^2$$

$$\text{If } d = 4600 \text{ km} \qquad N = \frac{32 \times 10^8}{\pi^2 \times (4600)^2} \approx \frac{32 \times 10^8}{10 \times 21 \times 10^6} \approx 15$$

$$\text{If } d = 390 \text{ km} \qquad N = \frac{32 \times 10^8}{10 \times 1.5 \times 10^5} \approx 2133$$

The soundings collected together in a batch were made over a period of about 5 days and Hawson's (1970) data shows that the correlation between winds measured at a single station 5 days apart is almost zero. Therefore, each batch of soundings contains two independent samples of the hemisphere. Thus if d = 4600 km the number of independent samples in a batch is about 30 and if d = 390 km every sounding in a batch is independent since the maximum number in a batch is about 3000.

The number of independent shears sampled by an aircraft flying a distance l in level flight is n = l/d. If n is put equal to the number of independent shears in a batch for a given d then l may be calculated. Figure 1 shows how n and l vary with d.

4. The use of extreme value theory to analyse the shear tables

The theory of the statistics of extremes is given by Jenkinson (1969). The arrangement of the data in the shear tables is convenient for extreme value analysis and figure 2 shows a plot of the shears for the layer from 100 mb to 70 mb. The return period is in batches and, therefore, equivalent to units of about 1.4×10^5 km of level flight. Figure 2 shows that positive and negative shears come from quite different populations at the lower values of shear but the extremes are similar for both.

The inhomogeneity of the data makes extrapolation very uncertain, the TSS 1-0 extreme shear could be met once every 200 batches or once every 5000 batches!

It has been stated (in conversation between Ruben and Sparks) that while the extreme shears in Cornford's tables are in reasonable agreement with those in Scientific Paper No 17 (Crossley 1962) the percentile values are not. This is so if one assumes that each measurement of shear in Cornford's tables is independent. However, one can work the other way and assume that the percentiles are the same and then obtain an estimate of the number of independent shears in a batch. Crossley's 1% shear for a layer 7,400 ft thick over London is about 18 m/s per km. Crossley did not distinguish between positive and negative shears so we must find the probability that neither a positive nor a negative shear of this magnitude will occur in a batch. From figure 2 that probability is about 0.27.

Let the probability of an event be p. Then if we take n independent samples the probability of that event not occurring is $(1-p)^n$. If n is the number of independent samples in a batch then $(0.99)^n = 0.27$, which gives $n = 258$. This number corresponds to a distance between independent shears of about 1575 km. This implies a flight distance of about 4×10^5 km to sample the same number of shears.

As a further check we may compare the 5% shear calculated from the tables, on the assumption that there are 258 independent shears, and Crossley's 5% shear for London. The values are 9 m/s per km and 13 m/s per km. The agreement is not good and to bring the 5% shears into agreement would require there to be only 64 independent shears in a batch. This implies a flight distance of 2×10^5 km to sample the same number of shears.

It is clear from the above figures that the distribution of shears over the whole of the northern hemisphere is not the same as over London but that consideration of the percentiles can refine the estimate of the distance which must be flown to sample the number of independent shears in a batch.

5. Conclusions

Calculations from published correlation coefficients of upper winds at various distances apart suggest that the distance that an aircraft must fly horizontally to sample the same number of independent shears as are sampled by one batch in the shear tables could vary from about 1.3×10^5 km. to 1.3×10^6 km, but it should be possible to reduce this range of uncertainty for a given layer by comparing the 1% and 5% shears with those for London.

REFERENCES

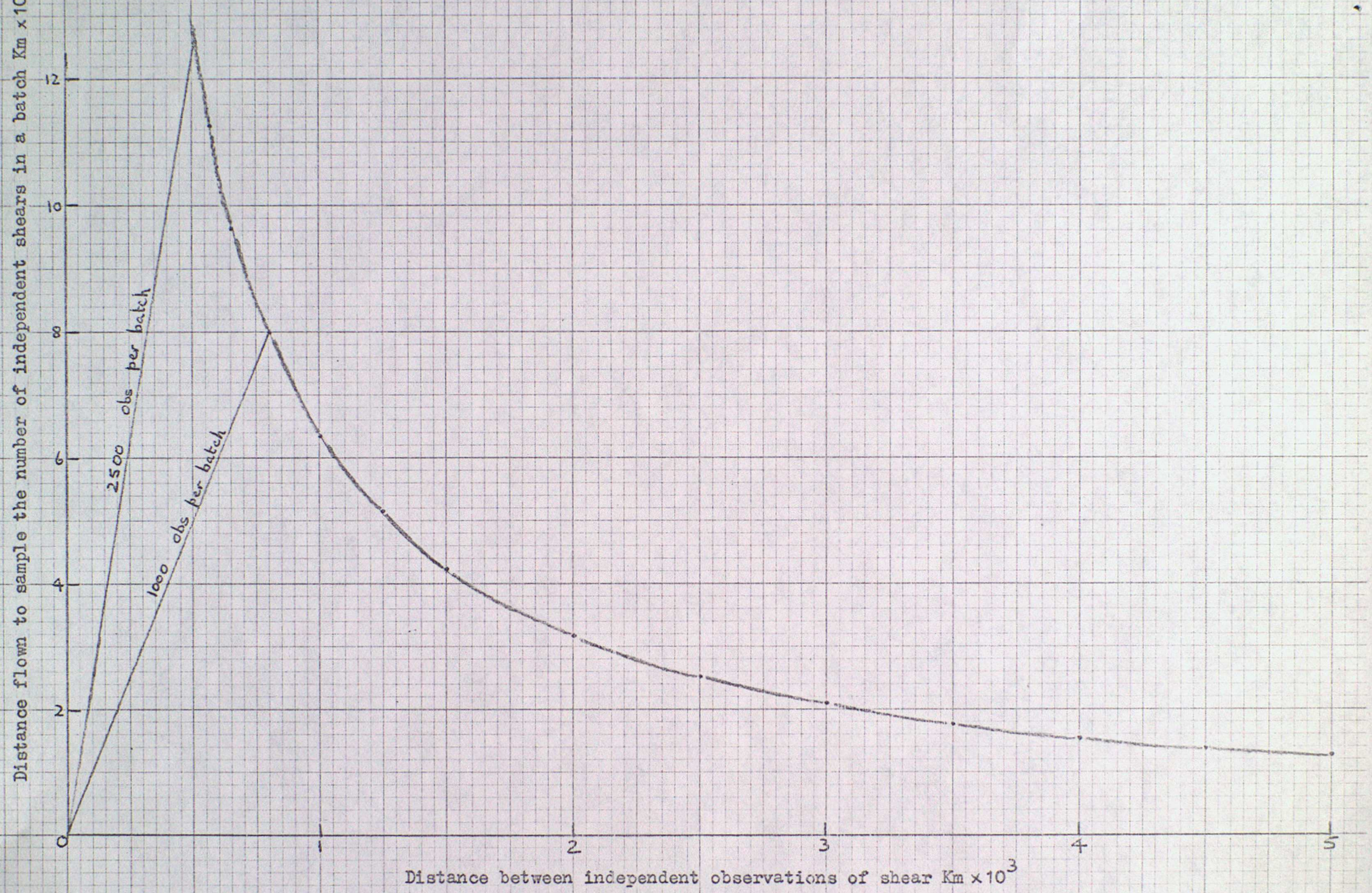
Brooks C E P and Carruthers N 1953. Handbook of Statistical Methods in Meteorology. MO 538 Meteorological Office London HMSO.

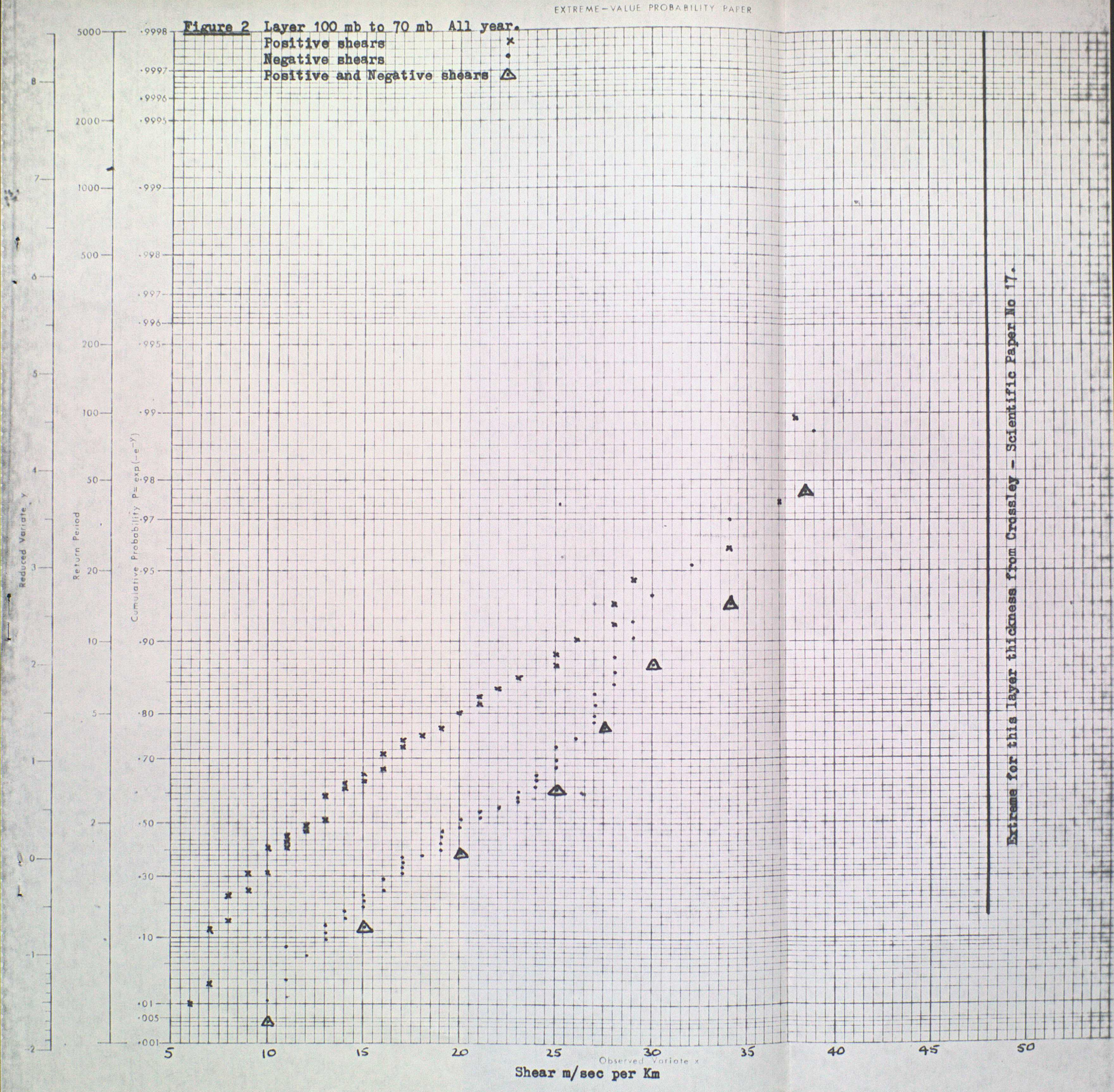
Hawson C L 1970. Performance Requirements of Aerological Instruments. An assessment based on atmospheric variability. WMO Tech Note No 112 WMO-No 267. TP. 151.

Lenhard R W. Variability of Wind Over a Distance of 16.25 km. Journal of Applied Meteorology Vol 12 No 6 Sept 1973 pp 1075-1078.

Crossley A F 1962, Extremes of Wind Shear.Meteorological Office Scientific Paper No 17. MO 730 HMSO London.

Figure 1


Distance flown to sample the number of independent shears in a batch Km x10^5
2500 obs per batch
1000 obs per batch
0
2
4
6
8
10
12
1
2
3
4
5
Distance between independent observations of shear Km x 10^3



METEOROLOGICAL OFFICE

Prepared and drawn in the Cartographic Section Met.O.18d Bracknell Met. O./Carto./D.O./Form File/ 161