

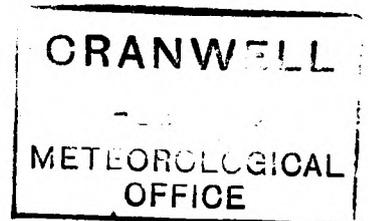
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METEOROLOGICAL OFFICE

GEOPHYSICAL MEMOIRS No. 42

*(Second Number of Volume V)*



The Time Interval  
BETWEEN  
Magnetic Disturbance  
AND  
Associated Sunspot Changes

By

J. M. STAGG, M.A., B.Sc.

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*Published by Authority of the Meteorological Committee*

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LONDON :

PUBLISHED BY HIS MAJESTY'S STATIONERY OFFICE

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1928

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# THE TIME INTERVAL BETWEEN MAGNETIC DISTURBANCE AND ASSOCIATED SUNSPOT CHANGES

## § 1—INTRODUCTION ; THEORETICAL ASPECTS

Although it is now many years since the occurrence of auroræ and magnetic disturbances on the earth were first attributed to the emission of charged particles from localized areas on the sun, little is definitely known of the physical characteristics of the particles. The early assumption of homogeneous streams of  $\alpha$  or  $\beta$  particles involved difficulties of dispersion due to mutual repulsion, to overcome which Lindemann suggested clouds of ionized atoms propelled outwards from the sun by radiation pressure. Further difficulties in the way of allowing particles sufficient speed and mass to carry them down into the earth's atmosphere far enough to account for the observed 80–100 km. level of auroræ still remained.

E. A. Milne<sup>1</sup> has more recently considered the possibility of the expulsion of heavier atoms at greater speeds by absorption of additional energy in the violet end of the spectrum and consequent exposure to increased radiation pressure. His results have been interpreted by S. Chapman<sup>2</sup> as suggesting that the expelled particles may reach the earth with a velocity of about 1,000 km. per second and take about four days to travel from sun to earth. It is of importance to see whether there is evidence in the actual sunspot and magnetic data for such a time lag between sunspot appearance and magnetic disturbance.

## § 2—RESUMÉ OF PREVIOUS STATISTICAL ANALYSES

In connexion with an earlier theory of Arrhenius which suggested two days as adequate for certain electricity carriers to reach the earth from the sun, C. Chree<sup>3</sup> has already made an examination of the question. Dealing with declination data for Kew Observatory for the eleven years 1890–1900 he extracted the ten days per month ( $n$ ) of largest absolute range and tabulated the daily values of sunspot projected areas as published in the *Greenwich Photo-Heliographic Results* from day  $n-3$  (the third day before day  $n$ ) to day  $n$ . The same was done for the ten days per month of least absolute range. The final mean results for the two groups of days are reproduced in Table I (a) which is extracted from Table XXIII, p. 238, of Chree's paper.

TABLE I—VARIATIONS OF SUNSPOT AREA ON DAYS ASSOCIATED WITH MAGNETIC DISTURBANCE AND QUIET

		Days of Largest Range						Days of Least Range					
		$n+1$	$n$	$n-1$	$n-2$	$n-3$	$n-4$	$n+1$	$n$	$n-1$	$n-2$	$n-3$	$n-4$
(a)	{ Means .. ..	—	+35	+39	+41	+48	—	—	-37	-38	-48	-49	—
	{ Percentages ..	—	+4.4	+5.1	+4.9	+6.8	—	—	-2.5	-2.9	-5.1	-6.4	—
(b)	Means .. ..	+4	+40	+59	+67	+72	+86	-52	-64	-59	-60	-32	-35

<sup>1</sup> E. A. Milne. *Monthly Notices, R.A.S.* 86 (1926), 459.

<sup>2</sup> S. Chapman. *Journal, London Math. Soc.* Vol. 2. Part 2 (1927), p. 134.

<sup>3</sup> C. Chree. *Phil. Trans. (A) R. Soc.* 208 (1908), p. 234.

To obtain the entries in the first line of this table, the algebraic excesses of the mean areas for the four days  $n-3$  to  $n$  over the mean daily area for each year separately have been added and meaned over the eleven years. The second line is the set of means obtained by expressing each year's results as a percentage of the average area for the year.

It was considered that the only hypothesis which was warrantably supported by these figures and the figures in the remainder of the complete table was one in which the time of propagation of the solar influence "varied in length from under a day to several days."

By restricting the tabulation of the magnetically disturbed and quiet days to the five days of greatest and five days of least absolute declination range, and extending the number of days examined from the fourth day before to the first day after the selected day  $n$ , the results reproduced in Table I (*b*) were obtained (extracted<sup>3</sup> from Table XXV, p. 240). From the evidence of these more restricted classes of days there was still no indication of an unvarying lag between sunspot area and magnetic range. For though in the data relating to days of largest range the fall from 40 to 4 units between days  $n$  and  $n+1$  pointed to some physical connexion, individual years showed remarkable divergences from this and other average results.

Further, detailed analysis of isolated cases, in which days selected for their abnormal range followed immediately after or were synchronous with days of no visible sunspots and *vice versa*, made it appear unlikely that any comprehensive conclusion as to the time required for the particles to pass from sun to earth could be reached from this material.

### § 3—SUNSPOT VARIABILITIES APPLIED TO FURTHER INVESTIGATIONS

(*a*) *The nature and use of day-to-day variabilities.*—While discussing the absolute ranges of declination at Kew Observatory<sup>4</sup> for the years 1901–10 it was considered advantageous to use the data to extend the examination carried out for the previous eleven years. Instead of using the projected areas of sunspots as tabulated in the Greenwich ledgers, however, the day-to-day differences of these areas were employed as the basis of the fresh investigation. That is, instead of tabulating the areas  $A_{n-1}, A_n, A_{n+1} \dots$  in the columns  $n-1, n, n+1 \dots$  corresponding with the selected disturbed (or quiet) days  $n$ , the algebraic differences of successive pairs of these areas  $A_{n-1}-A_{n-2}, A_n-A_{n-1}, A_{n+1}-A_n, \dots$  were entered. The object of this procedure was to discover whether there might not be a more intimate relation between the epoch of appearance or disappearance of sunspots and subsequent associated magnetic storms than between the latter and days on which sunspottedness was merely at a stationary maximum or minimum value.

A preliminary necessity was the tabulation of the day-to-day variabilities for each of the ten years 1901–10. The continued employment of projected areas rather than the areas corrected for foreshortening ensured that chief weight was given to the central area of the sun's disc.

(*b*) *Application to 78 days of large D range, 1901–10.*—As a basis of this re-examination the 78 days in the ten years which had an absolute range of not less than 30' were extracted and the sunspot variabilities for the six days before and one day after each of these selected days were tabulated.

In order to see how the net decreases and increases in sunspot area appeared separately, four sets of totals were formed.

1. By disregarding the individual signs of the constituent variabilities of each column, numbers representing the magnitude of the changes in sunspot area without reference to direction were obtained—the arithmetic variabilities.

<sup>3</sup> loc. cit. p. 3

<sup>4</sup> *Geophysical Memoirs*, No. 29, Vol. III. 1926.

2. By considering the entries of positive variabilities alone, *i.e.* only the occurrences of *increase* of sunspot area were recognized.
3. By considering the negative variability entries alone with the object of deriving information about the manner of *decrease* of the areas.
4. By algebraically adding the separate positive and negative variability means for each year and computing a general mean for the 35-year sequence.

The results of this investigation given in extenso in the publication cited<sup>4</sup> are reproduced in part here for convenience. The entries in Table II are in millionths of the sun's disc.

TABLE II—VARIABILITIES OF SUNSPOT AREA ON DAYS PRECEDING HIGHLY DISTURBED DAYS

	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$
Arithmetic Variabilities .. .. .	250	249	251	258	252	273	288	266
Positive Variabilities .. .. .	165	168	<b>180</b>	176	154	133	112	120
Negative Variabilities .. .. .	84	81	70	81	98	140	176	146
Mean Annual Algebraic Sums .. .. .	+81	+87	<b>+109</b>	+95	+55	-7	-64	-25

The table shows that whereas the arithmetic variability from the sixth to the second day prior to the selected disturbed day is practically steady, an increase amounting to 8 per cent of the previous average value appears on day  $n-1$  and is followed by an increase of about 15 per cent of that value on the selected day  $n$  itself.

The sets of separate positive and negative mean variabilities, however, suggest that the steady rate of change of sunspot area from day  $n-6$  to  $n-2$  is only apparent and is really a composite effect of the separate parallel variations of the two constituents. For the positive variabilities alone attain a distinct maximum at day  $n-4$  and the negative variabilities have a minimum value on the same day. The resultant algebraic sums (given in the fourth line) therefore increase from  $n-6$  to a maximum at  $n-4$ , decrease to  $n-2$  and from the first day before to the first day after the selected disturbed day, the mean change of sunspot area is in the direction of decrease.

(c) *Extension to 600 selected disturbed days.*—The extension of the investigation to the 600 days selected as the five days of greatest declination range at Kew Observatory from each month of the ten years 1901-10 produce the results given in Table XXI of the same *Memoir*. The final means for all 600 days point to  $n-5$  as the day of incidence of greatest positive variability; days  $n-6$  and  $n-5$  share the lowest mean negative variability values.

When the figures for individual years were examined, as great divergences from the mean result for the group of years appeared as in the earlier investigation. Hence before any general conclusion could be reached it seemed necessary to extend the analysis to other groups of years. This further work is now to be described.

#### § 4—SUBSEQUENT INVESTIGATIONS USING VARIABILITY STATISTICS

(a) *Extension to cover 35 years' data 1890-1924.*—In all, the 35 years 1890-1924 have now been examined. Up to and including 1910 the days selected as the basis of the analysis were the five days in each month of greatest declination range. From 1910 the international character figures assigned to each day replaced the range as a basis of selection.

The complete examination entailed the tabulation of the day-to-day sunspot projected area variabilities as defined above for the additional 25 years 1890-1900 and 1911-1924 and the tabulation of the eight consecutive values ( $n-6, n-5, \dots, n, n+1$ ) of these corresponding to each of the 60 selected days ( $n$ ) in each of the extra 25 years. Sums and means were formed for the total arithmetic variabilities and positive and negative variabilities. The former are regarded as measures of the tendency of the areas to change irrespective of the direction of the change, and the latter as measuring the separate tendencies to increase or decrease of area over the group of days associated with the magnetic disturbance. The final means for the complete set of 35 years, together with partial means for selected groups of years, are given in Table III. The entries are in millionths of the sun's disc.

TABLE III—SUNSPOT AREA VARIABILITIES ASSOCIATED WITH 35 YEARS' DISTURBANCE

	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Period Mean
<i>(a) Arithmetic Variabilities—</i>									
35 years .. .. .	166	172	172	168	166	169	166	165	160
11 S.-max. years .. .. .	93	99	98	95	93	93	91	90	88
11 S.-min. years .. .. .	14	14	13	13	13	14	13	12	13
Distribution of Maxima .. .	5½	5	7	2	4	6	1	4½	—
<i>(b) Positive Variabilities—</i>									
35 years .. .. .	91	93	92	84	82	81	76	73	80
11 S.-max. years .. .. .	153	162	164	144	141	135	125	124	141
11 S.-min. years .. .. .	23	24	22	22	18	19	18	16	20
Distribution of Maxima .. .. .	6	6½	8	2	4	3½	2	3	—
Distribution of Minima .. .. .	1	1	3	3½	5½	2½	6	12½	—
<i>(c) Negative Variabilities—</i>									
35 years .. .. .	76	79	80	84	84	89	90	91	80
11 S.-max. years .. .. .	144	152	148	159	154	161	166	163	140
11 S.-min. years .. .. .	21	21	21	21	23	25	24	23	20
Distribution of Maxima .. .. .	3	0	0	5	4½	6	8	8½	—
Distribution of Minima .. .. .	10½	3	5½	3	4	1½	3½	4	—

*(b) Discussion of arithmetic variability relations.*—The first section of the table relating to the arithmetic variabilities shows that for the complete 35 years the average tendency is to a maximum change of sunspot area about the fourth and fifth days prior to magnetic disturbance. The tendency for the visible spotted area to change falls off towards day  $n-2$  but increases again to form a secondary maximum on day  $n-1$ . The mean value of the variability on days  $n-4$  and  $n-5$  is  $7\frac{1}{2}$  per cent above the mean value for every day of the 35 years.

Of the 35 separate annual sets of means  $n-6, \dots, n+1$  which have gone to produce the final values reproduced in the first line of this section of the table, seven have their maximum on day  $n-4$ , and six on day  $n-1$ .

*(c) Results from groups of years of greater and less mean sunspot number.*—With the double purpose of testing whether these features of the final result are apparent in selected groups of years and, in addition, of testing whether there exists any indication of a displacement of the day of maximum value in years of high as compared with years of low sunspot frequency, eleven years of the highest and eleven years of the lowest Wolf-Wolfer (or mean Greenwich projected area) numbers were segregated and separate sums and means determined. These sets of eleven years are denoted respectively by S.-max. and S.-min. years and comprise 1892-5, 1905-7 and 1916-9 in the former set and 1890, 1899-1902, 1911-4 and 1922-3 in the latter group of S.-min. years.

The analysis showed that the trend of the projected sunspot area towards greatest change on the fourth and fifth days prior to the selected day was more prominent in the S.-max. than the S.-min. years. In the former group the mean

arithmetic variability reached a maximum value on day  $n-5$  of  $12\frac{1}{2}$  per cent above the mean day-to-day variability for the eleven years. In the S.-min. years the entries for days  $n-5$  and  $n-6$  both equalled that for day  $n-1$  but the common value for the three means was only one unit above the mean for every day of the group of years.

The evidence to be adduced from these summarized results for the arithmetic variabilities seems to be that on the average the fourth and fifth days prior to magnetic disturbance are days on which changes of the visibly disturbed areas on the sun are more pronounced than on any of the other days  $n-6$  to  $n+1$ . This feature, especially in respect of day  $n-5$ , is more noticeable in years of high than years of generally low solar activity.

(d) *Separate behaviour of positive and negative variabilities.*—When further information is sought from the results showing the manner of incidence of increase or decrease of disturbed solar area (Sections (b) and (c) of Table III) the above deductions are essentially confirmed. In particular, in the set of mean values of positive variabilities covering the 35 years, the greatest value appears on day  $n-5$ , the mean for the day being 16 per cent above that for all days of the 35 years. The excesses of the entry for day  $n-5$  over the entries for the two adjacent days are small compared with the difference between the average variability for these three days and that for the remaining five days in the interval examined. Further weight is given to the claims of the three earlier days and especially day  $n-4$  by the distribution of occurrences of maxima in individual years among the days  $n-6$  to  $n+1$ .

When the two classes, each of eleven years of high and low sunspot area, are formed, day  $n-4$  appears as a day of maximum value in S.-max. years and day  $n-5$  in S.-min. years. Especially in the former group is the maximum value well in excess of the average for all days of the 11 years.

The evidence of the negative variabilities representing the tendency to decrease of the sunspot area is not so definite. Over the 35 years the fifth and sixth days prior to the selected day have the smallest means, *i.e.* the sunspotted area appears to be liable to smaller decrease on these two days than on the six subsequent days. The figures showing the incidence of maxima in individual years, however, point to the fourth and fifth days before day  $n$  as being the days of least frequent incidence of maxima of negative variability.

It should be mentioned at this juncture that although days of greatest and least variability (either positive or negative) are spoken of as being themselves the days on which the tendency for sunspot areas to change is most pronounced, it is to be kept in mind that entries for any individual day are relative values dependent on the area for the day before as much as on the value of the area for the day itself. Hence a maximum of positive variability on day  $n-4$ , say, is really to be taken to mean that, in the aggregate, the increase of area from the fifth to the fourth days prior to the selected day is greater than the increase from any one of the other days to its immediate successor.

With this caveat, the results shown in the Table III, so far as they throw any light on the question for which information is being sought, may be taken to indicate that day  $n-4$  is the day about which there is the greatest tendency to increase of sunspot area in the period prior to the day selected for its magnetic disturbance. Further, this tendency appears to be most pronounced in years of greatest sunspot development.

#### §5—EXAMINATION OF RESULTS FOR INDIVIDUAL YEARS

(a) *Restriction to positive variabilities.*—The derivation of the final results presented above entailed, as previously explained, the previous construction of three separate tables giving the 35 separate yearly runs  $n-6$ ,  $n-5$ , . . . .  $n+1$  for the arithmetic, positive and negative variabilities. In addition, a fourth table giving the yearly algebraic variations of sunspot area was set up but the conclusions to be drawn from it differed little from those deducible from the other tables.

TABLE IV—MEAN DAILY VALUES OF POSITIVE VARIABILITY OF SUNSPOT AREA ON DAYS ASSOCIATED WITH MAGNETIC DISTURBANCE

Year	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Year's Mean
1890	20	<b>24</b>	16	16	<i>13</i>	20	21	<i>13</i>	24
1891	96	67	71	<i>47</i>	70	97	<b>99</b>	97	88
1892	175	175	<b>208</b>	133	143	112	102	<i>83</i>	155
1893	181	209	136	<b>219</b>	203	186	151	<i>106</i>	171
1894	154	161	<b>178</b>	<i>125</i>	153	164	169	145	153
1895	<b>131</b>	99	125	114	124	131	112	127	129
1896	92	89	<b>116</b>	111	97	77	<i>69</i>	90	89
1897	91	65	86	<i>61</i>	66	90	<b>102</b>	75	83
1898	88	105	78	97	<b>120</b>	85	89	97	66
1899	33	37	<b>42</b>	35	<i>22</i>	29	28	28	26
1900	<b>46</b>	39	18	30	33	30	22	<i>11</i>	22
1901	7	5	2	7	<b>10</b>	7	7	4	7
1902	<b>27</b>	25	21	19	16	15	14	9	16
1903	77	85	87	76	<b>92</b>	72	77	<i>55</i>	52
1904	53	71	70	59	69	<b>74</b>	<i>47</i>	60	76
1905	176	<b>210</b>	193	197	179	141	126	<i>115</i>	150
1906	<b>153</b>	121	119	93	81	88	72	79	105
1907	<i>140</i>	164	175	168	201	<b>202</b>	171	183	137
1908	<b>119</b>	109	<b>119</b>	98	<i>84</i>	101	112	99	103
1909	143	<b>153</b>	104	102	115	132	138	93	106
1910	57	37	52	51	<b>77</b>	67	39	<i>35</i>	46
1911	<b>14</b>	9	9	4	7	5	2	6	15
1912	<b>21</b>	20	13	9	<i>1</i>	8	14	10	11
1913	4	7	6	4	5	4	2	<i>1</i>	3
1914	22	19	23	<b>24</b>	23	17	14	8	32
1915	113	<b>154</b>	95	106	119	134	103	88	99
1916	149	125	153	136	114	<b>100</b>	139	<b>155</b>	113
1917	218	<b>237</b>	<b>237</b>	205	196	184	137	149	183
1918	100	153	<b>174</b>	127	<i>96</i>	116	116	142	127
1919	107	<b>131</b>	104	<i>64</i>	65	<i>64</i>	75	78	131
1920	159	143	<b>170</b>	166	105	74	67	90	103
1921	94	99	<b>104</b>	84	71	<i>65</i>	98	81	70
1922	46	55	76	74	<i>40</i>	51	59	<b>78</b>	50
1923	18	19	11	15	20	<b>25</b>	12	6	14
1924	48	51	36	54	<i>46</i>	58	55	<b>61</b>	49
35 years..	91	<b>93</b>	92	84	82	81	76	73	80

Table IV for the positive variabilities alone is shown as exemplifying the nature and degree of the divergences of individual years from the mean result for the complete period. The entries in each of the columns  $n-6, \dots, n+1$  represent for each year the mean net increase of sunspot area from the preceding day. The final column contains the average daily increase of area for all days of each year. The maximum value in each year is printed in heavy type, the least value in italics.

(b) *Tendency for years of similar variation to group.*—Inspection of the distribution of the maxima among the eight days suggests a tendency to concentrate towards days  $n-4$  and  $n-5$  in some groups of years and to be widely scattered among the other days in the intervening years. For example, in the years 1892, '94, '96 and '99 the greatest value appears in column  $n-4$ , but from 1900 to '12 only one year, 1908, has a maximum in that column and two years in column  $n-5$ . Again, except for 1914, in which year the maximum mean variability fell on day  $n-3$  and 1916 when the maximum occurred on day  $n+1$  though with an almost equal maximum on day  $n-4$ , every year from 1913 to 1921 inclusive attained the highest value on either the fourth or fifth day prior to that selected for its magnetic disturbance.

(c) *Results derived by grouping years of similar behaviour.*—This tendency for years of similar distribution of the maximum value among the days preceding the selected day is further illustrated in Table V. To form this table those years in which the maximum fell on one or other of the two days  $n-4$  or  $n-5$  have been grouped and separate means extracted for them and for the remaining years in which the day of incidence was distributed over the other six days. Seventeen years were included in the first group (A) and 18 in the second group (B).

TABLE V—POSITIVE VARIABILITIES OF SUNSPOT AREAS IN TWO SELECTED GROUPS OF YEARS

	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Group Mean
Group A ..	112	118	120	107	101	96	93	91	102
Group B ..	70	70	66	62	64	66	60	57	59

The series of means for group A makes it apparent that the incidence of maxima round day  $n-4$  is not merely a fortuitous concentration of isolated high values in the sequence of variabilities, but is the result of a real disposition to the formation at day  $n-4$  of a crest in the rise and fall of variabilities from the first to the last day examined. It is worthy of further note that the average value of the positive variability on day  $n-4$  in this selected group of years is some 20 per cent above the mean value for all days in the 17 years.

The figures for the remaining 18 years show no corresponding regularity in their variation from day  $n-6$  to  $n+1$ . The entries under days  $n-6$  and  $n-5$  are both about 16 per cent above the mean for the average day of the group of years, but instead of falling gradually towards the selected day  $n$  as in the previously discussed group of years there appears a tendency to formation of another maximum about day  $n-1$ .

It is to be noted that these results are not independent of the results derived by grouping years according to their sunspot development, for eight of the eleven years selected because of their high mean sunspottedness are included in the group of 17 years selected as being rich in maxima on days  $n-4$  and  $n-5$ . The same number of years from the S.-min. group are included among the 18 years of scattered maxima.

Although there are examples among the years of relatively good sunspot development, when the formation of a well-defined positive variability crest on one or other of the days previous to the selected day of magnetic disturbance is almost lacking, irregularity in the variations is more common in years of low sunspot frequency. The entries in Table VI are designed to illustrate this point.

TABLE VI—DISTRIBUTION OF 35 YEARS ACCORDING TO BEHAVIOUR OF POSITIVE VARIABILITIES OF SUNSPOT AREA ON DAYS PRIOR TO MAGNETIC DISTURBANCE

Years with							
Tendency to definite Maximum on			Tendency to definite Minimum on			Linear Run of Entries	No Characteristic
$n-4$	$n-3$ or $n-5$	Other days	$n-4$	$n-3$ or $n-5$	Other days		
1892	1893	—	—	1891	—	—	1890
1894	1896	1898	—	1897	—	—	1885
1899	—	1901	—	—	—	—	1900
—	—	1903	—	—	—	1902	—
—	1905	1907	—	—	—	1906	1904
1908	1909	1910	—	—	1912	1911	—
—	1913	—	—	—	—	—	—
—	1914	—	—	—	—	—	—
1916	1915	—	—	—	—	—	1916
1917	—	—	—	—	—	—	—
1918	1919	—	—	—	—	—	—
1920	—	—	—	—	—	—	—
1921	—	—	1923	—	—	—	—
1922	—	—	1924	—	—	—	1922
9	8		2	2	1	3	5

The table shows the distribution of the 35 years according to the character of the variation presented by the sequence of positive variability means in the period of eight days  $n-6, n-5, \dots, n, n+1$ . In the first two columns those years are noted which showed a fairly well defined crest on the fourth day or third or fifth days respectively prior to the selected day  $n$ . Years with maxima on days  $n-1$  or  $n-2$  are given in column 3. Where any set of yearly means were disposed to form a trough at some day previous to day  $n$ , the year was entered in columns 4, 5 or 6: no indication of a tendency to formation of crest or trough but rather a gradual rise or fall from day  $n-6$  to  $n+1$  resulted in the year being entered in column 7, and other years with no conspicuous characteristic tendencies were assigned to column 8. Dubious cases such as 1916 and 1922 might be put half in one column and half in another.

The resulting distribution as portrayed by Table VI shows that approximately 50 per cent of the 35 years have a maximum variability in one of the three days  $n-5, n-4$  or  $n-3$ . Only five of the 35 years have definite turning points at other days. Of the mainly indifferent years, only 1895 and 1916 belong to the group of years of good sunspot development; the others are years of relatively small sunspot frequency.

#### § 6—CONCLUSIONS FROM VARIABILITY DATA FOR 35 YEARS

Summarizing this part of the investigation which has dealt with the variabilities of sunspot area for the 35 years 1890–1924, it might be concluded:—

1. That there is a well marked tendency for the increase of sunspot area from the fifth to the fourth day prior to magnetic disturbance to exceed the increase on any other day of the group from six days before to one day after the selected day of magnetic disturbance.
2. This tendency is more pronounced in some years than in others, years of high sunspot frequency being on the whole more favourable to the phenomena than years of low sunspot frequency.
3. The results for positive variabilities indicate that the day of greatest increase of sunspot area tends to advance from the fourth to the fifth day prior to the selected disturbed day, in passing from years of high to years of low sunspot development.

#### § 7—SUNSPOT VARIATIONS ASSOCIATED WITH HIGH MAGNETIC DISTURBANCE

(a) *Difficulties underlying a general treatment; advantages of further selection of days.*—At this stage it will be seen that although the mean results from the treatment of the 35 years were sufficiently definite to indicate some measure of physical connexion between the solar and magnetic phenomena, the results from separate groups of years and the divergences of individual years within those groups demanded further examination. Indeed, even from the figures so far discussed, it seems unlikely that any definite and really comprehensive statement of the behaviour of sunspot and magnetic disturbance which will be valid for all cases can be made. For it is probable that some intermediate mechanism, such as variations in the ionization of the conducting layer(s) in the upper atmosphere of the earth, plays a large part in introducing a variable lag into the interval separating the appearance of an active sunspot from the subsequent magnetic storm with which it may be associated. Further, it may frequently be left for a visually insignificant spotted area to make the last contribution towards the production of a result for which the stage has already been prepared by its larger precursors.

Even though the period covered amounts to 35 years, material derived from treatment of five days per month must include many days of relatively small disturbance. The effect of the inclusion of these days can only serve to dilute the results from days of really large disturbance. Since, however, international magnetic

character figures have been allotted to each day since 1906, it was considered that by use of these a more restricted selection of days of recognized disturbance over the earth would ensure that the analysis rest on a more definite basis.

(b) *Use of 366 days of international character figure not less than 1.5.*—In order that the days selected might be at once days of universal and relatively great magnetic disturbance, those days of international character figure not less than 1.5 in the 20 years 1906-25 were extracted from the De Bilt lists. These, totalling 366, were tabulated in columns headed  $n$ . The algebraic sunspot variabilities were then entered in the appropriate columns  $n-6, n-5 \dots n+1$  corresponding with each selected day; separate sums and means of positive and negative variabilities were formed for individual years, for the 20 years together and for the selected groups of years. These results and the number of days in each year which have contributed to the year's means are given in Tables VII and VIII. The entries are in millionths of the sun's disc. The final columns in the two tables contain the mean variabilities, positive and negative respectively, for each year together with mean annual Wolf-Wolfer numbers in Table VII.

TABLE VII—POSITIVE VARIABILITIES OF SUNSPOT AREA ASSOCIATED WITH HIGHLY DISTURBED DAYS

Year	No. of days used	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Mean Variability for Year	Wolf-Wolfer Number
1906	20	102	57	85	45	53	<b>181</b>	103	64	105	53.8
1907	16	116	202	245	276	229	251	287	<b>387</b>	137	62.0
1908	17	<b>234</b>	171	156	63	38	48	76	139	103	48.5
1909	19	<b>209</b>	191	177	162	165	182	145	50	106	43.9
1910	16	86	25	71	31	<b>119</b>	54	55	49	46	18.6
1911	18	10	<b>11</b>	6	2	1	2	<b>3</b>	3	15	5.7
1912	5	26	20	5	16	6	20	<b>33</b>	31	11	3.6
1913	4	0	0	<b>19</b>	0	0	15	1	0	3	1.4
1914	8	39	36	18	20	35	<b>62</b>	14	5	32	9.6
1915	23	121	<b>133</b>	64	82	113	131	105	74	99	47.4
1916	22	175	173	180	<b>212</b>	89	49	67	139	113	55.4
1917	27	388	383	<b>391</b>	311	231	173	111	50	183	103.9
1918	30	101	166	<b>171</b>	141	108	130	151	157	127	80.6
1919	31	103	122	<b>123</b>	87	93	67	62	65	131	63.6
1920	19	256	216	243	<b>276</b>	241	156	95	91	103	37.6
1921	16	<b>199</b>	174	193	141	124	66	69	40	70	26.1
1922	15	61	154	<b>186</b>	148	42	15	9	9	50	14.2
1923	13	19	23	8	13	<b>25</b>	21	9	2	14	5.8
1924	21	<b>75</b>	59	48	58	29	20	22	25	49	16.7
1925	26	157	208	189	<b>213</b>	185	145	119	110	114	44.6
20 years ..	366	124	126	<b>129</b>	115	96	89	77	75	81	37.1
11 yrs. High W.-W. No.		178	<b>184</b>	<b>184</b>	170	140	138	120	121	120	58.3
9 yrs. Low W.-W. No.		57	56	<b>62</b>	48	42	31	24	18	32	11.3
6 yrs. Highest Number.		164	184	<b>197</b>	179	134	142	130	144	133	69.9
6 yrs. Lowest Number.		26	<b>41</b>	40	33	18	23	11	8	21	6.7

(c) *Behaviour of positive variabilities.*—Examination of the sequence of means of positive variabilities for the entire 20 years in the lower section of Table VII shows that day  $n-4$  is the day of incidence of greatest variability. For this comparatively restricted class of consistently great and world-wide disturbance, the greatest increase in sunspot area prior to the disturbed day therefore apparently takes place between the fifth and the fourth days preceding the selected day. The mean positive variability on day  $n-4$  is some 60 per cent above the mean for all days of the 20 years and the excess is little less for the adjacent days  $n-6$  and  $n-5$ .

The 20 years were divided into two classes according to the value of the Wolf-Wolfer sunspot frequency number, the 11 years 1906-09, 1915-20, and 1925 falling into the group of high frequency with a mean of 58.3 and the remaining 9 years forming the low frequency group with a mean of 11.3. The mean variabilities derived from each group are given in the third and fourth lines at the bottom of Table VII. They show that the prominence of day  $n-4$  as the day of greatest entry still remains unchallenged. In the group of 11 years, however, day  $n-5$  shares the maximum with  $n-4$ , both days having a value 51 per cent above the mean for all days of the group. The crest at  $n-4$  in the 9 years of low sunspot development is also well defined.

If a still more rigorous selection is made by taking only the six years of greatest and six years of least sunspot frequency, the means given in the last two lines of Table VII are obtained. The sets of means for the years 1906-7 and 1916-19 have gone to form the first line and those for 1911-14 and 1922-23 the second. In both of these restricted groups a prominent maximum in the variability figures occurs about day  $n-4$ . In the case of the group of highest sunspot frequency the maximum actually falls on day  $n-4$  with the entry for  $n-5$  next in magnitude. For the other group of years the positions of the two days are interchanged though there is only one unit difference between them.

(d) *Negative sunspot variabilities associated with high disturbance.*—Inspection of the negative variabilities in Table VIII allows similar though less well-defined conclusions to be drawn.

TABLE VIII—NEGATIVE VARIABILITIES OF SUNSPOT AREA ASSOCIATED WITH HIGHLY DISTURBED DAYS

Year	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Mean Variability for Year
1906	89	108	208	243	207	145	148	102	106
1907	140	186	141	37	84	128	121	135	142
1908	183	119	58	95	234	197	222	358	99
1909	111	48	68	113	121	175	129	200	107
1910	43	36	67	53	77	79	105	83	51
1911	28	24	18	30	38	15	9	7	15
1912	10	53	82	40	12	4	16	1	11
1913	0	0	0	1	14	4	1	9	3
1914	25	41	24	12	4	3	8	60	32
1915	54	39	66	80	119	73	107	75	95
1916	76	148	103	88	154	172	181	113	107
1917	159	131	236	251	276	313	328	365	167
1918	159	127	79	151	201	163	94	142	137
1919	139	134	123	150	167	170	139	138	123
1920	60	43	46	50	43	64	169	190	101
1921	35	98	84	66	104	127	125	163	72
1922	72	62	70	53	113	190	173	159	46
1923	3	1	2	4	16	18	11	13	19
1924	29	61	95	152	136	82	88	57	49
1925	70	23	46	42	73	135	171	182	100
20 years ..	74	74	81	86	110	113	117	128	79
11 years high W.-W. number.	113	101	107	118	153	158	164	182	117
9 years low W.-W. number.	26	42	49	46	56	58	59	61	47

On both the fifth and sixth days before the day  $n$  selected for its marked disturbance the mean negative variabilities are less than those of any of the six subsequent days and some 6 per cent less than the mean value of the quantity for the 20 years. The effect of grouping the years into the two classes comprising 11 years of higher and 9 years of lower mean sunspot frequency is to put the minimum value more definitely at day  $n-5$  in the former class but to leave the sixth day the day of

least entry for the years of low sunspot frequency. The sequence of variability means for the days  $n-6$  to  $n+1$  in this last group increases regularly from the first to the last with the exception of day  $n-3$  whose value is just less than that for day  $n-4$ .

It would therefore be reasonable to conclude that the results for this class of highly disturbed days confirm those from the more extensive data for the 35 years. This is especially true for the positive variabilities which, both for the complete 20 years and for selected groups within the period, indicate that the tendency for the sunspot areas to change is most pronounced on days  $n-4$  and  $n-5$ .

§ 8—ANALYSIS APPLIED TO 355 QUIET DAYS

(a) *Selection of quiet days.*—The nature of the investigation makes it advisable to collect evidence from as many independent sources as possible. It is therefore of interest to examine the behaviour of sunspot variabilities on days associated with selected quiet as well as disturbed magnetic conditions. For it might be expected as a corollary that if there were a real significance in the four or five day lag between sunspot appearance and subsequent magnetic disturbance, a similar interval should separate days of sunspot decay or disappearance and subsequent days of magnetic calm.

To test this, the 355 days ( $n$ ) of international magnetic character figure 0.0 in the 20 years 1906–25 were extracted from the De Bilt lists and the sunspot variabilities for the associated days  $n-6, \dots, n+1$  were tabulated as in the earlier parts of the examination. Separate positive and negative variability sums and means as well as their algebraic sums and means for each of the 20 years were formed, and from these 20-year and selected group means were set up. Tables IX and X contain the relevant material from the positive and negative variabilities.

TABLE IX—POSITIVE VARIABILITIES OF SUNSPOT AREA ASSOCIATED WITH QUIET DAYS

Year	No. of Days used	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Mean Variability for Year
1906	11	142	57	39	92	159	157	208	121	105
1907	6	82	111	156	203	258	228	245	260	137
1908	6	19	7	7	73	60	59	22	29	103
1909	13	47	85	114	193	69	84	48	121	106
1910	13	15	60	37	7	12	47	89	104	46
1911	16	3	2	2	6	35	19	23	21	15
1912	46	9	7	8	9	15	11	5	8	11
1913	36	1	3	6	7	1	4	6	5	3
1914	36	50	25	26	52	70	44	34	44	32
1915	34	83	72	73	73	43	46	57	71	99
1916	20	171	143	140	80	59	37	26	57	113
1917	22	283	322	257	181	226	194	147	212	183
1918	18	201	104	205	164	130	113	123	130	127
1919	19	155	87	127	146	136	92	119	111	131
1920	20	62	95	112	200	180	36	21	19	103
1921	16	99	55	49	81	55	10	22	72	70
1922	23	29	9	20	15	61	67	29	39	50
1923	27	20	19	16	17	11	23	21	22	14
1924	43	28	27	52	69	58	29	16	44	49
1925	47	137	104	99	125	91	175	117	76	114
20 years ..	355	82	70	77	90	86	74	69	78	81
11 years High W.-W. Number.		126	108	121	139	128	111	103	110	120
9 years Low W.-W. Number ..		28	23	24	29	35	28	27	40	32

(b) *Positive and negative variabilities on quiet days.*—Confining attention first to the positive variability changes it is seen that day  $n$  is itself the day of minimum value of the quantity but that the value for day  $n-5$  is only one unit greater. Both

values are approximately  $12\frac{1}{2}$  per cent less than the mean positive variability for the complete 20 years. In this respect that the selected quiet day itself shows the same tendency to a minimum increase of sunspot area as is shown by the previous fourth and fifth days, the selected disturbed and quiet days behave diversely. But the decrease from day  $n-6$  to  $n-5$  with subsequent rise to  $n-3$  is as well defined as the contrary tendency on the same days prior to disturbance.

The means for the two groups of years obtained by grouping the eleven years of high and nine years of low sunspot development (as enumerated above) show similar tendencies. In the former group the value for day  $n-5$  is 10 per cent below the mean for all days of the eleven years and in the years of low solar activity the decrease from  $n-6$  to  $n-5$  is conspicuous. In the means for both groups of years the fall to a secondary minimum at day  $n$  is a prominent feature.

TABLE X—NEGATIVE VARIABILITIES OF SUNSPOT AREA ASSOCIATED WITH SELECTED QUIET DAYS

Year	$n-6$	$n-5$	$n-4$	$n-3$	$n-2$	$n-1$	$n$	$n+1$	Mean Variability for Year
1906	78	59	81	96	<b>119</b>	<b>119</b>	93	33	106
1907	117	177	158	20	25	126	349	<b>357</b>	142
1908	66	25	50	13	42	<b>80</b>	53	63	99
1909	142	<b>223</b>	217	74	81	70	93	123	107
1910	36	30	49	<b>91</b>	55	19	11	12	51
1911	4	6	10	13	12	11	<b>18</b>	13	15
1912	<b>24</b>	13	5	7	7	6	11	16	11
1913	3	<b>6</b>	<b>6</b>	4	3	<b>6</b>	3	2	3
1914	40	34	33	35	42	36	<b>44</b>	37	32
1915	110	143	106	130	134	<b>156</b>	112	130	95
1916	49	83	96	<b>245</b>	151	214	145	116	107
1917	122	182	168	170	203	187	<b>266</b>	234	167
1918	97	109	<b>134</b>	130	93	110	127	126	137
1919	93	100	76	27	88	115	<b>140</b>	120	123
1920	50	94	70	63	85	103	<b>119</b>	100	101
1921	130	88	98	78	99	<b>157</b>	115	77	72
1922	24	21	11	15	20	23	<b>42</b>	40	46
1923	11	13	19	19	<b>24</b>	21	19	13	19
1924	46	<b>63</b>	46	34	21	40	43	45	49
1925	125	129	112	127	83	118	<b>158</b>	144	100
20 years ..	68	80	77	75	69	86	<b>98</b>	90	79
Distribution of Maxima ..	1	2 $\frac{1}{2}$	1 $\frac{1}{2}$	2	1 $\frac{1}{2}$	3	7	1	—
Alg. Means $\pm$ Vars.	+14	-10	0	+15	+17	-12	-29	-12	—

Examination of the results from the negative variabilities shows that both days  $n-5$  and  $n$  have greater decreases of sunspot area than the adjacent days, but while the mean for the selected quiet day  $n$  is some 25 per cent above the normal for the 20 years, that for day  $n-5$  is little over 1 per cent. Further evidence from the frequency of incidence of maximum on each of the eight days  $n-6$  to  $n+1$  in individual years, as shown in Table X, supports the claim of day  $n$  to pre-eminence. For seven of the 20 separate maxima fall on this day and between them days  $n$  and  $n-1$  alone have over 50 per cent of the total number.

The set of algebraic means inserted as the last line of Table X emphasises the combined results from positive and negative variabilities. Of the five days  $n-6$  to  $n-2$ , day  $n-5$  is the only day of net decrease of spotted area but the decrease on this day is only about 30 per cent of that on day  $n$  itself.

(c) *Inferences from quiet day data.*—The principal deductions to be made from the study of these data for quiet days would therefore seem to be that though a tendency exists for a net decrease of sunspot area on the fifth day and, to a less extent

on the fourth day, prior to magnetically quiet conditions, this tendency is more pronounced just previous to and on the selected quiet day itself. But the tables of results for individual years show that the divergences from this average tendency are just as large and as frequent as in the case of disturbance. This is especially true of years of relatively poor sunspot development.

§ 9—RESULTS FROM THE USE OF ACTUAL SUNSPOT AREAS

As explained earlier, the general result from the data relating to magnetic disturbances, *viz.* that the fourth day prior to the incidence of disturbance is on the average the day of greatest positive and least negative variability, was taken to mean that the tendency for the visibly disturbed areas on the sun to increase was more pronounced between the fifth and fourth days prior to magnetic disturbance than on any other of the days covered by the analysis.

Since the main part of the work here described was done, however, at least one other investigator has published the results of examinations carried out on similar lines. Employing Wolf-Wolfer sunspot numbers and material from the observatories of Parc Saint Maur and Val-Joyeux covering a period of 41 years, Prof. Ch. Maurain<sup>5</sup> has arrived at conclusions which differ in some details from those reached here. On the average year, according to Maurain's work, the maximum of sunspot area occurs some 2½ or 3 days prior to the day of magnetic disturbance.

The difference in the nature of the material used as a basis for the two investigations precludes direct comparison of the results arrived at by Prof. Maurain and those discussed here. The difficulty can be partly overcome, however, if that part of the work of this paper dealing with the disturbed days of international character not less than 1.5 is repeated using the projected areas of sunspots as tabulated in the Greenwich ledgers instead of their day-to-day differences. This has been done; Table XI shows the results obtained for each year separately, and also for the complete 20 years, groups being selected according to the average sunspottedness as detailed above.

TABLE XI—SUNSPOT PROJECTED AREAS ON DAYS ASSOCIATED WITH MAGNETIC DISTURBANCE

Year	Days Used	n-6	n-5	n-4	n-3	n-2	n-1	n	n+1
1906	20	1558	1457	1334	1136	987	1022	977	939
1907	16	1634	1650	1771	2017	2162	2284	2450	2702
1908	17	1721	1773	1872	1840	1645	1496	1351	1133
1909	19	1359	1502	1612	1661	1706	1713	1729	1578
1910	16	568	557	561	539	581	555	505	471
1911	18	148	136	124	96	58	45	39	35
1912	5	180	147	70	46	41	57	74	103
1913	4	0	0	19	18	4	15	15	7
1914	8	119	114	107	115	146	205	212	156
1915	23	796	876	874	877	871	928	927	926
1916	22	1086	1111	1188	1313	1249	1126	1012	1038
1917	27	3112	3360	3516	3576	3531	3391	3174	2858
1918	30	1445	1489	1578	1568	1505	1473	1529	1578
1919	31	1620	1608	1621	1558	1485	1383	1306	1233
1920	19	888	1061	1259	1485	1684	1776	1702	1583
1921	16	941	1017	1125	1199	1219	1155	1096	973
1922	15	704	796	913	1009	937	762	599	448
1923	13	34	57	63	73	82	85	83	72
1924	21	727	725	678	584	477	415	349	317
1925	26	1160	1346	1489	1660	1769	1736	1718	1642
20 years .. ..	366	990	1039	1089	1119	1107	1081	1042	990
11 years High W.-W. Number .. ..	250	1489	1567	1647	1699	1690	1666	1625	1565
9 years Low W.-W. Number .. ..	116	380	394	407	409	304	366	330	287
6 S.-max Years ..	146	1743	1779	1835	1861	1820	1780	1741	1725
6 S.-min. Years ..	63	197	208	216	226	211	195	170	137

<sup>5</sup> Paris, *Ann. Inst. Phys. Globe*, 8 (1927) pp. 86-108.

As was to be anticipated, comparison of this table with the corresponding table for positive variabilities shows that there exists no relation between the two sets of results which could have been previously forecasted. Not only do the individual days of maxima differ but the divergence of the means taken over the 20 years amounts to at least one day, the maximum for the projected areas falling approximately  $2\frac{3}{4}$  days and that for the variabilities 4 days before the selected day  $n$ . The means for the separate groups of years point to the same tendency to increase of the interval separating sunspots and disturbance with diminished solar activity as noted by Prof. Maurain.

Hence, while the results deduced from the use of 20 years' highly disturbed days agree with those based on entirely different material and covering a longer period, their divergence from the earlier variability results indicate that no fixed relation can be assumed to exist between the statistical means for changes of sunspottedness and the means of the sunspot areas themselves.

#### § 10—FINAL REMARKS

But other considerations would suggest that it is the interval between the day of *change* of sunspot area and the subsequent magnetically disturbed or quiet conditions that is the best measure of the time of passage of the electrical particles from sun to earth, rather than the time that elapses between a *stationary* condition of maximum or minimum and the magnetic results that may be associated with it. If this be correct the conclusions arrived at from the treatment of the sunspot *variabilities* (§ 4 to § 7 above) rather than those concerning the *direct values* of sunspot measures are those to be considered in relation to theoretically derived values of the time required for charged particles to reach the earth from locally disturbed solar areas.