

PRINCIPAL COMPONENT ANALYSIS - USE IN EVAPORATION
STUDIES

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INTRODUCTION

The method and objectives of Principal Component Analysis have been described thoroughly by Craddock (1968).

The method is very useful for studying relationships between several variables, even when we have few previous ideas about what these relationships are likely to be. It must be emphasised however that the method is purely mathematical; it is desirable at all stages in the analysis to look at the results and try to interpret them in a physically meaningful way, as otherwise we may be led to absurd conclusions.

The method may be applied to any assorted set of intercorrelated variables. In meteorology the set of variables used has often been the set of values of pressure or some other element taken at several points on a chart. In such an analysis all the variables have the same character, thus simplifying some matters such as the scaling of the variables. In the present application we shall be considering the use of variables measured in different units.

Principal component analysis can be performed (Craddock 1968) on:

- (1) the SSP matrix (raw data)
- (2) the covariance matrix (data transformed to anomalies relative to true means)
- (3) a modified covariance matrix (data transformed to anomalies relative to a set of artificial means)
- (4) the correlation matrix (data transformed to anomalies then scaled down so that all variables have the same variance).

In the present work we have used methods (2) and (4). The program used is detailed in Memo 10.

EXPERIMENTAL ANALYSIS

The present analysis was carried out on four variables, namely:

- (1) Penman estimate of evaporation using net radiation (PEN)
- (2) Piche' evaporimeter ($\frac{1}{2}$ metre height) measure of evaporation (Pi)
- (3) British pan (New site) measure of evaporation (Bn)
- (4) 3000 pan (West) measure of evaporation (3w)

Each variable was obtained daily at Kew during the period 1st June to 30th November 1968, with 7 dates omitted. The dates used are shown in Table 1.

The following analyses were performed:

- (A) Using the dates in odd-numbered columns in Table 1 (88 items).
- (B) Using the dates in even-numbered columns in Table 1 (88 items).
- (C) Using the first 90 dates.
- (D) Using the last 90 dates.
- (E) Using all 176 items.

We shall describe the results of (A) and (B).

Figure A1 (left) shows the mean and standard deviation of each variable for analysis A. The distributions are in fact different from Normal, with a few large values and many small values, so these may not be very meaningful statistics. The most significant result is that Piché has a higher mean and a higher standard deviation than the others. Thus, if it is sensible to regard all four as "measures of evaporation", then effectively Piché is measuring in different units. The authors have sometimes gained the impression, rightly or wrongly, that workers criticise Piché (and other methods of measuring evaporation) because the value given by Piché is very different from that given by, say, a lysimeter. But for Piché to be useful, what matters is not that the actual value is numerically close to the correct value, but that it is correlated with the correct value.

Results were produced by analysing first the covariance matrix, then the correlation matrix. Figure A1 (right) shows that in both methods the first eigenvector accounted for over 90% of the variance. Figure A2 shows the eigenvectors according to the two methods. Each eigenvector consists of four numbers, the lines joining these numbers having no significance.

Consider the covariance matrix first. Eigenvector (EV)1 shows each variable roughly in proportion to its SD; in other words, the variables agree except for scale. It seems reasonable, at least provisionally, to regard the coefficient (C1) of EV 1 as "the evaporation" on a given day. At any rate, it represents that part of the evaporation to which all four methods respond.

The other EVs show when one or more of the measures differed from the others. EV 2 indicates occasions when P1 differed from the other three. EV3 shows PEN differing from the pans, while EV 4 indicates variations between the two pans. The sequence of EVs shows that Piche was the "most individual" in its behaviour, while the two pans were the most consistent pair. The latter, at least, is to be expected.

We get a somewhat different picture if we use the EVs from the correlation matrix. EV1 is the same as before, but all the values are almost equal because the scales have been reduced. However, the reduction in the magnitude of variations in P1 shows up in EV2; the measure displaying most variance relative to the others is now PEN. EV3 then deals with occasions when P1 differs from the pans, and EV4 is unchanged.

Results of analysis B are shown in Figures B1 and B2. EV 1 and EV 4 are in both methods the same as in analysis A. However EV2 and EV3 are substantially different, suggesting that we should not place much reliance on conclusions drawn from them.

Figures C2, D2 and E2 show the eigenvectors from analyses C, D and E. Note the differences in EV2 and EV3.

In what follows, the EVs from the correlation matrix will be used.

COMPARISONS OF THE FOUR MEASUREMENTS

Suppose in analysis A that C2 was large and positive on a particular day. Then Penman was reading relatively high and the three evaporimeters were reading relatively low. This would imply one of the following:

- (a) Penman was in error on the high side, the evaporimeters being correct
- (b) The evaporimeters were all reading too low, and Penman was correct.

The method cannot determine which; although, since evaporimeters of two very different characters are involved, we would be inclined to think that (a) is generally the more likely. Conversely, any days on which (a) is true will have positive values of C2. At least, a study of those days on which C2 is numerically large could help us to understand the circumstances under which Penman differs from the evaporimeters. Similar remarks may be made about large values of C3 and C4.

TYPES OF DAYS

For analysis A the days were divided into types 1, 2+, 2-, 3+, 3-, 4+, 4- and hybrid, depending on the coefficients C2, C3 and C4. A day was called 2+ if $C2 \geq 0.5$, 3- if $C3 \leq -0.5$, etc. The value 0.5 was taken as the "qualifying value" for all three coefficients. A day was called 1 if all three coefficients were numerically ≤ 0.4 . Hybrid days were ones on which more than one coefficient was numerically ≥ 0.5 , and these were labelled as being members of two or three types.

For analysis B the same procedure was adopted, the type numbers being put in brackets. The characteristics of the types are summarised below:

1 and (1)	Penman estimate and evaporimeter readings agree				
2	PEN	v	Pi	Bn	3w
(2)	PEN Pi	v	Bn	3w	
3	Pi	v	Bn	3w	
(3)	PEN	v	Pi		
4 and (4)	Bn	v	3w		

(Measures on the same side of "v" agree but differ from opposing measures on the same line.)

For the purposes of this preliminary examination, meteorological variables readily available from records were tabulated as follows:

- Total amount of rainfall from midnight to midnight.
- Total duration of rainfall from midnight to midnight.
- Daily range of relative humidity (per cent).
- Total duration of bright sunshine.
- Day maximum temperature.
- Daily mean wind speed.

Tables 3 to 8 were prepared, showing the distribution of day types with respect to ranges of values of the chosen meteorological variables; for each range of each variable, the number of days and the percentage of all days of that type are given. Comparisons between percentages of types (and of "sub-types") and percentages of all days, regardless of type, corresponding to given ranges of meteorological variables give an idea of the importance of the distributions of values. Table 9 gives preliminary deductions based on the method of comparison of percentages for individual types and groups of types.

Scatter diagrams (available on request) were plotted giving the distribution of types against maximum temperature and mean wind speed. Broadly speaking, these showed that Penman estimates and evaporimeter readings agree at very low wind speeds and when low temperatures and lower wind speeds occur together.

CONCLUDING REMARKS

The deductions summarised in Table IX must be regarded as preliminary, because of the small data sample. The analysis would be worth repeating for a much longer data series. The "instability" of the second and third eigenvectors (with respect to independent data sets) suggests that no dominant coalition occurs between any two of: PEN; Pi; the pans.

It might be instructive to do a Principal Component Analysis on a larger number of variables, including other evaporimeters and also some of the weather variables discussed in this memo. This should highlight any associations there might be between the variables, and should indicate which measures of evaporation react to which weather variables. Difficulties would arise due to the fact that the variables would be in unrelated units, and it would not be obvious how many knots, for example, are equivalent to 10 mm rainfall. This means that method (4) (correlation matrix) would probably be the best to use.

ACKNOWLEDGEMENTS

Thanks are due to Mr. J. M. Craddock and Mr. C. R. Flood for advice.

NOTE

This memo is circulated for discussion purposes only. Comments are welcome and should be sent to the authors.

Met 0 8
Bracknell
November 1969

REFERENCES

CRADDOCK, J. M.

The use of eigenvector analysis in statistical meteorology.
Met 0 13 Branch Memo No. 27 1968.

TABLE 9 - DEDUCTIONS FROM TABLES 3-8

Penman agrees with Piche	With larger rainfalls and long duration. On warm days. With mean wind speeds roughly 6-10 kt.
Penman differs from Piche	On sunny days. On days with a large range of R.H. On dry or almost dry days - (perhaps).
Penman differs from Bn and 3w	With large rainfalls. With prolonged rainfall - (perhaps) On very warm days - (perhaps) With mean wind speeds roughly 6-10 kt.
Piche differs from Bn and 3w	On rainy days. On very warm days. With mean wind speeds roughly 6-10 kts.
Piche agrees with Bn and 3w	On partly sunny days - (perhaps). With mean wind speed < 11 kt - (perhaps). On dry days.
Bn differs from 3w	On dry or almost dry days - (perhaps) On days with a fairly large range of RH. On days with light winds.
Bn agrees with 3w	With mean wind speeds roughly 6-10 kt.

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO RAINFALL TOTALS

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO RAINFALL DURATION

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO DAILY RANGE OF RELATIVE
HUMIDITY

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO BRIGHT SUNSHINE

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO DAY MAXIMUM TEMPERATURE

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO
MEAN WIND SPEED

TABLE 1.

176 days used in Principal Component Analysis. 1968.

Column number

	1	2	3	4	5	6	7	8	9	10
Jun 1	Jun 2	Jun 3	Jun 4	Jun 5	Jun 6	Jun 7	Jun 8	Jun 9	Jun 10	
" 11	" 12	" 13	" 14	" 15	" 16	" 17	" 18	" 19	" 20	
" 21	" 22	" 23	" 24	" 25	" 26	" 27	" 28	" 29	" 30	
Jul 1	Jul 2	Jul 3	Jul 4	Jul 5	Jul 6	Jul 7	Jul 8	Jul 9	Jul 10	
" 11	" 12	" 13	" 14	" 15	" 16	" 17	" 18	" 19	" 20	
" 21	" 22	" 23	" 24	" 25	" 26	" 27	" 28	" 29	" 30	
" 31	Aug 1	A 2	A 3	A 4	A 5	A 6	A 7	A 8	A 9	
Aug 10	" 11	" 12	" 13	" 14	" 15	" 16	" 17	" 18	" 19	
" 20	" 21	" 22	" 23	" 24	" 25	" 26	" 27	" 28	" 29	
" 30	" 31	S 1	S 2	S 3	S 4	S 5	S 6	S 7	S 8	
Sep 9	" 10	" 11	" 12	" 13	" 14	" 16	" 17	" 18	" 19	
" 20	" 22	" 23	" 24	" 25	" 26	" 27	" 28	" 29	" 30	
Oct 1	O 2	O 3	O 4	O 5	O 6	O 7	O 8	O 9	O 10	
" 11	" 12	" 13	" 14	" 15	" 16	" 17	" 18	" 19	" 20	
" 21	" 22	" 23	" 24	" 25	" 26	" 27	" 28	" 29	" 30	
" 31	N 1	N 2	N 3	N 6	N 7	N 8	N 10	N 11	N 12	
Nov 13	" 14	" 16	" 17	" 19	" 20	" 21	" 22	" 23	" 24	
" 25	" 26	" 27	" 28	" 29	" 30					

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO RAINFALL TOTALS

RAINFALL IN MM.		NIL	0-1.9	2-3.9	4-7.9	8-11.9	12-19.9	≥ 20	No	
TYPE	MEANING	% No	% No	% No	% No	% No	% No	% No	/	
ALL	—	48% 96	25% 50	8% 15	11% 21	5% 9	2% 5	1% 2	193	
1		46% 44	28% 27	11% 11	10% 9	4% 3	1% 1		95	
2+	} PEN P _i B _N 3 _w	58% 7	17% 2	8% 1	17% 2				12	
2-		78% 7	22% 2						9	
2+2-		67% 14	19% 4	4% 1	10% 2				21	
(2+)	} PEN P _i B _N 3 _w	9% 1	18% 2		9% 1	18% 2	28% 3	15% 2	11	
(2-)		50% 6	35% 4		17% 2				12	
(2+2-)		30% 7	26% 6		13% 3	9% 2	13% 3	9% 2	23	
3+	} P _i B _N 3 _w	14% 1	14% 1	14% 1	28% 2	28% 2			7	
3-		88% 7	12% 1						8	
3+3-		54% 8	13% 2	7% 1	13% 2	13% 2			15	
(3+)	} PEN P _i	50% 5	21% 3	14% 2	8% 1	14% 2	7% 1		14	
(3-)		62% 10	19% 3		19% 3				16	
(3+3-)		50% 15	20% 6	7% 2	13% 4	6% 2	4% 1		30	
4+	} B _N 3 _w	63% 5	25% 2		12% 1				8	
4-		50% 3	50% 3						6	
4+4-		50% 8	30% 5		8% 1				14	
2 (2)	PEN	B _N 3 _w	48% 21	23% 10	2% 1	11% 5	5% 2	6% 3	5% 2	44
2 (3)	PEN	P _i	57% 29	20% 10	6% 3	11% 6	4% 2	2% 1		51
(2) 3	P _i	B _N 3 _w	33% 15	21% 8	3% 1	13% 5	11% 4	8% 3	5% 2	38
2 (2) 3		B _N 3 _w	40% 29	20% 12	3% 2	12% 7	8% 4	5% 3	3% 2	59

Table VI

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO BRIGHT SUNSHINE

Total duration of Bright Sunshine (hrs) =			0-1.9	2.0-5.9	6.0-10.9	≥ 11.0		
TYPE	MEANING		% No	% No	% No	% No	No	No
ALL	—		40% 79	32% 64	24% 47	4% 8		198
1	ESTIMATE & READINGS AGREE.		49% 47	28% 27	22% 20	1% 1		95
2+	} PEN	P _i , B _N , 3 _w	17% 2	50% 6	17% 2	16% 2		12
2-			45% 4	33% 3	11% 1	11% 1		9
2+2-			29% 6	43% 9	14% 3	14% 3		21
(2+)	} PEN, P _i	B _N , 3 _w	55% 6	27% 3	18% 2			11
(2-)			25% 3	25% 3	42% 5	8% 1		12
(2+2-)			39% 9	27% 6	30% 7	4% 1		23
3+	} P _i	B _N , 3 _w	29% 2	57% 4	14% 1			7
3-			50% 4	13% 1	25% 2	12% 1		8
3+3-			40% 6	33% 5	20% 3	7% 1		15
(3+)	} PEN	P _i	21% 3	36% 5	43% 6			14
(3-)			19% 3	44% 7	31% 5	6% 1		16
(3+3-)			20% 6	40% 12	37% 11	3% 1		30
4+	} B _N	3 _w	13% 1	30% 3	37% 3	13% 1		8
4-			67% 4	33% 2				6
4+4-			36% 5	36% 5	21% 3	7% 1		14
2 (2)	PEN	B _N , 3 _w	34% 15	34% 15	23% 10	9% 4		44
2 (3)	PEN	P _i	23% 12	41% 21	27% 14	5% 4		51
(2) 3	P _i	B _N , 3 _w	30% 15	29% 11	26% 10	6% 2		38
2 (2) 3.		B _N 3 _w	36% 21	34% 20	22% 13	8% 5		59

Table V

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO DAILY RANGE OF RELATIVE HUMIDITY

R.H. (MAX - MIN) % =		0-10	11-20	21-30	31-40	41-50	51-60		
TYPE	MEANING	% No.	% No.	% No.	% No.	% No.	% No.	No.	
ALL	—	4% 8	25% 49	27% 53	27% 54	15% 29	2% 5	198	
1	ESTIMATE & READINGS AGREE.	5% 5	32% 30	26% 25	21% 20	16% 15		95	
2+	} PEN P _i , B _N , S _w		9% 1	35% 4	33% 4	17% 2	8% 1	12	
2-			33% 3	23% 2	22% 2	11% 1	11% 1	9	
2+2-				19% 4	29% 6	28% 6	14% 3	10% 2	21
(2+)	} PEN, P _i B _N , S _w	18% 2	36% 4	27% 3	16% 1	9% 1		11	
(2-)				34% 4	35% 4	35% 4		12	
(2+2-)			9% 2	17% 4	30% 7	22% 5	22% 5		23
3+	} P _i B _N , S _w		29% 2	29% 2	42% 3			7	
3-			39% 3	12% 1	25% 2	12% 1	12% 1	8	
3+3-				35% 5	20% 3	35% 5	7% 1	7% 1	15
(3+)	} PEN P _i		14% 2	29% 4	43% 6	14% 2		14	
(3-)			6% 1	19% 3	13% 2	44% 7	12% 2	6% 1	16
(3+3-)			3% 1	18% 5	20% 6	43% 13	13% 4	3% 1	30
4+	} B _N S _w		12% 1	13% 1	50% 4	13% 1	12% 1	8	
4-				83% 5	17% 1			6	
4+4-				7% 1	43% 6	36% 5	7% 1	7% 1	14
2(2)	PEN B _N , S _w	5% 2	18% 8	29% 13	25% 11	15% 8	5% 2	44	
2(3)	PEN P _i	2% 1	18% 9	23% 12	37% 19	14% 7	6% 3	51	
(2)3	P _i B _N , S _w	8% 2	24% 9	26% 10	26% 10	16% 6	3% 1	38	
2(2)3	B _N S _w	4% 2	22% 13	27% 16	27% 16	15% 9	5% 3	59	

Table IV

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO RAINFALL DURATION

RAIN DURATION (HR)		0	0.1-1.9	2-3.9	4-7.9	8-11.9	≥ 12	No
TYPE	MEANING	% No	% No	% No	% No	% No	% No	—
ALL	—	49% 97	25% 50	12% 24	10% 19	3% 6	1% 2	198
1		47% 45	31% 29	11% 10	7% 7	2% 2	2% 2	95
2+	P _{EN} P _i B _N 3 _w	58% 7	17% 2	8% 1	17% 2			12
2-		78% 7	11% 1	11% 1				9
2+2-		67% 14	13% 3	10% 2	10% 2			21
(2+)	P _{EN} P _i B _N 3 _w	8% 1	28% 3	18% 2	18% 2	25% 3		11
(2-)		50% 6	25% 3	17% 2	8% 1			12
(2+2-)		3% 7	26% 6	18% 4	13% 3	13% 3		23
3+	P _i B _N 3 _w	14% 1	29% 2	14% 1	43% 3			7
3-		88% 7	12% 1					8
3+3-		54% 8	20% 3	7% 1	19% 3			15
(3+)	P _{EN} P _i	36% 5	25% 4	25% 4	8% 1			14
(3-)		62% 10	14% 3		13% 2	6% 1		16
(3+3-)		50% 15	13% 7	24% 4	10% 3	3% 1		30
4+	B _N 3 _w	63% 5		25% 2	12% 1			8
4-		50% 3	33% 2	17% 1				6
4+4-		56% 8	16% 2	21% 3	7% 1			14
2(2)	P _{EN} B _N 3 _w	48% 21	20% 9	14% 6	12% 5	6% 3		44
2(3)	P _{EN} P _i	57% 29	20% 10	11% 6	10% 5	2% 1		51
(2)3	P _i B _N 3 _w	39% 15	24% 9	13% 5	16% 6	8% 3		38
2(2)3	B _N 3 _w	49% 29	20% 12	12% 7	14% 8	5% 3		59

Table VII

DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO DAY MAXIMUM TEMPERATURE

MAX. TEMP °C		1-5.9		6.0-10.9		11.0-15.9		16.0-20.9		21.0-25.9		26.0-30.9		≥ 31.0		No.
TYPE	MEANING	%	No	%	No	%	No	%	No	%	No	%	No	%	No	198
ALL	—	0%	1	7%	14	16%	32	51%	101	22%	44	3%	5	1%	1	198
1	ESTIMATE 7 READINGS AGREE	4%	1	8%	8	16%	15	58%	55	17%	16					95
2+	PEN P _i B _N S _w							58%	7	42%	5					12
2-				11%	1	55%	4	33%	3	11%	1					9
2+2-				4%	1	19%	4	48%	10	29%	6					21
(2+)	PEN P _i B _N S _w					18%	2	64%	7	9%	1	9%	1			11
(2-)						8%	1	33%	4	50%	6	9%	1			12
(2+2-)							13%	3	43%	11	30%	7	9%	2		
3+	P _i B _N S _w					29%	2	57%	4	14%	1					7
3-				12%	1	13%	1	38%	3	13%	1	12%	1	12%	1	8
3+3-				6%	1	20%	3	47%	7	13%	2	7%	1	7%	1	15
(3+)	PEN P _i							64%	9	36%	5					14
(3-)				19%	3	31%	5	19%	3	31%	5					16
(3+3-)				10%	3	17%	5	40%	12	33%	10					30
4+	B _N S _w					12%	1	25%	2	38%	3	25%	2			8
4-				16%	1	17%	1	67%	4							6
4+4-				8%	1	14%	2	43%	6	21%	3	14%	2			14
2(2)	PEN B _N S _w			2%	1	16%	7	48%	21	29%	13	5%	2			44
2(3)	PEN P _i			8%	4	18%	9	43%	22	31%	16					51
(2)3	P _i B _N S _w			2%	1	16%	6	47%	18	24%	9	8%	3	3%	1	38
2(2)3	B _N S _w			4%	2	17%	10	47%	28	25%	15	5%	3	2%	1	59

Table VIII

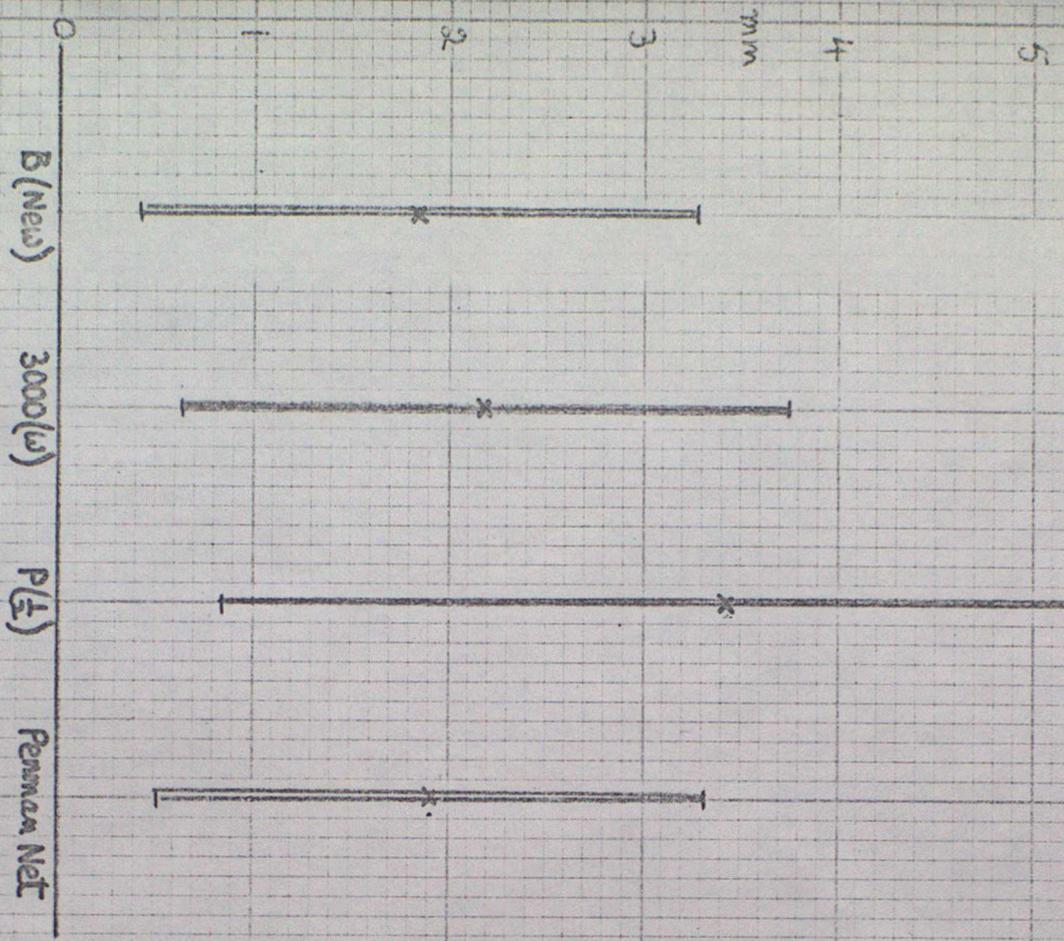
DISTRIBUTION OF "EVAPORATION DAYS" WITH RESPECT TO
MEAN WIND SPEED

MEAN WIND (KNOTS)		0-5.9	6-10.9	11-15.9	16-20	No
TYPE	MEANING	% No	% No	% No	% No	
ALL	—	49% 96	43% 86	7% 14	1% 2	198
1	ESTIMATE READINGS ACROSS	49% 47	40% 38	10% 9	1% 1	95
2+	P _{EN} P _i B _N S _W	58% 7	42% 5			12
2-		33% 3	67% 6			9
2+2-		48% 10	52% 11			21
(2+)	P _{EN} P _i B _N S _W	36% 4	64% 7			11
2-)		50% 6	42% 5	8% 1		12
(2+2-)		43% 10	52% 12	5% 1		23
3+	P _i B _N S _W	29% 2	71% 5			7
3-		50% 4	50% 4			8
3+3-		40% 6	60% 9			15
(3+)	P _{EN} P _i	69% 9	28% 4	8% 1		14
(3-)		25% 4	56% 9	12% 2	7% 1	16
(3+3-)		43% 13	47% 13	10% 3	4% 1	30
4+	B _N S _W	85% 7	15% 1			8
4-		50% 3	55% 2	17% 1		6
4+4-		71% 10	29% 3	8% 1		14
2 (2)	P _{EN} B _N S _W	45% 20	52% 23	3% 1		44
2 (3)	P _{EN} P _i	45% 23	47% 24	6% 3	2% 1	51
(2) 3	P _i B _N S _W	42% 16	55% 21	3% 1		38
2(2) 3	B _N S _W	44% 26	54% 32	2% 1		59

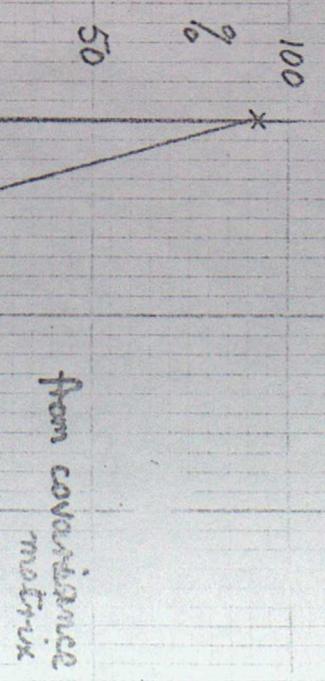
6 PRINCIPAL COMPONENT ANALYSIS OF 4 MEASURES OF EVAPORATION

(A) (1)

X MEAN
Ends of lines:
one s.d. on each side
of mean.

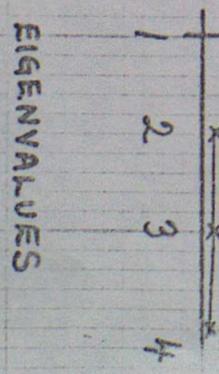


BASIC STATISTICS



from covariance matrix

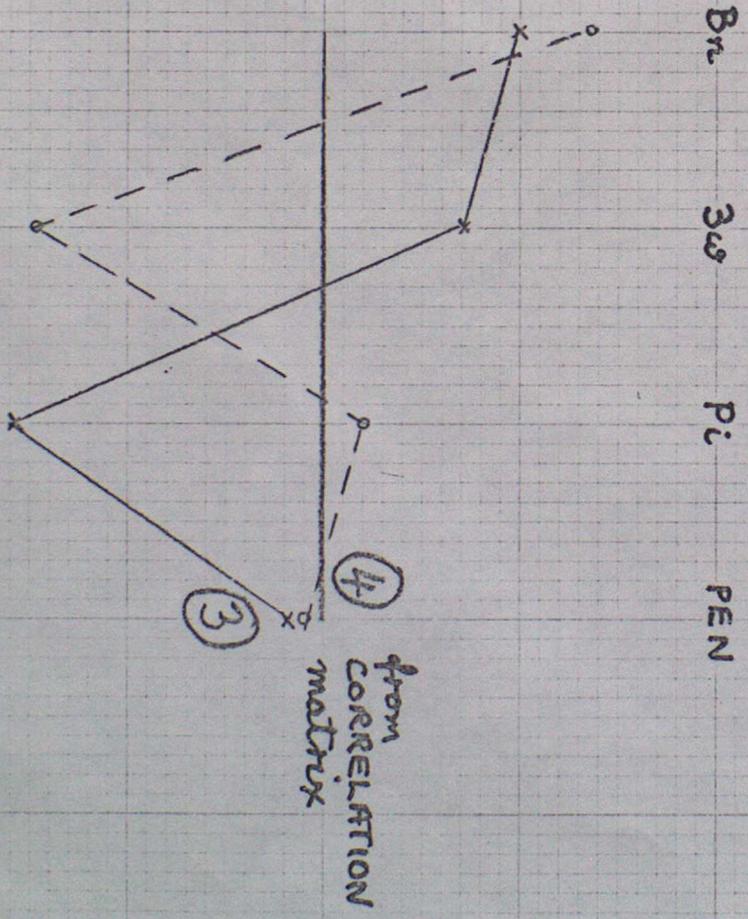
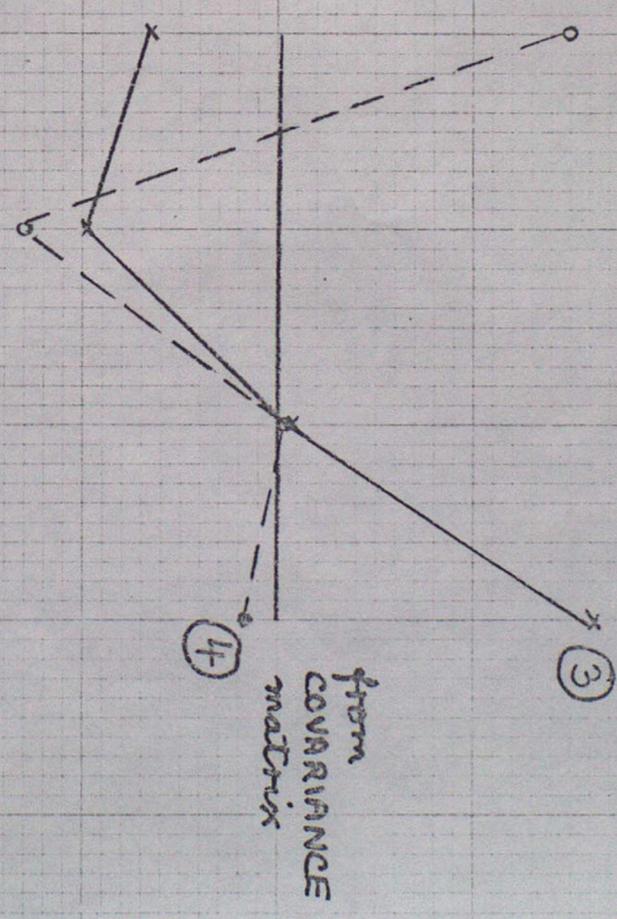
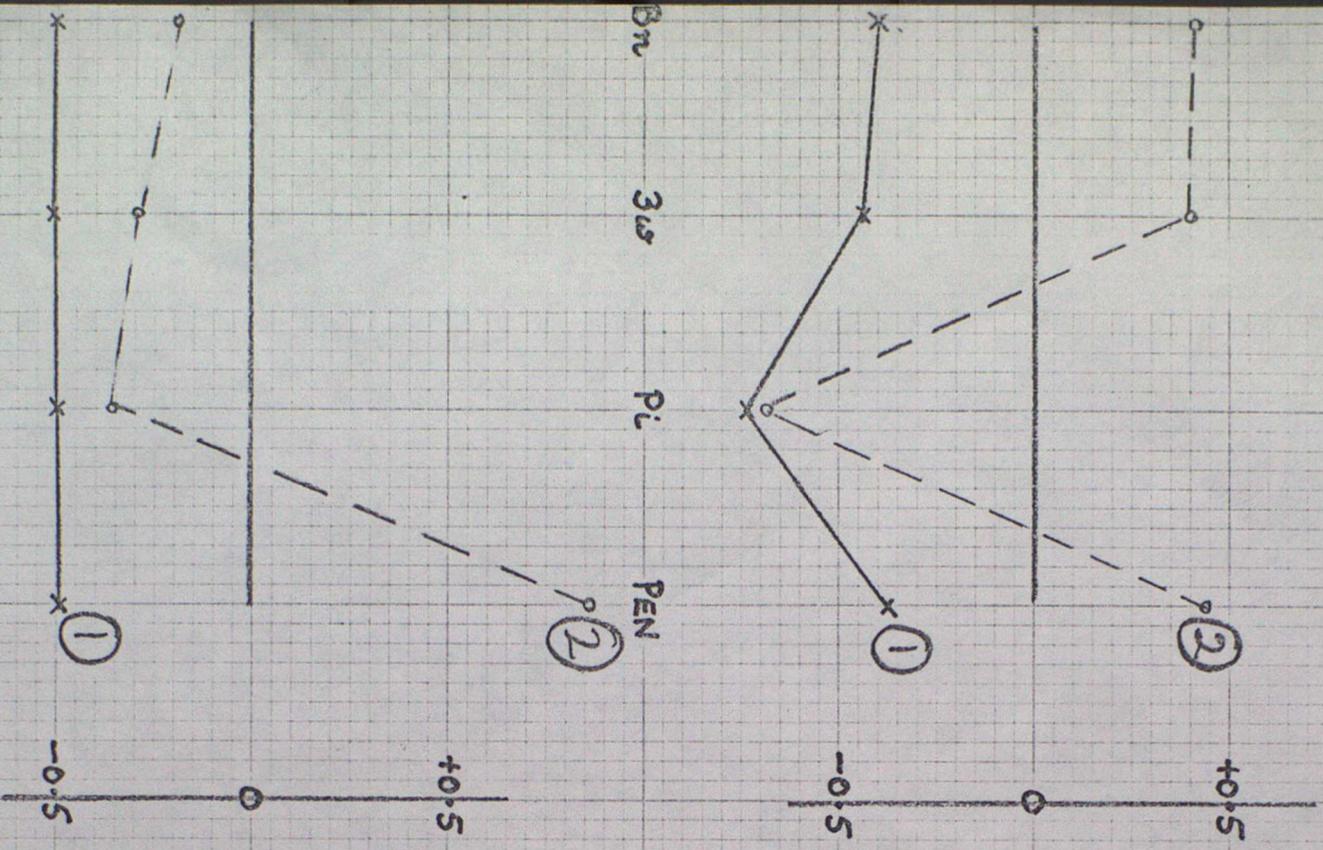
from correlation matrix



EIGENVALUES

FIGURE A2

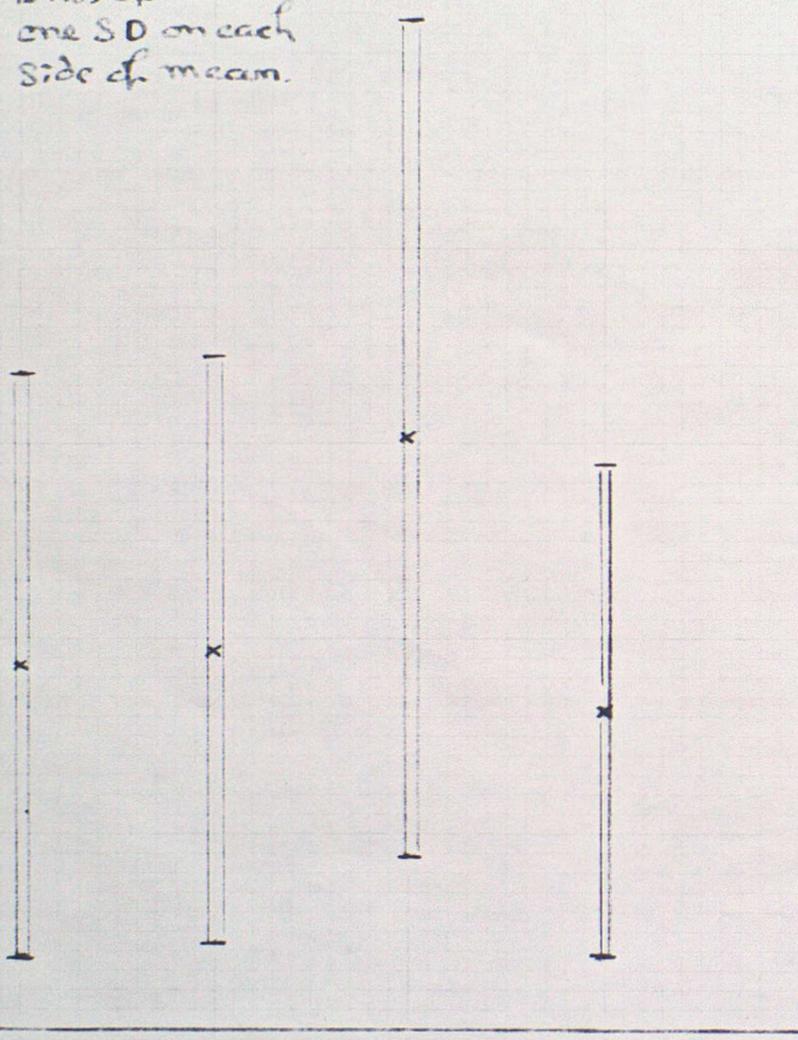
EIGENVECTORS (FROM ~~DATA~~-NUMBERED DATA)



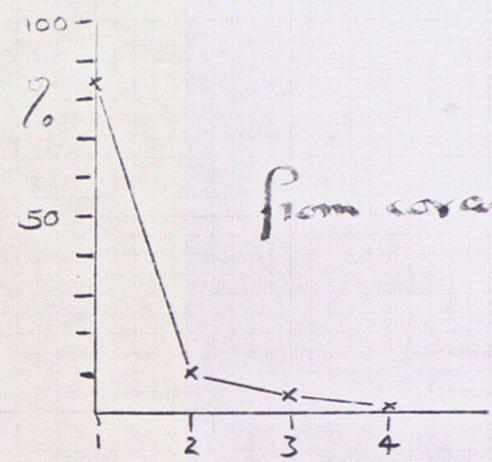
PRINCIPAL COMPONENT ANALYSIS OF 4 MEASURES OF EVAPORATION

(B) (I)

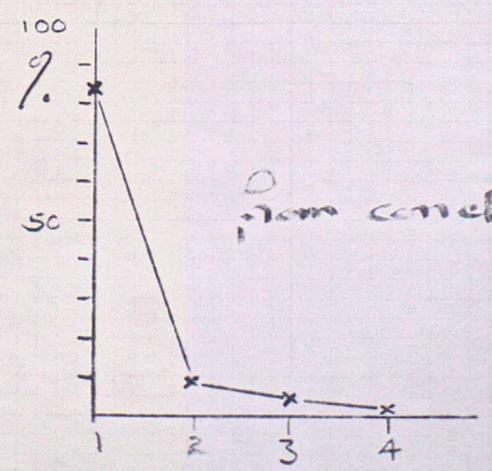
X MEAN
Ends of lines
one SD on each
side of mean.



BASIC STATISTICS.



from covariance matrix.

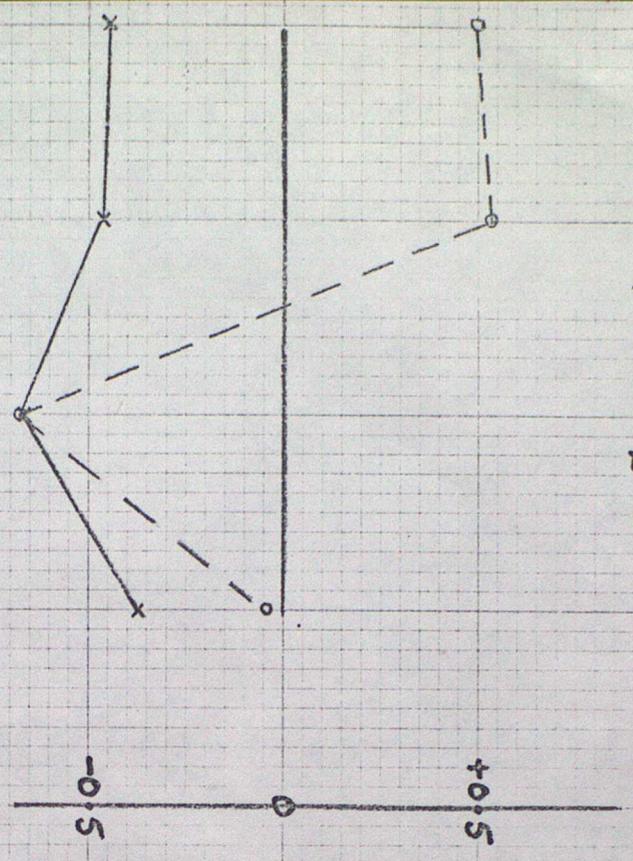


from correlation matrix.

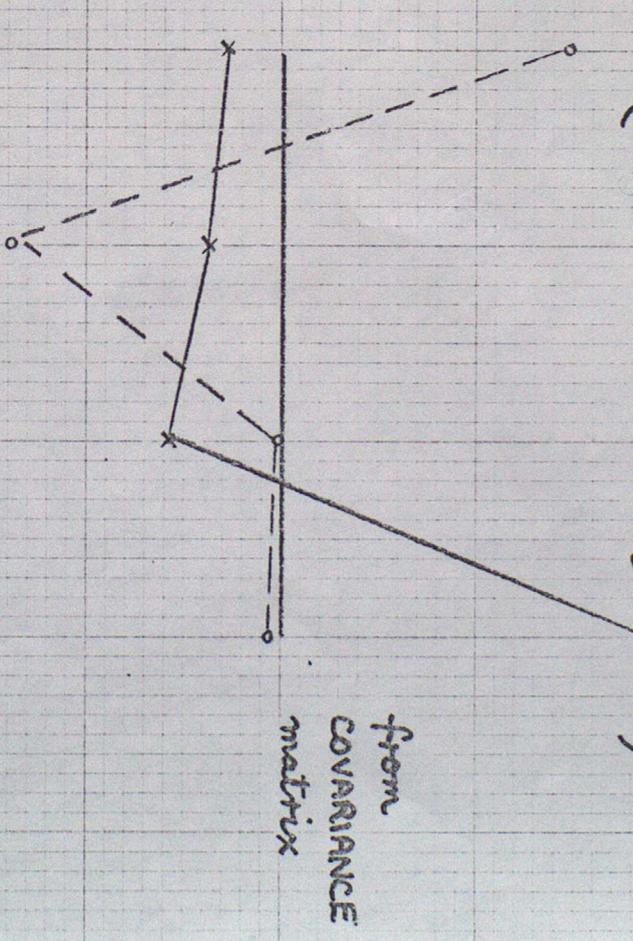
EIGENVALUES.

FIGURE B 2

EIGENVECTORS (FROM EVEN-NUMBERED DATA)



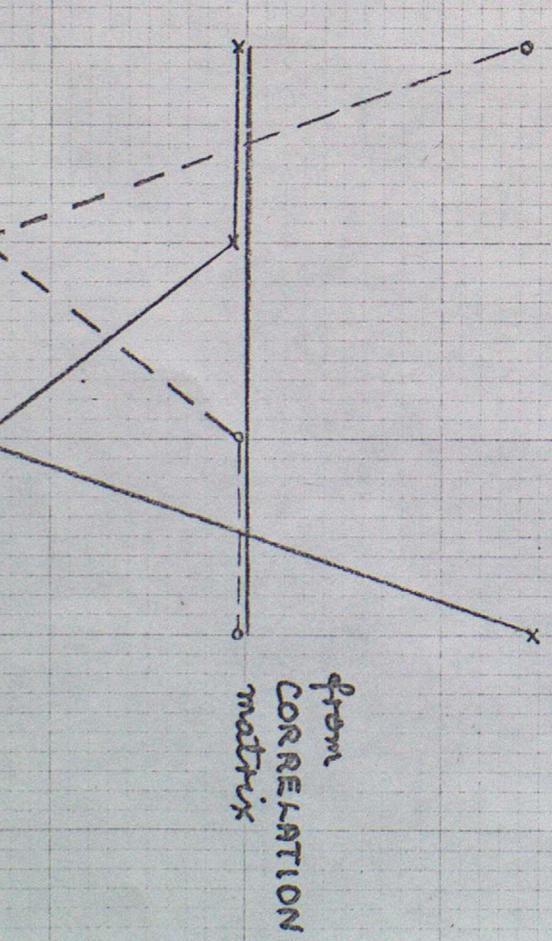
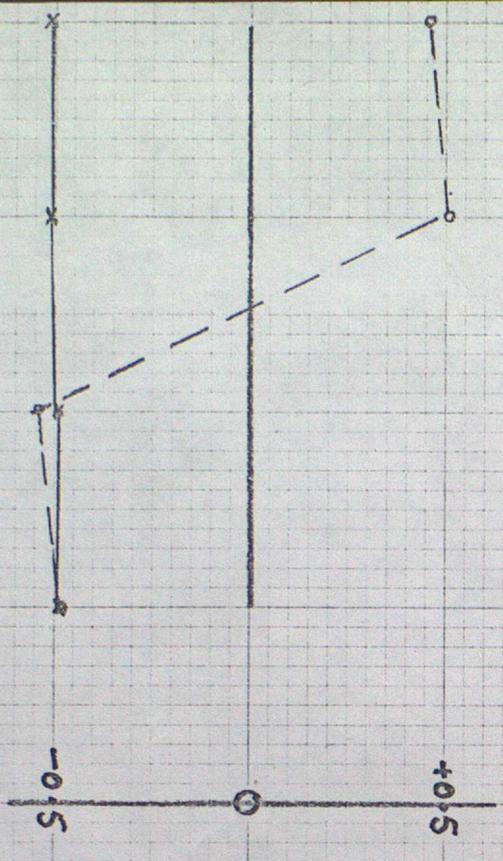
B_n 3ω P_i P_{EN}



B_n 3ω P_i P_{EN}

from
CORRELATION
matrix

from
COVARIANCE
matrix



B_n 3ω P_i P_{EN}

FIGURE C2

EIGENVECTORS (FROM FIRST 90 DAYS' DATA)

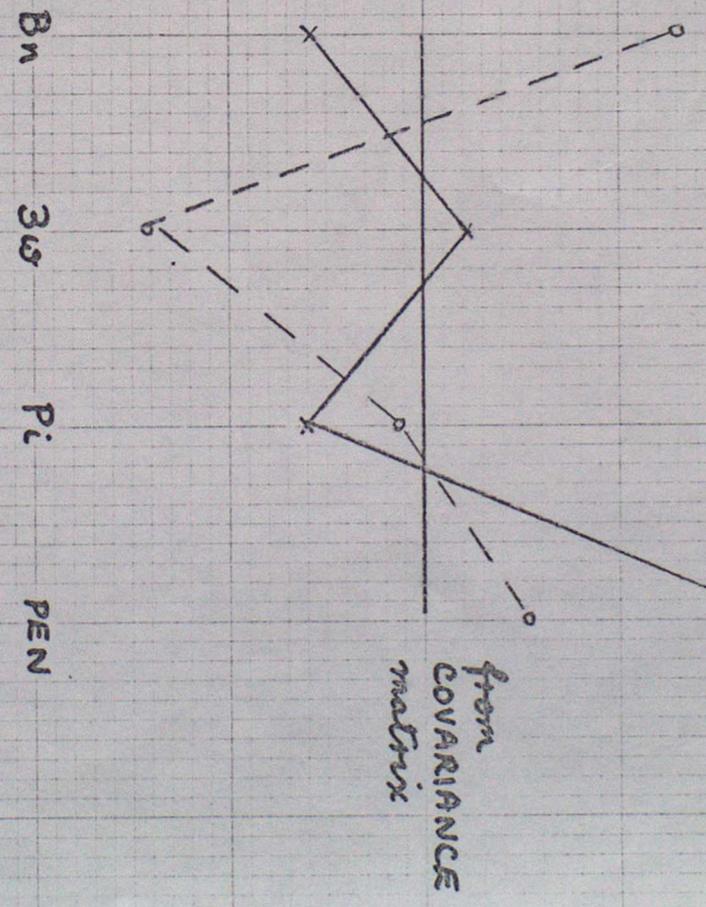
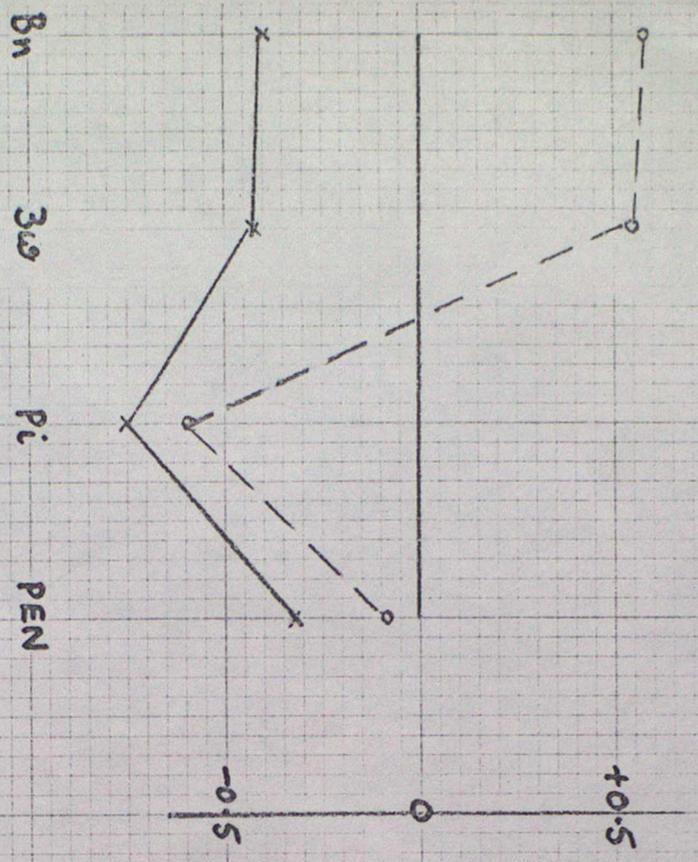
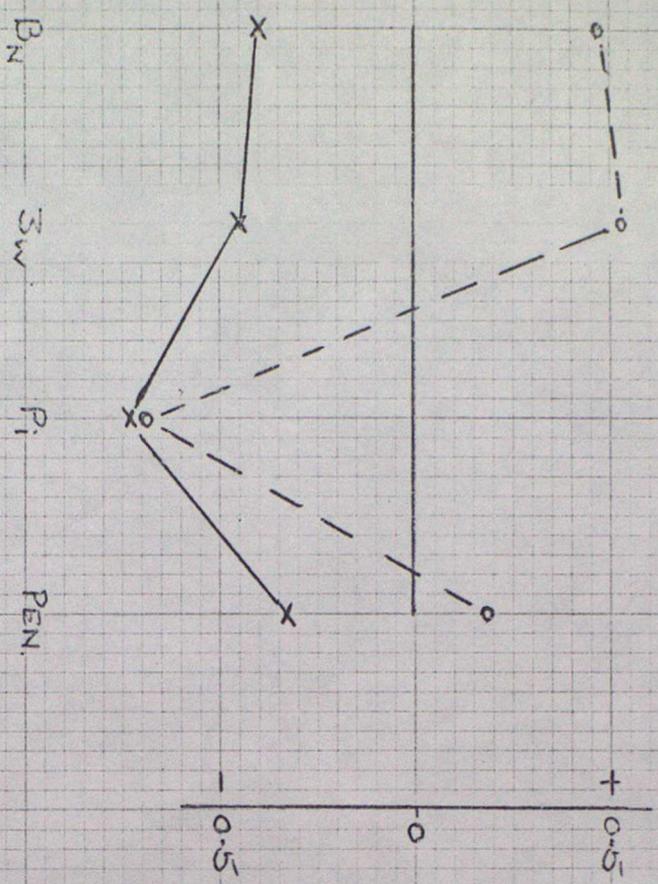
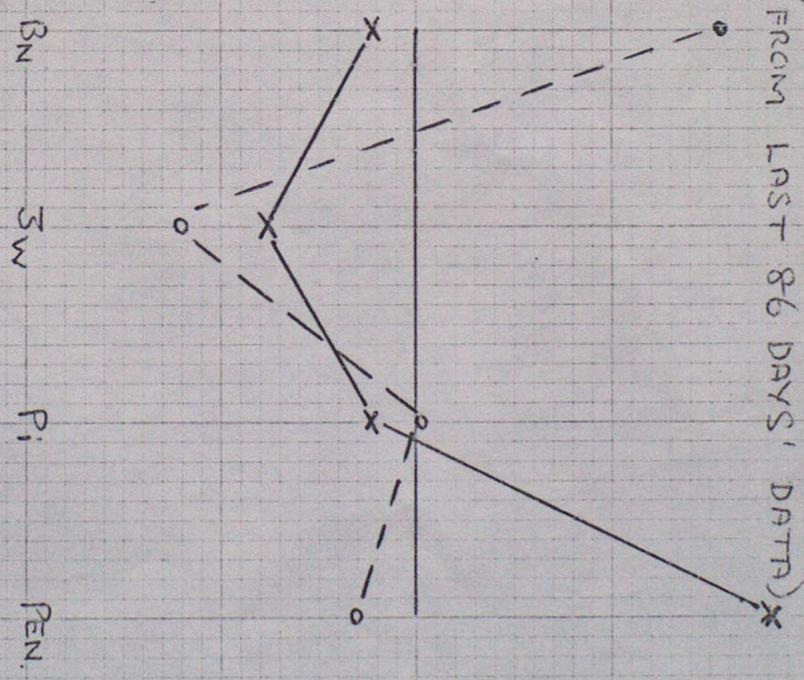


FIGURE D2 EIGENVECTORS

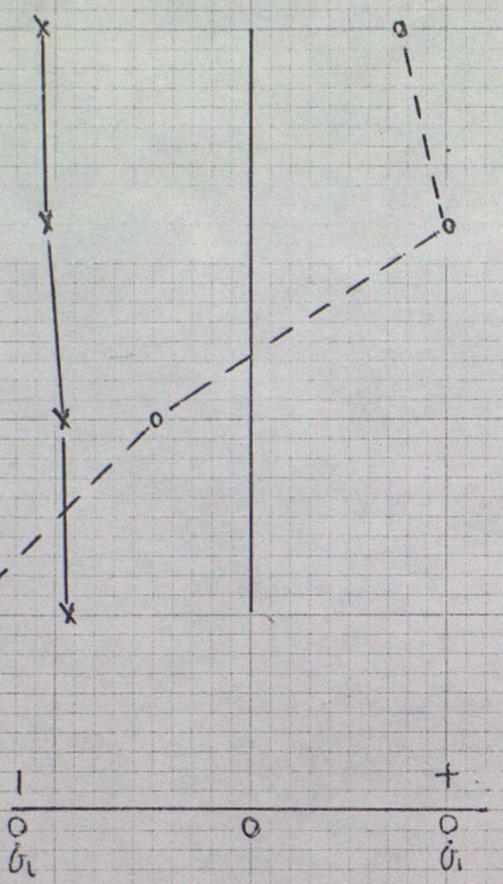
(FROM LAST 86 DAYS' DATA)*



From COVARIANCE matrix

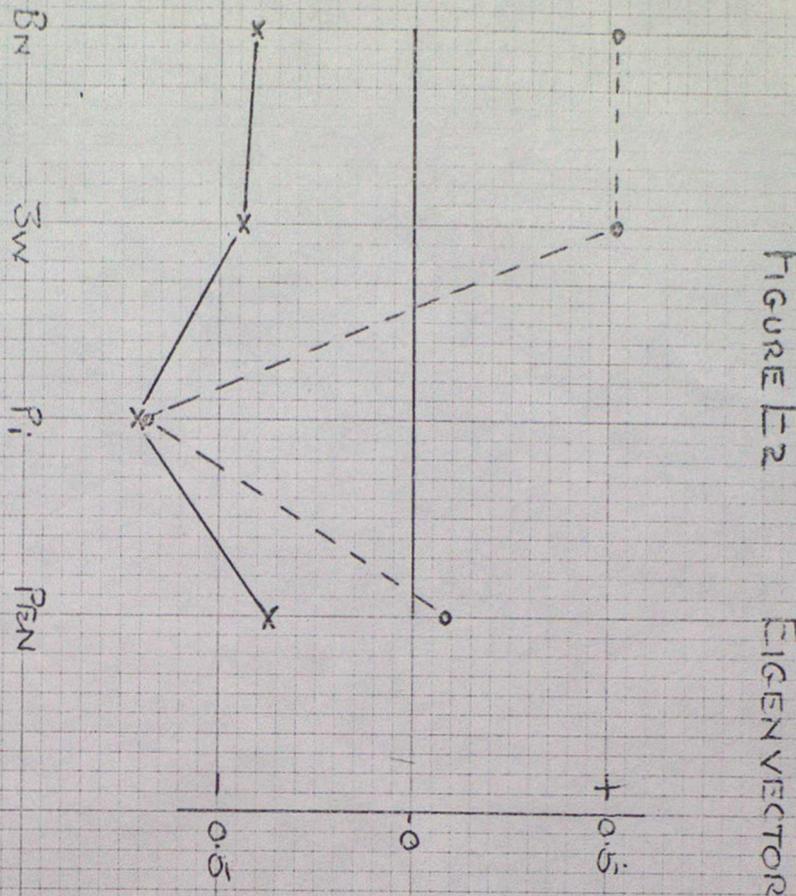


From CORRELATION matrix

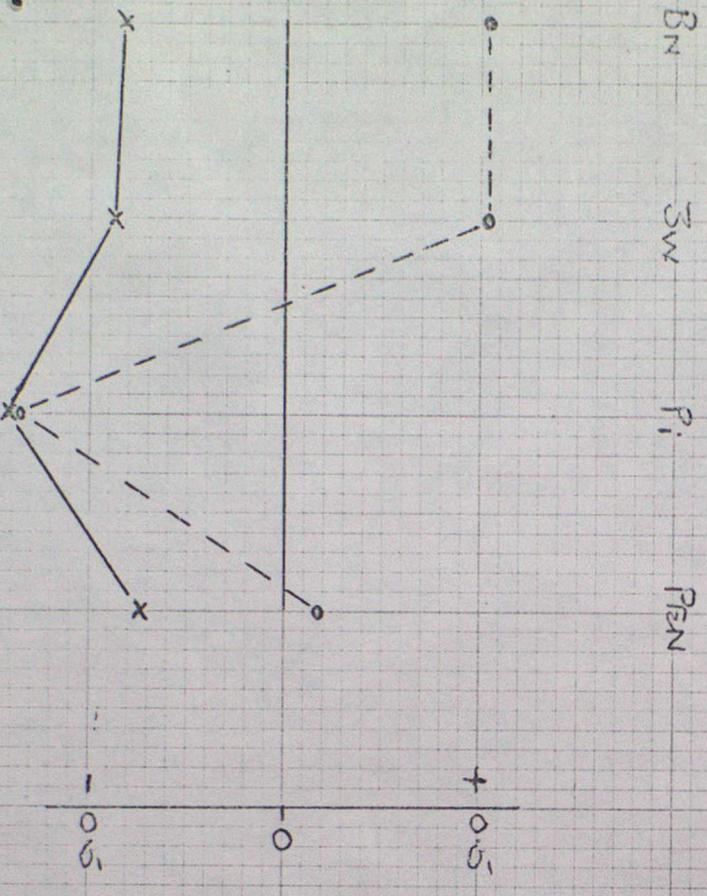


From COVARIANCE matrix

FIGURE E2 EIGENVECTORS (FROM 176 DAYS' DATA).



From COVARIANCE matrix



From CORRELATION matrix