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OPTIMUM CHOICE OF EQUIVALENT WAVENUMBER FOR A SATELLITE RADIOMETER WITH FINITE BANDWIDTH

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1. Introduction

Data from many satellite radiometers is nowadays used to provide information about atmospheric or sea surface temperatures. The proper use of this data requires some knowledge of how the radiant energy received at the detector of the radiometer is related to the energy emitted by the atmosphere or surface within the field of view. For the simple case of a radiometer capable of detecting radiation at only a single wavenumber ν viewing a scene with the emission characteristics of a blackbody at a temperature $T^{o}K$, the measured radiance is given by the Planck function,

$$B(\nu, T) = \frac{c_1 \nu^3}{\exp(c_2 \nu / T) - 1} , \qquad (1)$$

where c_1 and c_2 are known constants.

A radiometer with facilities for in-flight calibration observes scenes with blackbody emission characteristics when it views an internal blackbody at an independently measured temperature, and outer space to obtain a zero radiance reading. However, the filters on current spacecraft may have bandwidths of the order of tens of wavenumbers so the monochromatic case considered above is never achieved in practice. As a radiance by itself is useless for temperature sounding without some indication of filter wavenumber, it is usual to supply a 'mean' wavenumber for each channel of a radiometer. This may be simply the wavenumber at the peak or the centre of the filter profile, but, although large errors are unlikely, unnecessary errors may be introduced in retrieved temperatures if the channel wavenumber is not chosen properly. An alternative is to compute the radiance passed through the filter system as a function of scene temperature and use the Planck function in reverse to determine the wavenumber, but this makes the wavenumber dependent on radiance causing complications in temperature retrieval methods.

This report shows how a single wavenumber can be chosen for a radiometer channel to minimise the temperature errors deduced from measured radiances.

2. Optimum Wavenumber

Suppose a satellite radiometer channel having a filter profile $f(\nu)$ views a blackbody scene at temperature $T^\circ K$. The radiance observed by the instrument, $R(T)$, is given by

$$R(T) = \frac{\int f(\nu) . B(\nu, T) \, d\nu}{\int f(\nu) \, d\nu} , \tag{2}$$

where $B(\nu, T)$ is the Planck function given by equation (1) and the integrals are computed over the bandpass of the filter. As a special case, when the instrument views its internal blackbody at a constant temperature $T_B {}^\circ K$, the radiance will be $R(T_B)$.

If the channel wavenumber is taken as ν_o, the radiance computed from the output signal when the instrument is in blackbody view is $B(\nu_o, T_B)$. The fact that this is not necessarily equal to the 'true' radiance $R(T_B)$ is not important as the two radiances are associated with different wavenumbers. Thus, $B(\nu_o, T_B)$ is the 'correct' radiance to use with wavenumber ν_o while $R(T_B)$ is correct for some other wavenumber which we have not calculated and whose value depends on the filter profile and, in general, on T_B also.

The calibration procedure uses the signals from the internal blackbody and from space view (zero radiance) as two fixed points to determine the linear relationship between signal and radiance. For a blackbody scene at temperature $T^\circ K$, the signal is proportional to the quantity of energy reaching the detector (and therefore to $R(T)$) and the calibration converts this to a radiance $I(T)$ given by

$$\frac{R(T)}{R(T_B)} = \frac{I(T)}{B(\nu_o, T_B)} , \tag{3}$$

or, defining the constant k as

$$k = \frac{B(\nu_o, T_B)}{R(T_B)} , \tag{4}$$

this may be written simply as

$$I(T) = k\,R(T). \tag{5}$$

$I(T)$ is then the radiance deduced from the instrument signal and is associated with the wavenumber ν_o. $I(T)$ and $R(T)$ are not directly comparable because of the difference in associated wavenumbers, but we can compare $I(T)$ with the Planck function at wavenumber ν_o. The radiance difference ΔR defined by

$$\Delta R = I(T) - B(\nu_o, T) \tag{6}$$

estimates the radiance errors introduced by using the constant wavenumber ν_o. To estimate the effects on temperature we can convert $I(T)$ back to a temperature T' using the Planck function in reverse, i.e.,

$$T' = c_2 \nu_o \Big/ \log\left(\frac{c_1 \nu_o^3}{I(T)} + 1\right) \tag{7}$$

and define a temperature error ΔT by

$$\Delta T = T' - T. \tag{8}$$

The calibration procedure ensures that there is no error ($\Delta T = 0$) at $T = 0$ and $T = T_B$ but cannot ensure that ΔT vanishes at all other temperatures. The procedure we shall use to optimise the effective wavenumber is to choose ν_o such that ΔT vanishes for a temperature which approximates the mean scene temperature which the radiometer experiences in orbit. For example, if the radiometer is designed to observe atmospheric radiation from levels in the atmosphere for which the mean temperature is T_o, we choose ν_o such that ΔT (and therefore ΔR) vanishes for $T = T_o$. From equation (6) this condition is seen to be equivalent to

$$I(T_o) = B(\nu_o, T_o) \tag{9}$$

or, from equation (3)

$$\frac{R(T_o)}{R(T_B)} = \frac{B(\nu_o, T_o)}{B(\nu_o, T_B)}. \tag{10}$$

Expanding the Planck functions, this reduces to

$$\left(e^{c_2\nu_o/T_o} - 1\right) R(T_o) = \left(e^{c_2\nu_o/T_B} - 1\right) R(T_B) \tag{11}$$

which may be solved iteratively for ν_o.

How much does the value of ν_o obtained from equation (11) depend on the black-body temperature T_B? We can answer this question by differentiating equation (10) with respect to T_B which gives

$$\frac{d\nu_o}{dT_B} = -\left(\frac{\partial f}{\partial T_B}\right) \div \left(\frac{\partial f}{\partial \nu_o}\right) \tag{12a}$$

where

$$f = f(\nu_o, T_B) = R(T_o).B(\nu_o, T_B) - R(T_B).B(\nu_o, T_o) \tag{12b}$$

Since $R(T)$ and $B(\nu_o, T)$ are similar functions, the two terms in equation (12b) are of approximately the same magnitude and f is close to zero. Furthermore, a change in T_B will affect $R(T_B)$ and $B(\nu_o, T_B)$ in similar ways so that the changes in the terms compensate for each other and f remains close to zero. Thus $\frac{\partial f}{\partial T_B}$ is small, and as $\frac{\partial f}{\partial \nu_o}$ does not exhibit compensation to anything like the same extent, $\frac{d\nu_o}{dT_B}$ is small and the optimum wavenumber is only weakly dependent on T_B.

3. Example

As an example, consider the application of the above technique to the case of the high-pressure channel of a pressure modulator radiometer similar to the ones to be flown on TIROS-N satellites during the next few years. In this instrument radiation from the atmosphere passes through a broadband infra-red filter and then through a 1-centimetre cell containing carbon dioxide whose pressure is continuously cycled between about 80 and 120 millibars. The signal is taken from the AC output from the detector and therefore represents a difference in transmission rather than a straightforward transmission as would be the case with a conventional radiometer. We can allow for this by treating the cell as a second filter and using a transmission difference profile instead of a transmission profile for the carbon dioxide in the cell. The total filter profile $f(\nu)$ is therefore the product of the broadband filter profile and the effective cell filter profile, both of which are shown in figure 1. (As the profile for the cell reflects the very complicated absorption line structure of CO_2, the lower graph is presented as a histogram at 5 cm^{-1} intervals). In a conventional radiometer, $f(\nu)$ would be the profile for a single filter like the upper graph in figure 1. (More strictly, $f(\nu)$ should also contain the effects of any other optical components such as lenses or mirrors but these will be considered perfectly transmitting or reflecting for the purposes of this example. Also, the spectral response of the detector should be included.)

Knowing $f(\nu)$, we can compute $R(T)$ using equation (2). Figure 2 shows how $R(T)$ varies with T for temperatures between 75K and 310K. The effective wavenumber (the wavenumber for which the Planck function of temperature T is $R(T)$) is plotted against temperature in figure 3 which shows a variation of about four wavenumbers over the same temperature range.

The radiometer is designed to detect radiation from the lower stratosphere where the scene temperature is typically 214 - 230K; we choose a mean value of 222°K. Taking a typical value of internal blackbody temperature of 294K and using equation (2) (or reading off figure 2) we have,

$$\begin{aligned} T_o &= 222, \quad R(T_o) = 46.93 \\ T_B &= 294, \quad R(T_B) = 139.89 \end{aligned} \qquad (13)$$

The optimum wavenumber is now given by equation (11) as

$$46.93\,(e^{A\nu_o} - 1) = 139.89\,(e^{B\nu_o} - 1) \qquad (14a)$$

where

$$A = 6.4812 \times 10^{-3}, \quad B = 4.8940 \times 10^{-3} \qquad (14b)$$

Equation (14) may be solved by the following iteration procedure:

$$e^{A\nu_o^{(i+1)}} = 1 + 2.9809\,(e^{B\nu_o^{(i)}} - 1) \qquad (15)$$

where $\nu_o^{(i)}$ is the value of ν_o after the i^{th} iteration. Application of this method to the example we have been considering and iterating until convergence is achieved yields an optimum wavenumber of 672.338 cm^{-1}. The convergence is slow but adequate. Substitution of the relevant quantities into equation (12) reveals that the change in the optimum wavenumber is only about 0.002 cm^{-1} for a 1°K change in blackbody temperature.

4. Results

The pressure modulator radiometer used for the above example was designed to detect radiation in the 15-micron absorption band of CO_2 which is centred at 668 cm^{-1}. This would therefore seem to be a sensible wavenumber to use if no other information was available. If the function R(T) was known but no optimisation procedure was used, it would seem reasonable to use a wavenumber chosen from figure 3 such as the effective wavenumber at the blackbody temperature, in this case 673.78 cm^{-1}. We shall test the choice of optimum wavenumber by comparing it with these two alternative values.

First we examine the radiance errors ΔR calculated from equation (6). These are shown in figure 4 for the temperature range 75 - 310K for the three choices for ν_o. Over the range of temperatures plotted in this figure, the following statistics apply (radiances in mw/($m^2 cm^{-1}$ster)).

Wavenumber ν_o (cm^{-1})	668.0	673.78	672.338 (optimum)
Maximum radiance error in range 75 - 310K	-0.306	0.101	0.012
Maximum radiance error in range 214 - 230K	-0.305	0.101	0.003

The error of about 0.3 radiance units for 668 cm^{-1} amounts to about 0.6% of the total radiance. Thus, the errors are nowhere very large but may be considerably reduced and eliminated for all practical purposes by a proper choice of ν_o. In our example the error reduction involved in using the optimum value rather than 668 cm^{-1} is about a factor of 25 for the range plotted and 100 for typical scene temperatures of 214 - 230K.

The temperature errors given by equation (8) are plotted in figure 5 and show a similar reduction for the optimum wavenumber over the range 214-230K where temperature errors of up to $0.35^{\circ}K$ have been reduced to insignificant values. (The rise in ΔT at low temperatures is due to the much reduced value of $\frac{dR}{dT}$ at these temperatures - see figure 2.) We therefore conclude that for a satellite radiometer channel, a properly chosen wavenumber should eliminate radiance errors due to the finite bandwidth of the instrument, and temperature errors derived from these radiances. (Of course, other sources of error such as stray radiation from sources on the edge of the field of view may still be present.) Errors can be reduced for a particular temperature range by using the optimisation procedure developed in section 2. Furthermore, the optimum wavenumber is insensitive to changes in blackbody temperature.

Finally, figure 4 suggests a graphical interpretation of the optimisation procedure. As the wavenumber is increased from 668 cm^{-1} to 673.78 cm^{-1} the radiance errors change from negative to positive values. There is no wavenumber for which these errors are zero for all temperatures because the 'true' radiance R(T) does not follow a Planck function temperature dependence exactly. Therefore as the radiance errors change sign, the curve has an S-shaped form (remember ΔR is fixed at zero at temperatures of zero and 294K) which must cross the T-axis at some intermediate temperature. Small variations in ν_0 will cause this crossing point to move up and down the axis quite rapidly over a range of wavenumbers. We have chosen a value of ν_0 which makes this crossing point fall in the centre of the field-of-view temperatures observed by the satellite in orbit. In our case we took the mean temperature as 222K causing the radiance error to be zero at this point and therefore constraining the errors to be very small over a range of temperatures near this value.

5. TIROS-N SSU Radiometer

The example we have used was based on one of the three channels of the Stratospheric Sounder Unit (SSU) radiometer due to be flown on the next generation of operational meteorological satellites. Subsequently, wavenumber optimisations were performed for all three channels using the best available transmission data and including the full optics and not just the infra-red filter profile in the transmission profile (upper half of figure 1). The effective cell transmission profile for the other two channels is less broad than that shown for the 100-millibar cell (lower half of figure 1). For example, for the lowest pressure cell (10 mb), the contribution from outside the range 660-680 cm^{-1} is reduced relative to the peak by a factor of 4 or 5.

The wavenumbers obtained from the computations were

SSU channel 15 $\quad \nu_o = 669.988\ cm^{-1} \quad$ (222K)

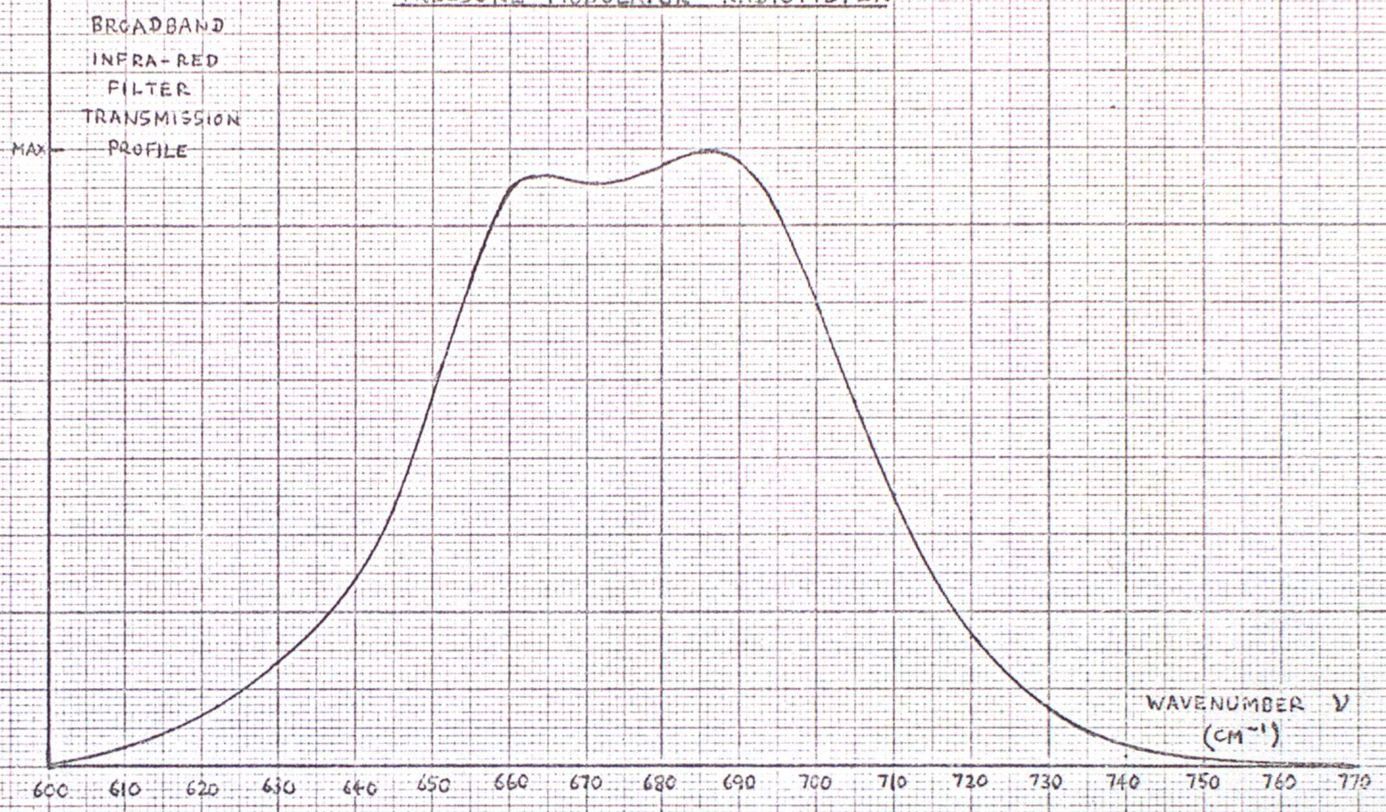
SSU channel 16 $\quad \nu_o = 669.628\ cm^{-1} \quad$ (233K)

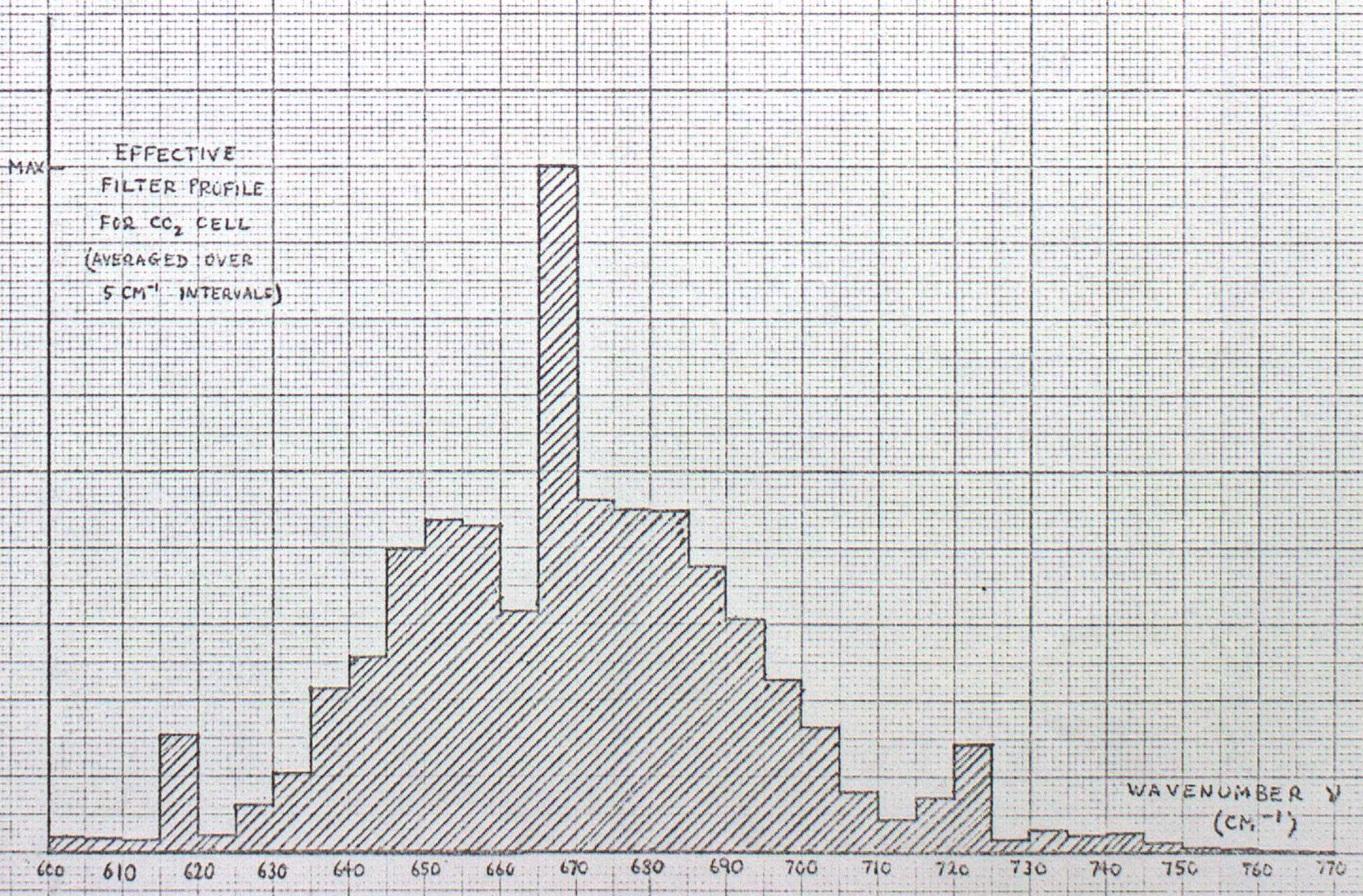
SSU channel 17 $\quad \nu_o = 669.357\ cm^{-1} \quad$ (248K)

The temperatures in brackets are the assumed mean scene temperatures and the internal blackbody temperature was taken as 293K. These wavenumbers (though, perhaps, not strictly accurate to the 6-figure accuracy quoted above), will be used in SSU software developed by the Meteorological Office. Once again it should be pointed out that they have been optimised only for errors due to the finite bandwidth of the radiometer.

FIGURE 1

TRANSMISSION PROFILES FOR A PRESSURE MODULATOR RADIOMETER





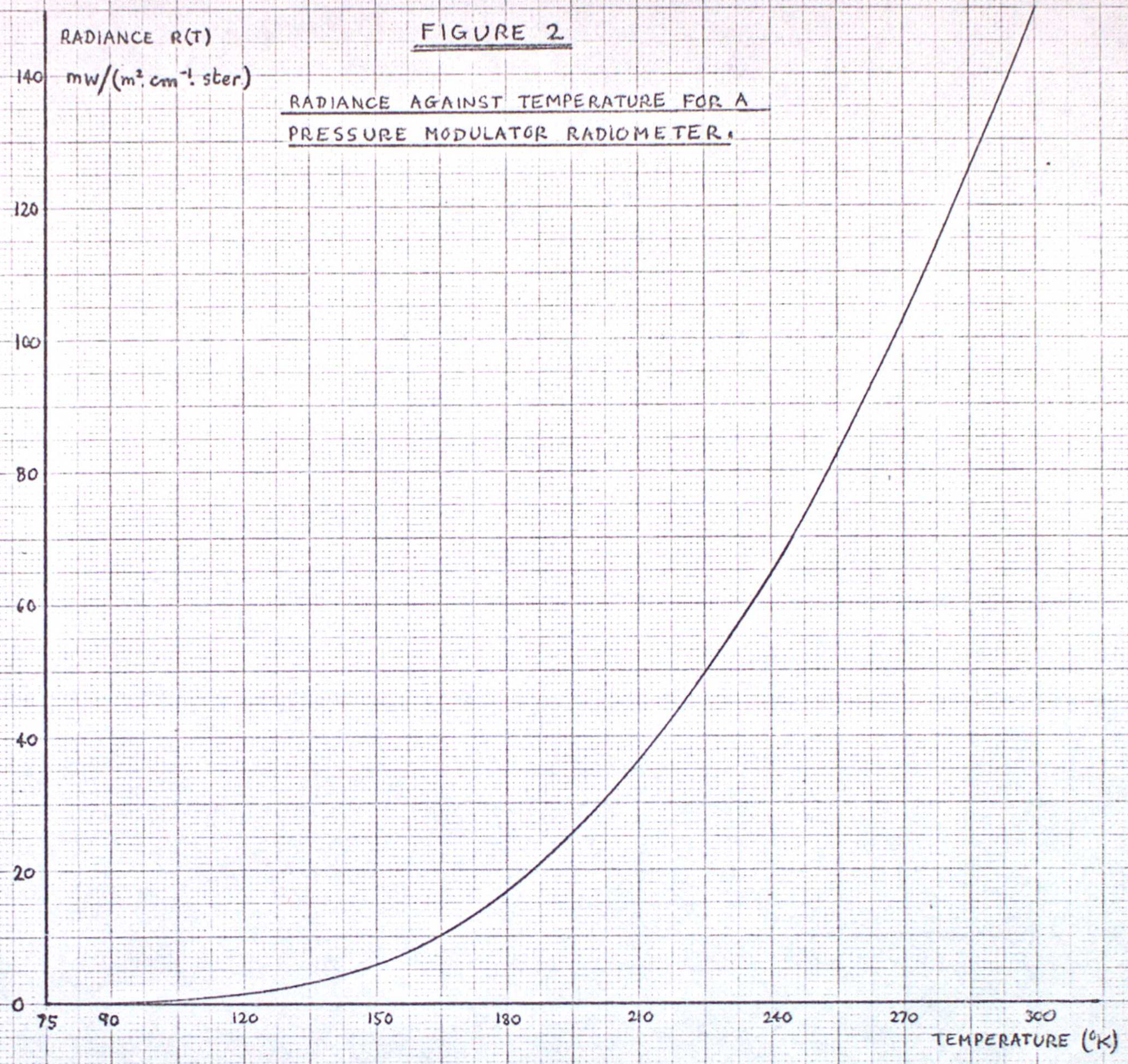


FIGURE 2

RADIANCE AGAINST TEMPERATURE FOR A PRESSURE MODULATOR RADIOMETER.

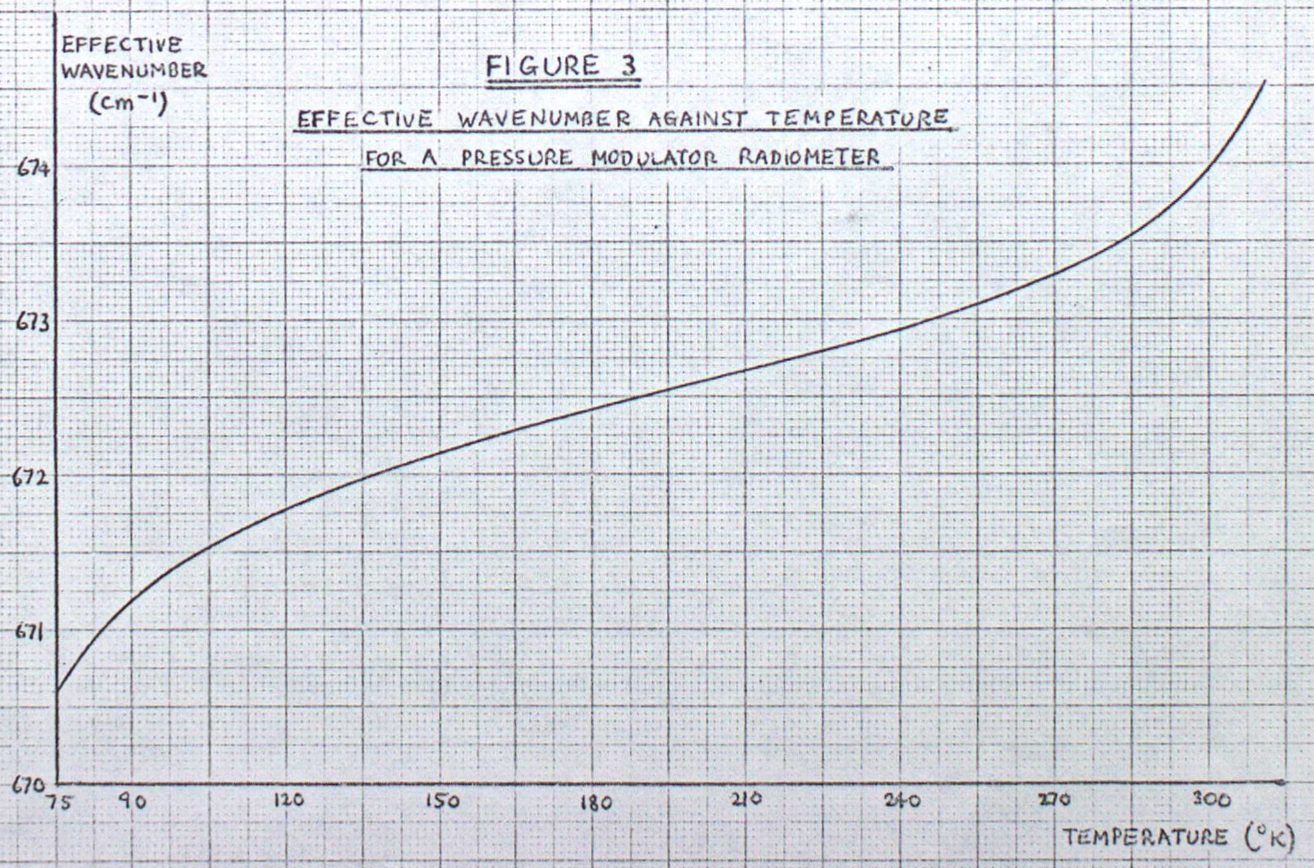


FIGURE 3

EFFECTIVE WAVENUMBER AGAINST TEMPERATURE FOR A PRESSURE MODULATOR RADIOMETER

FIGURE 4 RADIANCE ERRORS FOR THREE CHOICES OF ν_0.

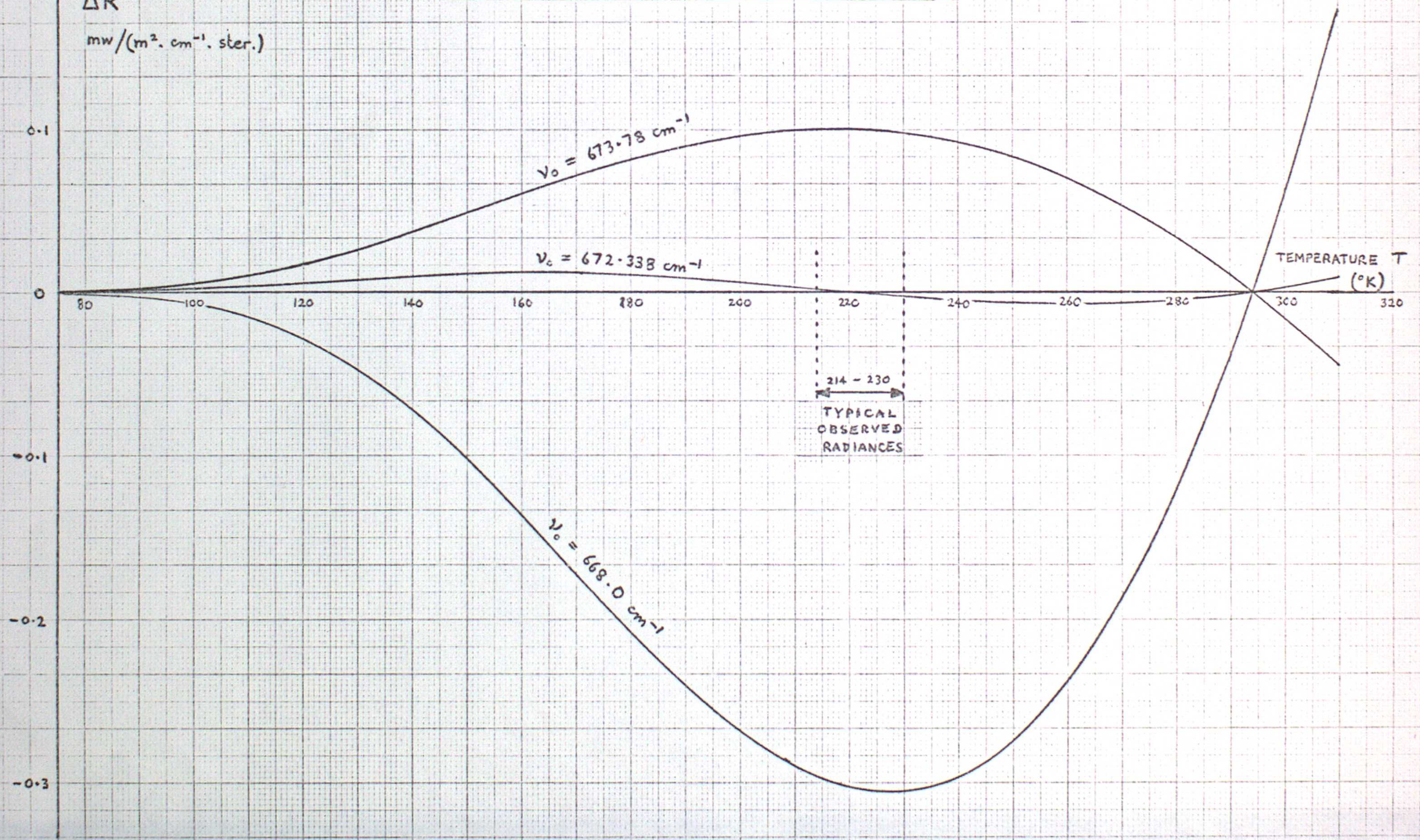


FIGURE 5 TEMPERATURE ERRORS FOR THREE CHOICES OF ν_0